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Prof. Pierpaolo Benigno

SUPERVISOR

Prof. Robert Kollmann

CO-SUPERVISOR

Marco D'Amico

CANDIDATE

Academic Year 2022/2023

Essays in Macroeconomics

Marco D'Amico[*]

Université Libre de Bruxelles and Luiss

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Thesis supervisor: Pierpaolo Benigno
Thesis supervisor: Robert Kollmann

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[*] Address: ECARES, Université Libre de Bruxelles, Avenue Franklin Roosevelt 42, 1050, Bruxelles, Belgium and Luiss Guido Carli, Department of Economics and Finance, viale Romania 32, 00197, Rome, Italy. Email: marco.damico@ulb.be.

Abstract

This thesis is composed of three chapters: in the first chapter, “Inequality and the Medium-Term Business Cycle”, I show that heterogeneous agents endogenous growth (HAEG) model economies display less deep recessions and more contained booms than their representative agent counterparts over the medium term business cycle. The smoothing in GDP standard deviation (over the medium term business cycle) ranges around estimates of 14% and 28%. This difference increases with (i) the presence of illiquid assets and (ii) the elasticity of R&D to the total stock of varieties. Nonetheless, at the business cycle frequency, HAEG economies behave as in Krusell and Smith (1998): output time series are as if generated by a representative household.

The second chapter, “Ruling Out Stagnation Traps”, joint with S. Nisitcò, presents empirical evidence in support of a non-monotonic relationship between employment and productivity growth. It studies the implications of such non-monotonicity in a model where growth occurs endogenously through both vertical innovation and reallocation into entrepreneurship, with nominal rigidities and monetary policy. It characterises the conditions for the existence of multiple steady-state equilibria featuring low growth and unemployment, and the role of economic policy to bring the economy towards the full-employment steady state. It shows that economic policies targeted at easing business creation activities can both prevent the occurrence of unemployment steady states and help the economy to get out of them.

Finally, the third chapter, “World Trade Stagnation”, presents recent empirical evidence showing that emerging economies (EMEs) currently account for most of the world GDP and global growth, their trade flows with advanced economies (AEs) doubled during the mid-80s till the 2012 and their investment in R&D increased by 26.4% per year between 1997-2008, almost catching-up with AEs productivity around the 2009 and displaying the same AEs TFP growth rate thereafter. These developments were accompanied by trade balance reversals and by a rapid increase in world trade (the so called “hyper-globalization”) which culminated in the Great Trade Collapse of the 2009 and a subsequent stagnation up to nowadays. I present a two-country model with firms heterogeneous in productivities à la Melitz (2003), endogenous growth and international R&D spillovers that accounts for these facts. It is found that innovation and trade dynamics are closely tied and trade balance reversals are consequences of asymmetric needs of funding stemming from innovation efforts. Trade stagnates after the 2008 because of stagnant proximity to the technological frontier of EMEs (with respect to AEs).

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Chapter I

Inequality and the Medium-Term Business Cycle

Abstract

I show that heterogeneous agents endogenous growth (HAEG) model economies display less deep recessions and more contained booms than their representative agent counterparts over the medium term business cycle. The smoothing in GDP standard deviation (over the medium term business cycle) ranges around estimates of 14% and 28%. This difference increases with (i) the presence of illiquid assets and (ii) the elasticity of R&D to the total stock of varieties. Nonetheless, at the business cycle frequency, HAEG economies behave as in Krusell and Smith (1998): output time series are as if generated by a representative household[1].

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1 Introduction

What is the role of households heterogeneity for medium-term fluctuations? The Great Recession (GR, 2008-09) poses naturally this question. On the one hand, the slow recovery and persistent deviation of output from its pre-crisis trend (output hysteresis) brought a revival in the literature linking the trend and the cycle: a slump in demand reduces productivity-enhancing investments (such as R&D), which, by their intrinsically sluggish dynamics, induce large negative swings in economic activity. On the other hand, the credit disruption that characterized the GR gave new light to models that incorporate households heterogeneity, for they allow to better understand the response of agents at different points in the wealth distribution, as opposed to models with a representative saver and borrower. Somewhat surprisingly, these two strands of the economic literature are so far disconnected. In this article I show that indeed, households heterogeneity and medium-term fluctuations are closely intertwined.

In order to derive a clear link between agents-heterogeneity and the long-run outcomes of the economy, I construct a tractable model in which heterogeneity stems from uninsurable idiosyncratic risk due to market incompleteness and borrowing constraints, and growth arises endogenously in the expanding variety fashion à la Romer (1990b) through investment in R&D. To that aim, I build on the work of Challe and Ragot (2016) (CR) which lay down a set of sufficient conditions to endogenously generate a wealth cross-sectional distribution with a limited number of states, and augment it with a mechanism that links the high-frequency fluctuations to the medium-term cycle in the spirit of Comin and Gertler (2006). This ensures a state-space representation of the model and standard perturbation techniques can be applied for the solution and simulation under aggregate uncertainty. I focus my analysis only on exogenous disturbances in total factor productivity (TFP), as in Krusell and Smith (1998). This allows mo to isolate the main channels through which inequality affects the medium term business cycle in a transparent way.

It is well known that real heterogeneous agents (HA) economies that do not match wealth inequality statistics (such as, *e.g.*, the baseline model in Krusell and Smith, 1998) generate aggregate time series that are "as if" were generated by a representative agent (or in a complete markets set up), when subjected to TFP shocks. One of the main results in Krusell and Smith (1998) (KS) is that allowing for a more empirically-consistent wealth distribution, the real business cycle model featuring households heterogeneity produces consumption amplification and investment smoothing with respect to its RA counterpart;

nonetheless, these two effects cancel out at the aggregate output level and the GDP behavior is the same across KS and RA model economies. That is, inequality is neutral as long as output time-series behavior is concerned. The same applies to the CR model on which I build.

My first result is that inequality affects the medium term business cycle for output in HA economies endowed with endogenous growth (HAEG), even though fluctuations at the high frequency in output are undistinguishable between HAEG and RAEG models. In other words, the Krusell and Smith (1998)'s result for output breaks down over the medium term cycle. In particular, the investment smoothing property of HA economies extends also to investment in R&D. The latter is the ultimate source of medium term oscillations in the economy as it shapes the dynamics of TFP, which is endogenous in my HAEG model. Hence, the HAEG economy displays a *smoothed* medium term business cycle with respect to its RAEG counterpart. Put differently, heterogeneous agents economies augmented with endogenous growth produce more contained booms and less deep recessions with respect to RAEG economies, if TFP shocks are the only source of exogenous disturbance.

The second finding of my work is that the degree of asset liquidity plays an important role in shaping the strength of the smoothing property featuring HA economies. Indeed, in the baseline HAEG model (and also in KS and CR), it is assumed that households have access only to capital stock holdings. Relaxing this assumption in favor of a more realistic environment in which the poorest can invest only in liquid bonds while the wealthiest both in bonds and illiquid capital stock holdings *amplifies* the smoothing property. The illiquidity of capital stock holdings raises marginal propensities to consume (MPCs) of the wealthiest, therefore even less resources are directed toward R&D in the face of a positive shock, while more are devoted to consumption.

The third result concerns the quantification of the role of inequality for the medium term business cycle. In the baseline model with only capital stock holdings as an investment vehicle, the difference in output standard deviations over the medium term business cycle between RAEG and HAEG is of 14%. This difference raises to 28% if the degree of asset liquidity is taken into account.

The magnitudes of the results are in line with the findings in Bilbiie et al. (2022) , even though they are topsy turvy. Indeed, Bilbiie et al. find that inequality (and in particular steady state inequality) amplifies the standard deviation of GDP growth with respect to the no-inequality counterfactual by 27%. Instead, I find the opposite, *i.e.* that inequality decreases the standard deviation of GDP through the investment smoothing property discussed above. This discrepancy is quite reasonable, for Bilbiie et al. (2022) estimate a medium-scale DSGE NK model. As I discuss in the body of the paper, my results are likely to be upside down in a New Keynesian environment with monetary policy shocks, as in that case inequality acts as an amplification mechanism at both the consumption and output level (see Auclert et al., 2020). Indeed, Bilbiie et al. find that most of the variation in GDP growth in their model is driven by demand shocks. Therefore I find my results complementary, rather than antagonist, to those in Bilbiie et al. (2022).

Related literature My work is related to several strands of the economic literature. I contribute to the literature linking high-frequency fluctuations to medium-frequency ones [2]. This literature includes, among the others, Comin and Gertler (2006) Kung and Schmid (2015), Anzoategui et al. (2019), Bianchi et al. (2019), Guerron-Quintana and Jinnai (2019), Queralto (2020) and Cozzi et al. (2021). The work of Comin and Gertler (2006) is the first to propose a unified framework that allows the joint study of high and medium-frequency fluctuations. They do so augmenting a standard RBC model with expanding varieties endogenous growth à la Romer (1990b) through investment in R&D and adoption. Anzoategui et al. (2019) instead build on the work of Comin and Gertler (2006) to estimate a model with R&D and adoption dynamics so as to investigate whether the productivity slowdown following the GR was due to an endogenous response to the observed contraction in demand. Bianchi et al. (2019) are the first to estimate an endogenous growth DSGE model with data on R&D, with a focus on the different effect of debt and equity financing for productivity dynamics. Guerron-Quintana and Jinnai (2019) instead estimate a model with endogenous growth à la Romer (1990b) and financial frictions à la Kiyotaki and Moore (2019). Queralto (2020) estimates a small open economy model with endogenous growth through expanding varieties tailored to analyze the South Korean 1997 financial crisis while Cozzi et al. (2021) assess the quantitative relevance of demand and supply factors in the post-GR U.S. growth slowdown estimating a model with creative destruction as in Aghion and Howitt (1992) and Nuño (2011). Kung and Schmid (2015) instead introduce endogenous growth in the spirit of Romer (1990b) to study the link between asset prices and aggregate risk premia and endogenous movements in long-term growth prospects. All these works are set-up in a representative-agent economy and thus abstract from the interactions between households-heterogeneity and medium term cycles. I innovate over this literature explicitly considering such interactions and showing that inequality acts as a smoothing mechanism for medium-frequency fluctuations, yielding

[2] For a great review of this literature see Cerra et al. (2020).

less deep recessions and smoothed booms with respect to RAEG economies.

This paper also contributes to the literature on households heterogeneity (stemming from uninsurable idiosyncratic income uncertainty) which dates back to Laitner (1979) and Bewley (1983) from whom many extensions followed, such as Deaton (1991), Huggett (1993), Aiyagari (1994) and Krusell and Smith (1998). More recent contributions are, among the others, Werning (2015), Guerrieri and Lorenzoni (2017) and Kaplan et al. (2018). In this literature effort has been put in understanding (i) how inequality changes the transmission mechanism of shocks or (ii) how it shapes macroeconomic aggregates behavior.

Relatively to (i), Kaplan et al. (2018) show that households heterogeneity allows models to generate a monetary policy transmission mechanism's decomposition, between direct and indirect effects, more in line with the empirical evidence. Auclert (2019) instead identifies three sources of redistribution induced by monetary policy innovations: a earnings heterogeneity channel, a Fisher channel (in that respect, see also Doepke and Schneider, 2006b and Doepke and Schneider, 2006a) and a interest rate exposure channel. He also argues that making a model coherent with sufficient statistics is essential in empirically disciplining consumption responses to shocks. Auclert et al. (2018) instead show that empirically estimated intertemporal marginal propensities to consume are inconsistent with the representative agent assumption and that they affect the transmission of fiscal policies. Instead, in relation to (ii) Krusell and Smith (1998) show that a real business cycle model augmented only with households heterogeneity behaves as a representative agent model at the aggregate level. Their main result is that, introducing also heterogeneity in thrift that allows a more empirically consistent wealth distribution, a sizable departure from representative agent behavior is observed. However, at the output level no difference is observed between HA and RA economies, even when HA ones feature an empirically consistent wealth distribution. Werning (2015) instead analyzes the effects of market incompleteness on aggregate demand while Krueger et al. (2016) study how households heterogeneity amplifies and propagates macroeconomic shocks. In particular, the latter find that, at the aggregate output level, there is no difference between HA and RA economies, as long as output is supply driven, in line with Krusell and Smith (1998). Guerrieri and Lorenzoni (2017) study the response of heterogeneous households to a credit crunch and underlie the importance of digging into the behavior of agents at different points of the wealth distribution. de Ferra et al. (2020) investigate the role of households heterogeneity for the transmission of foreign shocks in a small open economy. Auclert et al. (2020) show that heterogeneity amplifies macroeconomic shocks if it is embedded in a New Keynesian environment, in line with the intuition of Krueger et al. (2016) that, if output is partly demand-driven — as it is in New Keynesian economies — then inequality acts as an amplifier of shocks.

In this literature, I contribute in relation to objective (ii), *i.e.* in advancing the understanding of how heterogeneity shapes macroeconomic aggregates behavior. In particular, I show that, if the impact of heterogeneous MPCs on research and development is taken into account — as my HAEG model does — even in the baseline Krusell-Smith model without stochastic-β output behavior differs with respect to its representative agent counterpart, over the *medium-run*. Allowing for a more reasonable wealth distribution amplifies this difference even more.

I contribute to the literature that studies the quantitative role of inequality for aggregate time series behavior, such as Challe et al. (2017), Berger et al. (2019), Auclert et al. (2020), Bayer et al. (2020), Bilbiie et al. (2022). The novelty I bring is in quantifying the relevance of inequality for the medium term business cycle and thus for innovation dyamics, rather than focusing on the business cycle as these works do.

I am of course related to the work of Challe and Ragot (2016) who derive a set of sufficient conditions under which an economy populated by households heterogeneous in their wealth due to uninsurable idiosyncratic risk and a borrowing constraint can endogenously generate a wealth cross-sectional distribution with a limited number of states. I reformulate such conditions in the context of endogenous growth and extend the CR model with investment in R&D; this extension helps the model improve under several aspects as long as data fit is concerned.

I also bring new elements to the classics of endogenous growth, such as Romer (1990b), Aghion and Howitt (1992) and Grossman and Helpman (1991), incorporating productivity endogeneity in a DSGE model with uninsurable idiosyncratic risk due to market incompleteness and borrowing limits. My work is mostly related to Romer (1990b) as growth takes place through expanding varieties.

Structure The paper is organized as follows: Section 2 presents the baseline model; Section 3 presents the main results while Section 4 discusses the role of asset liquidity for the medium term business cycle. Section 5 concludes.

2 The Model

The model embodies a closed economy inhabited by a continuum of households uniformly distributed over the unit interval, monopolistic firms of endogenous mass producing a differentiated good and perfectly competitive innovators. The latter carry out research and development in order to discover new varieties, thus endogenously inducing long-run growth in the economy. Financial markets present frictions. For brevity, I will refer to the baseline model as HAEG (heterogeneous-agents, HA, endogenous growth, EG)[3].

2.1 Households

The structure of the household side of the economy follows Challe and Ragot (2016) (CR), except differences, to be described below, to make it consistent with the non-stationary nature of the economy . Households can be impatient or patient, distributed over the sub-intervals $[0, \Omega]$ and $(\Omega, 1]$ respectively, with $\Omega \in [0, 1)$. Households face idiosyncratic income uncertainty, stemming from changes in their employment status, which occur exogenously. Defining f the job-finding rate and s the job-loss rate, the law of motion for employment reads

$$n_t = (1 - n_{t-1}) f + (1 - s) n_{t-1}$$

thus I am focusing on the case of constant employment over the cycle; in particular $n_t = f/(f+s)$ for all t. Every household i is endowed with one unit of labour, which is supplied inelastically to firms, if the household is employed: in this case the household earns the net real wage, otherwise an unemployment benefit.

2.1.1 Impatient households

Impatient households maximize expected intertemporal utility from consumption,

$$\mathbb{E}_t \sum_{s=t}^{\infty} \left(\beta^I\right)^{s-t} u^I \left(c_s^i\right), \tag{1}$$

$i \in [0, \Omega]$, where $\beta^I \in (0, 1)$ is the subjective discount factor, $u^I(.)$ is the period utility function satisfying $u_c^I > 0$ and $u_{cc}^I \leq 0$ and c_t^i is consumption of the final good. Households do not have access to the complete set of Arrow-Debreu securities and enjoy only the

[3]The reader already familiar with the Challe and Ragot (2016) model, may want to skip this detailed presentation and instead read the shorter model description in Appendix G.

partial insurance provided by the unemployment benefit, $\delta^I_t > 0$. In addition, impatient households face an exogenous borrowing limit, $\mu_t \geq 0$. Both the unemployment benefit and the borrowing limit, are linear functions of the mass of adopted varieties N_{t-1} for stationarity reasons, *i.e.*, $\delta^I_t \equiv \delta^I N_{t-1}$ and $\mu_t \equiv \mu N_t$ where $\delta^I > 0$ and $\mu \geq 0$. The impatient household enters period t with a^i_{t-1} holdings of claims to the capital stock at the end of date $t-1$. It receives the ex-post gross return R_t on these holdings and the real wage w^I_t net of social contributions τ_t if it works, and the unemployment benefit δ^I_t otherwise. Also, I assume that net profits from innovation D_t are transferred to impatient households in amount proportional to their mass Ω; this ensures that in steady state inequality will not be affected by the presence of firm profits, which would otherwise make the calibration less flexible. The households' use of funds include consumption c^i_t and the purchase of claims on capital, a^i_t. The period budget constraint thus is

$$a^i_t + c^i_t = e^i_t w^I_t \left(1 - \tau_t\right) + \left(1 - e^i_t\right) \delta^I_t + D_t + R_t a^i_{t-1} \tag{2}$$

where $e^i_t = 1$ if the household is employed and 0 otherwise. The household maximizes life-time utility (1) subject to the period budget constraint (2) and to the debt limit $a^i_t \geq -\mu_t$. The Euler equation for this problem is

$$u^I_c\left(c^i_t\right) = \beta^I \mathbb{E}_t\left[u^I_c\left(c^i_{t+1}\right) R_{t+1}\right] + \varphi^i_t$$

where φ^i_t is the Lagrange multiplier associated with the borrowing constraint and $\varphi^i_t > 0$ if the latter is binding and $\varphi^i_t = 0$ otherwise.

2.1.2 Patient households

Patient households maximize expected intertemporal utility from consumption,

$$\mathbb{E}_t \sum_{s=t}^{\infty} \left(\beta^P\right)^{s-t} u^P\left(c^i_s\right), \tag{3}$$

where $\beta^P \in \left(\beta^I, 1\right)$ is their subjective discount factor while $u^P\left(c^i_t\right)$, $i \in (\Omega, 1]$, is their utility which is continuous, strictly increasing and strictly concave over $[0, \infty)$. The consumption basket c^i_t is defined in the same way as for impatient households. Contrary to impatient households, patient ones have access to the complete set of Arrow-Debreu securities. As a result, they behave as a large representative family of permanent-income consumers where the *pater familiæ* ensures each member an equal marginal utility of wealth. The period budget constraint thus reads

$$C_t^P + A_t^P = R_t A_{t-1}^P + (1-\Omega)\left[n_t w_t^P (1-\tau_t) + (1-n_t)\,\delta_t^P + D_t\right]$$

where C_t^P and A_t^P are consumption and asset holdings of the family, w_t^P and $\delta_t^P \equiv \delta^P N_{t-1}$ are the real wage and the unemployment benefit for the patient households, with $\delta^P > 0$ and D_t are profits from firms[4]. The Euler equation for patient households reads

$$u_c^P\left(\frac{C_t^P}{1-\Omega}\right) = \beta^P \mathbb{E}_t\left[u_c^P\left(\frac{C_{t+1}^P}{1-\Omega}\right) R_{t+1}\right].$$

Since patient households are all alike, they pin down a unique pricing kernel for assets in the economy, namely $M_{t+1} \equiv \beta^P \left(\frac{C_{t+1}^P}{1-\Omega}\right)^{-\sigma} / \left(\frac{C_t^P}{1-\Omega}\right)^{-\sigma}$.

2.2 Production

There is a continuum of monopolistically competitive firms, each producing a different variety $j \in \mathcal{N}$ of intermediate goods with symmetric elasticity of substitution $\theta > 1$. Firms combine labor from both impatient $n_t^I(j)$ and patient $n_t^P(j)$ households and capital $\tilde{k}_{t-1}(j)$ in a Cobb-Douglas fashion and share a common (stationary) stochastic aggregate productivity process z_t. Their production function thus is $y_t(j) = z_t\left(n_t^I(j) + \kappa n_t^P(j)\right)^{1-\alpha} \tilde{k}_{t-1}^{\alpha}(j)$, where $\kappa > 0$ is the relative efficiency of patient households' labour. Intermediate goods are then costlessly assembled through a CES technology $Y_t = \left(\int_{j\in\mathcal{N}} y_t^i(j)^{\theta/(\theta-1)}\,dj\right)^{\frac{\theta}{1-\theta}}$. Each period t, only a subset of varieties $\mathcal{N}_{t-1} \subset \mathcal{N}$ is available. As it will become clear below, the period mass N_{t-1} of the set $\mathcal{N}_{t-1}$, is an endogenous-trending variable and determines the growth rate of the economy along its balanced growth path (BGP). As shown in Dixit and Stiglitz (1977), the final good production-based price index is $P_t = \left(\int_{j\in\mathcal{N}_{t-1}} p_t(j)^{1-\theta}\,dj\right)^{\frac{1}{1-\theta}}$.

Firms maximize their period profits $d_t(j)$ subject to the residual demand curve they face, $y_t(j) = \left(p_t(j)/P_t\right)^{-\theta} Y_t$. All firms j set prices to a markup $\theta/(\theta-1)$ over marginal costs of production. For the problem faced by each firm is the same, they all set the same price and sell the same quantity, thus making identical profits, which amount to $d_t(j) = d_t = Y_t/\theta N_{t-1}$, $\forall j \in \mathcal{N}_{t-1}$ (see Appendix A.1 for details). Intuitively, profits

[4] Since patient households are all alike, a set-up in which they are allowed to trade in stock holdings would yield the very same budget constraint.

are increasing in total (stationary) demand Y_t/N_{t-1} and decreasing in the elasticity of substitution θ. The fact that profits are procyclical will make investment in R&D and varieties creation procyclical as well.

2.3 Research and Development

Each period, there is an unbounded mass of perfectly competitive innovators h that conduct R&D using the final good S_t in order to develop a new variety.

Let S_t^h be the total amount of R&D carried out by innovator h, ϑ_t her productivity that she takes as exogenous, N_{t-1}^h her total stock of innovations and $\phi \in (0,1)$ the probability of variety obsolescence. Then, the law of motion for varieties developed by innovator h is

$$N_t^h = \vartheta_t S_t^h + (1-\phi)\, N_{t-1}^h.$$

The innovator productivity ϑ_t is defined as

$$\vartheta_t = \frac{\chi N_{t-1}}{S_t^{1-\psi} N_{t-1}^{\psi}}$$

where $\chi > 0$ is a scale parameter and $\psi \in [0,1]$ is the elasticity of new varieties with respect to R&D. This technology features positive spillovers from the aggregate stock of varieties (innovations) $\partial\vartheta/\partial N > 0$, as in Romer (1990b), and a congestion externality $\partial\vartheta/\partial S < 0$ that raises the cost of developing new varieties as the *aggregate* level of R&D raises.

For an intermediate good producer, the value V_t of owning exclusive rights to produce variety j is the present discounted value of current and expected future monopoly profits, *i.e.*

$$V_t = \iota d_t + (1-\phi)\, \mathbb{E}_t\left[M_{t+1} V_{t+1}\right]$$

which does not depend on the specific variety j because of symmetry in equilibrium as of Section 2.2. The scale parameter $\iota \in (0,1)$ serves the only purpose of calibrating the model to match a R&D to GDP ratio of 2%, as in the data.[5]

Since all innovators face the same problem, given the linearity of the innovator's

[5]This necessity arises because, as it will be made clear below, in order to ensure balanced growth path stability, models of this fashion require that production function is homogeneous of degree one in the accumulating factors, thus binding the choice of the elasticity of substitution parameter θ to that of the capital share α. In my model this would lead to too large profits (due to too high mark-ups) and thus the R&D/GDP would be in the order of 27%.

technology and free entry, at the margin it must be that

$$\frac{1}{\vartheta_t} = (1-\phi)\,\mathbb{E}_t\left[M_{t+1}V_{t+1}\right] \tag{4}$$

i.e. the marginal cost of R&D (the LHS) must be equal to its expected discounted marginal benefit (the RHS). The procyclicality of monopolists profits d_t transfers to the cyclical pattern of firm's value V_t and, as a result, to R&D investment.

Since equation (4) is independent from innovators specific characteristics, aggregation among them yields the aggregate law of motion for varieties:

$$N_t = \vartheta_t S_t + (1-\phi)\,N_{t-1}. \tag{5}$$

2.4 Aggregate Production

As is is shown in Appendix (A.2), noticing that in equilibrium, by the law of large numbers, $n_t^I = \Omega n_t$ and $n_t^P = (1-\Omega)\,n_t$, the aggregate production function is of the familiar form

$$Y_t = \Xi_t K_{t-1}^{\alpha} L_t^{1-\alpha} \tag{6}$$

where $K_{t-1} \equiv \int_{j\in\mathcal{N}_{t-1}} k_{t-1}(j)$ is aggregate capital, $L_t \equiv n_t\,(\Omega + (1-\Omega)\,\kappa)$ is aggregate labour and

$$\Xi_t = z_t N_{t-1}^{1-\alpha} \tag{7}$$

is productivity. In order to ensure balanced growth, I have assumed $1/(1-\theta) = 1-\alpha$, which guarantees that the production function is homogeneous of degree one in the accumulating factors K_{t-1} and N_{t-1}[6]. Equation (7) suggests that the productivity process is driven by an exogenous stationary component, z_t, and by an endogenous *trending* one, N_{t-1}, which is pinned down by R&D choices. Through this formulation, exogenous fluctuations in the job-finding rate f_t, in the job-loss rate s_t and in z_t have permanent effects on productivity and thus on output.

[6]For a steady state to be feasible, permanent technical change must be expressible in a labor augmenting form, as stated in Swan (1964) and Phelps (1967) and stressed in King et al. (1988). The parameter restriction I am requiring is shared also with Kung and Schmid (2015) and Queralto (2020) who also have a DSGE model with endogenous growth of the expanding variety type.

2.5 Market Clearing

By the law of large numbers, effective labour is $n_t^I + \kappa n_t^P \equiv n_t \left(\Omega + (1-\Omega)\kappa\right)$ while the capital stock is $K_t = \left(\Omega + (1-\Omega)\kappa\right) n_t k_t$ (where k_t is aggregate capital in units of efficient labour). Moreover, firms' optimal choices imply that the price of one unit of effective labour is $w_t^I = {}^{(\theta-1)}/_{\theta} \left(1-\alpha\right) z_t k_{t-1}^{\alpha} N_{t-1}^{1-\alpha}$ while the ex-post gross return on capital holdings is $R_t = {}^{(\theta-1)}/_{\theta} \alpha z_t k_{t-1}^{\alpha-1} N_{t-1}^{1-\alpha} + 1 - \nu$, where $\nu \in (0,1)$ is the capital depreciation rate[7]. The market clearing for claims to the capital stock requires

$$A_t^P + \Omega \sum_{e=0,1} \int_{\tilde{a}=-\mu_t}^{\infty} a_t\left(\tilde{a}, e\right) F_t\left(\tilde{a}, e\right) d\tilde{a} = K_t$$

where $F_t\left(\tilde{a}, e\right)$ is the joint distribution of (beginning-of-period) wealth level $\tilde{a}$ and employment status e in the population and $a_{t-1}\left(\tilde{a}, e\right)$ is the associated optimal wealth level. Instead, goods market clearing requires

$$C_t^P + \Omega \sum_{e=0,1} \int_{\tilde{a}=-\mu_t}^{\infty} c_t\left(\tilde{a}, e\right) F_t\left(\tilde{a}, e\right) d\tilde{a} + I_t = Y_t$$

where $c_t\left(\tilde{a}, e\right)$ is optimal consumption of the household with employment status e and wealth $\tilde{a}$ while $I_t = K_t - (1-\nu) K_{t-1} + S_t$ is aggregate investment (in physical capital, R&D and adoption).

Moreover, the unemployment insurance scheme is balanced in every period:

$$\tau_t n_t w_t^I \left(\Omega + (1-\Omega)\kappa\right) = (1-n_t)\left(\Omega \delta_t^I + (1-\Omega)\delta_t^P\right).$$

It is now possible to define an equilibrium.

Definition 1. *An equilibrium in this economy is a sequence of quantities C_t^P, c_t^i, A_t^P, a_t^i, S_t,N_t,V_t, k_t, prices R_t, w_t^I, M_{t+1}, aggregate variables τ_t such that households' Euler equations and firms' optimal conditions are satisfied, discovered varieties follow the law of motion (5), innovators break-even, given the forcing process z_t, the initial wealth distribution $\left(A_{-1}^P, a_{-1}^i\right)_{i\in[0,\Omega]}$ as well as N_{-1} and k_{-1}, at any point in time t.*

[7] To obtain the expressions for w_t^I and R_t just combine equations (12) and (13) with (16).

2.6 Equilibrium with Limited Cross-Sectional Heterogeneity

In this section I lay down the assumptions under which it is possible to reduce the cross-sectional distribution of wealth to exactly two states. I then derive aggregation analytically and summarize the equilibrium conditions.

2.6.1 Aggregation

I follow the methodology of Challe and Ragot (2016) in order to obtain an equilibrium with limited cross-sectional heterogeneity. The advantage of this approach is that it allows analytical tractability and a state-space representation, thus making applicable perturbation techniques to solve and simulate the model.

In particular, I assume that the instant utility function of impatient households $u^I(c)$ is

1. continuous, increasing and differentiable over $[0, +\infty)$;
2. strictly concave with relative risk-aversion $\sigma^I(c) = {}^{-cu^I_{cc}}/_{u^I_c} > 0$ over $[0, c^*_t]$, where $c^*_t \equiv c^* N_{t-1}$ is an exogenous, positive threshold that drifts linearly with adopted varieties and
3. linear with slope $\eta_t \equiv \eta N_{t-1} > 0$ over $(c^*_t, +\infty)$.

Both c^*_t and η_t are linear functions of N_{t-1} for stationarity reasons. These assumptions ensure that relatively wealth impatient households (*i.e.* those with $c_t \in (c^*_t, +\infty)$) dislike more fluctuations causing a drop of their consumption level inside $[0, +\infty)$ then those causing a drop within $(c^*_t, +\infty)$.

For the derivations below, I assume

$$u^I(c_t) = \begin{cases} \begin{cases} \log c_t & \text{if } c_t \leq c^* N_{t-1} \\ (\log c^* - 1) + \eta N_{t-1}^{-\sigma}(c_t - c^*) & \text{if } c_t > c^* N_{t-1} \end{cases} & \text{if } \sigma = 1 \\ \begin{cases} {}^{(c_t^{1-\sigma}-1)}/_{(1-\sigma)} & \text{if } c_t \leq c^* N_{t-1} \\ \left[(c^*)^{1-\sigma} - 1\right] + \eta N_{t-1}^{-\sigma}(c_t - c^*) & \text{if } c_t > c^* N_{t-1} \end{cases} & \text{if } \sigma \neq 1 \end{cases}$$

i.e. a logarithmic utility in trending consumption when $\sigma = 1$ and a CRRA one when

$\sigma \neq 1$[8]. Then, combining these assumptions with

$$\text{Condition 1:} \forall i \in [0, \Omega], e_t^i = 1 \implies c_t^i > c_t^*, e_t^i = 0 \implies c_t^i \leq c_t^*$$

ensures that employed impatient households fear unemployment and thus make precautionary savings to limit the rise in marginal utility in case of losing their job. This mechanism is at work also in models with a complete cross-sectional heterogeneity, as in Bewley (1983), an proved key in understanding credit-tightening episodes as shown in Guerrieri and Lorenzoni (2017).

As an additional assumption, I require that the borrowing constraint is binding for all unemployed impatient households, *i.e.*

$$\text{Condition 2:} \forall i \in [0, \Omega], e_t^i = 0 \implies u_c^I\left(c_t^i\right) > \beta^I \mathbb{E}_t\left[u_c^I\left(c_{t+1}^i\right) R_{t+1}\right] \cup a_t^i = -\mu_t.$$

This characterization of the impatient households behaviour reduces their cross-sectional distribution of wealth to two states, implying that there are at most four types of impatient households[9]:

$$\begin{aligned} c_t^{ee} &= w_t^I(1-\tau_t) + R_t a_{t-1} - a_t + D_t & \qquad c_t^{eu} &= \delta_t^I + \mu_t + R_t a_{t-1} + D_t \\ c_t^{ue} &= w_t^I(1-\tau_t) - \mu_{t-1} R_t - a_t + D_t & \qquad c_t^{uu} &= \delta_t^I + \mu_t - \mu_{t-1} R_t + D_t \end{aligned}$$

where, for example, c_t^{ue} denotes consumption of an impatient household that was unemployed at $t-1$ and is employed at t. Aggregation then follows (details are relegated to the Appendix A.3). In particular, total claims on the capital stock by impatient households

[8]This is tantamount to requiring, in terms of de-trended consumption,

$$u_{\hat{c}}^I(\hat{c}_t) = \begin{cases} \begin{cases} \hat{c}_t^{-\sigma} & \text{if } \hat{c}_t \leq c^* \\ \eta & \text{if } \hat{c}_t > c^* \end{cases} & \text{if } \sigma \neq 1 \\ \begin{cases} \hat{c}_t^{-1} & \text{if } \hat{c}_t \leq c^* \\ \eta & \text{if } \hat{c}_t > c^* \end{cases} & \text{if } \sigma = 1 \end{cases}.$$

Notice that this is the very same assumption made in CR, adapted to an endogenous-growth environment. In other words, the resulting marginal utilities in detrended consumption in my framework have the same functional form of those in CR. This is what matters for the existence of the equilibrium with limited cross-sectional heterogeneity as perturbation techniques will be applied to the (balanced growth path) stationary equilibrium system of equations.

[9]For further details see Section 2.1 of Challe and Ragot (2016). See also Section 2.2 of Challe and Ragot (2016) for the existence conditions of this equilibrium. I do not report those conditions here for the sake of space; *mutatis mutandis*, they are isomorphic to those in CR and do not bring any value added to the paper.

amount to

$$A_t^I \equiv \Omega \sum_{e=0,1} \int_{\tilde{a}=-\mu_t}^{\infty} a_{t-1}(\tilde{a},e)\, F_t(\tilde{a},e)\, d\tilde{a} = \Omega \left[n_t a_t - (1-n_t)\,\mu_t \right]$$

while their total consumption is

$$\begin{aligned} C_t^I &\equiv \Omega \sum_{e=0,1} \int_{\tilde{a}=-\mu_t}^{\infty} c_t(\tilde{a},e)\, F_t(\tilde{a},e)\, d\tilde{a} = \\ &= \Omega \left[n_t w_t^I (1-\tau_t) + (1-n_t)\,\delta_t^I - (\mu_{t-1} R_t - \mu_t) + D_t \right] + \\ &+ \Omega \left[R_t n_{t-1} (a_{t-1} + \mu_{t-1}) - n_t (a_t + \mu_t) \right] \end{aligned} \tag{8}$$

Such a formulation is particularly convenient when it comes down to studying the behavior of impatient households' assets. Indeed, the Euler equation for an employed impatient household at time t is

$$\eta N_{t-1}^{-\sigma} = \beta^I \mathbb{E}_t \left\{ \left[(1-s)\,\eta N_t^{-\sigma} + s u_c^I \left(\delta_t^I + \mu_t + R_{t+1} a_t + D_{t+1} \right) \right] R_{t+1} \right\} \tag{9}$$

which pins down asset holdings as a function of aggregate variables only $(s, R_{t+1}, D_{t+1}, N_{t-1}$and $N_t)$. In case of a predicted rise in the the job-loss probability s, *cæteris paribus*, there is a rise in next period marginal utility $u_c^I\left(c_{t+1}^{eu}\right)$. To bring the Euler equation (9) back to equality, the household raises asset holdings a_t.

The equilibrium system of equations characterizing the model is reported in the Appendix A.4 and their stationary counterpart in A.5. For patient households I am assuming a standard CRRA utility function with curvature σ^P.

Existence conditions The equilibrium with limited cross-sectional heterogeneity exists, provided that the conditions outlined in Challe and Ragot (2016), *mutatis mutandis*, are verified. I lay them down in the following Proposition.

Proposition 1. *Assume that,*

(i) there are no aggregate shocks;

(i) unemployment insurance scheme is incomplete $\left(\text{i.e. } \delta^I < w^I (1-\tau)\right)$ *and*

(iii) the following inequality holds:

$$\eta\left[1+\frac{\beta^P-\beta^I}{\beta^I s}\right] > \max\left\{\frac{\beta^I}{\beta^P}\left(f\eta+(1-f)\left(\delta^I+D+\mu g-\mu R\right)^{-\sigma}\right)+\mu\left(g-1\right), \frac{\left(w^I\left(1-\tau\right)-\delta^I-D\right)}{1+\beta^P g^{1-\sigma}}+\delta^I+D+\mu g-\frac{\mu}{g^{-\sigma}\beta^P\left(1+\beta^P g^{1-\sigma}\right)}\right\}$$

then it is always possible to find a utility threshold c^*such that the limited-heterogeneity equilibrium described above exists.

3 Quantitative implications of inequality for the medium-term business cycle

In this section I explore the quantitative implications of inequality for the medium-term business cycle. I do that through the analysis of impulse responses (IRFs) to TFP shocks in several model variants and moments of the simulated series. Looking at IRFs helps understand the propagation mechanism of real disturbances in the model and give intuition on how those could affect the aggregate time series generated by the model. Instead, looking at moments helps isolate the quantitative relevance of such propagation mechanisms. Before doing that, I first discuss the calibration of the model.

3.1 Calibration

The model has 18 parameters. Those related to households heterogeneity are calibrated following CR. In particular, f and s are set to their quarterly averages, which are, respectively, 80.21% and 4.7%. As as result, the BGP value of n is 94.46%[10]. The replacement ratios δ^j/w^j for $j=I,P$ are set to 0.024 such that the consumption growth differential for the average household is 14.26%[11]. The replacement ratio in my model is smaller than that of CR (0.6) because my households receive insurance also from firms' profits. Nevertheless, setting the replacement ratio to a more standard value of 60% doesn't affect the results. I further set $\mu=0$.

[10] At the BGP, $n=f/(f+s)$.

[11] As in CR, this fall is $\Omega(c^e-c^{eu})/c^e$ where $c^e\equiv f\left(1-n\right)c^{ue}+\left(1-s\right)nc^{ee}$.

The share of impatient households is set to $\Omega = 0.6$ as in the baseline scenario in Challe and Ragot (2016) and in line with empirical evidence therein reported (estimates range between 15% and 60%). This implies that along the BGP the share of effectively borrowing constrained households is $\Omega(1-n) = 3.4\%$. The discount factor of patient households is set to $\beta^P = 0.99$ and the curvature of the utility function to $\sigma = 1$ (*i.e.* log utility in the baseline). The value of the marginal utility of *ee* impatient households in detrended consumption is set to $\eta = 1.75$ which ensures that they have the same marginal utility of patient households, thus minimizing differences in asset holding behaviour due to differences in instant utility functions. The impatient households discount factor β^I is set to 0.972 to produce a wealth share for the poorest 60% households of 0.30%, as it is in the distribution of liquid wealth in the Survey of Consumer Finances. The skill premium κ is set to 3.99 to yield a consumption share $C^I/(C^I+C^P)$ of 40.62% for the poorest 60% of households, in line with the Consumer Expenditure Survey[12].

Turning to production and R&D parameters, I set the capital share α to a standard value of $1/3$, the depreciation rate of capital ν to 2.5% and the obsolescence rate ϕ to 2%, in line with evidence in Caballero and Jaffe (1993). The value of α fixes that of the elasticity of substitution across varieties, θ, to 2.5, implying a markup of 1.67, slightly above common estimates[13] but in line with the one set in Comin and Gertler (2006) (1.6). Regarding the elasticities of new varieties with respect to R&D, ψ, I set it to 0.8, which is the mean of estimates reported in Griliches (1990). The scale parameter χ instead is set to 0.76 which yields a 2% annual growth rate for the economy. Through the patient households Euler equation, this, together with the choice of σ and β^P, implies a quarterly interest rate of 1.5%. I set the innovator's profits scale parameter ι to 0.07 such that the R&D to GDP ratio is 2%, the US post-war mean. Parameter values are summarized in Table 10 in the Appendix E.

Forcing process The only forcing process considered is an AR1 process for the exogenous component of TFP, z. Its persistence (0.71) and volatility (0.0061) are calibrated to match empirical estimates using a time series for TFP computed as in Ríos-Rull and Santaeulália-Llopis (2010) and filtered with the Christiano and Fitzgerald (2003) bandpass filter.

[12] Empirical estimates find a value for the skill premium that ranges between 1.3 and 1.9 (see Heathcote et al., 2010 and Acemoglu and Autor, 2011). My calibration features an unrealistically high skill premium to generate the observed consumption inequality because of the presence of firm profits in impatient households budget constraint.

[13] Recall that in Section 2.4 I required $1/(1-\theta) = 1 - \alpha$ to ensure balanced-growth.

3.2 Impulse responses

In this section I present impulse responses to a TFP shock z in order to shed light on the transmission mechanism of the baseline model and variations of it. Since the CR model can be though of as the "reduced heterogeneity version" of KS, I start discussing TFP shock propagation in the benchmark KS model and in its RA counterpart. This will help clarify the baseline mechanics of the KS structure and the difference with the more familiar RA model. I then turn to the basic CR model and outline differences/similarities with KS. Finally, I discuss HAEG IRFs with the objective of (i) outlining how it departs from the basic CR model and (ii) how the presence of heterogeneity affects the long-run.

KS vs RA The Appendix C contains all model equations and calibration of the KS model. Impulse responses are reported in Figure 1. The KS and RA models look indistinguishable, both in terms of output and in its components (consumption and investment). Indeed, one of the results in Krusell and Smith (1998) is that simulated time series from the baseline KS and its RA counterpart are alike, except for mean values. Even if Figure 1 suggests a little bit of amplification in consumption due to high marginal propensities to consume of constrained agents in the KS economy, this is accompanied by a milder response on impact in investment; the two effects cancel each other out and output responses between the two models are almost identical.

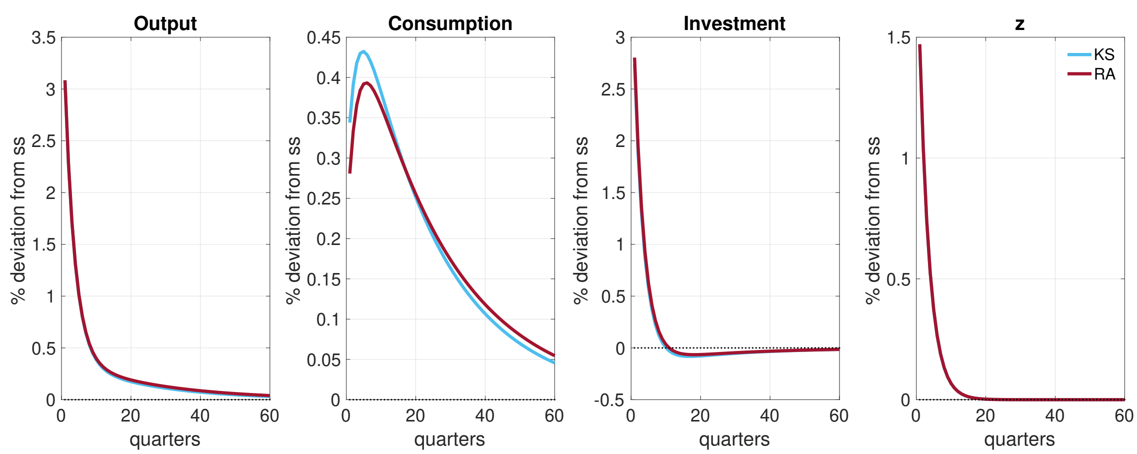

Figure 1: Impulse responses in the KS (light blue) and RA (red) model with a standard value for α, to a 1 standard deviation shock to TFP z.

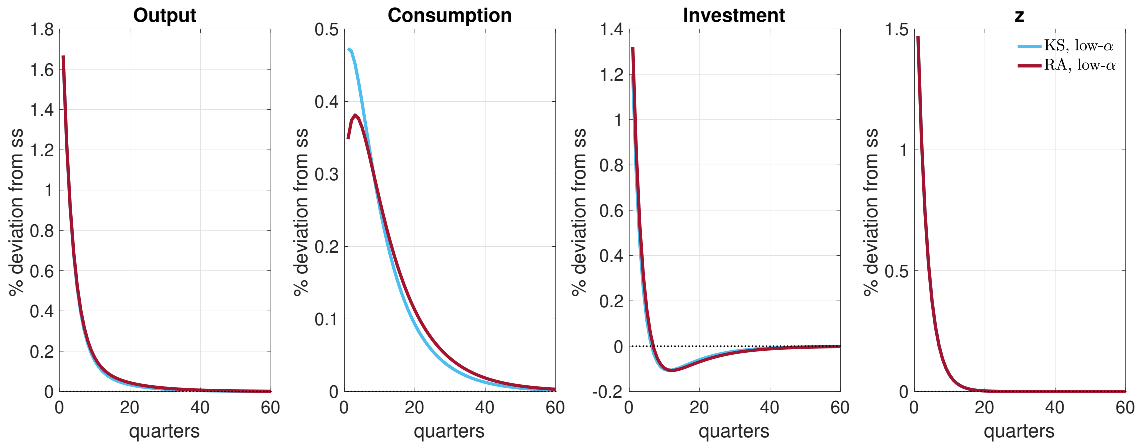

Figure 2: Impulse responses in the KS (light blue) and RA (red) model with a low value for α, to a 1 standard deviation shock to TFP z.

The reason why the two model economies are so similar lies in the wealth share of the poorest households, which is particularly high in KS. Figure 2 presents impulse responses in the KS and RA model with a lower wealth share for the poorest households, achieved via a reduction in the capital share α. Less room for self insurance induces *more* consumption amplification and investment smoothing, for the reasons outlined above. In this low-α case the KS and RA economies behave differently in terms of consumption and investment but at the aggregate output level are still identical.

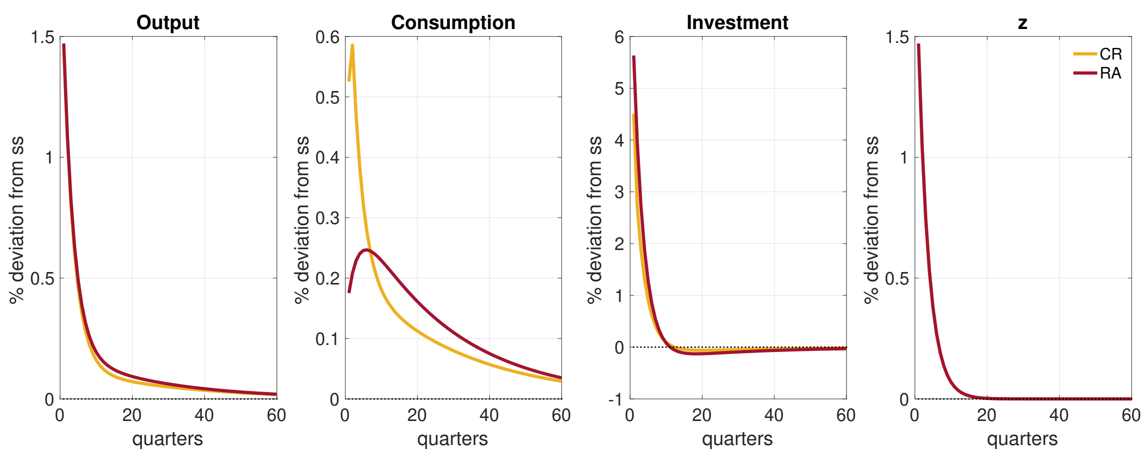

Figure 3: Impulse responses in the CR (yellow) and RA (red) model to a 1 standard deviation shock to TFP z.

CR vs RA and relation with KS Figure 3 presents IRFs to a one standard deviation increase in z in the baseline CR model and in its RA counterpart. Models equations are as in Challe and Ragot (2016) and reported in the Appendix B for convenience, together with calibrations.

IRFs show that consumption jumps much more on impact in the CR model than in the RA one, as a reflection of the higher marginal propensity to consume of impatient households given by their higher β^I; the other side of the medal is a milder increase in investment with respect to the RA model. This is the *investment smoothing* property that the CR model inherits from KS, having assumed a core structure similar to the latter. At the aggregate level, output responses are almost indistinguishable between the CR and RA model, as a reflection of the fact that the amplification in consumption is perfectly absorbed by the smoothing in investment.

Comparing the CR and KS impulse responses shows that the CR model is very similar to the low-α KS model. In particular, apart from a difference in magnitudes, the qualitative message of the KS model is present in CR: households near the borrowing limit have a high marginal propensity to consume which makes them respond very strongly to TFP shocks; this amplification in consumption produces a smoothing in investment, and output behaves as in the RA economy. The similarity between the CR and low-α KS model justifies the use of the former (and its variants) for isolating the role of inequality for the medium-term business cycle.

HAEG vs RAEG In order to understand the long-run effects of heterogeneity, I first discuss impulse responses of the stationary variables $\hat{x}$, which are reported in Figure 4. Regarding consumption, as it was the case in the CR model, it jumps on impact, much more than in the RAEG model (2.6 times), and quickly reverts back to steady state. Differently from the CR model, the HAEG one has 2 types of investment purposes, namely investment in physical capital and investment in R&D; importantly, the KS investment smoothing property applies to *both* investment's ends. In other words, in the HAEG model, not only investment in physical capital is smoothed with respect to the RAEG model, but also investment in R&D. This produces *long-run* differences between the two model economies. Indeed, looking at Figure 5 which reports IRFs for the original trending variables, it is possible to notice that the difference in R&D behavior cumulates in different varieties N_t and TFP Ξ_t dynamics, which in turn shape the long-run outcome. In particular, being R&D more responsive in the RAEG model, this will end-up in a steady state with 23% larger outcomes. Hence, if TFP shocks are the only source of exogenous fluctuations, the HAEG model displays milder booms and recessions than its RAEG counterpart.

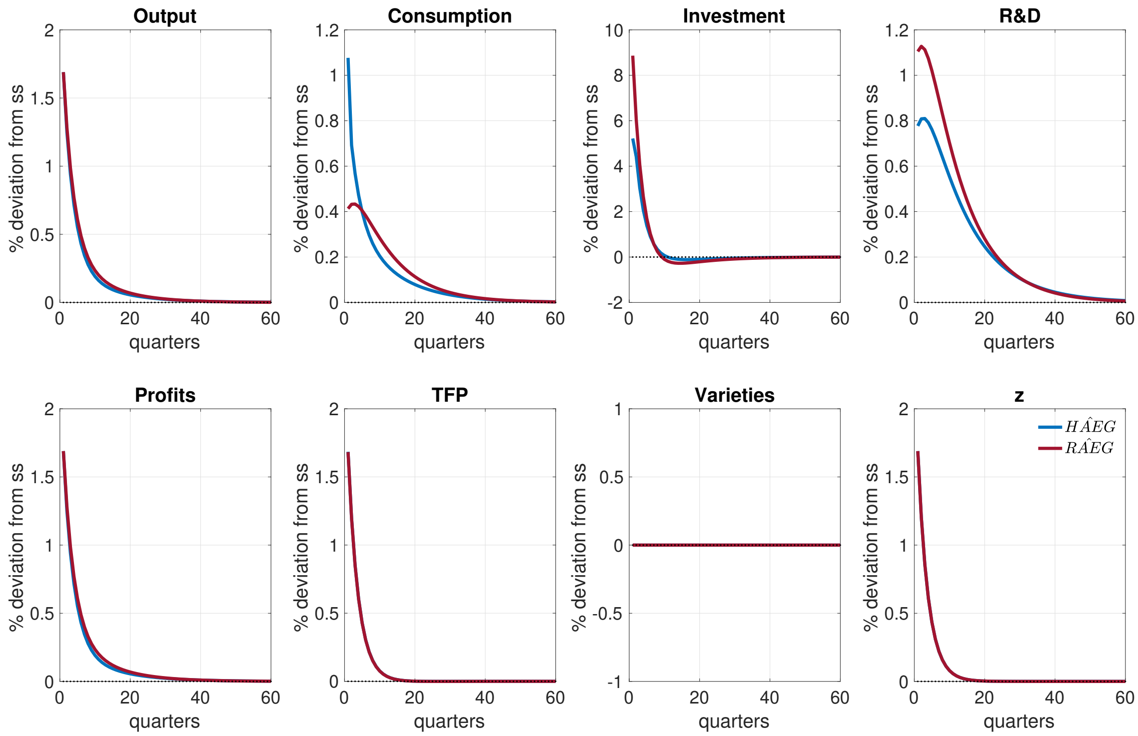

Figure 4: Impulse responses of detrended variables in the HAEG (blue) and RAEG (red) model to a 1 standard deviation shock to TFP z.

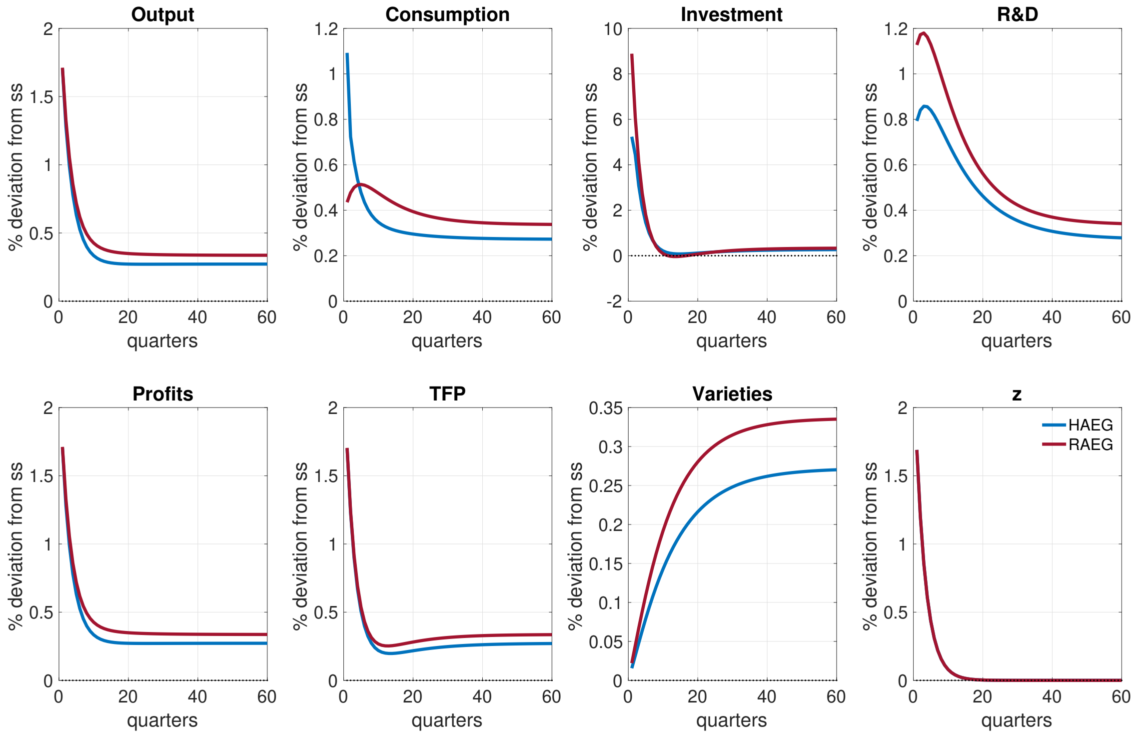

Figure 5: Impulse responses in the HAEG (blue) and RAEG (red) model to a 1 standard deviation shock to TFP z.

The IRFs behaviour of HAEG sheds light also on the moments generated by the model, which will be discussed in detail in Section 3.3. Indeed, for high-frequencies (*i.e.* business cycles) and at the *aggregate* level holds the KS result: HA economies behave much like RA economies; nonetheless, this result breaks down at lower frequencies (*i.e.* medium-run) as different short-run R&D dynamics trigger differences in medium-frequencies behavior at the aggregate level. This is a novel result which makes less innocuous abstracting from heterogeneity in macroeconomic modeling, especially if one looks at the medium/long-run behavior. This result calls for more research on the propagation mechanism of real disturbances in heterogeneous agent economies, both theoretically and empirically.

3.3 Moments

In this section I present moments of simulated series from several model specifications in order to understand the quantitative relevance of heterogeneity for high and medium frequencies fluctuations. The ultimate objective is not to discriminate among models based on overall performance; rather, it is to look for differences in models' performances and to understand where these come from.

In particular, I first compare moments from the CR and RA models to study differences at the aggregate level and in consumption and investment. This exercise will also give a sense of how much is gained/lost from switching from the RA to the CR model in terms of fit. I will then compare the CR model with the baseline HAEG model to check whether the introduction of endogenous growth improves or deteriorates overall performance. The last exercise will be to compare the HAEG and RAEG economies to isolate the role of heterogeneity for medium-frequencies fluctuations. Results are presented in Table 1.

CR vs RA The baseline CR model produces an output volatility (1.67) that is very close to that in the data (1.64) and is able to generate a substantial amount of consumption volatility for a given amount of output volatility (relative consumption standard deviation is 0.43 against 0.51 in the data). This result is mostly driven by the lower β of impatient households. Regarding correlations with output, the model overpredicts them for both consumption (0.95 in CR vs 0.79 in the data) and investment (0.99 in CR vs 0.88 in the data). In terms of autocorrelations instead, the model is close to the data as long as consumption is concerned (0.70 in CR vs 0.83 in the data) but it substantially misses those for output (0.56 in CR vs 0.84 in the data) and investment (0.48 in CR vs 0.79 in the data). The high consumption relative standard deviation is achieved, as suggested by IRFs (Figure 3), through a less volatile investment; indeed, the CR model predicted investment standard deviation is 4.87 against 7.42 in the data (and relative standard deviation is 2.92 against 4.52 in the data).

Turning to the comparison with its RA counterpart, as impulse responses (Figure 3) suggested, in terms of output volatility the two models perform in the same way (Table 1). They get right the latter at the business cycle frequency (HF standard deviation of output in CR is 1.67 against 1.66 in RA and 1.64 in the data) and underestimate it at the medium frequency (MF standard deviation of output in CR is 1.34 against 1.41 in RA and 2.39 in the data) by almost the same amount. Turning to consumption volatility, the CR model (0.72) is much closer to the data (0.84) than the RA one (0.37); as discussed in relation to impulse responses, this higher consumption volatility is obtained through a smoother investment time series. Nonetheless, the CR model misses investment volatility much less than the RA model misses the consumption one. This goes on also for relative standard deviations. In terms of correlations with output and first order autocorrelations the two models perform very similarly. Overall, the CR model fits slightly better the data (by a factor of 14%)[14].

[14] I compute a moments-loss function of the type, $\mathcal{L} \equiv \sum_{i=1}^{I} \left[\left(m_i^{model} - m_i^{data} \right) / m_i^{data} \right]^2$, for a generic moment m

Table 1: Medium-term business cycle statistics

	U.S.											
	Standard deviation (%)			Relative standard deviation			Correlation with output			Autocorrelation (1st order)		
	MT	MF	HF	MT	MF	HF	MT	MF	HF	MT	MF	HF
Y	2.92	2.39	1.64							0.95	0.99	0.84
C	2.01	1.82	0.84	0.69	0.76	0.51	0.75	0.76	0.79	0.97	1	0.83
I	10.34	7.15	7.42	3.54	2.99	4.52	0.76	0.72	0.88	0.89	1	0.79
	CR											
Y	2.17	1.34	1.67							0.73	0.99	0.56
C	1.13	0.85	0.72	0.52	0.64	0.43	0.93	0.94	0.95	0.87	0.99	0.70
I	5.91	3.21	4.87	2.73	2.40	2.92	0.97	0.96	0.99	0.64	0.99	0.48
	RA											
Y	2.21	1.41	1.66							0.74	0.99	0.56
C	0.83	0.78	0.27	0.37	0.55	0.16	0.62	0.74	0.68	0.97	1.00	0.80
I	7.92	4.40	6.45	3.59	3.12	3.88	0.96	0.92	1.00	0.70	0.99	0.55
	HAEG											
Y	2.52	1.59	1.92							0.73	0.99	0.55
C	1.69	1.22	1.13	0.67	0.77	0.59	0.97	0.96	0.99	0.75	0.99	0.47
I	7.2	3.97	5.88	2.86	2.5	3.06	0.96	0.91	0.99	0.74	0.99	0.62
	RAEG											
Y	2.67	1.79	1.94							0.76	0.99	0.56
C	1.53	1.39	0.61	0.57	0.77	0.31	0.78	0.87	0.82	0.95	1.00	0.75
I	10.89	5.59	9.18	4.09	3.14	4.73	0.90	0.81	0.98	0.67	0.99	0.54

Notes: Entries are filtered with the bandpass filter proposed in Christiano and Fitzgerald (2003). The medium term cycle is defined as in Comin and Gertler (2006) and includes frequencies between 2 and 200 quarters. The high-frequency component includes frequencies between 2 and 32 quarters (the standard notion of business cycles). The medium-frequency component includes frequencies between 32 and 200 quarters.

Unsurprisingly, focusing on the medium-term business cycle only, the CR and RA model economies perform almost in the same way, with a difference as low as 4%. This follows from the fact that in such models is absent a mechanism that endogenously propagates short-run disturbances to the medium-run *and* that is affected by heterogeneity; this mechanism is instead present in the HAEG and RAEG model economies (*i.e.* R&D) and will produce a very different medium-run outcome than CR-RA do.

HAEG vs CR Turning to HAEG, the introduction of endogenous growth allows the basic CR model to better capture medium term cycles (MT) and medium frequency (MF) fluctuations, although something is lost in accuracy at the business cycle frequency. The HAEG model is better at capturing relative standard deviations at all frequencies while in terms of correlation with output and first order autocorrelation the two models share the same performance. Where the HAEG model is particularly bad is in terms of consumption autocorrelation which appears to be somewhat low (0.47) compared to that of the data (0.83) and of CR (0.70). Nonetheless, the HAEG model improves over the CR one by 27% (and by 46% looking only at the medium term business cycle). This suggests that the introduction of the endogenous growth mechanism allows the CR model to improve along several dimensions and is thus an important mechanism to fit the data at both high and medium frequencies.

HAEG vs RAEG Comparing HAEG and RAEG model economies allows to isolate the role of heterogeneity for fluctuations at both the high and medium frequency. The RAEG model does not feature the consumption amplification/investment smoothing property of the HAEG one, thus is better in capturing investment volatilities and less so in terms of consumption ones. RAEG investment relative standard deviation is 4.73 against 3.06 in HAEG and 4.52 in the data while the consumption one is 0.31 against 0.59 in HAEG and 0.51 in the data. The same ordering of fit holds also at the medium frequency and over the medium term cycle. The RAEG model improves over the HAEG one in terms of high frequency consumption correlation with output (0.82 in RAEG against 0.99 in HAEG and 0.79 in the data) and in terms of medium frequency and medium term cycle consumption and investment correlations with output. In terms of autocorrelations, RAEG performs better in relation to consumption but looses in terms of investment, thus overall they perform similarly.

of variable *i*. Thus, "fitting better the data by a factor of 14%" means that the loss function from switching from RA to CR diminishes by 14%.

It is also important to notice that, for a given amount of high frequency volatility, the RAEG model implies larger medium frequency fluctuations and medium term cycles than HAEG. This follows from the analysis made looking at impulse responses in Figure 5: the HAEG model features investment smoothing both on investment in physical capital *and* on investment in R&D, thus generates lower growth (and thus medium frequency fluctuations and medium term cycles) than its RAEG counterpart[15].

Of interest, and in stark contrast with the CR/RA result, is that switching from HAEG to RAEG implies a gain in model fit of 78% (and of 79% looking only at the medium term business cycle). This underlies two important aspects: first, contrary to the case of CR, the RAEG counterpart of the heterogeneous agents, endogenous growth model performs better in terms of moments fit; second, the difference between the RAEG and HAEG model economies is *sizable*, both overall *and* at the medium term business cycle frequency. This is a reflection of the impulse responses analysis: without endogenous growth, looking at a model endowed with households heterogeneity is as if one is looking at a representative agent economy, for impulse responses and generated time series moments are almost undistinguishable. In particular, there is no difference at all between the KS and RA models while there is a *small* difference between the CR and RA models. Yet, absent a mechanism that endogenously connects the short and medium run, even such small differences do not propagate to the medium term business cycle, which is the very same in CR and RA model economies. Instead, introducing into the picture endogenous growth in HA models, while leaving high frequency behavior almost unaffected (in terms of output, consumption and investment), induces much different medium run outcomes. Different responses on impact to short run disturbances translate in different slow moving evolutions of TFP Ξ_t which in turn shape differently the medium term business cycle. This suggests that HA model economies should be embraced not only because they allow the study of distributional specific issues (*e.g.* the effect of inequality for aggregate time series fluctuations) as argued in Krusell and Smith (1998), but also because they imply very different medium-run behaviors.

[15]Neither the HAEG model nor the RAEG one are able to generate the observed medium frequency amplification. In the influential paper of Comin and Gertler (2006) it was shown that, in order to capture observed medium run dynamics, a model needs investment in R&D, endogenous technology adoption and endogenous entry and exit. For simplicity, I introduced only R&D into the CR framework, and therefore the model is not able to match this feature of the data. Nonetheless, it is a good starting point as its simplicity allows to elicit more clearly the role of endogenous growth and which mechanism accounts for it.

3.4 Simulations with historical productivity shocks

Up to now, the analysis of impulse responses and of theoretical moments, showed that the HAEG and RAEG model economies behave similarly in terms of output at the business cycle frequency, but also that the induced medium-term cycle is quite different. Even though the percentage change in model's fit is a good measure to understand the difference between the two models, it is of difficult practical interpretation. It is thus useful to feed in the models the historical productivity shocks for the period 1948Q2-2007Q4, and compare the generated time series. This task I tackle in the present section.

Figure 6 shows historical time series for HAEG and RAEG together with the observed ones in the US, at the business cycle frequency. Consistently with the IRFs and moments analysis, the output time series generated by HAEG and RAEG are remarkably similar. Indeed, the difference in standard deviations is just 2%. The differences are instead much larger for consumption (76%) and investment (36%), as suggested by the consumption amplification and investment smoothing properties of the HAEG model already discussed.

Figure 8 instead plots historical series at the medium-term cycle frequency. The difference between the HAEG and RAEG output series is now more evident. Indeed, the standard deviation of output is 7% larger in RAEG than in HAEG. This suggests that the investment smoothing observed at the high frequency in Figure

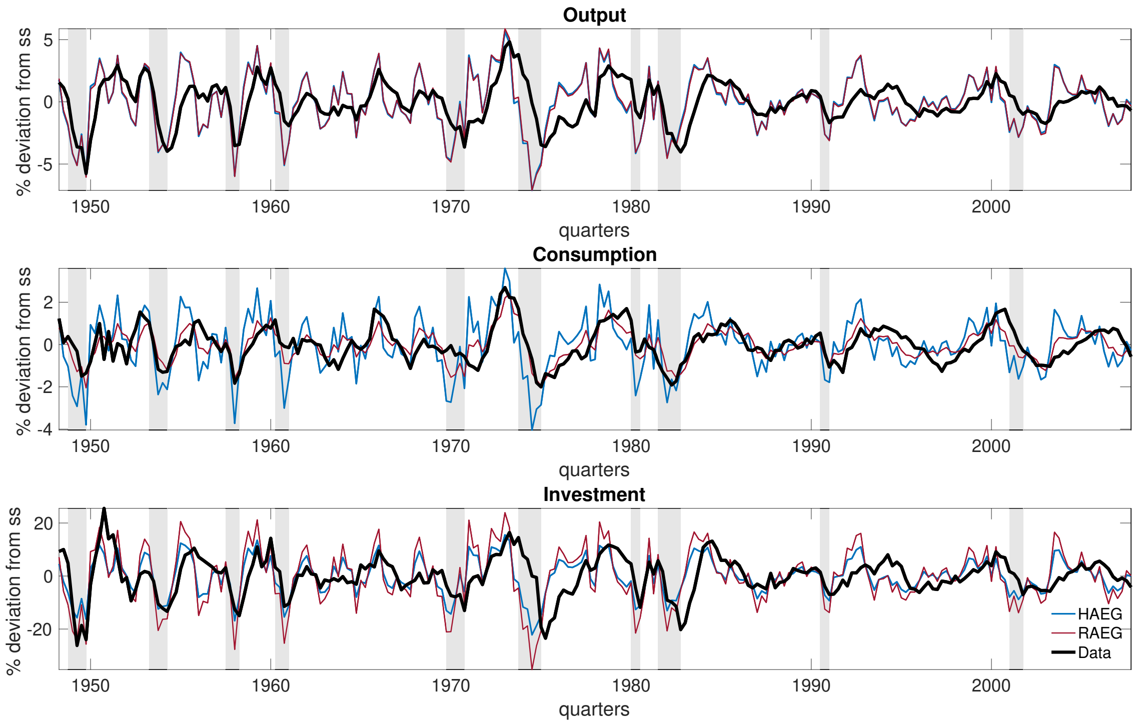

Figure 6: Simulated model's (HAEG in blue and RAEG in red) time series at the high frequency (2-32 quarters bandpassed) with estimated innovations and data (in black). Grey bars denote NBER recessions. Correlations for the HAEG model with data are, respectively for output, consumption and investment, 0.57, 0.47 and 0.52. That for RAEG instead are 0.60, 0.75 and 0.47.

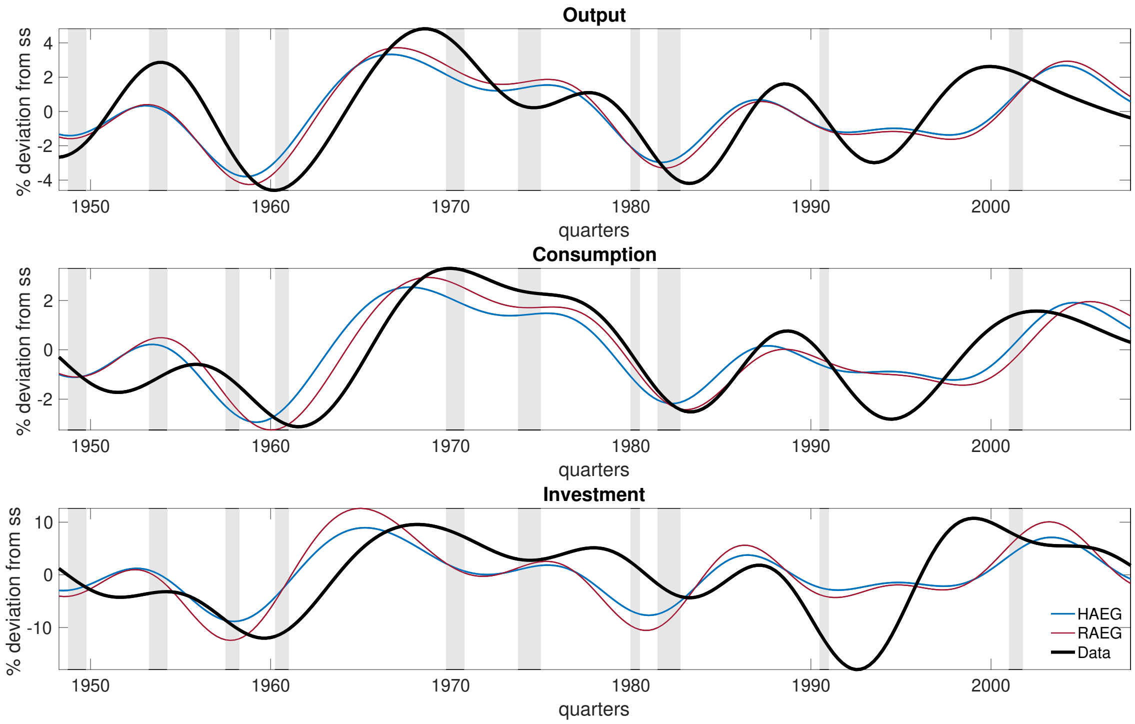

Figure 7: Simulated model's (HAEG in blue and RAEG in red) time series at the medium-frequency (32-200 quarters bandpassed) with estimated innovations and data (in black). Grey bars denote NBER recessions. Correlations for the HAEG model with data are, respectively for output, consumption and investment, 0.70, 0.80 and 0.52. That for RAEG instead are 0.73, 0.85 and 0.50.

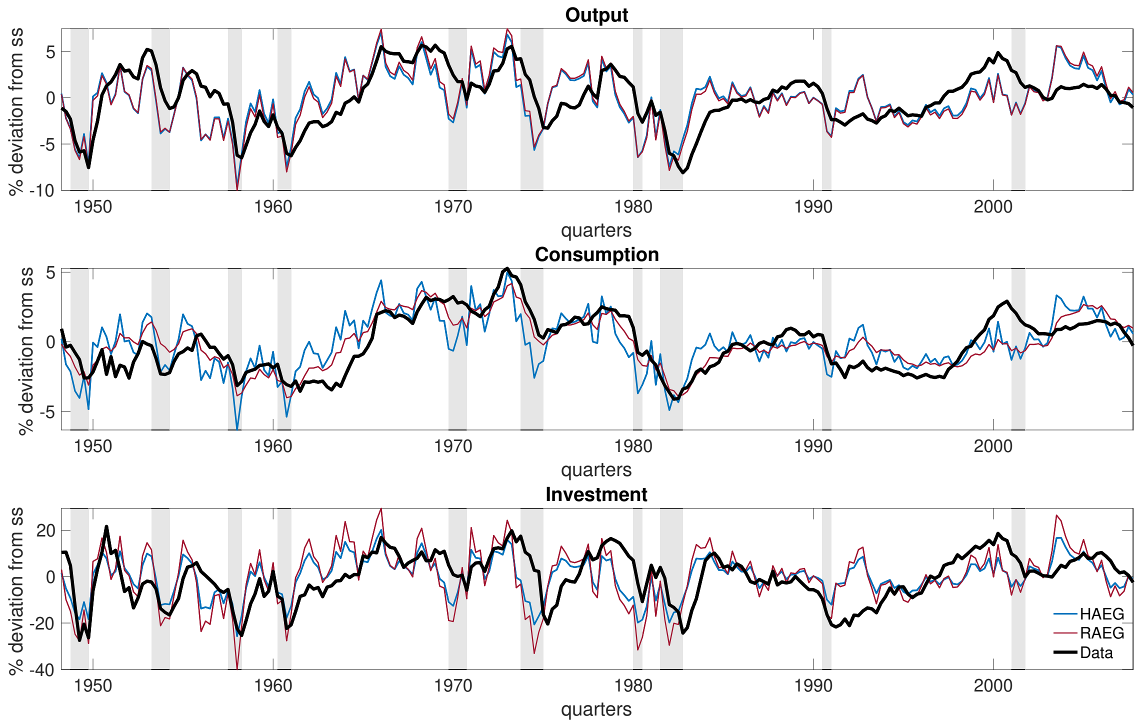

Figure 8: Simulated model's (HAEG in blue and RAEG in red) time series at the medium-term cycle frequency (2-200 quarters bandpassed) with estimated innovations and data (in black). Grey bars denote NBER recessions. Correlations for the HAEG model with data are, respectively for output, consumption and investment, 0.63, 0.67 and 0.50. That for RAEG instead are 0.67, 0.84 and 0.46.

6 in HAEG induced different slow-moving dynamics with respect to RAEG, thus yielding different medium-term business cycles. This intuition is confirmed looking at Figure 7 which shows historical time series at the medium-frequency. The induced medium-frequency oscillations of RAEG are 10% more volatile than that of HAEG as long as output is concerned; regarding consumption and investment instead, volatilities are, respectively, 10% and 27% higher in RAEG. This confirms that there is a sizable difference between medium-term oscillations between heterogeneous agents and representative agents model economies, even if at the business cycle frequency the two models look very similar.

3.5 Sensitivity analysis

I now investigate how robust are the results so far presented. In particular, of interest is to what extent the output time series discrepancy between HAEG and RAEG changes as deep parameters change. I analyze output, rather than also consumption and investment because this is the main focus of the paper. In particular, the fact that consumption and investment time series differ also at the medium term business cycle is not of particular interest, as this was already implied by the analysis of Krusell and Smith (1998) (with a more realistic wealth share for the poorest) and Challe and Ragot (2016); instead differences in output at the medium term business cycle are the distinctive feature of my HAEG model economy. Nonetheless, for completeness I present a sensitivity analysis based on overall performance in Appendix F.

I inspect parameters that affect the ability of impatient households to self-insure against idiosyncratic shocks, for these increase/decrease the difference brought by households heterogeneity, as well as parameters relative to research and development, as they control how much the short-run affects the medium-run. I do that for HAEG and RAEG. As the room for self-insurance increases, one expects the heterogeneous agent model to behave much more like its representative agent counterpart, and thus the difference in output behavior should decline. Results are reported in Table 2.

I start with the change in the mass of impatient households, Ω. A fall in Ω means that there are fewer households facing idiosyncratic shocks in an incomplete market set up, thus the overall population is better self-insured and the heterogeneous agent economies behave more like RA ones. Table 2, column (b), confirms this intuition: the difference in output volatilities declines from 7% to 6%.

Table 2, columns (c) and (d), display results under a replacement ratio $\delta^{I,P}/w^{I,P}$ of, respectively, 0.45 and 0.6. A higher replacement ratio reduces the necessity for self-insurance. Simulations suggest that this parameter affects only marginally the aggregate time series properties: in both the cases considered, the difference in output volatilities remains at 7%, as in the baseline.

The skill premium parameter κ affects the ability to self-insure as well as the consumption inequality in the economy. Setting κ to 1 thus makes consumption more equally distributed (through also a raise in the replacement ratio) and pushes HA models to behave more as RA ones; at the same time, impatient households consume more than in the baseline case in steady state, which pushes toward a greater difference between HA and RA economies. In my simulations the latter effect dominates and output volatility is 8% larger in RAEG than in HAEG (column (e)).

The impatient households discount factor β^I controls the extent to which impatient households are impatient and their ability to self-insure. Increasing its value to 0.9773 doesn't affect much the results (column (f)). The same applies to raising the borrowing limit μ to 0.1 (column (g)).

Regarding the risk aversion coefficient, $\sigma^{I,P}$, I explore values of it of 1.5 and 0.5 Results are in columns (h) and (i). In the 1.5 case, higher risk aversion induces higher savings which in turn requires a lower β^I to match the 0.30% liquid wealth target. Thus there are two contrasting effects: on the one hand the higher σ implies higher risk aversion and thus smoother and more autocorrelated consumption while on the other hand the lower β^I induces more reactive consumption and less autocorrelation. In my simulations the second effect dominates and the difference between HA and RA economies is strengthened: the RAEG model produces an output volatility 7.3% larger than HAEG, thus only slightly larger than in the baseline. With a σ of 0.5 the results are topsy turvy and the RAEG-HAEG difference declines to 5%.

I also analyze the role of ψ which controls the elasticities of new varieties with respect to R&D. Estimates for this parameter reported in Griliches (1990) range from 0.6 and 1. I therefore simulate the 0.6 and 1 cases and results are reported in columns (j) and (k) respectively. A higher value for ψ means that new varieties N_t are more responsive to changes in investment in R&D S_t thus suggesting that short-run disturbances should have larger medium-run consequences. This is indeed the case: the $\psi = 1$ case displays a output standard deviation that is 14% bigger in RAEG than in HAEG (against a 7% in the baseline case). For the $\psi = 0.6$ case, the difference lowers to 6%.

This analysis suggests that the main result of the paper, *i.e.* that inequality substantially affects the medium-term business cycle, is robust. Estimates around empirically plausible parameter values suggest that differences in output volatilities due to inequality range between 5% and 14%. However, I show below (Section 4) that the degree in asset liquidity plays an important role in shaping the medium-term effects of inequality for aggregate time series. Moreover, as already discussed, the value for ψ chosen in the baseline calibration, 0.8, doesn't allow the model (neither HAEG nor RAEG) to generate empirically plausible medium frequencies oscillations. A value of 1 for ψ is much more realistic, thus the effect of inequality leans more toward the 14% neighborhood than toward the 5% one.

Table 2: Inequality and the medium term business cycle

	baseline	$\Omega =$ 0.3	$\delta^{I,P}/w^{I,P} =$ $\{0.45, 0.6\}$	$\kappa = 1$	$\beta^I =$ 0.9773	$\mu = 0.1$	$\sigma = 1.5$	$\sigma = 0.5$	$\psi = 0.6$	$\psi = 1$
	(a)	(b)	(c), (d)	(e)	(f)	(g)	(h)	(i)	(j)	(k)
σ^Y	7%	6%	7%	8%	7%	7%	7%	5%	6%	14%

Notes: Entries refer to percentage deviations of output standard deviations between the HAEG and RAEG model economies fed with historical shocks over the period 1948Q2-2007Q4.

3.6 The role of monetary policy shocks

The influential work of Kaplan et al. (2018) showed that households heterogeneity is essential for capturing the right transmission of monetary policy to consumption. The standard representative agent model features a transmission of monetary policy that relies only on direct effects (the change in consumption due to a change in the interest rate) governed by the Euler equation; at the same time, empirically this channel is small, and more importance is indeed attached to indirect effects (the change in consumption due to general equilibrium effects that the change in the interest rate brought). The HANK model proposed in Kaplan et al. (2018) gets the decomposition between direct and indirect effects right, and households heterogeneity is shown to be essential to achieve that. As a result, consumption responses to monetary policy shocks are amplified in the baseline HANK with respect to its RANK counterpart. Subsequent work of Auclert et al. (2020) showed that also investment acts as an amplification mechanism for monetary policy, *i.e.* the output response to monetary policy shocks is amplified in HA economies when investment is added to the model with respect to the baseline case with no investment and with respect to the RA counterpart. On the other hand, Smets and Wouters (2003) and Smets and Wouters (2007) reported that monetary policy shocks play an important role for explaining output variations in the Euro Area for the period 1970-2000 and less so for the US, where indeed productivity shocks and wage mark-up shocks are the main drivers of output over the period 1966-2004.

This discussion suggests that my analysis may be missing an important aspect through which inequality may affect the medium-run behavior of model economies. One may conjecture that, since heterogeneity amplifies output response to monetary policy shocks – and even more so in the presence of investment – it should be the case that HA economies display more responsive investment in R&D than their RA counterparts, as innovation incentive is intimately related to expected discounted future profits, which are simply a fraction of GDP, as showed in Section 3. Thus the results for TFP shocks in Sections 3.2 and 3.3 may be topsy-turvy for monetary policy shocks, in the sense that heterogeneity amplifies medium-frequencies fluctuations. However, the introduction of monetary policy shocks and the analysis of their quantitative contribution for the medium-term business cycle in an estimated model such as Auclert et al. (2020) or Bilbiie et al. (2022) is outside the scope of this work and I leave it to future research.

4 The role of asset liquidity

So far, an assumption underlying the KS structure, and thus also that of CR and HAEG, is that households form precautionary savings through capital stock holdings. In reality, holdings of the capital stock are illiquid and precautionary savings are more realistically formed through bonds. In this section I discuss the implications of introducing a distinction between liquid (bonds) and illiquid (capital stock holdings) assets for the transmission of real shocks in the heterogeneous agents economies (CR and HAEG) and their respective RA counterparts. In both the CR and HAEG model, I assume that impatient households can invest only in liquid bonds while patient households have access to both liquid bonds and illiquid capital stock holdings. The latter's degree of liquidity is governed by investment adjustment costs à la Christiano et al. (2005). The model set up and equilibrium conditions are laid down in the Appendix D, together with their respective calibrations.

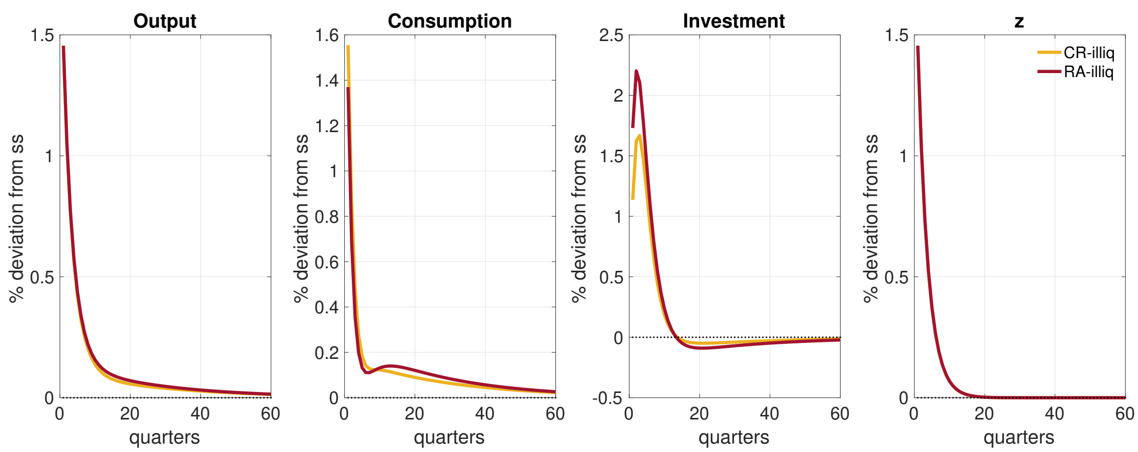

Figure 9: Impulse responses to a 1 standard deviation shock to TFP z in the CR (yellow) and RA (red) models augmented with liquid and illiquid assets.

CR Figure 9 shows impulse responses to a one standard deviation TFP shock z in the CR and RA models augmented with liquid and illiquid assets. As it is well known from the RBC literature, investment adjustment costs make, at the same time, investment less and consumption more responsive to shocks. In particular, comparing the RA response with adjustment costs in Figure 9 with that without in Figure 3, one notices that the RA model behaves much more like the CR model; in other words, being less able to smooth consumption over time, the RA economy with illiquid assets compensates that with a higher jump in consumption, thus displaying a MPC similar to that of the CR model. The CR model instead behaves similarly as before, except for the higher magnitude of responses; this is due to the fact that the patient households populating the CR model behave much more like the impatient ones, amplifying consumption fluctuations with respect to the baseline scenario. This, together with the fact that investment behaves very similarly in the CR and RA models, implies that output responses are the same. In the Appendix E in Figure 14 I conduct the same exercise with infinite investment adjustment costs – thus making investment irresponsive to shocks – and find that the similarity between CR and RA is strengthened. This conclusion is in line with the "as if" result of Werning (2015), *i.e.* that when there is no liquidity and income risk is acyclical (as it is the case in CR with infinitely illiquid assets) the heterogeneous agent economy behaves as if the economy was populated by a representative households (or markets were complete).

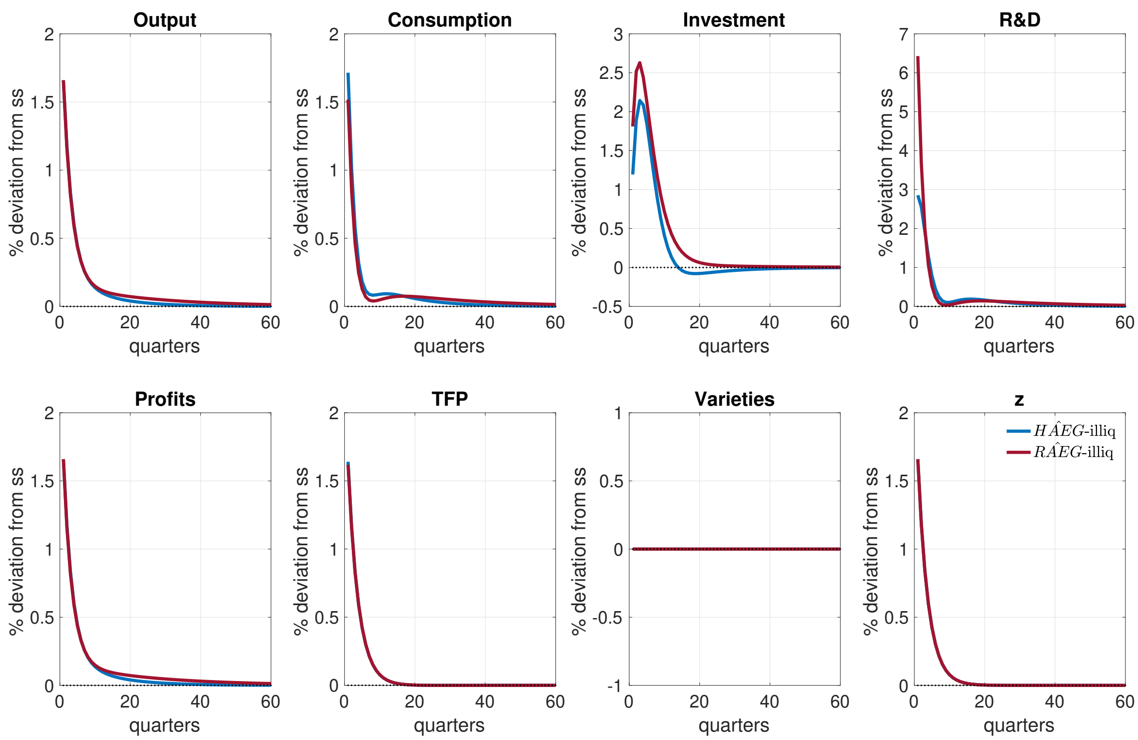

Figure 10: Impulse responses of detrended variables to a 1 standard deviation shock to TFP z in the HAEG (blue) and RAEG (red) models augmented with liquid and illiquid assets.

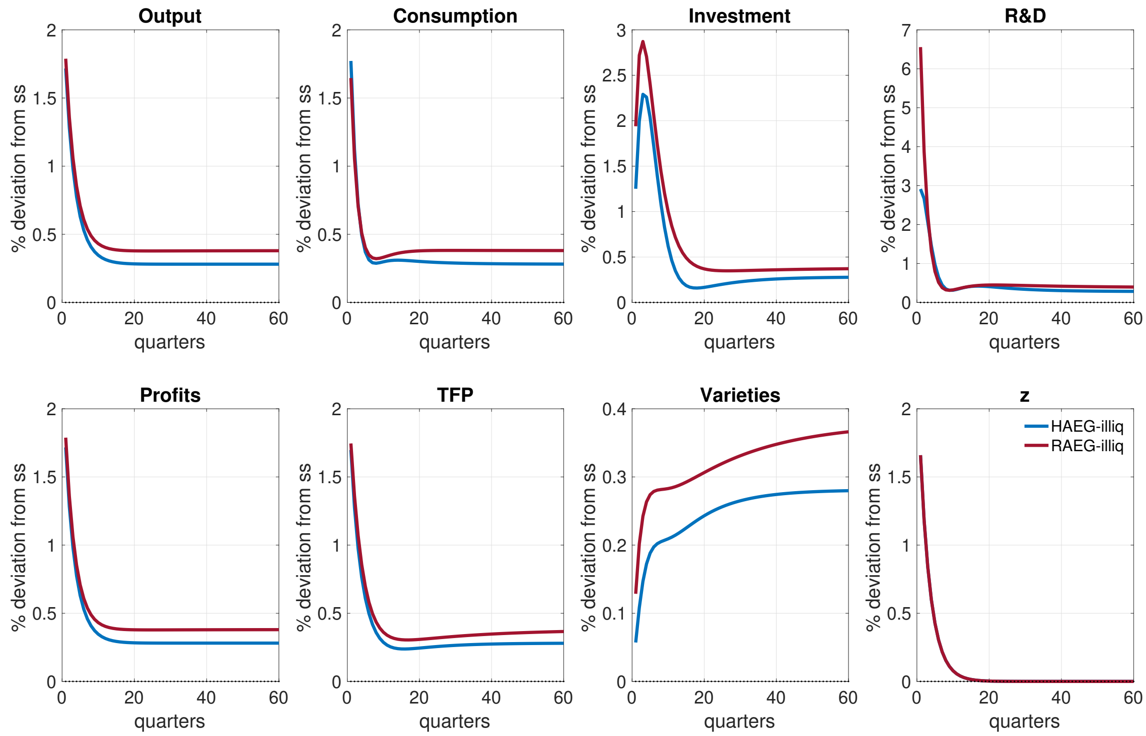

Figure 11: Impulse responses to a 1 standard deviation shock to TFP z in the HAEG (blue) and RAEG (red) models augmented with liquid and illiquid assets.

HAEG For the HAEG model economy, I consider the case $\psi = 0.8$ and the more realistic one $\psi = 1$. I start with the former. Results are reported in Table 3. I again present impulse responses of detrended (Figure 10) and trending (Figure 11) variables. Comparing the responses of the model economy without illiquid assets in Figure 4 with that with in Figure 10, similarly to what happens for the CR-RA model, also the RAEG model displays a response in consumption closer to that of the HAEG model. There still is a sizable difference on impact consumption responses in the latter case though, for the RAEG economy directs part of the additional resources brought by the TFP shock toward R&D. Put differently, the HAEG investment smoothing property is strengthened by the presence of illiquid assets, leading to lower investment in R&D, and therefore to a larger long-run difference with the RA economy (the new steady state reached by the RAEG economy is now 28% times higher than that reached by HAEG, against a 23% difference in the baseline scenario), as it is shown by IRFs of trending variables in Figure 11. Also, feeding the model with historical shocks produces a difference in standard deviations of 8% (against a baseline of 7%). Regarding investment instead, the on impact difference that was present in the baseline scenario and vanished quickly is now much more persistent. Being capital an accumulating variable as varieties, this difference will also build up more amplified medium term business cycles in output.

Table 3: Inequality and the medium term business cycle with illiquid assets

	baseline	$\psi = 0.8$
σ^Y	28%	8%

Notes: Entries refer to percentage deviations of output standard deviations between the HAEG and RAEG model economies with liquid and illiquid assets fed with historical shocks over the period 1948Q2-2007Q4.

The more realistic case of $\psi = 1$ features IRFs on the same qualitative vein of the case just analyzed, and are thus omitted in the interest of space (of course, magnitudes are larger and the new steady state reached after the shock is 56% higher in RAEG than it is in HAEG). What is interesting is looking at historical output time series (Figure 12). First, it is possible to notice how both the presence of illiquid assets and a more realistic elasticity of varieties to R&D ψ allows both model versions to better capture the observed time series properties. Indeed, HAEG output time series correlation with that in the data is 0.75 while for RAEG is 0.76[16]. This suggests that the degree of asset liquidity is important not only to model more realistically the households portfolio choice, but also to better capture the medium term properties of the economy[17].

Second, the smoothing effect inequality brings in the HAEG model is now more visible, and the standard deviation of output is 28% larger in RAEG than in HAEG, suggesting that asset liquidity amplifies the smoothing effect.

As capital stock holdings become more illiquid, the RAEG economy consumption behavior approaches that of the HAEG one, thus reducing the amplification in R&D response; moreover, the investment responses become closer between HAEG and RAEG, further attenuating medium-term differences; in the case of irresponsive investment, there still is long-run amplification of 6% in RAEG (Figures 15 and 16 in Appendix E).

[16]Figure 17 plots also consumption and investment time series. Correlations for the HAEG model with data are, respectively for consumption and investment, 0.71 and 0.67. That for RAEG instead are 0.78 and 0.65.

[17]The ratio of standard deviations at the medium frequency to the high frequency for output is, in the data, 1.46. In the baseline economy, this ratio was 0.92 and raised to 1.23 in the $\psi = 1$ case. Instead, with adjustment costs this ratio is 1.43, much more in line with the data.

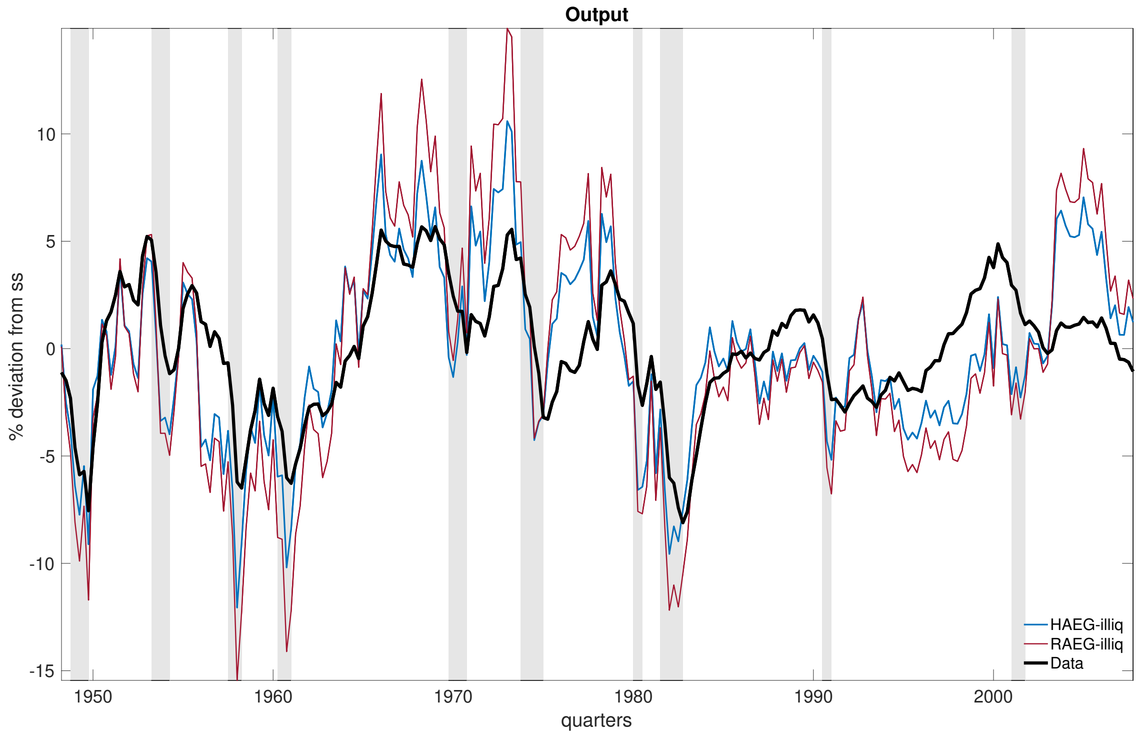

Figure 12: Simulated model's (HAEG in blue and RAEG in red, with liquid and illiquid assets) output at the medium-term cycle frequency (2-200 quarters bandpassed) with estimated innovations and data (in black). Grey bars denote NBER recessions. Correlations for the HAEG model with the data is 0.75 while that of RAEG 0.76.

5 Conclusion

In this paper I studied the link between households heterogeneity, stemming from uninsurable idiosyncratic risk due to market incompleteness and borrowing constraints, with medium-term cycles. I did so building a tractable heterogenous-agents model which, provided certain conditions hold, endogenously generates a limited cross-sectional distribution of wealth, and growth arises endogenously in the expanding variety fashion through investment in R&D.

I found that the Krusell and Smith (1998)'s result of aggregate time series for output of HA economies being "as if" were generate by a RA model breaks down over the medium term business cycle. I show that the investment smoothing property characterizing HA economies subjected to exogenous disturbances in TFP extends also to investment in R&D, when endogenous growth is added to the picture. In particular, heterogeneous agents endogenous growth models feature less deep recessions and more contained booms than their representative agents counterparts do. My simulations show that the standard deviation of GDP over the medium term business cycle is decreased by a factor in the range 14%-28% in HAEG with respect to RAEG. The strength of the smoothing effect is increased by the presence of illiquid assets. Results could be topsy turvy in the presence of monetary policy shocks and nominal frictions.

These findings call for more research on the link between inequality and the medium term business cycle. Particularly promising is quantifying the role of households heterogeneity for medium term oscillations in a medium-scale DSGE model in the spirit of Auclert et al. (2020) and Bilbiie et al. (2022).

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Appendix

A Appendix to the baseline HAEG model

A.1 Intermediate good firms problem

Intermediate good firms solve

$$\begin{aligned}
&\max_{p_{t,j}} \frac{p_{t,j}}{P_t} y_{t,j} - w_t^I n_t^I - w_t^P n_t^P - (R_t - 1 + v)\,\tilde{k}_{t-1,j} \\
&\text{s.t. } y_{t,j} = \left(\frac{p_{t,j}}{P_t}\right)^{-\theta} Y_t \\
&y_{t,j} = z_t \left(n_{t,j}^I + \kappa n_{t,j}^P\right)^{1-\alpha} \tilde{k}_{t-1,j}^{\alpha}.
\end{aligned}$$

The problem can be rewritten as

$$\min_{n_{t,j}^P, n_{t,j}^I, k_{t-1,j}} w_t^I n_{t,j}^I + w_t^P n_{t,j}^P + (R_t - 1 + v)\,\tilde{k}_{t-1,j} \tag{10}$$

$$\text{s.t. } z_t \left(n_{t,j}^I + \kappa n_{t,j}^P\right)^{1-\alpha} \tilde{k}_{t-1,j}^{\alpha} \geq \bar{y}$$

where $MC_{t,j}$ is the lagrange multiplier associated to this problem. First order conditions are:

$$\frac{w_t^P}{\kappa} = MC_{t,j}\,(1-\alpha)\,z_t \left(n_{t,j}^I + \kappa n_{t,j}^P\right)^{-\alpha} \tilde{k}_{t-1,j}^{\alpha} \tag{11}$$

$$w_t^I = MC_{t,j}\,(1-\alpha)\,z_t \left(n_{t,j}^I + \kappa n_{t,j}^P\right)^{-\alpha} \tilde{k}_{t-1,j}^{\alpha} \tag{12}$$

$$R_t - 1 + \nu = MC_{t,j}\,(1-\alpha)\,z_t \left(n_{t,j}^I + \kappa n_{t,j}^P\right)^{1-\alpha} \tilde{k}_{t-1,j}^{\alpha-1} \tag{13}$$

and (11) with (12) imply $w_t^I = {}^{w_t^P}/_{\kappa}$. Substituting (11), (12) and (13) into (10) yields

$$w_t^I n_{t,j}^I + w_t^P n_{t,j}^P + R_t \tilde{k}_{t-1,j} = MC_{t,j} y_{t,j}$$

which allows to rewrite the initial maximization problem as

$$d_{t,j} \equiv \max_{p_{t,j}, y_{t,j}} \frac{p_{t,j}}{P_t} y_{t,j} - MC_{t,j} y_{t,j} \tag{14}$$

$$\text{s.t. } y_{t,j} = \left(\frac{p_{t,j}}{P_t}\right)^{-\theta} Y_t$$

whose FOC is

$$\frac{p_{t,j}}{P_t} = \frac{\theta}{\theta-1} MC_{t,j}. \tag{15}$$

Now divide (11) by (13) to get

$$\frac{w_t^I}{R_t - 1 + \nu} = \frac{1-\alpha}{\alpha} \frac{\tilde{k}_{t,j}}{\left(n_{t,j}^I + \kappa n_{t,j}^P\right)}$$

and substitute back in (11) to get

$$MC_t = z_t^{-1} \left(\frac{w_t^I}{1-\alpha}\right)^{1-\alpha} \left(\frac{R_t - 1 + \nu}{\alpha}\right)^{\alpha}$$

where the j subscript has been dropped because the marginal cost is independent from firm specific characteristics.

Using the FOC (15) in (14) yields an expression for profits as a function of marginal costs:

$$d_t = \frac{1}{\theta} \left(\frac{\theta}{\theta-1} MC_t\right)^{1-\theta} Y_t.$$

Now compute the aggregate price index using (15) into its definition:

$$P_t = \left(\int_{j \in \mathcal{N}_{t-1}} p_t(j)^{1-\theta} dj\right)^{\frac{1}{1-\theta}} \implies MC_t = N_{t-1}^{\frac{1}{\theta-1}} \frac{\theta-1}{\theta} \tag{16}$$

and use it in (15) to get

$$\frac{p_{t,j}}{P_t} = N_t^{\frac{1}{\theta-1}}.$$

The latter can now be substituted in the definition of profits, (14) together with $y_{t,j} = \left(p_{t,j}/P_t\right)^{-\theta} C_t$ to yield

$$d_t = \frac{1}{\theta} \frac{Y_t}{N_{t-1}}$$

which is the same equation described in Section 2.2 in the main text.

A.2 Aggregate production function

Notice that by the law of large numbers $n_t^I = \Omega n_t$ and $n_t^P = (1-\Omega) n_t$. The final output aggregator implies

$$Y_t = \left(\int_{j \in \mathcal{N}_{t-1}} y_t(j)^{\theta/(\theta-1)} dj \right)^{\frac{\theta}{1-\theta}} = N_{t-1}^{\frac{\theta}{\theta-1}} \tilde{y}_t$$

and $\tilde{y}_t = z_t \tilde{l}_t^{1-\alpha} \tilde{k}_{t-1}^{\alpha}$, where $\tilde{l}_t = n_t^I + \kappa n_t^P$ and $\tilde{k}_{t-1}$ are, respectively, the optimal levels of labour and capital for the individual intermediate good producer (and are symmetric across firms). The aggregate production function thus is

$$Y_t = N_{t-1}^{\frac{\theta}{\theta-1}} z_t \tilde{l}_t^{1-\alpha} \tilde{k}_{t-1}^{\alpha}. \tag{17}$$

Now use the input market clearing conditions (in the production sector) to get

$$K_{t-1} \equiv \int_{j \in \mathcal{N}_{t-1}} \tilde{k}_t(j)\, dj = N_{t-1} \tilde{k}_{t-1}$$

$$L_t \equiv \int_{j \in \mathcal{N}_{t-1}} \left[\left(n_t^I(j) + \kappa n_t^P(j) \right) \right] dj = N_{t-1} n_t \left[\Omega + (1-\Omega)\kappa \right]$$

which can be substituted in (17) to get

$$Y_t = \Xi_t K_{t-1}^{\alpha} L_t^{1-\alpha}$$

with $\Xi_t = z_t N_{t-1}^{1-\alpha}$ which is equation (6) in the main text.

Notice that, for a symmetric comparison with Challe and Ragot (2016), I write equilibrium conditions in terms of aggregate capital in units of efficient labour, $k_t \equiv K_t / (\Omega + (1-\Omega)\kappa) n_t$, as they do. The equilibrium system of equations for the non-stationary model can be found in Appendix A.4 and their stationary counterpart in A.5.

A.3 Aggregation with limited cross-sectional heterogeneity

Defining ω^{ij} as the measure of impatient households of type ij at time t. Then they amount to:

$$\omega_t^{ee} = \Omega(1-s_t)\left(\omega_{t-1}^{ee} + \omega_{t-1}^{ue}\right) \qquad \omega_t^{eu} = \Omega s_t \left(\omega_{t-1}^{ee} + \omega_{t-1}^{ue}\right)$$

$$\omega_t^{uu} = \Omega(1-f_t)\left(\omega_{t-1}^{eu} + \omega_{t-1}^{uu}\right) \qquad \omega_t^{ue} = \Omega f_t \left(\omega_{t-1}^{eu} + \omega_{t-1}^{uu}\right)$$

then it is possible to use them to compute equilibrium aggregates. Namely, for the impatient households assets

$$A_t^I \equiv \Omega \sum_{e=0,1} \int_{\tilde{a}=-\mu_t}^{\infty} a_{t-1}\left(\tilde{a},e\right) F_t\left(\tilde{a},e\right) d\tilde{a} = \Omega\left[\omega_t^{ee} a_t + \omega_t^{ue} a_t + \omega_t^{uu}\left(-\mu_t\right) + \omega_t^{eu}\left(-\mu_t\right)\right]$$

$$\implies A_t^I = \Omega\left[n_t a_t - \left(1-n_t\right)\mu_t\right]$$

and, for consumption

$$C_t^I \equiv \Omega \sum_{e=0,1} \int_{\tilde{a}=-\mu_t}^{\infty} c_t\left(\tilde{a},e\right) F_t\left(\tilde{a},e\right) d\tilde{a} =$$

$$= \Omega \left\{ \begin{array}{cc} \omega_t^{ee}\left[w_t^I\left(1-\tau_t\right) + R_t a_{t-1} - a_t\right] + & \omega_t^{ue}\left[w_t^I\left(1-\tau_t\right) - \mu_{t-1} R_t - a_t\right] \\ \omega_t^{eu}\left[\delta_t^I + \mu_t + R_t a_{t-1}\right] + & \omega_t^{uu}\left[\delta_t^I + \mu_t - \mu_{t-1} R_t\right] + D_t \end{array} \right\}$$

$$= \Omega\left[n_t w_t^I\left(1-\tau_t\right) + \left(1-n_t\right)\delta_t^I - \left(\mu_{t-1} R_t - \mu_t\right) + D_t\right] + \Omega\left[R_t n_{t-1}\left(a_{t-1} + \mu_{t-1}\right) - n_t\left(a_t + \mu_t\right)\right]$$

which are the expressions in the main text.

A.4 Summary of the baseline model

The equilibrium system of equations characterizing the model is:

$$\eta N_{t-1}^{-\sigma} = \beta^I \mathbb{E}_t\left\{\left[\left(1-s_{t+1}\right)\eta N_t^{-\sigma} + s_{t+1}\left(\delta_{t+1}^I + \mu_{t+1} + R_{t+1} a_t + D_t\right)^{-\sigma}\right] R_{t+1}\right\}$$

$$C_t^I + A_t^I = \Omega\left[n_t w_t^I\left(1-\tau_t\right) + \left(1-n_t\right)\delta_t^I + D_t\right] + R_t A_{t-1}^I$$

$$A_t^I = \Omega\left[n_t a_t - \left(1-n_t\right)\mu_t\right]$$

$$\left(\frac{C_t^P}{1-\Omega}\right)^{-\sigma} = \beta^P \mathbb{E}_t\left[\left(\frac{C_{t+1}^P}{1-\Omega}\right)^{-\sigma} R_{t+1}\right]$$

$$C_t^P + A_t^P = \left(1-\Omega\right)\left[\kappa n_t w_t^I\left(1-\tau_t\right) + \left(1-n_t\right)\delta_t^P + D_t\right] + R_t A_{t-1}^P$$

$$w_t^I = \frac{\theta-1}{\theta} z_t k_{t-1}^{\alpha}\left(1-\alpha\right) N_{t-1}^{1-\alpha}$$

$$R_t = \frac{\theta-1}{\theta} z_t k_{t-1}^{\alpha-1} \alpha N_{t-1}^{1-\alpha} + 1 - \nu$$

$$A_t^P + A_t^I = [\Omega + (1-\Omega)\kappa] n_{t+1} k_t$$

$$\tau_t n_t w_t^I (\Omega + (1-\Omega)\kappa) = (1-n_t) \left(\Omega \delta_t^I + (1-\Omega)\delta_t^P\right)$$

$$n_t = (1-n_{t-1}) f_t + (1-s_t) n_{t-1}$$

$$Y_t = \Xi_t n_t (\Omega + (1-\Omega)\kappa) k_{t-1}^{\alpha}$$

$$\frac{1}{\vartheta_t} = (1-\phi)\mathbb{E}_t [M_{t+1} V_{t+1}].$$

$$\vartheta_t = \frac{\chi N_{t-1}}{S_t^{1-\psi} N_{t-1}^{\psi}}$$

$$N_t = \vartheta_t S_t + (1-\phi) N_{t-1}.$$

$$V_t = \iota d_t + (1-\phi)\mathbb{E}_t [M_{t+1} V_{t+1}]$$

$$D_t = d_t - S_t$$

$$d_t = \frac{Y_t}{\theta N_{t-1}}$$

$$Y_t = C_t + I_t + S_t$$

$$M_{t+1} = \beta^P \left(\frac{C_{t+1}^P}{1-\Omega}\right)^{-\sigma} / \left(\frac{C_t^P}{1-\Omega}\right)^{-\sigma}$$

where $k_{t-1} \equiv \int_{j\in\mathcal{N}_{t-1}} k_{t-1}(j) / \left(n_t^I + \kappa n_t^P\right)$ is aggregate capital in units of efficient labour.

A.5 Summary of the baseline stationary model

The equilibrium system of equations characterizing the BGP stationary model is:

$$\eta = \beta^I \mathbb{E}_t \left\{ \left[(1-s_{t+1})\eta + s_{t+1}\left(\delta^I + \mu g_{t+1} + R_{t+1}\hat{a}_t + \hat{D}_t\right)^{-\sigma} \right] R_{t+1} g_t^{-\sigma} \right\}$$

$$\hat{C}_t^I + \hat{A}_t^I g_t = \Omega \left[n_t \hat{w}_t^I (1-\tau_t) + (1-n_t)\delta^I + \hat{D}_t \right] + R_t \hat{A}_{t-1}^I$$

$$\hat{A}_t^I = \Omega [n_t \hat{a}_t - (1-n_t)\mu]$$

$$\left(\frac{\hat{C}_t^P}{1-\Omega}\right)^{-\sigma} = \beta^P \mathbb{E}_t \left[\left(\frac{\hat{C}_{t+1}^P}{1-\Omega}\right)^{-\sigma} R_{t+1} g_t^{-\sigma} \right]$$

$$\hat{C}_t^P + \hat{A}_t^P g_t = (1-\Omega)\left[\kappa n_t \hat{w}_t^I (1-\tau_t) + (1-n_t)\delta^P + \hat{D}_t \right] + R_t \hat{A}_{t-1}^P$$

$$\hat{w}_t^I = \frac{\theta-1}{\theta} z_t \hat{k}_{t-1}^{\alpha} (1-\alpha)$$

$$R_t = \frac{\theta-1}{\theta} z_t \hat{k}_{t-1}^{\alpha-1} \alpha + 1 - \nu$$

$$\hat{A}_t^P + \hat{A}_t^I = [\Omega + (1-\Omega)\kappa] n_{t+1} \hat{k}_t$$

$$\tau_t n_t \hat{w}_t^I (\Omega + (1-\Omega)\kappa) = (1-n_t)\left(\Omega \delta^I + (1-\Omega)\delta^P\right)$$

$$n_t = (1-n_{t-1}) f_t + (1-s_t) n_{t-1}$$

$$\hat{Y}_t = z_t n_t (\Omega + (1-\Omega)\kappa) \hat{k}_{t-1}^{\alpha}$$

$$g_t = \chi \hat{S}_t^{\psi} + (1-\phi)$$

$$\frac{\hat{S}_t^{1-\psi}}{\chi} = (1-\phi) \mathbb{E}_t [M_{t+1} V_{t+1}]$$

$$V_t = \iota d_t + (1-\phi) \mathbb{E}_t [M_{t+1} V_{t+1}]$$

$$D_t = d_t - \hat{S}_t$$

$$d_t = \frac{\hat{Y}_t}{\theta}$$

$$\hat{Y}_t = \hat{C}_t + \hat{I}_t + \hat{S}_t$$

$$M_{t+1} = \beta^P \left(\frac{\hat{C}_{t+1}^P}{1-\Omega}\right)^{-\sigma} / \left(\frac{\hat{C}_t^P}{1-\Omega}\right)^{-\sigma} g_t^{-\sigma}$$

A.6 RAEG counterpart

The RAEG economy obtains as $\Omega^{RA} \to 0$. The calibration is the same as in HAEG (Table 10) except for the skill premium parameter, κ^{RA}, which is calibrated to $\kappa^{RA} = \Omega + (1-\Omega)\kappa^{HA} = 2.20$ in order to induce the same steady state.

The equilibrium system of equations is:

$$\left(C_t^P\right)^{-\sigma} = \beta^P \mathbb{E}_t \left[\left(C_{t+1}^P\right)^{-\sigma} R_{t+1}\right]$$

$$C_t^P + A_t^P = \left[\kappa n_t w_t^I (1-\tau_t) + (1-n_t)\delta_t^P + D_t\right] + R_t A_{t-1}^P$$

$$w_t^I = \frac{\theta-1}{\theta} z_t k_{t-1}^{\alpha} (1-\alpha) N_{t-1}^{1-\alpha}$$

$$R_t = \frac{\theta - 1}{\theta} z_t k_{t-1}^{\alpha-1} \alpha N_{t-1}^{1-\alpha} + 1 - \nu$$

$$A_t^P = \kappa n_{t+1} k_t$$

$$\tau_t n_t w_t^I \kappa = (1 - n_t)\, \delta_t^P$$

$$n_t = (1 - n_{t-1})\, f_t + (1 - s_t)\, n_{t-1}$$

$$Y_t = \Xi_t n_t \kappa k_{t-1}^{\alpha}$$

$$\frac{1}{\vartheta_t} = (1 - \phi)\, \mathbb{E}_t \left[M_{t+1} V_{t+1} \right].$$

$$\vartheta_t = \frac{\chi N_{t-1}}{S_t^{1-\psi} N_{t-1}^{\psi}}$$

$$N_t = \vartheta_t S_t + (1 - \phi)\, N_{t-1}.$$

$$V_t = \iota d_t + (1 - \phi)\, \mathbb{E}_t \left[M_{t+1} V_{t+1} \right]$$

$$D_t = d_t - S_t$$

$$d_t = \frac{Y_t}{\theta N_{t-1}}$$

$$Y_t = C_t + I_t + S_t$$

$$M_{t+1} = {\scriptstyle \beta^P \left(C_{t+1}^P\right)^{-\sigma} / \left(C_t^P\right)^{-\sigma}}$$

where $k_{t-1} \equiv {\scriptstyle \int_{j \in \mathcal{N}_{t-1}} k_{t-1}(j) / \left(\kappa n_t^P\right)}$ is aggregate capital in units of efficient labour.

B The Challe and Ragot (2016) model

The CR model is calibrated following the same strategy adopted for HAEG discussed in Section 3.1. Parameter values are summarized in Table 4. The only parameters that differ in values from the HAEG calibration are η, κ and the replacement ratio ${}^{\delta^j}/_{w^j}$ for $j = I, P$. The latter is now 0.6, since the absence of firms profits decreases the level of insurance of impatient households.

Model's equations are:

$$\eta = \beta^I \mathbb{E}_t \left\{ \left[(1 - s_{t+1}) \eta + s_{t+1} \left(\delta^I + \mu + R_{t+1} a_t \right)^{-\sigma} \right] R_{t+1} \right\}$$

$$C_t^I + A_t^I = \Omega \left[n_t w_t^I (1 - \tau_t) + (1 - n_t) \delta^I \right] + R_t A_{t-1}^I$$

$$A_t^I = \Omega \left[n_t a_t - (1 - n_t) \mu_t \right]$$

$$\left(\frac{C_t^P}{1 - \Omega} \right)^{-\sigma} = \beta^P \mathbb{E}_t \left[\left(\frac{C_{t+1}^P}{1 - \Omega} \right)^{-\sigma} R_{t+1} \right]$$

$$C_t^P + A_t^P = (1 - \Omega) \left[\kappa n_t w_t^I (1 - \tau_t) + (1 - n_t) \delta^P \right] + R_t A_{t-1}^P$$

$$w_t^I = z_t k_{t-1}^{\alpha} (1 - \alpha)$$

$$R_t = z_t k_{t-1}^{\alpha - 1} \alpha + 1 - \nu$$

$$A_t^P + A_t^I = \left[\Omega + (1 - \Omega) \kappa \right] n_{t+1} k_t$$

$$\tau_t n_t w_t^I \left(\Omega + (1 - \Omega) \kappa \right) = (1 - n_t) \left(\Omega \delta^I + (1 - \Omega) \delta^P \right)$$

$$n_t = (1 - n_{t-1}) f_t + (1 - s_t) n_{t-1}$$

$$Y_t = z_t n_t \left(\Omega + (1 - \Omega) \kappa \right) k_{t-1}^{\alpha}$$

$$Y_t = C_t + I_t + S_t$$

$$M_{t+1} = \beta^P \left(\tfrac{C_{t+1}^P}{1 - \Omega} \right)^{-\sigma} / \left(\tfrac{C_t^P}{1 - \Omega} \right)^{-\sigma}.$$

Table 4: Parameter values for the CR model

Parameter	Description	Value	Source/Target
β^P	Patient households subjective discount factor	0.99	standard
β^I	Impatient households subjective discount factor	0.972	Liquid share of bottom Ω% is 0.30%, Survey of Consumer Finances
$\sigma^{I,P}$	Inverse of elasticity of int. substitution	1	baseline scenario
Ω	Share of impatient households	0.6	Challe and Ragot (2016)
η	Marginal utility of *ee* impatient households	1.91	Ensure *ee* have same marginal utility of patient households
κ	Skill premium	1.73	Consumption share of bottom Ω% is 40.62% , Consumer Expenditure Survey
δ^j/w^j	Replacement ratio	0.59	Consumption growth differential for the average household is 14.26%
μ	Borrowing constraint	0	baseline scenario
α	Capital share	1/3	standard
ν	Capital depreciation rate	2.5%	standard
f	BGP value of job-finding rate	80.21%	U.S. post-war mean
s	BGP value of job-separation rate	4.7%	U.S. post-war mean
n	BGP value of employment rate	94.46%	Implied by f and s

C The Krusell and Smith (1998) model

The KS model is the baseline version presented in Krusell and Smith (1998). This is an otherwise standard stochastic growth model (RBC) with households facing idiosyncratic income risk and borrowing limits. There are n_e idiosyncratic states and transitions between any two states e and e' are governed by exogenous probabilities $P(e,e')$ with stationary distribution $\pi(e)$. The individual households problem is:

$$V_t(e,k_{t-1}) = \max_{c_t,k_t} u(c_t) + \beta \sum_{e'} V_{t+1}(e',k_t)\, P(e,e') \tag{18}$$

$$\text{s.t.} c_t + k_t = (1+R_t)\,k_{t-1} + w_t e l \tag{19}$$

$$k_t \geq 0. \tag{20}$$

Denote by $k_t(e, k_{t-1})$ the policy function that solves the above problem and by $D_t(e, k_{t-1})$ the mass of households with idiosyncratic state e and capital holdings k_{t-1}, then

$$\mathcal{K}_t(R_t, w_t) = \sum_e \int_{k_{t-1}} k_t(e, k_{t-1}) \, D_t(e, k_{t-1}) \, dk_{t-1}$$

is the capital function, *i.e.* a function that maps the rental rate of capital R_t and the real wage w_t into the aggregate level of capital holdings supplied by households. Supply of capital from households must meet demand from firms, thus in equilibrium $\mathcal{K}_t(R_t, w_t) = K_t$.

Firms are perfectly competitive and produce the final good combining capital K_{t-1} and labor L_t in a Cobb-Douglas fashion and are subject to exogenous TFP disturbances, z_t.

It then follows that model equilibrium equations are:

$$Y_t = z_t K_{t-1}^{\alpha} L_t^{1-\alpha}$$

$$R_t + \nu = \alpha z_t K_{t-1}^{\alpha-1} L_t^{1-\alpha} \tag{21}$$

$$w_t = (1-\alpha) \, z_t K_{t-1}^{\alpha} L_t^{-\alpha} \tag{22}$$

$$L_t = \sum_e \pi(e) \, e$$

$$\mathcal{K}_t(R_t, w_t) = K_t. \tag{23}$$

Calibration I assume that idiosyncratic states e_{it} follow $\log e_{it} = \rho^e \log e_{it-1} + \sigma^e \epsilon_{it}$ discretized using the Rouwenhorst method over 7 states (Kopecky and Suen, 2010). The number of points on the asset grid is set to 500. I first take a uniformly spaced grid. Then I double-exponentially transform such a grid in order to obtain an asset grid where there are more points near the borrowing limit (because this is where the policy function is more nonlinear in assets). The persistence parameter ρ^e is set to 0.966 and the standard deviation σ^e to 0.92. The capital depreciation rate ν is set to 0.025 and the inverse of intertemporal elasticity of substitution σ to 1. I then consider two cases: (i) one in which the capital share α is set to a standard value of 0.33, which then requires a discount factor β of 0.98 to hit a real rate R of 1%, and a second one (ii) in which instead α is set to 0.11, in order to limit the ability of households to self-insure against idiosyncratic shocks; this implies a β of 0.97 to hit a real rate of 1%. The latter case is the same considered in Auclert et al. (2021).

Solution method for the steady state For the steady state of the model, the solution methodology is standard. It involves a backward iteration on the marginal value function to obtain the steady state policy function. This is done with the endogenous gridpoints method of Carroll (2006), which I briefly describe.

Notice that the problem (18)-(20) has first order condition

$$u_c\left(c_t\left(e,k\right)\right) \geq \beta \mathbb{E}\left[V_{k,t+1}\left(e',k'_t\left(e,k\right)\right) \mid e\right] \tag{24}$$

and envelope condition

$$V_{k,t}\left(e,k\right) = \left(1+R_{t-1}\right) u_c\left(c_t\left(e,k\right)\right). \tag{25}$$

The code proceeds as follows:

1. for each point on the grid, compute corresponding cash on hand $coh_t = \left(1+R_t\right) k_{t-1} + w_t el$; guess consumption is 5% of cash on hand and compute guessed marginal value function with the envelope condition (25);

2. compute consumption on the endogenous gridpoint using (24), *i.e.*, $c_t^{endog}\left(e,k_t\left(e,k'\right)\right) = u_c^{-1}\left(\beta \mathbb{E}\left[V_{k,t+1}\left(e',k'_t\left(e,k\right)\right) \mid e\right]\right)$;

3. since c_t^{endog} must be consistent with the cash on hand coh_t, linearly interpolate coh_t and c_t^{endog} on the grid for k to obtain k', for each state e;

4. make sure $k' \geq 0$ and back out consumption from the budget constraint (19): $c_t = coh_t - k'$.

5. use (25) to get $V_{k,t}\left(e,k\right)$

6. if guessed $V_{k,t}\left(e,k\right)$ in step (2) is different from that computed in step (5), go back to step (3), now using the c_t from step (4) and $V_{k,t}$ from step (5) ; stop otherwise.

Notice that prices R_t and w_t are inputs in the code; this is simply due to the fact that equilibrium amount of capital K is endogenous and adjusts so that the inputed R_t and w_t are consistent with equilibrium. In the presence of a government instead (which then bounds the total amount of assets K to be equal to government bonds B), a parameter (*e.g.* β) should be calibrated to meet a targeted price R_t or w_t.

The code then performs a forward iteration to obtain the steady state distribution. This simply applies iteratively the law of motion of the distribution

$$D_{t+1}\left(e', K\right) = \sum_{e} D_t \left(e, k_t^{*-1}\left(e, K\right)\right) P\left(e, e'\right)$$

using lotteries (to make sure assets policy k' is on the grid) and the Markov transition matrix P until convergence is reached. Aggregate quantities are then obtained aggregating steady state individual policies $c_{ss}\left(e, k\right)$ and $k_{ss}\left(e, k\right)$ with the steady state distribution $D_{ss}\left(e, k\right)$ $\left(i.e.\ A = \sum_{i=1}^{n_a} \sum_{j=1}^{n_e} a_{ss}\left(i, j\right) D\left(a_i, e_j\right) \text{ and } C = \sum_{i=1}^{n_a} \sum_{j=1}^{n_e} c_{ss}\left(i, j\right) D\left(a_i, e_j\right)\right)$.

Solution method for impulse responses The method and algorithm is explained in Auclert et al. (2021) in detail. Here I briefly discuss key elements. The procedure relies on the fact that an equilibrium in the space of perfect-foresight-sequences can be expressed as a solution to a nonlinear system

$$\mathbf{F}\left(\mathbf{X}, \mathbf{Z}\right) = 0 \tag{26}$$

where $\mathbf{X}$ represents the time path of endogenous variables and $\mathbf{Z}$ that of exogenous shocks. Impulse responses, to first order, are then given by

$$d\mathbf{X} = -\mathbf{F}_{\mathbf{X}}^{-1} \mathbf{F}_{\mathbf{Z}} d\mathbf{Z} \tag{27}$$

which requires computing the Jacobians $\mathbf{F}_{\mathbf{X}}$ and $\mathbf{F}_{\mathbf{Z}}$.

A way to express models in form (26) is to write them as a set of blocks along a directed acyclic graph. A block is simply a function mapping sequences of inputs into sequences of outputs using a subset of equilibrium conditions. An example for the KS model is provided in Figure 13.

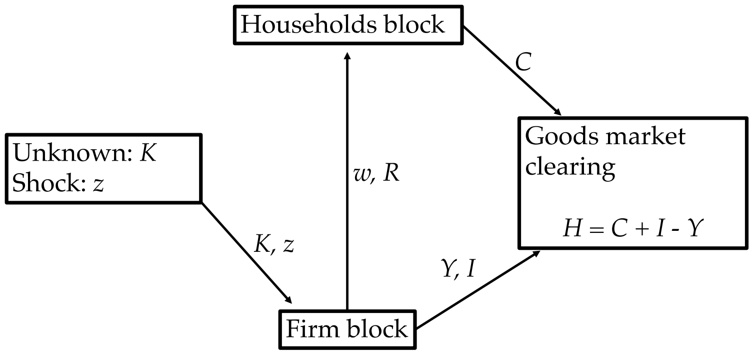

Figure 13: Directed acyclic graph for the Krusell-Smith model.

In the KS case $\mathbf{X} = \mathbf{K}$ and $\mathbf{Z} = \mathbf{z}$. The function $\mathbf{F}$ is the asset market clearing condition (23), which can be rewritten as

$$H_t(\mathbf{K}, \mathbf{z}) = \mathcal{K}_t\left(\left\{\underbrace{\alpha z_s\left(\frac{K_{s-1}}{\sum \pi(e)\,el}\right)^{\alpha-1} - \nu}_{\text{by}(21)}, \underbrace{(1-\alpha)\,z_s\left(\frac{K_{s-1}}{\sum \pi(e)\,el}\right)^{\alpha}}_{\text{by}(22)}\right\}_{s\geq 0}\right) - K_t$$

where $\mathbf{K} = \{K_s\}_{s=0}^{T}$ and $\mathbf{z} = \{z_s\}_{s=0}^{T}$ are sequences of endogenous (K) and exogenous (z) variables while T is the truncation horizon, which I set to 300, as in Auclert et al. (2021). The impulse response of output $d\mathbf{Y}$ to a TFP shock $d\mathbf{z}$ is then obtained as

$$d\mathbf{Y} = \mathcal{J}^{Y,K} d\mathbf{K} + \mathcal{J}^{Y,z} d\mathbf{z} \tag{28}$$

where $\mathcal{J}^{Y,K} \equiv \left\{\frac{\partial Y_t}{\partial K_s}\right\}_{s=0}^{T}$ and $\mathcal{J}^{Y,z} \equiv \left\{\frac{\partial Y_t}{\partial z_s}\right\}_{s=0}^{T}$ are the Jacobians capturing the effect, respectively, of K on Y and of z on Y while $d\mathbf{K} = \left(\frac{\partial \mathbf{H}}{\partial \mathbf{K}}\right)^{-1}\frac{\partial \mathbf{H}}{\partial \mathbf{z}} d\mathbf{z}$ is computed as in (27). The way in which Jacobians $\frac{\partial \mathbf{H}}{\partial \mathbf{K}}$ and $\frac{\partial \mathbf{H}}{\partial \mathbf{z}}$ are computed is chaining block Jacobians along the directed acyclic graph (Figure 13). As an example: $\frac{\partial \mathbf{H}}{\partial \mathbf{K}} = \mathcal{J}^{C,R}\mathcal{J}^{R,K} + \mathcal{J}^{C,w}\mathcal{J}^{w,K} + \mathcal{J}^{I,K} - \mathcal{J}^{Y,K}$.

To compute impulse response (28) one needs to compute Jacobians. A possibility would be to do one backward iteration from the shock at $T-1$ to get the effect on steady state policies aggregated with steady state distribution. Then shift this to get the effect at each horizon s (the idea is that what matters is time left until the shock). However, this is not computationally efficient because it needs to simulate distribution from 0 to $T-1$ for all T columns of the Jacobian. Auclert et al. (2021) propose an alternative way. It is possible to build a Jacobian matrix starting from a simpler matrix: the "fake news matrix", which is defined as $\mathbf{F}_{t,s} \equiv \mathbf{J}_{t,s} - \mathbf{J}_{t-1,s-1}$ and represents the extra effect of anticipating, at date 0, that there will a shock in date s. In this way the code computes only one simulation of the distribution from 0 to $T-1$. An additional contribution of Auclert et al. (2021) is to compute only one forward iteration on the distribution and then use expectation vectors to update it from 1 to $T-1$.

The code then proceeds as follows:

1. do one backward iteration from the shock at $T-1$ to get the effect on steady state policies aggregated with steady state distribution, for each horizon s;

2. do one period forward iteration on distribution at each horizon s;

3. compute, for each policy, expectations iteration to get expectation vectors a at horizons $s \in [1, T-2]$;

4. compute fake news matrix $\mathbf{F}_{t,s}$ for each policy and shock (so at $t=0$ it is simply what the code gets from step (1) while for $t>0$ it applies expectation vectors to the distribution computed in step (2));

5. build Jacobian $\mathbf{J}$ for each policy and shock summing along the diagonals of the matrix $\mathbf{F}$.

D Models with liquid and illiquid assets

D.1 The HAEG model with liquid and illiquid assets

Impatient households Impatient households are modeled as in the baseline HAEG model, with the agreement that investment is made only in liquid bond holdings a_t^i with gross return R_{t-1}. The period budget constraint thus is as before, namely

$$a_t^i + c_t^i = e_t^i w_t^I \left(1 - \tau_t\right) + \left(1 - e_t^i\right) \delta_t^I + D_t + R_{t-1} a_{t-1}^i \tag{29}$$

where $e_t^i = 1$ if the household is employed and 0 otherwise.

Patient households Patient households have access to both liquid bonds A_t and to holdings of the capital stock, K_t with net return given by R_t^K. Their budget constraint is then

$$C_t^P + A_t^P + I_t = R_{t-1} A_{t-1}^P + R_t^K K_t + (1 - \Omega) \left[n_t w_t^P \left(1 - \tau_t\right) + \left(1 - n_t\right) \delta_t^P + D_t \right]$$

where $I_t \equiv K_t + \Phi\left(I_t, I_{t-1}\right) - (1 - \nu) K_{t-1}$ is investment in capital which is subject to investment adjustment costs à la Christiano et al. (2005). In particular, $\Phi\left(I_t, I_{t-1}\right) \equiv \frac{\Phi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 I_t$ with $\Phi > 0$.

Patient households optimality conditions for the baseline model of Section 2.1.2 are augmented with standard equations from the q-theory defining the relative price of capital in terms of consumption (q_t) and the Euler equation for capital; they are, respectively

$$q_t = M_{t+1} \left(R_{t+1}^K + (1 - \nu)\, q_{t+1} \right)$$

$$1 = q_t \left(1 - \frac{\Phi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \Phi \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right) + M_{t+1} q_{t+1} \Phi \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 .$$

Government The government is assumed to follow a balanced budget rule as in the baseline. Besides taxes to finance unemployment benefits, the government supplies bonds in a fixed amount $A_t \equiv A N_t$, $A > 0$, which is assumed to drift with varieties N_t over time for stationary reasons. Its budget constraint thus is

$$\tau_t n_t w_t^I (\Omega + (1-\Omega)\kappa) + A_t = R_{t-1}A_{t-1} + (1-n_t)\left(\Omega\delta_t^I + (1-\Omega)\delta_t^P\right).$$

Market clearing It then follows that bonds holdings by patient and impatient households must clear each period with the government's supply:

$$A_t^I + A_t^P = A_t = AN_t$$

and, in stationary form

$$\hat{A}_t^I + \hat{A}_t^P = A.$$

Calibration The calibration is as in the baseline model (Table 10). The investment adjustment cost parameter Φ is set to 1.

D.1.1 Equilibrium system of equations

It then follows that the equilibrium system of equations is:

$$\eta N_{t-1}^{-\sigma} = \beta^I \mathbb{E}_t \left\{ \left[(1-s_{t+1})\eta N_t^{-\sigma} + s_{t+1}\left(\delta_{t+1}^I + \mu_{t+1} + R_{t+1}a_t + D_t\right)^{-\sigma} \right] R_{t+1} \right\}$$

$$C_t^I + A_t^I = \Omega\left[n_t w_t^I (1-\tau_t) + (1-n_t)\delta_t^I + D_t\right] + R_t A_{t-1}^I$$

$$A_t^I = \Omega\left[n_t a_t - (1-n_t)\mu_t\right]$$

$$\left(\frac{C_t^P}{1-\Omega}\right)^{-\sigma} = \beta^P \mathbb{E}_t\left[\left(\frac{C_{t+1}^P}{1-\Omega}\right)^{-\sigma} R_{t+1}\right]$$

$$C_t^P + A_t^P + I_t = R_{t-1}A_{t-1}^P + R_t^K K_t + (1-\Omega)\left[n_t w_t^P (1-\tau_t) + (1-n_t)\delta_t^P + D_t\right]$$

$$I_t = K_t + \frac{\Phi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 I_t - (1-\nu)K_{t-1}$$

$$w_t^I = \frac{\theta-1}{\theta} z_t K_{t-1}^{\alpha}(1-\alpha)\left[(\Omega + (1-\Omega)\kappa)n_t\right]^{-\alpha} N_{t-1}^{1-\alpha}$$

$$R_t^K = \frac{\theta-1}{\theta} z_t K_{t-1}^{\alpha-1}\alpha\left[(\Omega + (1-\Omega)\kappa)n_t\right]^{1-\alpha} N_{t-1}^{1-\alpha}$$

$$A_t^P + A_t^I = A_t$$

$$\tau_t n_t w_t^I \left(\Omega + (1-\Omega)\kappa\right) + A_t = R_{t-1} A_{t-1} + (1-n_t)\left(\Omega \delta_t^I + (1-\Omega)\,\delta_t^P\right)$$

$$n_t = (1-n_{t-1})\,f_t + (1-s_t)\,n_{t-1}$$

$$Y_t = \Xi_t n_t \left[(\Omega + (1-\Omega)\,\kappa)\,n_t\right]^{1-\alpha} K_{t-1}^{\alpha}$$

$$\frac{1}{\vartheta_t} = (1-\phi)\,\mathbb{E}_t\left[M_{t+1}V_{t+1}\right].$$

$$\vartheta_t = \frac{\chi N_{t-1}}{S_t^{1-\psi} N_{t-1}^{\psi}}$$

$$N_t = \vartheta_t S_t + (1-\phi)\,N_{t-1}.$$

$$V_t = \iota d_t + (1-\phi)\,\mathbb{E}_t\left[M_{t+1}V_{t+1}\right]$$

$$D_t = d_t - S_t$$

$$d_t = \frac{Y_t}{\theta N_{t-1}}$$

$$Y_t = C_t + I_t + S_t$$

$$M_{t+1} = \beta^P \left(\frac{C_{t+1}^P}{1-\Omega}\right)^{-\sigma} / \left(\frac{C_t^P}{1-\Omega}\right)^{-\sigma}$$

$$q_t = M_{t+1}\left(R_{t+1}^K + (1-\nu)\,q_{t+1}\right)$$

$$1 = q_t \left(1 - \frac{\Phi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \Phi\left(\frac{I_t}{I_{t-1}} - 1\right)\frac{I_t}{I_{t-1}}\right) + M_{t+1} q_{t+1} \Phi \left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2$$

D.2 The CR model with liquid and illiquid assets

The set-up follows that for the extended HAEG model and the calibration is as in the baseline CR model (Table 4) with $\Phi = 1$. The resulting equilibrium equations are then:

$$\eta = \beta^I \mathbb{E}_t \left\{ \left[(1-s_{t+1})\,\eta + s_{t+1}\left(\delta^I + \mu + R_{t+1} a_t\right)^{-\sigma} \right] R_{t+1} \right\}$$

$$C_t^I + A_t^I = \Omega\left[n_t w_t^I (1-\tau_t) + (1-n_t)\,\delta^I\right] + R_t A_{t-1}^I$$

$$A_t^I = \Omega\left[n_t a_t - (1-n_t)\,\mu_t\right]$$

$$\left(\frac{C_t^P}{1-\Omega}\right)^{-\sigma} = \beta^P \mathbb{E}_t \left[\left(\frac{C_{t+1}^P}{1-\Omega}\right)^{-\sigma} R_{t+1}\right]$$

$$C_t^P + A_t^P + I_t = R_{t-1} A_{t-1}^P + R_t^K K_t + (1-\Omega)\left[n_t w_t^P (1-\tau_t) + (1-n_t)\,\delta^P\right]$$

$$I_t = K_t + \frac{\Phi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 I_t - (1-\nu)\,K_{t-1}$$

$$w_t^I = z_t K_{t-1}^{\alpha} (1-\alpha)\left[(\Omega + (1-\Omega)\,\kappa)\,n_t\right]^{-\alpha}$$

$$R_t^K = z_t K_{t-1}^{\alpha-1} \alpha \left[(\Omega + (1-\Omega)\,\kappa)\,n_t\right]^{1-\alpha}$$

$$A_t^P + A_t^I = A_t$$

$$\tau_t n_t w_t^I (\Omega + (1-\Omega)\,\kappa) + A_t = R_{t-1} A_{t-1} + (1-n_t)\left(\Omega \delta^I + (1-\Omega)\,\delta^P\right)$$

$$n_t = (1-n_{t-1})\, f_t + (1-s_t)\, n_{t-1}$$

$$Y_t = z_t n_t \left[(\Omega + (1-\Omega)\,\kappa)\,n_t\right]^{1-\alpha} K_{t-1}^{\alpha}$$

$$Y_t = C_t + I_t$$

$$M_{t+1} = \beta^P \left(\tfrac{C_{t+1}^P}{1-\Omega}\right)^{-\sigma} / \left(\tfrac{C_t^P}{1-\Omega}\right)^{-\sigma}$$

$$q_t = M_{t+1}\left(R_{t+1}^K + (1-\nu)\, q_{t+1}\right)$$

$$1 = q_t \left(1 - \frac{\Phi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \Phi\left(\frac{I_t}{I_{t-1}} - 1\right)\frac{I_t}{I_{t-1}}\right) + M_{t+1} q_{t+1} \Phi \left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2$$

E Additional Tables and Figures

Table 5: Parameter values

Parameter	Description	Value	Source/Target
β^P	Patient households subjective discount factor	0.99	standard
β^I	Impatient households subjective discount factor	0.972	Liquid share of bottom Ω% is 0.30%, Survey of Consumer Finances
$\sigma^{I,P}$	Inverse of elasticity of int. substitution	1	baseline scenario
Ω	Share of impatient households	0.6	Challe and Ragot (2016)
η	Marginal utility of *ee* impatient households	1.75	Ensure *ee* have same marginal utility of patient households
κ	Skill premium	3.99	Consumption share of bottom Ω% is 40.62% , Consumer Expenditure Survey
δ^j/w^j	Replacement ratio	0.024	Consumption growth differential for the average household is 14.26%
μ	Borrowing constraint	0	baseline scenario
α	Capital share	$1/3$	standard
ν	Capital depreciation rate	2.5%	standard
ϕ	Obsolescence rate	2%	Caballero and Jaffe (1993)
θ	Elasticity of substitution across varieties	2.5	implied by BGP restriction $1/(1-\theta) = 1-\alpha$
ψ	Elasticity of new varieties to R&D	0.8	Griliches (1990)
χ	R&D scale parameter	0.57	2% annual growth rate
ι	Innovator's profits scale parameter	0.07	$R\&D/GDP = 2\%$, as in the data.
f	BGP value of job-finding rate	80.21%	U.S. post-war mean
s	BGP value of job-separation rate	4.7%	U.S. post-war mean
n	BGP value of employment rate	94.46%	Implied by f and s

Table 6: Inequality and the medium term business cycle, overall performance

	baseline (a)		$\Omega = 0.3$ (b)		$\delta^{I,P}/w^{I,P}$ (c), (d)		$\kappa = 1$ (e)		$\beta^I = 0.9773$ (f)		$\mu = 0.1$ (g)		$\sigma = 1.5$ (h)		$\sigma = 0.5$ (i)		$\psi = 0.6$ (j)		$\psi = 1$ (k)	
	MT	Total	MT	Total	MT	Total	MT	Total	MT	Total	MT	Total	MT	Total	MT	Total	MT	Total	MT	Total
CR/RA	4%	14%	1%	6%	3%	13%	2%	5%	4%	14%	4%	14%	10%	18%	4%	9%	4%	14%	4%	14%
HAEG/RAEG	79%	78%	76%	14%	75%	72%	51%	57%	77%	77%	77%	77%	76%	82%	73%	56%	39%	49%	150%	170%

Notes: Entries refer to percentage deviations in the moments matching loss function switching from RA to CR (first row) and from RAEG to HAEG (second row). The column labelled "MT" refers to the loss function computed on the medium term cycle while the one labelled "Total" refers to the loss function computed on the medium term cycle, medium-frequencies and business cycles.

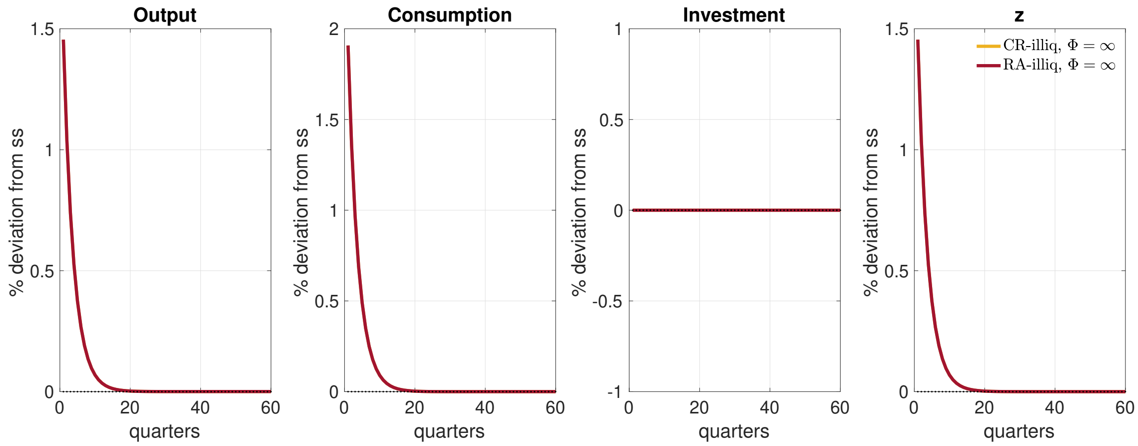

Figure 14: Impulse responses to a 1 standard deviation shock to TFP z in the CR (yellow) and RA (red) models augmented with liquid and perfectly illiquid assets.

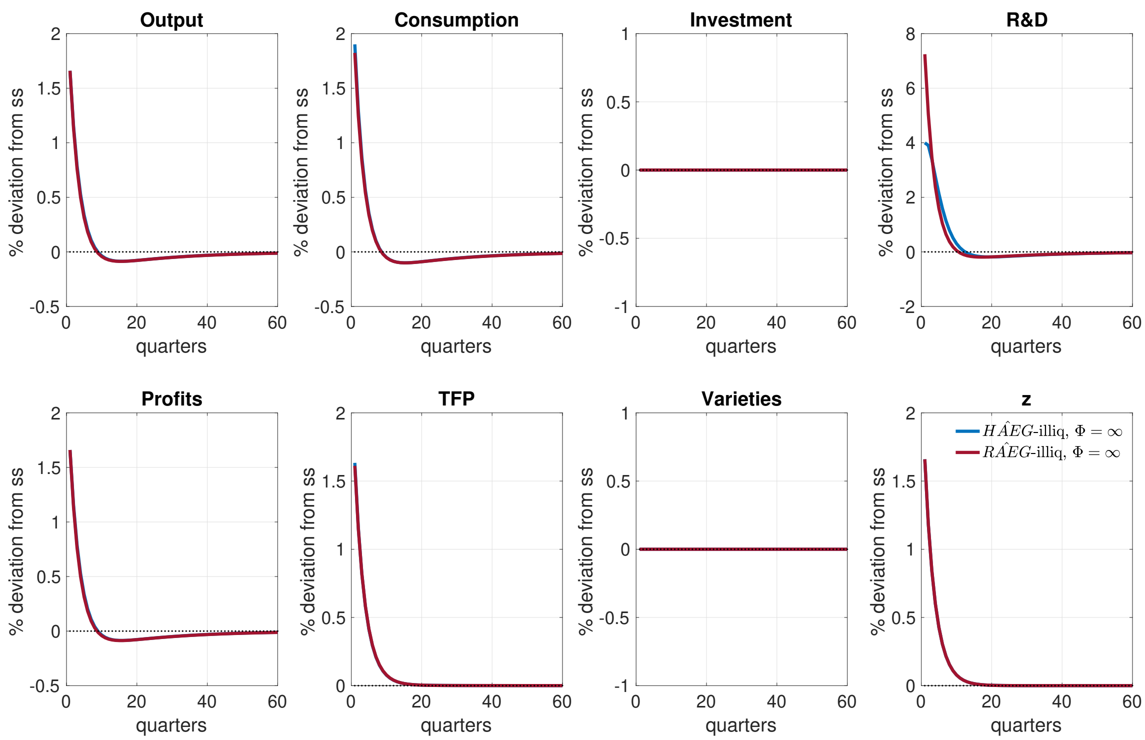

Figure 15: Impulse responses of detrended variables to a 1 standard deviation shock to TFP z in the HAEG (blue) and RAEG (red) models augmented with liquid and perfectly illiquid assets.

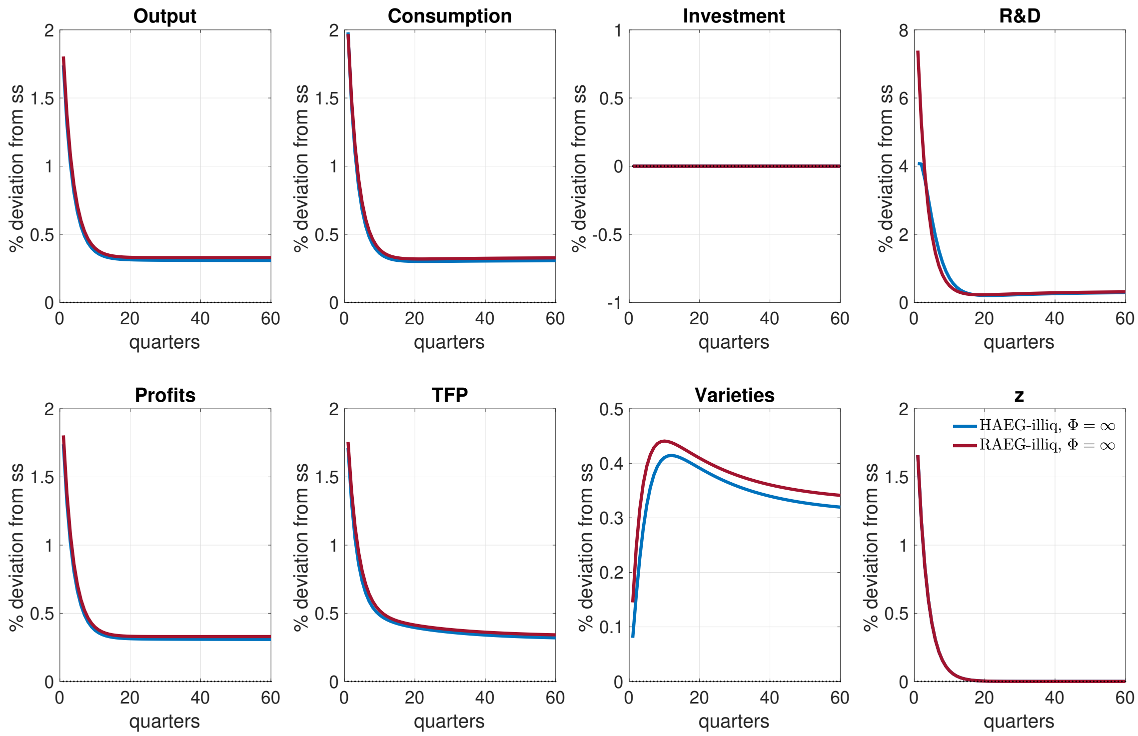

Figure 16: Impulse responses to a 1 standard deviation shock to TFP z in the HAEG (blue) and RAEG (red) models augmented with liquid and perfectly illiquid assets.

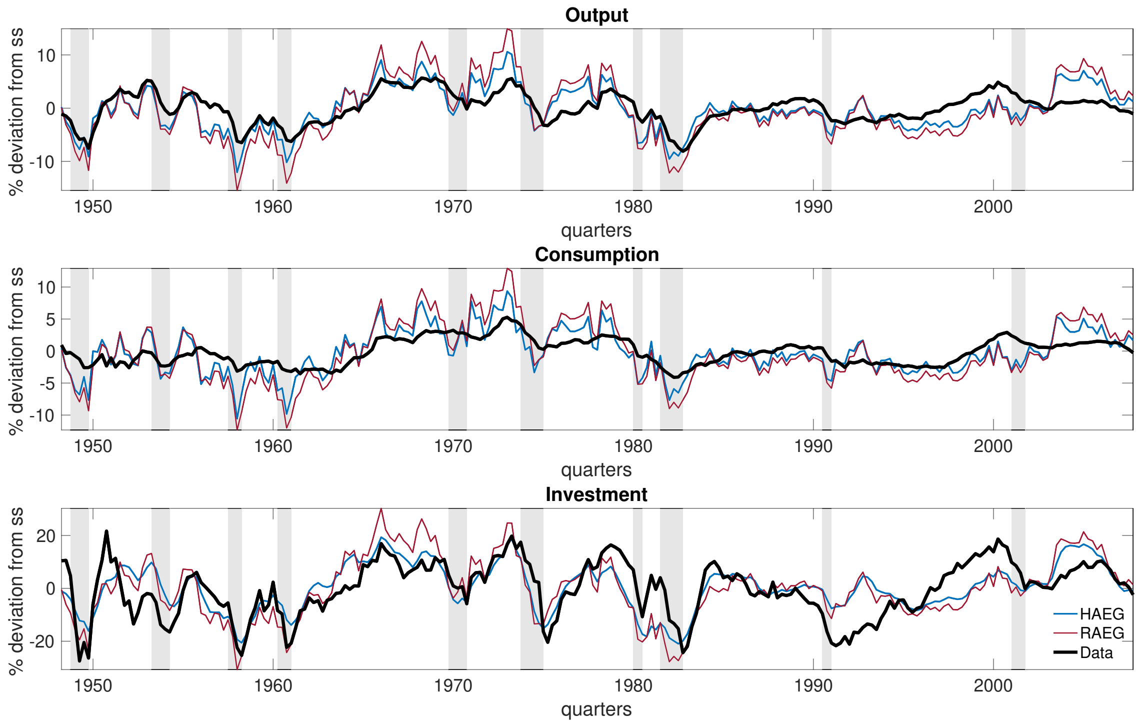

Figure 17: Simulated model's (HAEG in blue and RAEG in red, with liquid and illiquid assets) time series at the medium-term cycle frequency (2-200 quarters bandpassed) with estimated innovations and data (in black). Grey bars denote NBER recessions. Correlations for the HAEG model with data are, respectively for output, consumption and investment, 0.75, 0.71 and 0.67. That for RAEG instead are 0.76, 0.78 and 0.65.

F Sensitivity (based on overall model's performance)

I now investigate how robust are the results so far presented looking at overall models' performance. I therefore focus on changes in model's fit, rather than output volatilities, since it captures changes in volatilities, relative volatilities, correlations and autocorrelations, thus proving a broader measure of performance. I do that for CR, RA, HAEG and RAEG. As the room for self-insurance increases, one expects the CR and HAEG models to behave much more similarly to their respective RA counterparts. Still, the discrepancy between HAEG and RAEG should always be greater than that between CR and RA, because of the different medium-run dynamics implied by a different R&D response to shocks. Arguments are the on the same lines of Section 3.5 and repeated here for convenience. Results are reported in Table 6.

I start with the change in the mass of impatient households, Ω. A fall in Ω means that there are fewer households facing idiosyncratic shocks in an incomplete market set up, thus the overall population is better self-insured and the heterogeneous agent economies behave more like RA ones. Table 6, column (b), confirms this intuition. Importantly, the CR and RA model economies are almost identical (the difference in model fit is 6%) while there still remains a relatively sizable difference (14%) between HAEG and RAEG.

Results for the 4 model variants under a replacement ratio $\delta^{I,P}/w^{I,P}$ of, respectively, 0.45 and 0.6, are in column (c) and (d). A higher replacement ratio reduces the necessity for self-insurance. Simulations suggest that this parameter affects only marginally the aggregate time series properties: in the case of a replacement ratio set at 0.6 the difference between the CR and RA model is 14%, as it is in the baseline scenario while that between HAEG and RAEG is lowered at 71% from 78% in the baseline; lowering the replacement ratio to 0.45 leaves the statistics almost unchanged.

The skill premium parameter κ affects the ability to self-insure as well as the consumption inequality in the economy. Setting κ to 1 thus makes consumption more equally distributed (through also a raise in the replacement ratio) and pushes HA models to behave more as RA ones; in particular, the difference between CR and RA declines to 5% while that between HAEG and RAEG to 51%.

The impatient households discount factor β^I controls the extent to which impatient households are impatient and their ability to self-insure. Increasing its value to 0.9773 doesn't affect much the results. The same applies to raising the borrowing limit μ to 0.1.

Regarding the risk aversion coefficient, $\sigma^{I,P}$, I explore values of it of 1.5 and 0.5. In the 1.5 case, higher risk aversion induces higher savings which in turn requires a lower β^I to match the 0.30% liquid wealth target. Thus there are two contrasting effects: on the one hand the higher σ implies higher risk aversion and thus smoother and more autocorrelated consumption while on the other hand the lower β^I induces more reactive consumption and less autocorrelation. In my simulation the second effect dominates and the difference between HA and RA economies is strengthened: the CR economy improves over the RA one by 18% (against 14% in the baseline) while the RAEG model's fit is 82% better than the HAEG one (against a 78% in the baseline). With a σ of 0.5 the results are topsy turvy and model economies are more similar: CR-RA difference is 9% while RAEG-HAEG is 57%.

I also analyze the role of ψ which controls the elasticities of new varieties with respect to R&D. Estimates for this parameter reported in Griliches (1990) range from 0.6 and 1. I therefore simulate the 0.6 and 1 cases. A higher value for ψ means that new varieties N_t are more responsive to changes in investment in R&D S_t thus suggesting that short-run disturbances should have larger medium-run consequences. This is indeed the case: the $\psi = 1$ case displays a difference between the RAEG and HAEG model economies as high as 161% (against a 78% in the baseline) with a permanent change in steady state of 26% following a 1 standard deviation rise in the exogenous component of TFP z (against a 23% in the baseline) , while, for the $\psi = 0.6$ case, the difference lowers to 49% with a permanent change in steady state of 22% following a 1 standard deviation rise in the exogenous component of TFP z.

This analysis suggests that the main result of the paper, *i.e.* that inequality substantially affects the medium-term business cycle, is robust. In particular, even in the extreme cases where CR-RA models are almost identical (such as in the $\Omega = 0.3$, $\kappa = 1$ and $\sigma = 0.5$ cases), the difference between HAEG and RAEG model economies is still sizable, and at least of 49%.

G Shorter description of the model

In this section I briefly present the main blocks of the model. Since the core structure is based on Challe and Ragot (2016), the complete (and self-contained) description of the model is relegated to the Appendix 3.

The model embodies a closed economy inhabited by a continuum of households that can be impatient or patient, distributed over the sub-intervals $[0, \Omega]$ and $(\Omega, 1]$ respectively, with $\Omega \in [0, 1)$. The assumption of two types of households eases matching distributional statistics and is not necessary for the results below. There are monopolistic firms of endogenous mass producing a differentiated good and perfectly competitive innovators. The latter carry out research and development in order to discover new varieties, thus endogenously inducing long-run growth in the economy. Financial markets present frictions. For brevity, I will refer to the baseline model as HAEG (heterogeneous-agents, HA, endogenous growth, EG).

Impatient households Impatient households can be though of as the households in the Krusell and Smith (1998) model with only 2 idiosyncratic states: employment or unemployment. They are "impatient" because their discount factor β^P is assumed to be smaller than that of patient households: $\beta^P < \beta^I$. Impatient households can trade holdings of the capital stock and face a borrowing limit $a_t^i \geq -\mu_t$. Differently from Krusell and Smith (1998), they receive partial insurance from unemployment benefits $\delta_t^I > 0$. The period budget constraint thus is

$$a_t^i + c_t^i = e_t^i w_t^I (1 - \tau_t) + \left(1 - e_t^i\right) \delta_t^I + D_t + R_t a_{t-1}^i$$

where D_t are net profits from innovation transferred to impatient households which ensures that in steady state inequality will not be affected by the presence of firm profits, which would otherwise make the calibration less flexible.

Patient households Patient households have access to the full set of Arrow-Debreu securities and, therefore, behave like the standard permanent income consumer. Since they are then homogeneous, they also pin down the pricing kernel of all securities: $M_{t+1} \equiv \beta^P \left(\frac{C_{t+1}^P}{1-\Omega}\right)^{-\sigma} / \left(\frac{C_t^P}{1-\Omega}\right)^{-\sigma}$.

Production There is a continuum of monopolistically competitive firms, each producing a different variety $j \in \mathcal{N}$ of intermediate goods with symmetric elasticity of substitution $\theta > 1$. They have the following production function:

$$y_t(j) = z_t \left(n_t^I(j) + \kappa n_t^P(j)\right)^{1-\alpha} \tilde{k}_{t-1}^{\alpha}(j),$$

where $\kappa > 0$ is the relative efficiency of patient households' labour and z_t is a common (stationary) stochastic aggregate productivity process.

Standard arguments imply that firms make identical profits, which amount to $d_t(j) = d_t = Y_t/\theta N_{t-1}$, $\forall j \in \mathcal{N}_{t-1}$ (see Appendix A.1 for details). Intuitively, profits are increasing in total (stationary) demand Y_t/N_{t-1} and decreasing in the elasticity of substitution θ. The fact that profits are procyclical will make investment in R&D and varieties creation procyclical as well.

Research and development Each period, there is an unbounded mass of perfectly competitive innovators h that conduct R&D using the final good S_t in order to develop a new variety.

Let S_t^h be the total amount of R&D carried out by innovator h, ϑ_t her productivity that she takes as exogenous, N_{t-1}^h her total stock of innovations and $\phi \in (0,1)$ the probability of variety obsolescence. Then, the law of motion for varieties developed by innovator h is

$$N_t^h = \vartheta_t S_t^h + (1-\phi) N_{t-1}^h.$$

The innovator productivity ϑ_t is defined as

$$\vartheta_t = \frac{\chi N_{t-1}}{S_t^{1-\psi} N_{t-1}^{\psi}}$$

where $\chi > 0$ is a scale parameter and $\psi \in [0,1]$ is the elasticity of new varieties with respect to R&D. This technology features positive spillovers from the aggregate stock of varieties (innovations) $\partial\vartheta/\partial N > 0$, as in Romer (1990b), and a congestion externality $\partial\vartheta/\partial S < 0$ that raises the cost of developing new varieties as the *aggregate* level of R&D raises.

For an intermediate good producer, the value V_t of owning exclusive rights to produce variety j is the present discounted value of current and expected future monopoly profits, *i.e.*

$$V_t = \iota d_t + (1-\phi)\mathbb{E}_t\left[M_{t+1}V_{t+1}\right]$$

which does not depend on the specific variety j because of symmetry in equilibrium as of Section 2.2. The scale parameter $\iota \in (0, 1)$ serves the only purpose of calibrating the model to match a R&D to GDP ratio of 2%, as in the data.[18]

Since all innovators face the same problem, given the linearity of the innovator's technology and free entry, at the margin it must be that

$$\frac{1}{\vartheta_t} = (1 - \phi) \mathbb{E}_t [M_{t+1} V_{t+1}] \tag{30}$$

i.e. the marginal cost of R&D (the LHS) must be equal to its expected discounted marginal benefit (the RHS). The procyclicality of monopolists profits d_t transfers to the cyclical pattern of firm's value V_t and, as a result, to R&D investment.

Since equation (30) is independent from innovators specific characteristics, aggregation among them yields the aggregate law of motion for varieties:

$$N_t = \vartheta_t S_t + (1 - \phi) N_{t-1}. \tag{31}$$

Aggregate production The aggregate production function is of the familiar form

$$Y_t = \Xi_t K_{t-1}^{\alpha} L_t^{1-\alpha} \tag{32}$$

where

$$\Xi_t = z_t N_{t-1}^{1-\alpha} \tag{33}$$

is productivity. Equation (33) suggests that the productivity process is driven by an exogenous stationary component, z_t, and by an endogenous *trending* one, N_{t-1}, which is pinned down by R&D choices. Through this formulation, exogenous fluctuations in z_t have permanent effects on productivity and thus on output.

Equilibrium with limited cross-sectional heterogeneity Up to now, even if there are only 2 possible idiosyncratic states, households can take asset positions on the continuum $[-\mu_t, \infty)$ which makes impossible to solve analytically for equilibrium. Instead, following the Challe and Ragot (2016) approach, it is possible to reduce the cross-sectional distribu-

[18]This necessity arises because, as it will be made clear below, in order to ensure balanced growth path stability, models of this fashion require that production function is homogeneous of degree one in the accumulating factors, thus binding the choice of the elasticity of substitution parameter θ to that of the capital share α. In my model this would lead to too large profits (due to too high mark-ups) and thus the R&D/GDP would be in the order of 27%.

tion of wealth to two states, allowing analytical aggregation. This allows the following definition:

Definition 2. *An equilibrium with limited cross-sectional heterogeneity in this economy is a sequence of quantities* $C_t^P, c_t^i, A_t^P, a_t^i, S_t, N_t, V_t, k_t$*, prices* R_t, w_t^I, M_{t+1}*, aggregate variable* τ_t *such that households' Euler equations and firms' optimal conditions are satisfied, discovered varieties follow the law of motion* (31)*, innovators break-even, given the forcing processes* z_t, f_t, s_t*, the initial wealth distribution* $\left(A_{-1}^P, a_{-1}^i\right)_{i\in[0,\Omega]}$ *as well as* N_{-1} *and* k_{-1}*, at any point in time* t*. Moreover, all market clears, i.e.*

$$A_t^P + A_t^I = K_t$$

and

$$Y_t = C_t + I_t$$

where $A_t^I = \Omega\left[n_t a_t - (1-n_t)\,\mu_t\right]$, $C_t^I = \Omega\left[n_t w_t^I (1-\tau_t) + (1-n_t)\,\delta_t^I - (\mu_{t-1}R_t - \mu_t) + D_t\right] + \Omega\left[R_t n_{t-1}(a_{t-1}+\mu_{t-1}) - n_t(a_t+\mu_t)\right]$ *and* $I_t = I_t = K_t - (1-\nu)\,K_{t-1} + S_t$.

Chapter II

Ruling Out Stagnation Traps

Abstract

This paper presents empirical evidence in support of a non-monotonic relationship between employment and productivity growth. It studies the implications of such non-monotonicity in a model where growth occurs endogenously through both vertical innovation and reallocation into entrepreneurship, with nominal rigidities and monetary policy. It characterises the conditions for the existence of multiple steady-state equilibria featuring low growth and unemployment, and the role of economic policy to bring the economy towards the full-employment steady state. It shows that economic policies targeted at easing business creation activities can both prevent the occurrence of unemployment steady states and help the economy to get out of them[19].

JEL codes: E32, E43, E52

Keywords: Stagnation Traps, Horizontal Innovation, Endogenous Growth, Hysteresis, Liquidity Traps.

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1 Introduction

In the literature on entrepreneurship it is widely accepted that recessions are not only suppressive episodes, but also times when great innovation can be produced. Strangler (2009) reports that of the firms listed in *Fortune 500*, collecting the fastest-growing firms companies in the U.S. as of 2009, over 50% were established during a major recession or a bear market[20]. Several studies have shown that period of economic downturns are powerful triggers of *entrepreneurship*. Rees and Shah (1986) and Evans and Leighton (1989) emphasise that self-employment can be a refuge for individuals who experience employment difficulties in the wage/salary sector, as documented in Le (1999). The same evidence is also presented in Farber (1999) and Krashinsky (2005). Babina (2019) shows that incumbent firms' financial distress has a positive and economically significant effect on employee departures to entrepreneurship. The relationship between financial distress and entrepreneurship has been analysed in models of incomplete contracting as Grossman and Hart (1986), Hart and Moore (1994) and Rajan and Zingales (2001), where it is emphasised how financial distress is able to spur entrepreneurship. Fairlie (2013), using the 1996-2009 Current Population Survey's micro-data, found a positive relationship between the unemployment rate and the probability that individuals start businesses while Bell and Blanchflower (2011) find the same evidence in the UK during the Great Recession and Koellinger and Roy Thurik (2012) report that the entrepreneurial cycle is positively affected by the national unemployment cycle in a cross-country panel of 22 OECD countries for the period 1972-2007. Moreover, Borjas (1986) reported that labour market experience increases self-employment probabilities in the US, in line with the finding in a recent work of Azoulay et al. (2020) that most successful founders are middle-aged and with both closer and longer experience in the specific industrial sector of the start-up. These findings concert with the one of Bhide (2000) whom reported that 71% of founders replicated or modified an idea from a previous job.

There have been studies that empirically show the reverse causality between entrepreneurship and the business cycle. Koellinger and Roy Thurik (2012) using a cross-country panel of 22 OECD countries for the period 1972 to 2007 show that entrepreneurship Granger-causes the cycles of the world economy and that the entrepreneurial cycle is positively affected by the national unemployment cycle whilst Congregado et al. (2012) find evidence of entrepreneurship hysteresis in Spain, *i.e.*, the cyclical component of entrepreneurship has persistent effects on the natural rate of entrepreneurship. Morevoer,

[20] As an example, IBM, HP, FedEx and Microsoft were all started during a recession.

they present evidence for the counter-cyclicality of entrepreneurship which is even stronger when own-account self-employees are considered[21]. This evidence of counter-cyclical entrepreneurship was embedded in the theoretical models of Francois and Lloyd-Ellis (2003) and Faria (2014). It is also well known that entrepreneurs create disproportionate numbers of innovations and jobs (Acs and Audretsch, 1990; Audretsch, 2003; Haltiwanger, 2009) and stimulate economic growth (Audretsch and Keilbach, 2004; van Stel et al., 2005; Foster et al., 2008; Lentz and Mortensen, 2008).

Overall, this evidence suggests the existence of a resilience mechanisms that can be triggered by recessions – particularly the deep ones – which is able to lay the foundation for the subsequent recovery ("The recuperative powers of capitalism", Schumpeter, 1934)[22]. Nonetheless, the legacy of the Great Recession has been a long period characterized by low growth, high unemployment, policy rates at their zero lower bound (or close to it) and persistent slumps (Cerra and Saxena, 2017; Fatas and Mihov, 2013; Ball, 2014; Rawdanowicz et al., 2014; Reifschneider et al., 2015).

This paper explores the role of this resilience mechanism and its policy implications in an endogenous growth model with nominal rigidities and monetary policy.

To this aim, we first test on empirical groundings the well established positive and linear relationship between employment and productivity growth theoretically underpinned by the framework of Aghion and Howitt (1992). We find evidence that such a relationship is better captured by a (concave-up) parabola, that thus allows for different slopes at different levels of employment.

We then build on and extend the framework of Benigno and Fornaro (2018), who study the role of endogenous growth spurred by vertical innovation and its interplay with the zero-lower bound on nominal interest rates to generate stagnation traps featuring low growth and unemployment. Our main point of departure from this framework is in the engine of endogenous growth, which complements the intensive margin of vertical innovation trough quality ladders with the extensive one of horizontal innovation induced by business creation and reallocation into self-employment.

In particular, introducing a margin that allows recessions to spur entrepreneurship as documented by the aforementioned literature, implies a counter-cyclical component in the process of innovation formation and technological progress, that interacts with the

[21] *I.e.* entrepreneurs who work on their own.

[22] Evidence on the positive feedback of downturns on the subsequent recovery and long-run growth rate of the economy is presented in Beaudry and Koop (1993); Pesaran and Potter (1997) and Altissimo and Violante (2001).

pro-cyclical intensive margin and implies a non monotonic relationship between growth and employment. This will allow the existence of multiple "stagnation traps" and the possibility to reconcile the empirical evidence of a positive entry rate with recession periods. Expectations will play a crucial role in selecting the prevailing equilibrium and demand-driven short-run fluctuations can have permanent effects on long-run growth (*i.e.* output hysteresis) . We show the role of entrepreneurship in determining the conditions under which this multiple stagnation traps exist, and the additional dimension of policy options that this extensive margin implies for policy makers seeking to rule out these unemployment equilibria, or steer the economy out of them.

1.1 Related Literature

Our work is related to the literature born in the late 80s that started to consider the interaction between business cycles and long-run growth (Stadler, 1986, 1990; Stiglitz, 1993; Martin and Rogers, 1997; **?**; Comin and Gertler, 2006; Anzoategui et al., 2019; Bianchi et al., 2019; Garga and Singh, 2020; Benigno and Fornaro, 2018; Comin, 2009; Schmitt-Grohe and Uribe, 2017; Eggertsson et al., 2019)[23]. We extend this literature allowing recessions to have both negative and positive effects on long-run growth, depending on how deep they are. The presence of the zero lower bound on monetary policy makes possible for demand-driven shocks à la **?** or expectations revisions à la Benigno and Fornaro (2018) to cause permanent scars on long-run GDP growth because it may impair the counter-cyclical component of innovation. A new role for industrial policies will naturally emerge.

We are also related to Aghion and Saint-Paul (1991), Gali and Hammour (1992), Caballero and Hammour (1994), Hall (1991), Saint-Paul (1997), Aghion and Saint-Paul (1998b), DeLong (1990) and King and Robson (1989) whom build models in which recessions have positive, cleansing effects, mainly trough an intertemporal substitution effect (or opportunity cost); a theory on the "recuperative powers of capitalism" (Schumpeter, 1934) that dates back at least to Schumpeter (1939; 1942). These works find empirical support in Davis and Haltiwanger (1992), Blanchard et al. (1990), Aghion and Saint-Paul (1998a), Bean (1990), Burnside et al. (1993), Basu and Fernald (1995), Dunne et al. (1996),Nickell et al. (2001) and **?** and more recently in Aghion et al. (2010; 2012) and Fernald and Wang (2016). Nonetheless, this empirical literature is not clear-cut on whether firms tend to innovate more or less during recession periods and it is therefore not possible to reject one theory in favor of the other. We complement this literature taking a different standpoint. Instead of

[23]For a complete historical review of this literature the reader is referred to Cerra et al. (2020)

debating on whether firms' innovation is pro- or counter-cyclical, we allow our framework to encompass both, with the agreement that pro- or counter-cyclicality will depend on the depth of the recession[24]. Moreover, if one closely looks at the aforementioned works, would struggle to understand why, if recessions are good, we do not observe their positive effects on GDP growth. We will argue that constraints to monetary policy, and in particular the presence of the ZLB, can make more difficult to exploit the counter-cyclical side of innovation.

This paper is related also to the expanding varieties literature such as Romer (1990a), Comin and Gertler (2006), Comin (2009) and Bilbiie et al. (2012). These works share a pro-cyclical entry/creation of firms/varieties because they relate it to pro-cyclical R&D. Our model instead relates firms/varieties entry/creation to counter-cyclical entrepreneurship and therefore inherits its cyclical properties.

We are linked to the literature showing the positive effects of entrepreneurship on innovation (Acs and Audretsch, 1990; Audretsch, 2003; Haltiwanger, 2009) and on economic growth (Audretsch and Keilbach, 2004; van Stel et al., 2005; Foster et al., 2008; Lentz and Mortensen, 2008) as in our model entrepreneurship contributes to the advancement of the innovation frontier, productivity growth and, as a novelty, to escape from stagnation traps.

Our work is also related to the literature that studies the countercyclicality of entrepreneurship (Fairlie, 2013; Congregado et al., 2012; Koellinger and Roy Thurik, 2012; Bell and Blanchflower, 2011; Francois and Lloyd-Ellis, 2003; Rees and Shah, 1986; Krashinsky, 2005; Farber, 1999; Evans and Leighton, 1989; Caballero and Hammour, 1994; Ben-ner, 1988; Parker, 2018; Perotin, 2006). It should be noticed that, among the empirical works cited, most of them use self-employment as a measure of entrepreneurship but, as argued in Congregado et al. (2012), "it typically under-samples Schumpeterian innovative entrepreneurs relative to "replicative" or "me too" entrepreneurs (Audretsch and Fritsch, 2002; Baumol et al., 2009)". We contribute to this literature linking the business creation intensity to the level of managerial ability. This naturally delivers a countercyclical pattern of entrepreneurship because as the recession deepens in severity the average managerial ability needed to profitably start a new business decreases.

This paper is also related to the literature that links entrepreneurial activity and the business cycle. Francois and Lloyd-Ellis (2003) build a model in which there exists a business cycle that is interlinked with the economy's growth process and in which cyclical fluctuations are induced by decentralised entrepreneurs. Faria (2014) instead, extends the

[24]We will show this intuition in Section 3.7 and in the Appendix H.2.

Ramsey's model introducing entrepreneurship dynamics related to unemployment and output dynamics, generating limit cycles in which unemployment and entrepreneurial cycles cause business cycles. We contribute to this literature because our model features counter-cyclical entrepreneurship and short-run fluctuations that affect it and will have permanent effects on productivity growth.

Gourio et al. (2016) show that entry of firms acts as a propagation mechanism and has significant and persistent effects on GDP and productivity. Our model is consistent with this finding in that an increase in the death rate of firms or a decrease in business creation can slow down horizontal innovation with a persistent effect on GDP, altering the growth rate of productivity and putting the economy in a stagnation trap.

We are also related to works that show how financial distress spurs entrepreneurship (Grossman and Hart, 1986; Hart and Moore, 1994; Rajan and Zingales, 2001; Babina, 2019) as we allow for profound recessions to ease entrepreneurship.

Finally, we are related to the classics of endogenous growth (Segerstrom et al., 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992) as we extend their framework introducing entrepreneurship, monetary policy and nominal rigidities.

2 Empirical evidence

In the theoretical literature on endogenous growth, starting with Aghion and Howitt (1992), a key relation links employment with productivity growth spurred by innovation, describing the supply-side of the macroeconomic system. This relation is typically implied to be monotonic and upward-sloping. Indeed, entrepreneurs invest in R&D so as to acquire future expected monopolists profits related the ownership of the newly created firm that, after innovation, produces a variety with higher associated quality. Since expected profits are an increasing function of employment, this will pin down a positive relationship between the level of employment and innovation intensity (where the latter pins down the productivity growth rate). More recently, Benigno and Fornaro (2018) show that a supply-side of this kind, once interacting with a demand-side that explicitly accounts for an effective lower-bound on nominal interest rates, gives rise to multiple equilibria, one of which features the so-called "stagnation trap", where very low levels of employment are associated to low productivity growth.

In particular, in the specification of Benigno and Fornaro (2018), the optimal innovation intensity combined with the households' stochastic discount factor yields

$$(g_{t+1}-1)\left(1-\beta\mathbb{E}_t\left[\left(\frac{c_t}{c_{t+1}}\right)^{\sigma} g_{t+1}^{-\sigma}\chi\gamma\varpi L_{t+1}\right]\right)=0 \tag{34}$$

where g_{t+1} is the (gross) productivity growth rate (between period t and $t+1$), c_t is consumption normalized by the productivity index, L_{t+1} is employment, σ is the inverse of the elasticity of intertemporal substitution and χ, γ and ϖ are positive parameters. Equation (34) determines the optimal R&D effort by entrepreneurs and implies a monotonically upward-sloping relationship between innovation-led growth g_{t+1} and employment L_{t+1}.

On the other hand, in the entrepreneurship literature (Section 1), the link between innovation and the economic cycle is less clear cut. This literature emphasizes the cleansing and sullying effects that may be triggered during recessions (Haltiwanger et al., 2021), thus implicitly suggesting that this link may in fact be non-monotonic, with a downward slope arising during deep recessions and very low levels of employment.

In this Section we scrutinize empirically this relationship for the US, while in Section 3 we will analyze its theoretical implications.

2.1 Data description

Our sample contains quarterly observations on labour productivity growth, consumption growth and employment over the period 1948Q1-2021Q1. We will focus first on the period spanning from the Great Moderation (GM, 1985Q1-2007Q3), as we believe that, if resilience mechanisms are indeed present, they are actually facilitated by digital technologies. It should then be easier to detect these forces at play in the GM-onward window.

Quarterly data on labour productivity of the business sector are from Fernald (2014). The data present in the database are annual percentage change (natural logarithm change times 400) thus we adjust them to be consistent with our quarterly specification.

Regarding the employment rate, we take the complement to one of the unemployment rate, quarterly and seasonally adjusted, from the FRED database. We scale it by its maximum so as to get a number between 0 and 1 whose maximum is 1, as in our model specification. We do so for both the monthly and the quarterly time series. From the same database we also take real personal consumption expenditures per capita, chained 2012 dollars, quarterly and seasonally adjusted.

We also employ monthly data on new patents assignments in the Santa Clara county (also known as Silicon Valley) from the USPTO. They are available since August 1980. We end our sample in that case in December 2014 because, as reported in the FRED database, thereafter there is a big drop in the time series "likely due to the time it takes to grant a patent, currently averaging 24 months"[25]. Data are normalized by the highest observation. We chose Santa Clara as a county of reference because, as it is widely known, most of the developments of the IT revolution occurred there; the new patents assignments in the Silicon Valley are thus a good proxy for IT and overall innovation.

2.2 Econometric specification

Our approach is to estimate a linear approximation of equation (34) which involves a simple time-series regression of the form

$$\ln g_t = \alpha + \beta_1 L_t + \beta_c controls_t + \epsilon_t \tag{35}$$

where we are using the same notation as before and $controls_t$ includes several controls that for the moment are relegated to consumption growth at time t (defined as $\Delta \ln C_t$). A positive estimate of β_1 would be consistent with the relationship implied by the aforementioned literature.

As a first step, we estimate regression (35) over the Great Moderation period (1985Q1 through 2007Q3) and over the period between WWII and the Great Financial Crisis (GFC, 1948Q1 through 2007Q3). Since our conjecture is that the resilience mechanism activated by deep recessions may have become increasingly facilitated by the recent IT and digital revolution, we expect the β_1 to be positive over both samples. This is the historical window in which the Aghion and Howitt (1992)'s framework should be a reasonable (simplified) description of reality. Figure 18 displays the scatter plot of the data, and a simple linear interpolation to show that, indeed, a (very weak) positive unconditional correlation seems to characterize the growth-employment relationship, in particular over the Great-Moderation sample, although the amount of dispersion is still very high, particularly once we look as back as the post-WWII period.

Around the 2005-2007, there were at least two major events that may have impacted the relationship between innovation-driven productivity growth and employment. On the one hand, the Fourth Industrial Revolution (FIR) accelerate, expanding the set of digital technologies available for the production of goods and services and the extent to which these technologies can be readily adopted by new firms. On the other hand, the GFC

[25] https://fredblog.stlouisfed.org/2018/03/of-places-and-patents/FRED.

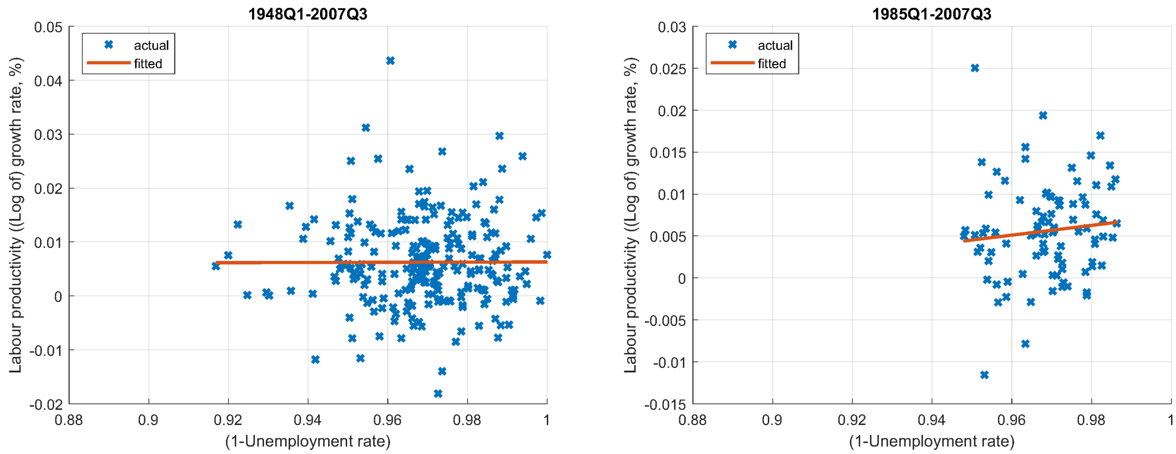


Figure 18: Linear interpolation of the growth rate of labour productivity with employment. Left panel: 1948Q1-2007Q3. Right panel: 1985Q1-2007Q3.

brought about falls in employment levels that are among the deepest in the post-WWII sample.

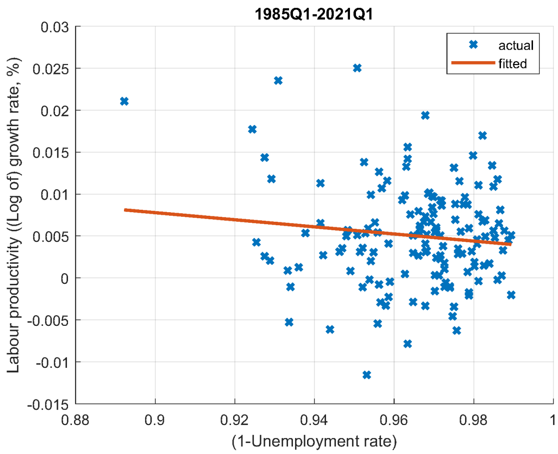
We want to test whether deep slumps – thank to the new technological regime allowed by the FIR – are able to trigger cleansing powers of some sort that allow for high levels of innovation and productivity growth despite the low levels of employment, while during the "normal" state of affairs of the economy, the standard, upward-sloping relationship theoretically underpinned by the Aghion and Howitt (1992)'s mechanism, holds.

So to study whether the combination of these two events marked a change in the structural relationship between innovation-driven productivity growth and employment, we estimate regression (35) over the period 2005Q1-2021Q1. Moreover, to further scrutinize the role of FIR and the GFC for this relationship, we also run a dummy regression of the type

$$\ln g_t = \alpha + \beta_1 L_t + \beta_2 dummy + \beta_3 L_t * dummy + \beta_c controls_t + \epsilon_t \tag{36}$$

where the *dummy* variable will be associated with the post-2004 sample or with particularly deep recessions, depending on the specification. Therefore, the coefficient β_3 should capture the change in the slope between the alternative regimes.

To study the role of recessions and deep recessions, in particular, we estimate regression (36) in two different specifications: (i) where the *dummy* takes value 1 if employment hits a value in the lower decile of its post-WWII distribution and zero otherwise; we label this cases as "deep recessions". (ii) where the *dummy* takes value 1 if employment hits a value in the second lower decile of its post-WWII distribution and zero otherwise; we label this



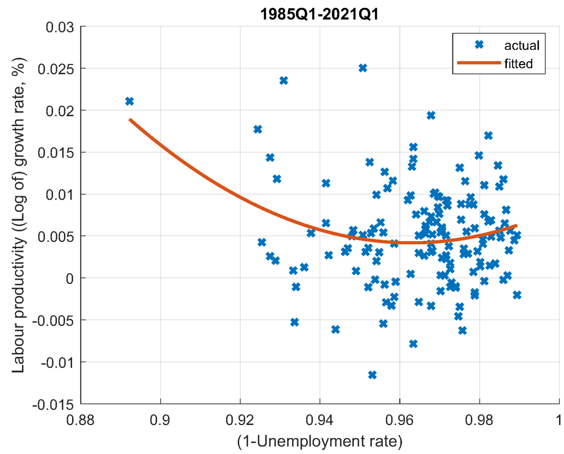


Figure 19: Simple interpolations of the growth rate of labour productivity with employment over the period 1985Q1-2021Q1. Left panel: linear interpolation. Right panel: quadratic interpolation.

cases as "recessions". We should then expect the β_2 to be negative in both cases and more negative in case (i) with respect to case (ii).

Next, we evaluate the ability of a non-linear regression model to explain the data over the period 1985Q1-2021Q1. This is a natural exercise in that, if there is a change in the productivity-employment relationship within the sample period, then a non-monotone relationship should better capture the link between the two variables. Figure 19 displays simple interpolations over the period 1985Q1-2021Q1 to show that, indeed, while a monotonically increasing relations seems at best challenged by the data overall, a non-monotonic U-shaped relation looks to better fit the evidence.

To better scrutinize this descriptive evidence, we thus estimate

$$\ln g_t = \alpha + \beta_1 L_t + \beta_2 L_t^2 + \beta_c controls_t + \epsilon_t \tag{37}$$

and look for a positive β_2 and then confront it with the estimates of regression (35) over the same period.

As we believe the underlying driving force of this negative relationship between productivity and employment is a surge in innovation activities, we perform the same exercises just described but using new patents assignments in Santa Clara as dependent variable. The resulting evidence should go hand in hand with what we expect for the productivity-growth-exercises. In particular, the linear relationship between new patents assignments and employment should be positive during the 1985M1-2007M9 period. Figure 20 shows the scatter plot of the data and a linear interpolation. As it was the case for

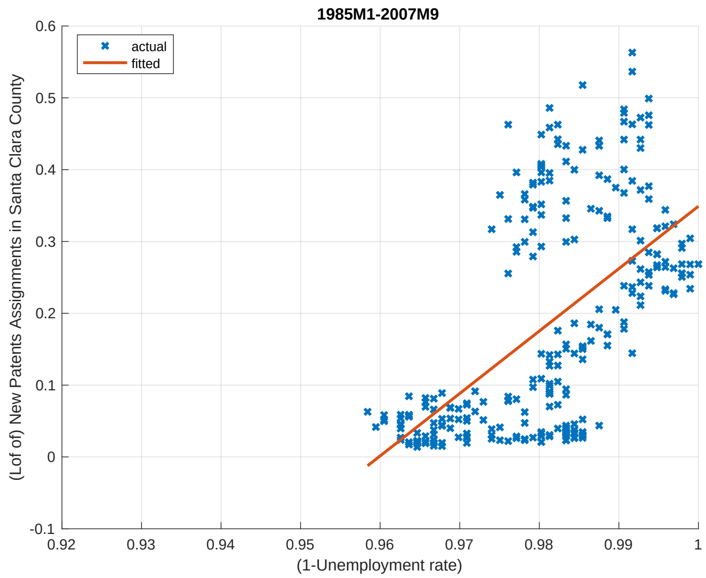


Figure 20: Linear interpolation of the growth rate of (log of) new patents assignment in Santa Clara county with employment, 1985M9-207M9.

productivity growth, also for innovation the positive link with employment characterizes reasonably well the period over the Great Moderation. This relationship is then expected to change around the 2005-2007 period for the reasons already discussed. Figure 21 reports interpolations in linear and quadratic form that make this point clear.

2.3 Results

Results relative to the productivity growth rate regressions are reported in Table 7. As expected, the regression (35) over both the GM and the 1948Q1-2007Q3 period, supports the positive relationship between growth and employment ($\beta_1 > 0$), even though it is not statistically significant. It could therefore been argued that the Aghion and Howitt (1992)'s mechanism holds during "normal" times.

Running the same regression on 2005 toward the end of the sample instead yields a negative and significant (at the 3% level) relationship between growth and employment ($\beta_1 < 0$). The regression (36) with a dummy on the post-2004 period supports the latter finding. Indeed, in 2005 there seems to be a discontinuity, with a change in slope of $\beta_3 = -.172$ significant at the 2% level, with respect to the pre-2005 period.

Columns (5) and (6) help understand the role of recessions and deep recessions in generating this change in slope. Regression (36), (i), predicts a change in the slope of $-.281$

Table 7: Estimation of regressions (35), (36), and (37) where the dependent variable is (log of) labour productivity growth rate.

	(35) GM	(35) 1948-2007	(35) 2005-2021	(36) 1985-2021	(36) 1985-2021	(36) 1985-2021	(35) 1985-2021	(37) 1985-2021
Constant	$-.050$	-0.12	$.077^{**}$	$-.0896$	$-.0173$	$-.0391$	$.0274$	2.201^{**}
	(.057)	(.033)	(.033)	(.061)	(.0494)	(.063)	(.0291)	(1.05)
L_t	.0571	.0179	$-.076^{**}$	.0982	.023	.0451	$-.0241$	-4.571^{**}
	(.058)	(.034)	(.034)	(.063)	(.051)	(.064)	(.0301)	(2.191)
L_t^2								2.38^{**}
								(1.144)
post-2004				$.1641^{**}$				
				(.0692)				
$L_t *$ post 2004				$-.1719^{**}$				
				(.072)				
1st decile					$.263^{**}$			
					(.111)			
$L_t *$ 1st decile					$-.281^{**}$			
					(.12)			
2nd decile						$.196^{**}$		
						(.095)		
$L_t *$ 2nd decile						$-.206^{**}$		
						(.0991)		
$\Delta \ln C_t$	.002	$.002^{*}$	.001	$.001^{*}$	.001	$.001^{**}$	$.001^{*}$	$.001^{*}$
	(.001)	(2.753)	(.001)	(.0004)	(.0004)	(.0004)	(.0004)	(.0004)
R^2	.0365	.0311	.145	.116	.0922	.0854	.0563	.0843

Notes: * indicates significance at 10% level, ** at 5% and *** at 1%. GM: Great Moderation (1985Q1-2007Q3). Standard errors in parentheses.

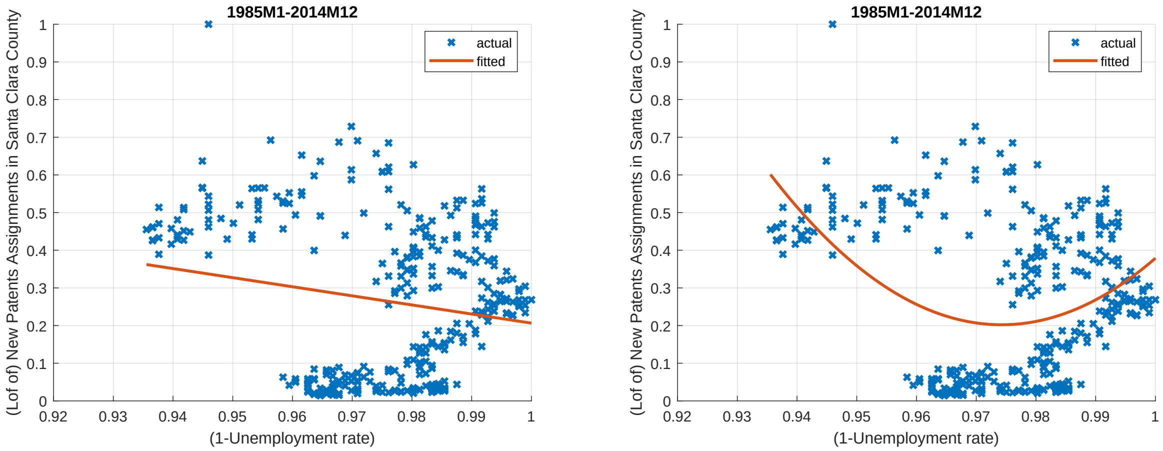


Figure 21: Simple interpolations of the log of new patents assignments in Santa Clara with employment over the period 1985M1-2014M12. Left panel: linear interpolation. Right panel: quadratic interpolation.

(significant at the 2% level) when the level of employment takes on values in the 1st decile of its post-WWII distribution; we label these occurrences as deep recessions. If instead one considers not so deep recessions (2nd decile), the change in the slope is still there and significant at the 4% level, but less pronounced ($-.21$), consistent with the idea that deep slumps are the events that trigger a stronger positive response in productivity.

The last exercise conducted in this section relative to the growth-employment relationship, is to evaluate the performance of the non-linear regression model to fit the data relative to the linear one, over the period 1985Q1-2021Q1. Last two columns of Table (7) report results of this exercise. The parameters governing the shape of the parabola in regression (37) are all significant at the 4% level while the slope parameter of the linear regression (35) is negative but not statistically significant. Moreover, even if the R^2 are generally low for all regressions, as a result of the high dispersion in the data, the one associated with the non-linear model is about 50% higher than the one associated to the linear model.

Results of the same exercises done for the new patents assignments in Santa Clara as a dependent variable are in the exact same line of what already discussed and are relegated in the Appendix 9.

To summarize, the findings of this section indicate that the Aghion and Howitt (1992)'s positive link between productivity growth and employment seems broadly consistent with the data if one looks at periods until the 2005-2007 (even though the amount of noise in the data is very large). Looking at the full recent sample that includes the last two deep recessions, on the other hand, a non-monotonic U-shaped relationship seems to better fit the link between innovation-drive productivity growth and employment. The same holds true for the link between innovation and employment. In the next section we study the implications of adding this non-monotonicity in an endogenous growth model with nominal rigidities.

3 The Model

We consider a closed economy, inhabited by households, firms and a policy maker.

3.1 Final good sector

This sector produces the consumption good Y_t under perfect competition, using a Cobb-Douglas technology that combines labor L_t and a continuum of intermediate inputs $y_t(j)$ whose mass is N_t, indexed by $j \in [0, N_t]$ and with associated productivity/quality $A_t(j)$:

$$Y_t = (L_t)^{1-\alpha} \int_0^{N_t} A_t(j)^{1-\alpha} y_t(j)^{\alpha} dj, \tag{38}$$

where $\alpha \in (0,1)$. Final good firms will maximize their profits choosing the optimal quantities of intermediate goods and labour. The solution of this problem is characterized by the following first order conditions,

$$p_t(j) = \alpha P_t (L_t)^{1-\alpha} y_t(j)^{\alpha-1} A_t(j)^{1-\alpha}, \forall j \in [0, N_t] \tag{39}$$

$$W_t = (1-\alpha) P_t (L_t)^{-\alpha} \int_0^{N_t} A_t(j)^{1-\alpha} y_t(j)^{\alpha} dj \tag{40}$$

where $p_t(j)$ is the price of the intermediate good j and P_t the price of final good.

3.2 Intermediate good sector

Following Aghion and Howitt (2008), the intermediate good in sector j is produced by a monopolist using the final good $x_t(j)$, with a linear technology,

$$y_t(j) = x_t(j), \tag{41}$$

for $j \in [0, N_t]$. This implies a one to one relationship between intermediate goods, sectors and firms and we will refer to them interchangeably throughout the text. The monopolist chooses price and quantity of the intermediate good to solve

$$\Pi\left[A_t(j)\right] \equiv \max_{p_t(j), x_t(j)} p_t(j)\, y_t(j) - P_t x_t(j)$$
$$\text{subject to (39) and (41)}$$

which implies the equilibrium quantity of intermediate good j

$$y_t(j) = \alpha^{\frac{2}{1-\alpha}} A_t(j)\, L_t \tag{42}$$

and the equilibrium price of intermediate good j

$$p_t(j) = \frac{1}{\alpha} P_t.$$

Accordingly, the equilibrium aggregate amount of intermediate goods

$$X_t^y \equiv \int_0^{N_t} y_t(j)\, dj = \alpha^{\frac{2}{1-\alpha}} Q_t L_t \tag{43}$$

where $Q_t \equiv \int_0^{N_t} A_t(j)\, dj$ is the aggregate productivity index, while the monopolist's equilibrium profits in sector j are

$$\Pi\left[A_t(j)\right] = \psi P_t A_t(j)\, L_t \tag{44}$$

with $\psi \equiv (1-\alpha)\, \alpha^{\frac{1+\alpha}{1-\alpha}}$. Using (42) in (38) gives the equilibrium level of final output

$$Y_t = \alpha^{\frac{2\alpha}{1-\alpha}} Q_t L_t.$$

3.3 Vertical innovation through quality ladders

As in the seminal work of Grossman and Helpman (1991) and **?**, the engine of vertical innovation is underpinned by quality ladders, in the spirit of the Schumpeterian idea of creative destruction (Schumpeter, 1942). In each period t and in each sector j, an entrepreneur has the opportunity to attempt an innovation. If she succeeds, the innovation creates a new version of the intermediate good with an increased associated productivity. In particular, the productivity of the improved intermediate good will go from last period's value $A_{t-1}(j)$ to $A_t(j) = (1+\gamma)\, A_{t-1}(j)$, where $\gamma > 0$ is the innovation step size. If she fails, the intermediate product will be the same that was used in $t-1$ and therefore $A_t(j) = A_{t-1}(j)$ and will be produced by another randomly chosen monopolist. To summarize:

$$A_t(j) = \begin{cases} (1+\gamma) A_{t-1}(j) & \text{with prob. } z_t(j) \\ A_{t-1}(j) & \text{with prob. } 1 - z_t(j) \end{cases}$$

In order to generate a successful innovation with probability $z_t(j)$, an entrepreneur must invest in research and development, according to the following production function:

$$z_t(j) = \left[2\delta \frac{R_t(j)}{A_t(j)}\right]^{\frac{1}{2}}, \tag{45}$$

where $R_t(j)$ denotes the amount of final good used in R&D activity. This production function implies that is more costly to innovate towards more advanced products. The entrepreneur will maximize her expected profits from innovation:

$$d_t(j) \equiv \max_{z_t(j)} z_t(j) \Pi[(1+\gamma) A_{t-1}(j)] - R_t(j) P_t$$
$$\text{subject to (44) and (45)}$$

leading to the equilibrium innovation intensity in sector j

$$z_t(j) = \delta\psi L_t = z_t \tag{46}$$

for all $j \in [0, N_t]$. To ensure $z_t \in [0,1]\, \forall t$ it is sufficient that $\delta \leq 1/\psi$.[26]

The growth rate of sectoral productivity will be $1 + g_t^A \equiv \frac{A_t}{A_{t-1}} = 1 + z_t\gamma$, where

$$g_t^A = z_t\gamma = \delta\psi\gamma L_t \tag{47}$$

and the definition of aggregate productivity implies

$$Q_t = N_t A_t.$$

[26] Therefore, the equilibrium probability of vertically innovating is common across firms. Assuming a uniform initial cross-sectional distribution of sectoral productivities $A_{-1}(j) = A_{-1}$ for all $j \in [0, N_{-1}]$ implies symmetry ex-post: $A_t(j) = A_t$, for all j.

Using (46), the equilibrium expected profits for the innovator are

$$d_t = \frac{1}{2\delta} z_t^2 A_t P_t.$$

3.4 Households

This economy is populated by a unit mass of households deriving utility from consumption of a final good. Their lifetime utility is

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} \right) \right],$$

where C_t denotes consumption in units of the final good, $\beta \in (0, 1)$ the subjective discount factor and σ is the relative risk aversion coefficient (and the the inverse of the elasticity of intertemporal substitution). Within each household there is a unit measure of members that are willing to supply labor services in the labor market. Within each household there is perfect consumption sharing.

As in Benigno and Fornaro (2018), households face idiosyncratic unemployment risk: at the beginning of each period, each household will be unemployed with constant probability p. If the household is unemployed, all members of the household are out of a job, and the household receives an unemployment subsidy that makes its income equal to b times the income of the employed household, with $b < 1$. If the household is employed, on the other hand, its members supply inelastically labor services on the labor market. Because of nominal rigidities, however, the equilibrium may imply a labor demand from firms that is insufficient to absorb all labor supply and a measure $1 - L_t \in (0, 1)$ of the household members may be unemployed. The unemployed members of the employed household, however, will enjoy the same level of consumption of the employed ones, due to the perfect risk-sharing mechanism at work within the household.[27]

Households have access to risk free bonds B_t that pay the nominal interest rate i_t and own all the firms, from which they receive dividends D_t.

[27] Given this structure, there are two types of unemployment. The first type is "exogenous", of mass p. The second type is instead "endogenous", as it is generated by the equilibrium labor demand, and it is measured by the mass $(1-p)(1-L_t)$. Hence, households accounting is summarised as follows:

$$1 = \underbrace{p}_{\text{exogenous unemployed}} + \underbrace{(1-p)(1-L_t)}_{\text{endogenous unemployed}} + \underbrace{(1-p)\, L_t}_{\text{employed}}.$$

Thus, there will be two period by period budged constraints,

$$P_t C_t + \frac{B_{t+1}}{1+i_t} = \mathbb{I} W_t L_t + B_t + D_t + T_t$$

where the indicator function $\mathbb{I}$ takes value 1 if the household is employed and 0 otherwise, while the transfer T_t will be a subsidy for the unemployed and a tax for the employed[28].

The household maximizes her expected lifetime utility subject to the period by period budget constraint, a no-borrowing constraint, $B_{t+1} \geq 0$, no trade in firms' shares and the standard transversality condition. Noticing that the borrowing constraint binds only for unemployed households, this problem leads to the following first order conditions:

$$\mu_t = \frac{(C_t^e)^{\sigma}}{P_t} \tag{48}$$

$$\mu_t = \beta (1 + i_t) \mathbb{E}_t \mu_{t+1} \tag{49}$$

together with the standard transversality condition $\lim_{T\to\infty} \mathbb{E}_t \beta^T \frac{Uc,T}{Uc,t} B_T$, where C_t^e denotes consumption of employed households, and by the assumption on unemployed households' income, $C_t^{ne} = bC_t^e < C_t^e$.

3.5 Nominal rigidities and monetary policy

We introduce nominal frictions so that there will be a role for monetary policy in this framework and involuntary unemployment will be possible. Following Benigno and Fornaro (2018), we start by assuming that wages evolve according to constant inflation,

$$W_t = \bar{\pi}^w W_{t-1} \tag{50}$$

and will relax later this assumption introducing a Wage Phillips curve.

Monetary policy will be conducted through a truncated interest rate rule, taking into consideration the zero lower bound (*i.e.* $i_t \geq 0$),

$$1 + i_t = \max\left\{ \left(1 + \bar{i}\right) L_t^{\phi}, 1 \right\} \tag{51}$$

[28] In particular, for the employed $T_t = -\frac{p}{1-p} \frac{bW_t L_t + (b-1) N_t d_t}{1 + \frac{bp}{1-p}}$ and for the unemployed $T_t = \frac{bW_t L_t + (b-1) N_t d_t}{1 + \frac{bp}{1-p}}$.

where $\phi > 0$ and $\bar{i} \geq 0$. Under this specification and combining (40) and (42), prices are

$$P_t = \omega^{-1} \frac{W_t}{Q_t} \tag{52}$$

where $\omega^{-1} \equiv \frac{1}{1-\alpha} \alpha^{\frac{2\alpha}{\alpha-1}}$ while stationary real wages are $\frac{W_t}{P_t Q_t} \equiv \omega$. Combining the equation for prices (52) and the law of motion of wages (50) one gets the price inflation

$$\pi_t \equiv \frac{P_t}{P_{t-1}} = \frac{\bar{\pi}^w}{1 + g_t} \tag{53}$$

where $1 + g_t \equiv {}^{Q_t}/_{Q_{t-1}}$.

3.6 Horizontal innovation through business creation and selection into entrepreneurship

In this section we introduce entrepreneurship, building on the expanding product variety model of Romer (1990a) and the entrepreneurial-decision model of Lucas (1978).

Each period, an unbounded mass of outside entrepreneurs can engage in start-up activities. In order to create a new firm, a would-be entrepreneur has to carry R&D efforts related to vertical innovation, so as to get the most up-to-date quality of the new product, which depends on the equilibrium vertical innovation intensity z_t defined by equation (46): thus, the cost of horizontal innovation is $R_t(z_t) P_t$. If the innovation is successful, the horizontal entrepreneur will enjoy the monopolist profits, $\Pi(A_t)$[29]. The horizontal innovation is successful with probability z_t^h, which we assume to be increasing in the managerial ability of the entrepreneur

$$z_t^h = \frac{x_t}{1 + x_t}, \tag{54}$$

where $x_t > 0$ is the managerial ability of the individual to be drawn from a fixed distribution $\Gamma : \mathbb{R}^+ \rightarrow \mathbb{R}^+$, with CDF $G(x)$. Under perfect competition among the prospective entrepreneurs, the free entry condition requires that the expected benefit of setting-up a new firm equals its cost, namely

$$z_t^h \Pi(A_t) = R_t(z_t) P_t. \tag{55}$$

[29]We use this specific characterization of the horizontal innovator problem to keep the resource constraint simple.

Accordingly all and only the entrepreneurs whose probability of success is greater than $\underline{z}_t^h = {}^{\delta\psi L_t}/_2$, and whose associated managerial ability is greater than

$$\underline{x}_t = \frac{\delta\psi L_t/2}{1 - \delta\psi L_t/2}. \tag{56}$$

will find it optimal to create a firm in a new sector and enjoy the corresponding monopolist profit. This implies that the newly created firms at time t are

$$N_t^e = \left[\int_{\underline{x}_t}^{\infty} \Gamma\left(x\right) dx\right] N_t. \tag{57}$$

In using a fixed distribution we are implicitly assuming that, however deep is the recession, the distribution of managerial abilities among potential entrepreneurs is always the same. If one turns to the data, would see that this is not the case. In the work of Hoynes et al. (2012), the authors show that not all workers experience the same effects during recessions and that these differences in the cyclicality across demographic groups are stable across three decades of time (from 1980 until 2009) and across recessionary versus expansionary periods. As one may reasonably expect, the raise in the unemployment rate decreases with the level of education, both in the 1980 and the 2007 recession. This in turn led to an increase in the creation of new firms because empirically there is a U-relationship between education and entrepreneurial ability[30]. To be consistent with this evidence, we would have had assumed a varying distribution of managerial abilities that shifts toward the left as the recession deepens in severity because more and more low skill workers get fired relatively to high skill ones. This pattern can be obtained assuming heterogeneity among workers. Nonetheless, there is a more parsimonious way of achieving the same empirical regularity, a way we prefer for the sake of tractability of the model. Indeed, assuming a fixed distribution of managerial abilities with a *varying* threshold ability is isomorphic to assuming a distribution that shifts toward the left as the recession deepens, but for the greater simplicity of the former. The reason is that as the recession deepens in severity (L_t decreases) the level of ability needed to break-even in the creation of a new firm decreases because the costs to create it decrease more relatively to the decrease that occurs in the expected profit from such a creation. This induces an increases in the creation of firms through an increase in the mass of active entrepreneurs, matching the reported

[30]This evidence is presented in Le (1999), Fairlie (2013) and Lohmann and Luber (2004). For an interesting summary discussion about the relationship between entrepreneurship and education, the curious reader is referred to Parker (2018, chapter 4.3) and the works cited therein.

evidence.

In order to derive the complete law of motion of firms, consistently with the firms dynamics literature, such as Ghironi and Melitz (2005) and Bilbiie et al. (2012) among the others, let us assume that at the beginning of each period an exogenous constant "death shock" $\Delta \in (0,1)$ hits the existing firms[31]. Hence, the law of motion for the number of firms is

$$N_t = (1-\Delta)\, N_{t-1} + N_t^e.$$

Given this specification, the rate of growth for the number of firms is

$$1+g_t^N \equiv \frac{N_t}{N_{t-1}} = \frac{1-\Delta}{1-\frac{N_t^e}{N_t}} = \frac{1-\Delta}{G\left(\underline{x}_t\right)}. \tag{58}$$

and we have used the fact that $1-\frac{N_t^e}{N_t} = 1-\int_{\underline{x}_t}^{\infty} \Gamma(x)\,dx = \int_0^{\underline{x}_t} \Gamma(x)\,dx = G\left(\underline{x}_t\right)$. It is easy to see that as $L_t \to 0$ the threshold ability $\underline{x}_t \to 0$, thus making the growth rate of firms $1+g_t^N$ to diverge. Clearly, this is a region never visited by an economy in the reality and can be disregarded. What is important to notice is that the growth rate of the number of firms is increasing in unemployment, capturing the reallocation into self-employment that was observed during the Great Recession[32] and the countercicliclality of entrepreneurship[33]. Instead, as $L_t \to 1$ the entrepreneurial engine slows down because it gets harder to create new firms ($\underline{x}_t$ increases) and the growth rate of firms becomes $1+g_t^N = {}^{(1-\Delta)}/_{[G(\underline{x}_t(L_t=1))]}$ reaching its minimum over the domain $L_t \in [0,1]$. Such an effect may be summarised by the medieval adage "*beati monoculi in terra caecorum*[34]": when the economy is in a recession and thus far from its full potential, there are a lot of needs that emerge and consequently a large share of opportunities for would-be entrepreneurs to be exploited; as the economy reaches its full potential the number of opportunities reduces because most of the innovative ideas have been already exploited and thus it is much harder to become a successful entrepreneur.

It should be noticed that the refuge into self-employment translates into a negative relationship between employment and the growth rate of firms, $\left(g_t^N\right)_{L_t} < 0$, a counter-

[31] For evidence suggesting this is a reasonable starting assumption look Broda and Weinstein (2010) and Lee and Mukoyama (2007).

[32] Fairlie (2013); Bell and Blanchflower (2011).

[33] Fairlie (2013); Congregado et al. (2012); Koellinger and Roy Thurik (2012); Bell and Blanchflower (2011); Francois and Lloyd-Ellis (2003); Rees and Shah (1986); Krashinsky (2005); Farber (1999); Evans and Leighton (1989); Caballero and Hammour (1994); Ben-ner (1988); Parker (2018); Perotin (2006).

[34] In English: Blessed are the one-eyed in the land of the blind.

cyclical mechanism that will interact with the pro-cyclical one of vertical innovation, resulting in a non-monotone relationship between aggregate productivity and employment. This aspect is crucial to our analysis as it introduces another channel through which the economy can end up in an adverse steady state. This in turn has very important implications in the analysis of recessions and the industrial policies a government may undertake to steer an economy out of a recession.

3.7 Equilibrium

The resource constraint implies that the final output is used either to consume, to produce the intermediate good or employed in research activities for vertical innovation[35],

$$Y_t = C_t + \int_0^{N_t} y_t\left(j\right) dj + \int_0^{N_t} R_t\left(j\right) dj.$$

Using (42), (45) and (46) one gets

$$C_t = \Psi L_t Q_t - \frac{\delta}{2}\left(\psi L_t\right)^2 Q_t,$$

where $\Psi \equiv \alpha^{\frac{2\alpha}{1-\alpha}}\left[1-\alpha^{\frac{2-2\alpha}{1-\alpha}}\right]$. Defining stationary variables as $\tilde{x}_t = {}^{x_t}/_{Q_t}$ the resource constraint can be written as

$$\tilde{C}_t = \Psi L_t - \frac{\delta}{2}\left(\psi L_t\right)^2. \tag{59}$$

The equilibrium firm dividends are

$$d_t = \int_0^{N_t} d_t\left(j\right) dj = d_t N_t = \psi Q_t L_t P_t\left[1 - \frac{\delta}{2}\psi L_t\right] \tag{60}$$

[35]To get this start from the budget constraint of households, notice that in equilibrium $b_{t+1} = 0\,\forall t$ and that $D_t =$ final good firms profits + intermediate good firms profitsand that intermediate good firms profits encompass both vertical innovators profits and horizontal ones:

$$P_t C_t = W_t L_t + \underbrace{P_t Y_t - W_t L_t - \int_0^{N_t} P_t\left(j\right) y_t\left(j\right) dj}_{\text{Final good firm profits}} + \underbrace{\int_0^{N_{t-1}} \left(P_t\left(j\right) y_t\left(j\right) - P_t y_t\left(j\right) - R_t\left(j\right) P_t\right)}_{\text{Int. good, vertical innovators profits}} +$$
$$+ \underbrace{\int_{N_{t-1}}^{N_t} \left(P_t\left(j\right) y_t\left(j\right) - P_t y_t\left(j\right) - R_t\left(j\right) P_t\right)}_{\text{Int. good, horizontal innovators profits}}.$$

where we have used the households budget constraint and the fact that in equilibrium $B_t = 0$ for all t.

Turning to households, combining (48), (49) and (53), and using the fact that $C_t = pC_t^{ne} + (1-p)\,C_t^e$ and that by the assumption on unemployed households' income $C_t^{ne} = bC_t^e < C_t^e$, the Euler equation is

$$\bar{\pi}^w \tilde{C}_t^{-\sigma} = \beta\,(1+i_t)\,\rho\mathbb{E}_t\left\{\tilde{C}_{t+1}^{-\sigma}\,(1+g_{t+1})^{1-\sigma}\right\} \tag{61}$$

where $\rho = 1 + p(b^{-\sigma} - 1) > 1$, and we assume $\sigma > 1$ so that the income effect dominates and there is a positive relationship between consumption today $\tilde{C}_t$ and productivity tomorrow g_{t+1}.

The rate of growth of aggregate productivity will be the result of the interaction of the two productivity driving forces in this economy: i) the intensive margin of innovation through quality ladders, captured by (47), and ii) the extensive margin of innovation through reallocation into entrepreneurship, captured by (58). Thus, combining the two we get

$$1 + g_t = (1-\Delta)\,\frac{1+\psi\delta\gamma L_t}{G\left(\underline{x}_t\right)} \tag{62}$$

and it is clear that it inherits the two contrasting effects of the two margins: for low values of L_t it will be a decreasing function of L_t while for bigger values it will be increasing. Indeed, equation (62) is the combination of a convex decreasing function in labour L_t, $1+g_t^N$, for the horizontal innovation margin, and a linear increasing function in labour L_t, $1+g_t^A$, for the vertical innovation one. As a result it will be a convex decreasing function of L_t for low values of L_t, reaching a minimum, and an increasing function of L_t thereafter[36]. We will refer to equation (62) as GG throughout the text.

This analysis leads to the following definition:

Definition 3. *A stationary equilibrium in this economy is a sequence $\left\{\tilde{C}_t, g_t, L_t, i_t, \underline{x}_t\right\}_{t=0}^{\infty}$ that, given parameter values $\sigma, \bar{\pi}^w, \rho, \beta, \Psi, \delta, \gamma, \Delta, \psi, \omega, b, p, \lambda, \bar{\imath}, \phi$ and a distribution $\Gamma(x)$, satisfies* (59), (61), (62), (51) *and* (56)*and $L_t \leq 1\,\forall t$.*

3.8 Steady State Analysis

Turning off the uncertainty and focusing on constant values for consumption $\tilde{C}$, labour L, aggregate productivity growth g and for the nominal interest rate i, equilibria are

[36] Further details on its shape are provided in the Appendix H.2

characterized by the solutions to the following system of equations:

$$(1+g)^{\sigma-1} = \frac{\rho\beta\,(1+i)}{\bar{\pi}^{w}} \tag{63}$$

$$\tilde{C} = \Psi L - \frac{\delta}{2}\,(\psi L)^2 \tag{64}$$

$$1+g = (1-\Delta)\,\frac{1+\psi\delta\gamma L}{G\,(\underline{x})} \tag{65}$$

$$\underline{x} = \frac{\delta\psi L/2}{1-\delta\psi L/2} \tag{66}$$

$$1+i = \max\left[\left(1+\bar{i}\right)L^{\phi}, 1\right]. \tag{67}$$

The system (63)-(67), can be described by two relevant equations. Combining the Euler equation (63) with the monetary policy rule (67) one gets an aggregate-demand type of relationship between productivity growth and labour:

$$1+g = \max\left\{\left(\frac{\rho\beta}{\bar{\pi}^{w}}\right)^{\frac{1}{\sigma-1}}, \left[\frac{\rho\beta}{\bar{\pi}^{w}}\left(1+\bar{i}\right)L^{\phi}\right]^{\frac{1}{\sigma-1}}\right\} \tag{68}$$

and it is easy to see that when $i > 0$[37] the element on the right in the max dominates and the relationship between labour L and productivity growth g is positive; when instead $i = 0$[38] the relationship between labour L and aggregate productivity g becomes flat because the central bank is not anymore able to respond to unemployment decreasing further the policy rate. We will refer to equation (68) as AD. The other relevant equation is the GG evaluated at the steady state (equation 65).

The system (63)-(67) gives rise to multiple equilibria, under some conditions. In particular a full employment steady state emerges naturally and with few assumptions while multiple unemployment steady states can emerge if horizontal innovation has some properties (or, to anticipate it, if business creation is not relatively strong enough).

3.8.1 Full employment Steady State

A full employment steady state is characterized by $L^F = 1$. From (65) we get

$$g^F = (1-\Delta)\,\frac{1+\psi\delta\gamma}{G\left(\underline{x}^F\right)} - 1 \tag{69}$$

[37] Which happens when $L > \bar{L} \equiv \frac{\bar{\pi}^{w}}{\rho\beta}\frac{1}{(1+\bar{i})}$.

[38] Or, equivalently, $L < \bar{L}$.

while (63) gives

$$i^F = \left[(1-\Delta) \frac{1+\psi\delta\gamma}{G\left(\underline{x}^F\right)} \right]^{\sigma-1} \frac{\bar{\pi}^w}{\rho\beta} - 1$$

and the monetary policy equation (67) ensures the sustaining of the full employment steady state setting $\bar{\imath} = i^F$. Lastly, from the resource constraint (64) we can recover the consumption level

$$\tilde{C}^F = \Psi - \frac{\delta}{2}\psi^2.$$

Assumption 1. *The parameters satisfy:*

$$\bar{\imath} = \left[(1-\Delta) \frac{1+\psi\delta\gamma}{G\left(\underline{x}^F\right)} \right]^{\sigma-1} \frac{\bar{\pi}^w}{\rho\beta} - 1 > 0 \tag{70}$$

$$\delta \leq \frac{1}{\psi} \tag{71}$$

$$(1-\Delta) \frac{1+\psi\delta\gamma}{G\left(\underline{x}^F\right)} - 1 > 0 \tag{72}$$

$$\phi > (\sigma - 1)\,\varphi. \tag{73}$$

where $\varphi = \frac{\psi\delta\gamma}{1+\psi\delta\gamma} - \frac{2\delta\psi}{(2-\delta\psi)^2} \frac{G_{\underline{x}^F}}{G(\underline{x}^F)}$.

Proposition 2. *(Existence, uniqueness and local determinacy of full employment steady state) Suppose Assumption 1 holds. Then there exists a unique full employment steady state characterised by* $L^F = 1$, $g^F > 0$, $i^F > 0$ *and* $\tilde{C}^F > 0$ *and it is locally determinate*[39].

To give and intuition, the inequality in assumption (71) ensures that consumption in full employment is positive and that the innovation probability lies between zero and one ($z_t \in [0,1]$). Assumption (70) guarantees that the monetary policy target is consistent with the existence of a full employment steady state while assumption (73) that the central bank responds sufficiently strongly to fluctuations in employment so that the full employment steady state is locally determinate,as it is common in the New Keynesian literature (see Galí, 2015). Notice that the assumption (73) for local determinacy is milder with respect to the one in Benigno and Fornaro (2018) suggesting that if the horizontal margin of innovation (*i.e.* business creation activities) is active then the monetary policy strength in the responses to fluctuations in employment can be relaxed. Moreover, the bigger the

[39] Proof in Appendix (H.1).

relative strength of horizontal innovation over the vertical one the more the response in monetary policy to fluctuations in employment can be relaxed. To conclude, assumption (72) ensures that growth in full employment is positive.

3.8.2 Unemployment Steady States

An unemployment steady state is characterised by $i = 0$ which, by equation (63) implies

$$g^U = \left(\frac{\rho\beta}{\bar{\pi}^w}\right)^{\frac{1}{\sigma-1}} - 1.$$

In our framework, due to the non-linearity of the GG equation, it is not obvious neither the existence of an unemployment steady state nor its uniqueness. A complete discussion of the matter is provided in the Appendix (H.2). For our analysis, it is sufficient to understand that at least one unemployment steady state will exist provided that the minimum of the GG (with respect to L) will be smaller than the level of growth at the zero lower bond consistent with the horizontal portion of the AD, g^U, and that at $L = 1$ the slope of the GG is smaller than the one of the AD. Such an unemployment steady state will thus be characterised by $L^U \in (0, \bar{L})$ and by a real interest rate smaller than the one in the full employment steady state. Let us summarise this result in the following proposition.

Proposition 3. *(Existence, multiplicity and local indeterminacy of full employment steady state) Suppose assumption 1 holds together with*

$$\rho\beta > \bar{\pi}^w \tag{74}$$

$$\min_L (1-\Delta)\frac{1+\psi\delta\gamma}{G\left(\underline{x}^F\right)} - 1 \leq g^U \tag{75}$$

then there exist at least one unemployment steady state, and at most two, characterised by $g^U < g^F$, $0 < \tilde{C}^U < \tilde{C}^F$, $i^U = 0$ and $L^U \in (0, \bar{L}) < L^F = 1$, where $\bar{L} = \frac{\bar{\pi}^w}{\rho\beta}\frac{1}{(1+\bar{i})}$, and each unemployment steady state will be locally indeterminate[40].

Assumption (73) ensures that the upward sloped portion of the AD lies below the GG over a left neighbourhood of $L = 1$ while assumption (75) guarantees that the GG schedule has a minimum below or on the horizontal portion of the AD so that at least one unemployment steady state exists. Assumption (74) ensures productivity growth

[40]Proof in Appendix (H.2).

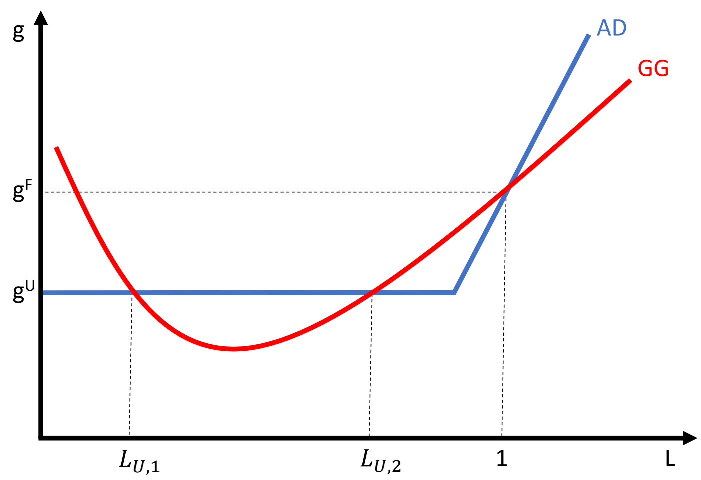


Figure 22: Multiple stagnation traps.

to be positive in the unemployment steady states. Assumption (71) instead ensures that consumption is positive $\forall L^U \in (0, \bar{L})$.

Being the monetary policy constrained by the zero lower bound the unemployment steady states are locally indeterminate and the central bank cannot prevent the economy from being trapped in them whenever negative expectations or sunspot shocks occur, as it is shown in Figure 22.

However, notice also that assumption (75) depends on the particular shape of horizontal innovation. From the behaviour of the GG, it is easy to see that the more it is easier to start a new business the higher the relative strength of the horizontal margin over the vertical one, over the whole domain of labour L_t and, more importantly, over recessionary values of it. This suggests that, if the CDF of ability $G\left(\right)$ is smaller for any value of the threshold ability $\underline{x}_t$ or if the threshold ability $\underline{x}_t$ is itself smaller for any value of labour L_t, *ceteris paribus*, the GG schedule would result in a north-east shifted and clockwise rotated curve, making harder for assumption (75) to be verified and for stagnation traps to exist[41]. This also means that any intervention that loosens the link between labour L_t and the minimum threshold ability $\underline{x}_t$ needed to create a new firm, would avoid the formation of stagnation traps or allow the economy to escape from them, as we show in Section 5.

[41] To see this notice that $1 + g_t^N$ diverges faster as $L_t \to 0$ if $G\left(\right)$ is smaller for any value of the threshold ability $\underline{x}_t$ or if $\underline{x}_t$ is itself smaller for any value of labour L_t. This in turn means that the minimum of the new GG schedule is on the right and above the minimum of the old GG.

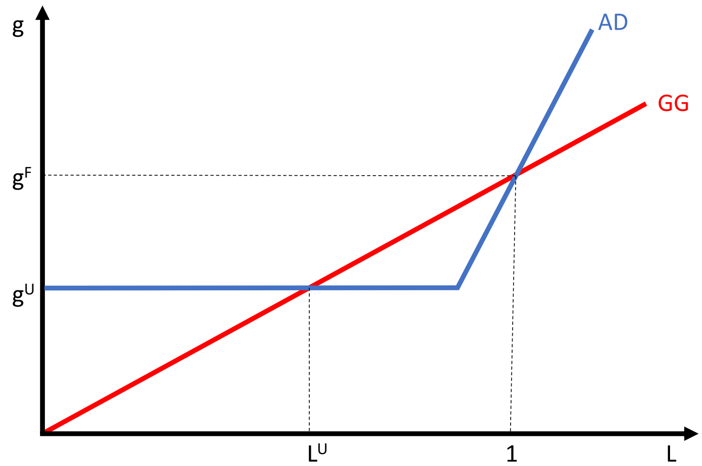


Figure 23: Non-stochastic steady states without horizontal innovation

3.8.3 Graphical Analysis

We are now ready to make a parallel with Benigno and Fornaro (2018). First of all, let us shut down horizontal innovation assuming for the moment that $\Delta, \Gamma(x) = 0 \, \forall x$ so that $g^N = 0$ and the GG becomes

$$1 + g = 1 + g^A = 1 + \psi\delta\gamma L.$$

This is the case that resembles Benigno and Fornaro (2018). In Figure 23 we plot the AD and the BF-equivalent version of the GG:

Under some calibrations, 2 steady states exist: the unemployment steady state, represented by the couple $\left(L^U, g^U\right)$, and the full employment steady state, $\left(1, g^F\right)$. At this point, it is important to notice that the existence of the unemployment steady state depends critically on the intercept of the GG and on its shape. Indeed, if we turn on the horizontal innovation mechanism, allowing for some $\Delta > 0$ and a non-degenerate distribution $\Gamma(x)$, we can easily see that, as represented in Figure 24, the unemployment steady state does not necessarily exist anymore[42]. Notice also that we set $\Delta = \int_{\underline{x}_t(L_t=1)}^{\infty} \Gamma(x)\, dx$ so that the full employment steady state is the same one represented in Figure 23.

The horizontal innovation engine that is activated when L_t decreases starts to dominate

[42]We will make an analytical discussion of this statement. For the moment we are just showing the results of several calibrations.

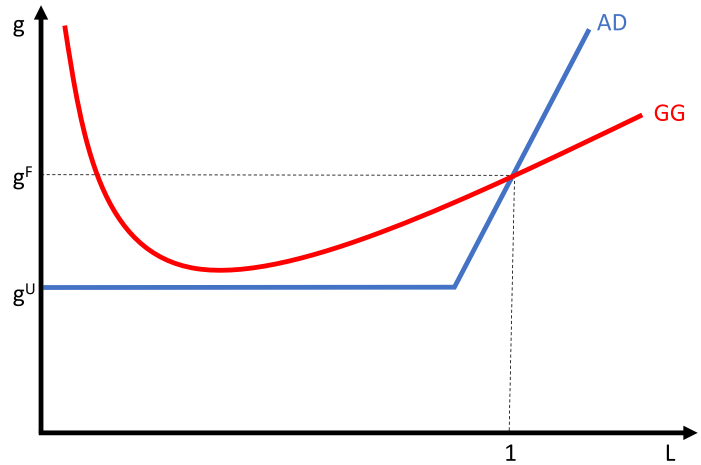


Figure 24: Non-stochastic steady states with horizontal innovation. Same calibration as in Figure 23 but allowing for $\Delta > 0$ and a non-degenerate distribution $\Gamma(x)$.

over the vertical one and prevents the economy from falling in any non-desirable steady state. With this, we are not saying that the economy cannot find itself in a protracted situation of low growth, low interest rates and low employment: we are saying that such a situation is not necessarily a steady state and therefore, if the economy is pushed around the value of labor L^Uthat characterizes the unemployment steady state in the model without horizontal innovation, the economy will endogenously escape from it converging back to the full employment steady state. Hence, the fundamentals of the economy, and in particular the ones describing entrepreneurship, are crucial as it is any kind of economic policy that influences them.

4 Extensions and calibration

We now move away from the simplistic form of wage rigidity assumed in Section 3.5 and introduce downward rigidity which gives rise to a wage Phillips curve:

$$\frac{W_t}{W_{t-1}} = \pi_t^w \geq \theta(L_t)$$

with $\theta_{L_t} > 0$ and $\theta(1) = \bar{\pi}^w$. For $\pi_t^w > \bar{\pi}^w$ output is at potential, instead, in the opposite case, there is a positive relationship between inflation and the output gap.

Now the central bank responds to deviations of wage inflation from a target π^*, still following a truncated interest rate rule:

$$1 + i_t = \max \left[(1 + \bar{i}) \left(\frac{\pi_t^w}{\pi^*} \right)^{\phi}, 1 \right] \tag{76}$$

where $\pi^* \geq \bar{\pi}^w$ so that when the central bank hits the target the economy is at full employment. Moreover, we assume the interest rate target to be $1 + \bar{i} = \frac{\pi^*}{\rho\beta} \left(1 + g^F\right)^{\sigma - 1}$, where g^F is the full employment productivity growth rate[43].

In this set up, a steady state is characterized by solutions to a system described by (65), (76) evaluated at the steady state, (64) and

$$(1 + g)^{\sigma - 1} = \frac{\rho\beta (1 + i)}{\pi^w} \tag{77}$$

$$\pi^w \geq \theta (L) . \tag{78}$$

The proof for existence and uniqueness of the full employment steady state is similar to the one in the baseline model and the reader is referred to Appendix H.1 for a guidance. Notice also the the full employment steady state is still locally determinate if the monetary policy is strong enough. Combining (77), (76) and (78), the new AD is

$$1 + g = \max \left\{ \left(\frac{\rho\beta}{\theta (L)} \right)^{\frac{1}{\sigma - 1}}, \left[\frac{\rho\beta}{\theta (L)} (1 + \bar{i}) \left(\frac{\theta (L)}{\pi^*} \right)^{\phi} \right]^{\frac{1}{\sigma - 1}} \right\} \tag{79}$$

which is characterized by a negative relationship between productivity growth and employment for values of labour L low enough so that the zero lower bound binds[44].

As we did in Section (3.8), we start the analysis focusing on the model without horizontal innovation. We calibrate the model so as to replicate the full employment steady state and the unemployment steady state of Benigno and Fornaro (2018). The parameters common to our model and their are the same. We set the productivity of vertical innovation δ at a value of 0.047 so as to match a full employment productivity growth rate of $1 + g^F = 1.02$

[43] This guarantees that when $\pi^w = \pi^*$ then by (76) $i = \bar{i} \implies g = g^F$ and by (65) $L = 1$.

[44] It also presents a new kink in correspondence of $\bar{L}_P = \theta^{-1} \left[\left(1/(1+\bar{i}) \right)^{\frac{1}{\phi}} \pi^* \right]$ and another one at $L = 1$ because nominal rigidity is defined only downward and above $L = 1$ wages are perfectly flexible and the Phillips curve becomes vertical and so does the AD.

Table 8: Parameters

	Value	Source
Elasticity of int. substitution	$1/\sigma = 0.5$	Standard
Discount factor	$\beta = 0.96^4$	Standard
Idiosyncratic risk	$\rho = 1.067$	Benigno and Fornaro (2018)
Wage inflation at full employment	$\pi^* = 1.04$	Benigno and Fornaro (2018)
Share of labour in gross output	$1 - \alpha = 0.83$	Standard
Vertical innovation step	$1 + \gamma = 1.55$	Benigno and Fornaro (2018)

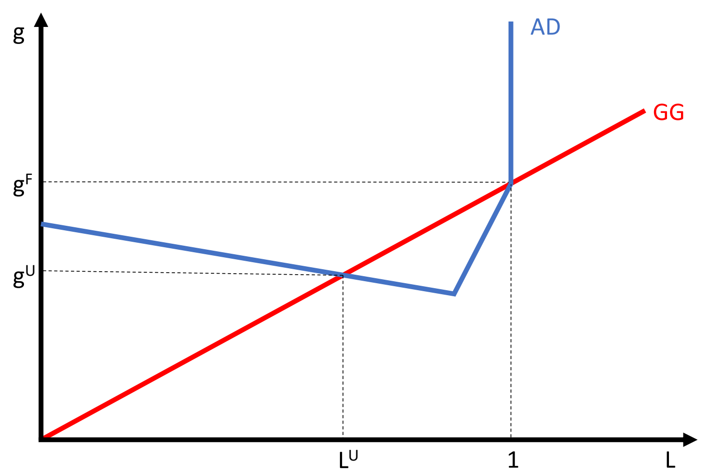


Figure 25: Non-stochastic steady states in the Phillips curve extension without horizontal innovation.

and the slope of our linear Phillips curve[45] to 0.01 to match an unemployment productivity of $1 + g^U = 1.0167$ and we target the same output gap of $1 - L^U = 0.075$. The results of this calibration are represented in Figure 25.

We can see how the introduction of the wage Phillips curve always guarantees the existence of the unemployment steady state (L^U, g^U) in the case of a linear GG. Moreover, as argued in Benigno and Fornaro (2018), higher wage flexibility in this scenario (characterized by a steeper negative portion of the AD) leads to better outcomes in terms of productivity growth in unemployment steady states. Both this features are not necessarily true once we introduce horizontal innovation.

Keeping the same calibration presented in Table 8, we now introduce horizontal innovation allowing $\Delta > 0$, a non-degenerate distribution $\Gamma(x)$, and in particular we set

[45]We are assuming $\theta(L) = \pi^* L$.

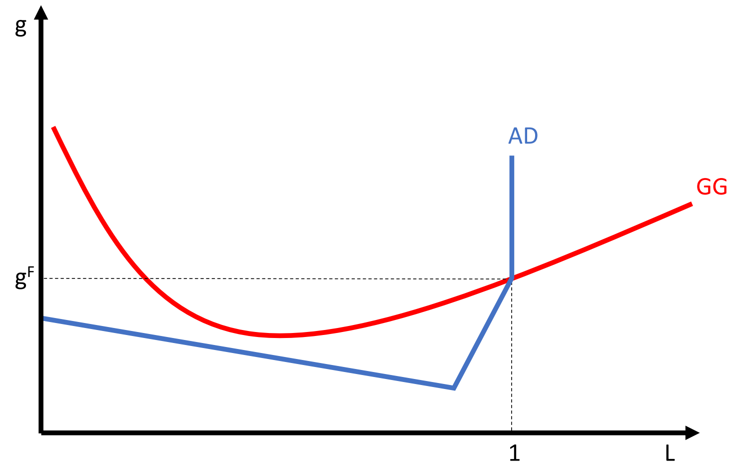


Figure 26: Non-stochastic steady states in the Phillips curve extension with horizontal innovation

$\Delta = \int_{\underline{x}_t(L_t=1)}^{\infty} \Gamma(x)\, dx$ so that at full employment the only engine active is the one of vertical innovation and the productivity growth associated to it is the same as the one in Figure 25. This allows us to clearly see what is the effect of the introduction of horizontal innovation.

Now the unemployment steady state does not exist anymore. As it happened in the case without the Phillips curve, the horizontal innovation mechanism that starts to dominate for low values of employment L, prevents the economy from falling in the unemployment steady state.

With this being said, we would like to stress that our findings support and complement the theory of stagnation traps of Benigno and Fornaro (2018). Indeed, we are introducing a dimension that is crucial for both the existence of and the escape from stagnation traps: firm dynamics. In the next section we will see how such a dimension can be a channel trough which stagnation traps may vent their way and also how economic policy can influence entrepreneurship so that the escape from a trap or the prevention of it is possible.

5 Stagnation Traps and Economic Policy

As anticipated *supra*, our framework introduces a dimension trough which the policy maker can intervene to prevent stagnation traps or to escape faster from them. Such a dimension is firm dynamics and in particular the fostering of entrepreneurship. Given the large evidence that individuals tend to refuge into self-employment when they experience difficulties in the wage/salary sector (Rees and Shah, 1986; Evans and Leighton, 1989; Le, 1999; Farber, 1999; Krashinsky, 2005; Babina, 2019; Fairlie, 2013), it seems promising to ease this transition both in nefarious economic times to allow the economy to move away from the trap faster and in normal ones to prevent it.

To see this, assume the policy maker introduces a constant subsidy $s \in (0,1)$ on the creation of new firms, financed entirely with a lump-sum tax on households[46]. The free entry condition (55) would become

$$z_t^h \Pi(A_t) = (1-s) R_t(z_t) P_t$$

yielding to a minimum horizontal innovation intensity to profitably create a new firm to be $\underline{z}_t^h = (1-s) \frac{\delta}{2} \psi L_t$ which is clearly decreasing in the subsidy s and lower than before $\forall L_t$. This is also reflected in the threshold managerial ability

$$\underline{x}_t = \frac{(1-s)\delta\psi L_t}{2-(1-s)\delta\psi L_t}$$

which is decreasing in the subsidy s and lower than its ex-subsidy counterpart (56) $\forall L_t$. This in turn means that the introduction of a subsidy to horizontal innovation makes it easier for would-be entrepreneurs to create a new firm because the minimum managerial ability required to profitably start a business is now lower. This translates into an upward shift and a clockwise rotation of the GG equation making it hard for assumption (75) to be verified and hence more remote the possibility to end up in a stagnation trap[47]. The movement of the GG is accompanied with a reduction in the kink of the new AD schedule (*i.e.* the domain over which the zero bound on the interest rate binds becomes

[46]This obviously does not change the resource constraint because

$$P_t C_t = W_t L_t - T_t + \underbrace{P_t Y_t - W_t L_t - \int_0^{N_t} P_t(j)\, y_t(j)\, dj}_{\text{Final good firm profits}} +$$
$$+ \underbrace{\int_0^{N_{t-1}} \left(P_t(j)\, y_t(j) - P_t y_t(j) - R_t(j) P_t\right)}_{\text{Int. good, vertical innovator profits}} + \underbrace{\int_{N_{t-1}}^{N_t} \left(P_t(j)\, y_t(j) - P_t y_t(j) - (1-s) R_t(j) P_t\right)}_{\text{Int. good, horizontal innovators profits}}$$

and $T_t = sR_t P_t (N_t - N_{t-1})$.

[47]To see this, notice that the subsidy makes the $1+g_t^N$ to diverge faster as $L_t \to 0$ and to converge slowly to $1-\Delta$ as $L_t \to \infty$. This in turn means that the minimum of the new GG schedule occurs to the right and above the minimum of the old GG. In addition, the reduction in the point of kink in the AD reduces the domain over which the ZLB binds, which is also the domain over which the $\arg\min_L GG$ must occur to have an unemployment steady state. This makes harder for assumption (75) to be satisfied.

smaller) and by a steepening in the positively inclined portion of it[48]. Moreover, was the economy already in a stagnation trap, it should be clear that the introduction of a subsidy to the reallocation of labour into entrepreneurship helps the economy to get out from it bringing the it back to the full employment steady state, as shown in Figure 27. It should be noticed also that the introduction of a subsidy allows the economy to reach a new full employment steady state characterised by an higher productivity growth. This can easily be seen looking at equation (69) defining the full employment growth rate. It depends negatively on $G(\underline{x})$ which in turn depends negatively on the subsidy s[49]. This analysis shows that not only subsidising business creation can prevent the economy from falling into stagnation traps and escaping from them but also that in prosperous times it ensures a better standard of living sustaining a higher productivity growth rate.

Clearly, the introduction of the subsidy influences the behaviour of the GG as we move among the domain $L_t \in [0, 1]$ making it steeper before its minimum and flatter after it (see footnote 47) . In other words, the subsidy strengthens the relative weight of the horizontal margin of innovation over the vertical one on the whole domain $L_t \in [0, 1]$, thus also, and perhaps even more importantly, on low values of L_t. This is because the built-in resilience mechanism of our economy activated trough the reallocation into self-employment is more smooth being the cost of setting-up a new business partly alleviated by the subsidy. This both allows unlucky households to escape from a situation of poverty due to unemployment and lucky ones to give a chance to their entrepreneurial ideas and to Orazio's Odi recommendation[50].

It is important at this point to focus on the differences of our framework with horizontal innovation from Benigno and Fornaro (2018), in the behaviour under a subsidy policy. Benigno and Fornaro (2018) introduce an additive subsidy that is contingent on growth (or employment) in order to induce the strict convexity (and non-monotonicity, in the case of wage Phillips curve) in the GG schedule that is needed in order to rule out the stagnation trap for any value of employment. While both features are powerful in delivering the result that a stagnation trap can be ruled out, they make the ultimate policy guidance not immediately clear.

In our framework, on the other hand, the extensive margin of horizontal innova-

[48] This occurs because the introduction of the subsidy changes the natural rate of interest consistent with full employment, $d\left(i_s^F\right) > 0$, and therefore the central bank has to adjust its policy rate $\bar{i}$ to sustain the new equilibrium.

[49] Because $\underline{x}_s = -\frac{2\delta\psi L_t}{[2-(1-s)\delta\psi L_t]^2} < 0$.

[50] *Carpe diem quam minimum credula postero.*

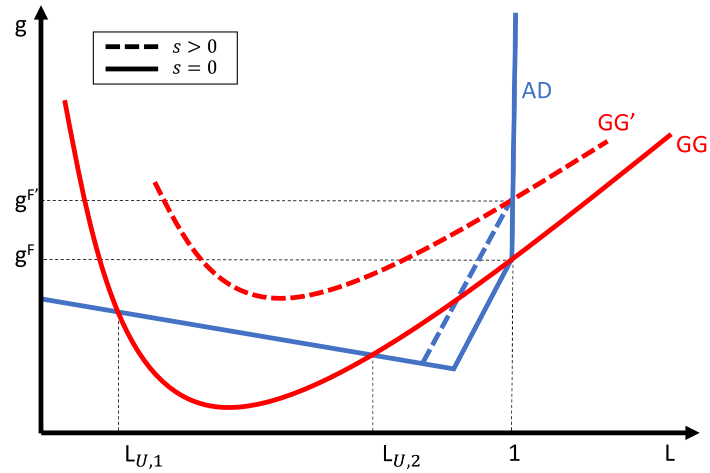


Figure 27: The effect of the introduction of a subsidy on the existence of stagnation traps. The GG schedule in solid line is the one before the introduction of the subsidy and it induces two stagnation traps. The GG' schedule in dashed lines is the one resulting after the introduction of the subsidy; is it just an upward shift and a clockwise rotation of the previous GG. It shows that if the subsidy is strong enough, it can bring the economy out from stagnation traps and bring it to a new, higher, full employment steady state.

tion that captures the reallocation into self-employment and entrepreneurship, naturally delivers a strictly convex and non-monotonic growth schedule, without the need for a state-contingent subsidy to induce it. As a result, a simple constant and multiplicative subsidy, which has a clear interpretation in terms of economic policy, is able to rule out the (multiple) stagnation traps.

5.1 Firms Entry and Exit

Consistently with the evidence reported in Table **??**, our framework can allow for stagnation traps induced by an increase in the exit rate of firms, as we observed in the aftermath of the Great Recession. This in our model is achieved with an increase in the (momentarily) exogenous firms death shock Δ. The effect is a downward shift of the GG schedule, making stagnation traps more likely[51], as shown in Figure 28, (a).

Moreover, our framework also allows to analyse the effect of a decline in the entry rate of firms, as it occurred during the Great Recession and reported in Table **??**. Such a mechanism vents through an increase in the CDF of ability $G()$ for any value of the

[51] It is easy to see that assumption (75) is easier to be verified the bigger is Δ as a $d\Delta > 0$ shifts downwards and rotates counter-clockwise the GG schedule.

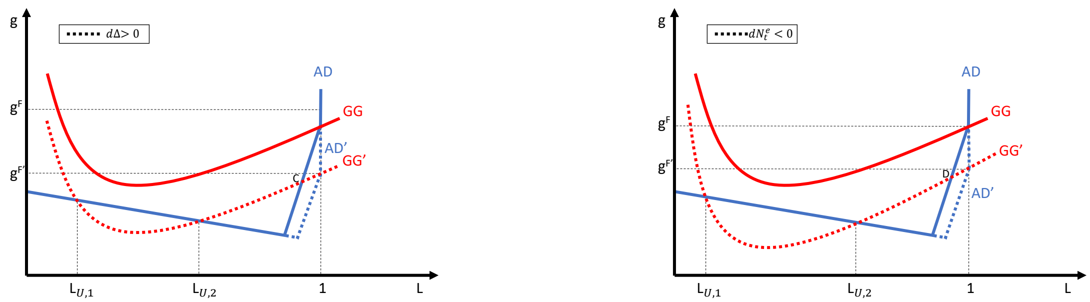


Figure 28: (a) Increase in death rate. (b) Decrease in firms entry rate.

threshold ability $\underline{x}_t$ or through the threshold ability $\underline{x}_t$ if it is itself higher for any value of labour L_t, leading to a GG schedule south-west shifted and counter-clockwise rotated[52], as shown in Figure 28, (b).

Both in the increase in the firms death rate Δ and in the decrease in the entry rate N_t^e, the monetary policy needs to adjust to the new natural rate of interest $i^{F,'}$ consistent with the new, smaller, full employment growth rate $g^{F,'}$. This has an effect on the AD schedule which, in addition to have a kink moved to the right, has a positively sloped portion less steep than before. Notice also that, should the central bank not adjust its policy to the new target consistent with full employment, the economy may end up in a situation characterised by low growth and high unemployment but outside from liquidity traps, thus also stagnations alone are possible when the horizontal margin of innovation is considered. This is shown in point C in Figure 28, (a) and in point D in Figure 28, (b).

The interpretation of this mechanics can be done through the "missing generation of firms" theory of Gourio et al. (2016). They show that entry of firms acts as a propagation mechanism and has significant and persistent effects on GDP and productivity. Indeed, lower entry means that the total number of firms is lower (if the exit rate is constant or increased) and also aggregate productivity will be lower as well. This is so because if fewer firms enter the market then there will be fewer firms that innovate and that replace less-productive incumbents. Then, using the same parallel that the authors use, just like a missing generation of births during wartime causes a "missing generation" in population demographics, then the same is true for firms: if firms death rate increases (or if the entry

[52]This behaviour meakes easier for assumption (75) to be verified. To see this just notice that $1 + g_t^N$ diverges slower as $L_t \to 0$ if $G\,()$ is bigger for any value of the threshold ability $\underline{x}_t$ or if $\underline{x}_t$ is itself higher for any value of labour L_t. This in turn means that the minimum of the new GG schedule is on the left and above the minimum of the old GG.

rate decreases, or a combination of the two) then this leads to a "missing generation" of firms, which has persistent effects on GDP and productivity. The persistence comes from the fact that newly birth firms, conditional on survival, tend to grow and to invest in innovation. If a fraction of them was never born, then this is a hole in R&D spending not only in the period in which the generation of firms misses but also in the subsequent ones. Our model is well suited to study the effects of a missing generation of firms. If the death rate increases or if the business creation is affected by the events described at the beginning of this section, then this would lead to a generation of firms to be missing. In turn, the effect would be of a reduction in business creation activities, weakening the strength of the horizontal innovation margin and making stagnation traps possible. Gourio et al. (2016) show that the effect of a change in the entry rate of firms leads to a 1-1.5 percent change in real GDP and can last 10 years or longer. Our model reconciles this empirical evidence showing that a missing generation of firms can make periods of low growth, high unemployment possible, together with policy rates close to their zero lower bound, *i.e.*, stagnation traps.

6 Conclusion

We have provided empirical evidence that favors a non-monotone relationship between productivity growth and employment over the period 1985Q1-2021Q1. The long established positive and linear link is instead better suited to study periods that span from the 1948Q1 til the 1984Q4, the idea being that the digital revolution has made easier for the recuperative powers of capitalism to emerge during slumps and in particular during deep slumps.

We have explored the role and policy implications of a counter-cyclical component in the process of innovation formation and technological progress, *entrepreneurship*, in a model of endogenous growth with nominal rigidities and monetary policy. We found the interaction of the counter-cyclical business creation with the pro-cyclical intensive margin of innovation to produce a non-monotonic relationship between growth and employment. We showed the conditions under which this allows existence of multiple stagnation traps. In particular, if the horizontal margin is not relatively strong enough or, in other words, if the reallocation of labour into self-employment is not smooth enough, multiple stagnation traps will be possible. Consequently, any policy that eases entrepreneurship, is a good candidate to avoid the existence of stagnation traps or to bring an economy out from them.

This paper shows that a constant, multiplicative subsidy to business creation activities achieves this.

H Proofs

H.1 Proof of Proposition 2

Proof. Let us firstly prove existence. Setting $L^F = 1$ in the system (63)-(67) and using (65) implies

$$g^F = (1-\Delta)\,\frac{1+\psi\delta\gamma}{G\left(\underline{x}^F\right)} - 1$$

which is positive thanks to assumption (72). Equation (63) then implies

$$i^F = \left[(1-\Delta)\,\frac{1+\psi\delta\gamma}{G\left(\underline{x}^F\right)}\right]^{\sigma-1}\frac{\bar{\pi}^w}{\rho\beta} - 1$$

which is positive thanks to assumption (70) and (67) implies $\bar{i} = i^F$ so that the central bank sustains the full employment steady state setting the policy rate consistent with it. Using (64) one gets

$$\tilde{C}^F = \Psi - \frac{\delta}{2}\psi^2$$

and it is positive by the inequality in assumption (71) because $\tilde{C}^F > 0 \iff \delta < \frac{2\Psi}{\psi}$ and the last inequality is ensured by assumption (71) because $\frac{2\Psi}{\psi} > \frac{1}{\psi}$. Thus a full employment steady state exists. It is also unique because, given $L^F = 1$ equation (65) implies only one value of g^F consistent with it and so does equation (63) given g^F.

Concerning determinacy, the system (63)-(67) has been log-linearly approximated around the full employment steady state, yielding:

$$\hat{C}_t = \hat{L}_t\frac{\Psi - \delta\psi^2}{\Psi - \frac{\delta}{2}\psi^2} \tag{80}$$

$$\sigma\hat{C}_t = -\hat{i}_t + \sigma\mathbb{E}_t\hat{C}_{t+1} - (1-\sigma)\,\mathbb{E}_t\hat{g}_{t+1} \tag{81}$$

$$\hat{g}_t = \left(\frac{\psi\delta\gamma}{1+\psi\delta\gamma} - \frac{2\delta\psi}{(2-\delta\psi)^2}\frac{G_{\underline{x}^F}}{G\left(\underline{x}^F\right)}\right)\hat{L}_t \tag{82}$$

$$\hat{i}_t = \phi\hat{L}_t \tag{83}$$

where $G_{\underline{x}^F} \equiv \left[{}^{\partial G(\underline{x}_t)}/{}_{\partial \underline{x}_t}\right]_{L=1}$ and $\hat{x} = \frac{x_t - x^F}{x^F}$ for every variable x except for $\hat{i}_t = \frac{\hat{i}_t - i^F}{1+i^F}$ and

$\hat{g}_t = \frac{g_t - g^F}{1+g^F}$. The system can be summarised by

$$\hat{L}_t = \xi_1 \mathbb{E}_t \hat{L}_{t+1} + \xi_2 \mathbb{E}_t \hat{g}_{t+1}$$
$$\hat{g}_t = \xi_3 \mathbb{E}_t \hat{L}_{t+1} + \xi_4 \mathbb{E}_t \hat{g}_{t+1}$$

where

$$\xi_1 = \frac{\sigma\left(\Psi - \delta\psi^2\right)}{\sigma\left(\Psi - \delta\psi^2\right) + \phi\left(\Psi - \frac{\delta}{2}\psi^2\right)}$$
$$\xi_2 = -\frac{(1-\sigma)\left(\Psi - \delta\psi^2\right)}{\sigma\left(\Psi - \delta\psi^2\right) + \phi\left(\Psi - \frac{\delta}{2}\psi^2\right)}$$
$$\xi_3 = \varphi\xi_1$$
$$\xi_4 = \varphi\xi_2$$
$$\varphi = \frac{\psi\delta\gamma}{1+\psi\delta\gamma} - \frac{2\delta\psi}{(2-\delta\psi)^2} \frac{G_{\underline{x}^F}}{G\left(\underline{x}^F\right)}.$$

This system is determinate if and only if[53]:

$$|\xi_1\xi_4 - \xi_2\xi_3| < 1 \tag{84}$$
$$|\xi_1 + \xi_4| < 1 + \xi_1\xi_4 - \xi_2\xi_3. \tag{85}$$

It is easy to see that condition (84) is always verified while condition (85) is verified if

$$\phi > \frac{\left(\Psi - \delta\psi^2\right)(\sigma - 1)\varphi}{\Psi - \frac{\delta}{2}\psi^2}$$

which is guaranteed by assumption (73) and hence the full employment steady state is locally determinate. □

[53] Woodford (2011).

H.2 Proof of proposition 3

Proof. We start by proving existence. Setting $i = 0$ in (63) we get

$$g^U = \left(\frac{\rho\beta}{\bar{\pi}^w}\right)^{\frac{1}{\sigma-1}} - 1$$

which is positive by assumption (74). To understand whether one or more unemployment steady states exist we should firstly understand the behaviour of the GG equation (62). Clearly, as $L \searrow 0$, $1 + g \to \infty$ because $1 + g^A = 1$ and $1 + g^N \to \infty$. Instead, as $L \to \infty$ then $1 + g \to \infty$ because $1 + g^A \to \infty$ and $1 + g^N \to 1 - \Delta$. Moreover, notice that $1 + g^A$ monotonically increases and $1 + g^N$monotonically decreases over the domain $L \in (0, \infty)$ and that $1 + g = \left(1 + g^A\right)\left(1 + g^N\right)$ is a C^2 function that will have a global minimum on the domain $L \in (0, \infty)$. Assumption (75) guarantees that such a minimum is below the horizontal portion of the AD schedule while (73) that at $L = 1$ the slope of the upward-sloped portion of the AD is bigger than the one of the GG so that the GG must cross the horizontal portion of the AD at least one time[54]. To see this just notice that, being the GG monotonically increasing to the right of its minimum, if at $L = 1$ the slope of the GG is smaller than the one of the AD, then it must be that in a left neighbourhood of $L = 1$ the GG lies above the AD while in a right neighbourhood of $L = 1$ it lies below it (recall that by Proposition 3 the GG and the AD cross at $L = 1$). This in turn implies that, if assumption (75) is satisfied, then it must be that there exist at least one intersection between the increasing portion of the GG and the horizontal portion of the AD.

Let us prove that $L^U \in (0, \bar{L})$ by contradiction. Suppose $L^U \notin (0, \bar{L})$. Then, by the monotonicity of the AD to the right of $\bar{L}$ and by the monotonicity of the GG to the right of its minimum, and since by Proposition 3 the GG and the AD cross at $L = 1$, then it must be that $\frac{\partial GG}{L}\mid_{L=1} > \frac{\partial AD}{L}\mid_{L=1}$ which requires $\phi < (\sigma - 1)\varphi$. But this is a contradiction because assumption (73) guarantees $\phi > (\sigma - 1)\varphi$.

If assumption (75) is satisfied with equality there will be only one unemployment steady

[54]Notice that

$$\frac{\partial GG}{L}\mid_{L=1} < \frac{\partial AD}{L}\mid_{L=1} \iff$$

$$\iff \left[(1-\Delta)\frac{1+\psi\delta\gamma}{G\left(\underline{x}^F\right)}\left[\frac{\frac{\psi\delta\gamma}{1+\psi\delta\gamma}G\left(\underline{x}^F\right) - G_{\underline{x}^F}\frac{2\delta\gamma}{(2-\delta\psi)^2}}{G\left(\underline{x}^F\right)}\right]\right]_{L=1} < \left[(1-\Delta)\frac{1+\psi\delta\gamma}{G\left(\underline{x}^F\right)}\right]\frac{\phi}{\sigma-1}L^{\frac{1+\phi-\sigma}{\sigma-1}}\mid_{L=1} \iff$$

$$\iff \phi > (\sigma - 1)\varphi.$$

state while there will be two if it is satisfied with the inequality and zero if it is not satisfied at all. Notice also that the solution to $\min_L (1-\Delta)\frac{1+\psi\delta\gamma}{G(\underline{x}^F)} - 1$ will in general depend on the particular shape (and governing parameters) one assumes for the distribution of ability as it must satisfy

$$\frac{\partial(1+g)}{\partial L} = \psi\delta\gamma G(\underline{x}(L)) - (1+\psi\delta\gamma L)\frac{\partial G(\underline{x}(L))}{\partial\underline{x}(L)}\frac{2\delta\psi}{(2-\delta\psi L)^2} = 0.$$

Clearly, once we get the value of $L^U$from equation (65), we substitute it in (64) to get

$$\tilde{C}^U = \Psi L^U - \frac{\delta}{2}\left(\psi L^U\right)^2$$

which is guaranteed positive by the inequality in assumption (71) noticing that $L^U \in (0,1)$.

Moreover, being $i^F > 0$, $g^U < g^F$ then we get $1 + r^U = \frac{1}{\pi^U} = \frac{g^U}{\bar{\pi}^w} < \frac{(1+i^F)g^F}{\bar{\pi}^w} = \frac{(1+i^F)}{\pi^F} = 1 + r^F$ and this patently shows why the real interest rate in the unemployment steady state is lower then the one in full employment.

Regarding determinacy, proceeding as in Appendix (H.1) it is easy to notice that condition (85) cannot be satisfied as in a neighborhood of an unemployment steady state $\hat{i}_t = 0$. □

	(35)	(35)	(36)	(36)	(36)	(35)	(37)
	GM	2005-2021	1985-2021	1985-2021	1985-2021	1985-2021	1985-2021
Constant	-8.34^{***}	1.231^{***}	-6.367^{***}	0.769	-4.890^{***}	2.621^{***}	253.08^{***}
	(.7159)	(.431)	(.649)	(.832)	(.954)	(.671)	(33.554)
L_t	8.689^{***}	$-.76^{*}$	6.516^{***}	$-.529$	5.221^{***}	-2.414^{***}	-519.13^{***}
	(.729)	(.446)	(.663)	(.85)	(.973)	(.688)	(69.215)
L_t^2							266.42^{***}
							(35.687)
post-2004			7.598^{***}				
			(.808)				
$L_t *$ post 2004			-7.411^{***}				
			(.828)				
1st decile				-11.955			
				(13.671)			
$L_t *$ 1st decile				12.93			
				(14.542)			
2nd decile					5.310^{*}		
					(.3.2)		
$L_t *$ 2nd decile					-5.154		
					(3.373)		
R^2	.343	.0241	.725	.0721	.255	.0333	.164

Notes: * indicates significance at 10% level, ** at 5% and *** at 1%. GM: Great Moderation (1985Q1-2007Q3). Standard errors in parentheses.

Table 9: Estimation of regressions (35), (36), and (37) where the dependent variable is (log of) new patents assignments in Santa Clara.

	(35) GM	(35) 2005-2021	(36) 1985-2021	(36) 1985-2021	(36) 1985-2021	(35) 1985-2021	(37) 1985-2021
Constant	-8.34^{***}	1.231^{***}	-6.367^{***}	0.769	-4.890^{***}	2.621^{***}	253.08^{***}
	(.7159)	(.431)	(.649)	(.832)	(.954)	(.671)	(33.554)
L_t	8.689^{***}	$-.76^{*}$	6.516^{***}	$-.529$	5.221^{***}	-2.414^{***}	-519.13^{***}
	(.729)	(.446)	(.663)	(.85)	(.973)	(.688)	(69.215)
L_t^2							266.42^{***}
							(35.687)
post-2004			7.598^{***}				
			(.808)				
$L_t *$ post 2004			-7.411^{***}				
			(.828)				
1st decile				-11.955			
				(13.671)			
$L_t *$ 1st decile				12.93			
				(14.542)			
2nd decile					5.310^{*}		
					(.3.2)		
$L_t *$ 2nd decile					-5.154		
					(3.373)		
R^2	.343	.0241	.725	.0721	.255	.0333	.164

Notes: * indicates significance at 10% level, ** at 5% and *** at 1%. GM: Great Moderation (1985Q1-2007Q3). Standard errors in parentheses.

]Results of regressions of Section 2 for new patents assignments in Santa Clara

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Chapter III

World Trade Stagnation

Abstract

Emerging economies (EMEs) account for most of the world GDP and global growth nowadays, their trade flows with advanced economies (AEs) doubled during the mid-80s till the 2012 and their investment in R&D increased by 26.4% per year between 1997-2008, almost catching-up with AEs productivity around the 2009 and displaying the same AEs TFP growth rate thereafter. These developments were accompanied by trade balance reversals and by a rapid increase in world trade (the so called "hyper-globalization") which culminated in the Great Trade Collapse of the 2009 and a subsequent stagnation up to nowadays. I present a two-country model with firms heterogeneous in productivities à la Melitz (2003), endogenous growth and international R&D spillovers that accounts for these facts. It is found that innovation and trade dynamics are closely tied and trade balance reversals are consequences of asymmetric needs of funding stemming from innovation efforts. Trade stagnates after the 2008 because of stagnant proximity to the technological frontier of EMEs (with respect to AEs)[55].

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1 Introduction

Emerging economies (EMEs) account for most of the world GDP and global growth (Arezki and Liu, 2020), and their trade flows with advanced economies (AEs) doubled during the mid-80s till the 2012, accounting for 40% of world trade since 2012, surpassing the contribution of within AEs trade flows[56]. EMEs investment in R&D grew at an impressive annual average rate of 26.4% over the period 1997-2008, more than 5 times that of AEs, almost leading to a technology catch-up around the Global Financial Crisis (GFC, 2008-09, Figure 29, left); AEs and EMEs shared the same TFP growth rate thereafter[57]. These developments were accompanied by trade balance reversals (Figure 30) and by a rapid increase in world trade (the so called "hyper-globalization") which culminated in the Great Trade Collapse of the 2009 and a subsequent stagnation up to nowadays (Figure 29, right). This paper sheds light on these developments studying the international links between AEs and EMEs through the lens of a two-country model with firms heterogeneous in productivities à la Melitz (2003), growth occurring endogenously via expanding varieties (Romer, 1990b) and international R&D spillovers (Barro and Sala-i Martin, 1997).

It is found that the innovation activities in both countries groups are closely intertwined with the pattern of trade and the sluggish dynamics of the EMEs' proximity to the technological frontier transfers to that of trade. Conditional on shocks to the exogenous component of the TFP, the model successfully reproduces the observed pattern of trade and proximity - where their autocorrelation is governed by the international R&D spillover parameter - and trade balance reversals. Intuitively, following a shock, firms' profits increase, decreasing the export productivity cut-off. Forward looking entrepreneurs engage in innovation activities, with the aim of acquiring the increased profits through the development of new varieties. Since historical shocks had been larger in EMEs than in AEs, the larger demand coming from EMEs triggers investment in R&D also in AEs and via international R&D spillovers this reinforces the innovation activity in EMEs. The stock of varieties grows faster in EMEs than in AEs, due to the larger shock size and EMEs get closer to the technological frontier. As there is a larger mass of varieties and a greater fraction of them is exported, trade increases faster than GDP, closely tracking the dynamic behaviour of the EMEs proximity to the technological frontier. The whole dynamics is accompanied by an initial trade balance deterioration in AEs, as borrowing finances the increased R&D spending which, on impact, exceeds the increased output. As

[56] Author's calculation on IMF, Direction of Trade Statistics data.

[57] The relative productivity behaviour is robust to countries classification. See Figure 32.

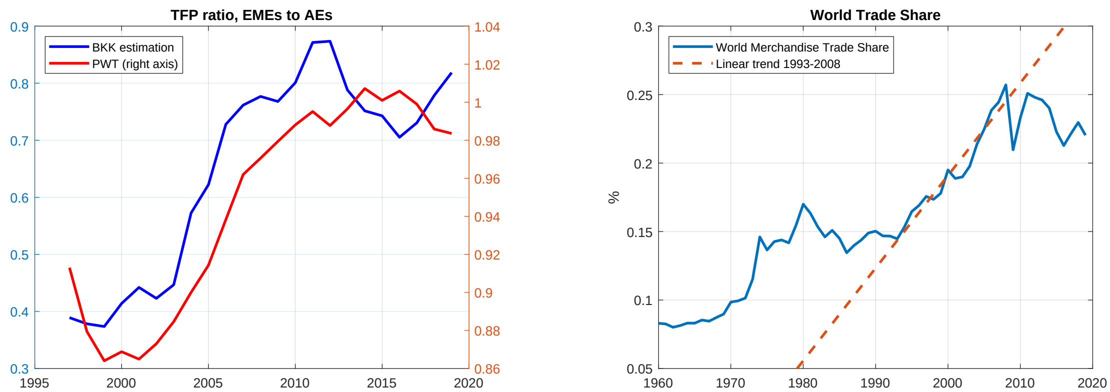

Figure 29: (Left) EMEs TFP relative to that of AEs, 1997-2019. See the Appendix I.1 for details. (Right) World merchandise trade as a share of world GDP, 1960-2019. Source: World Bank.

time goes by, trade balances dynamics reverse as the initial desire to invest in R&D in AEs becomes sustained by the increased output; the opposite occurs in EMEs, where the shock propagates more persistently, and their international position deteriorates, consistently with the observed pattern.

Since this work is among the first to study advanced and emerging economies in a joint model, I provide business cycle statistics for AEs and EMEs and check the ability of the model to be consistent with these, in the spirit of Backus et al. (1992). Business cycles across AEs and EMEs are found to be highly correlated and macroeconomic variables are generally more volatile in EMEs than in AEs (with the exception of investment). The model is roughly able to match these findings even though the unusual negative international comovement in physical investment is a lacuna. This model thus serves as a useful starting point to the joint analysis of advanced and emerging economies. In doing so, it provides micro-foundations for international trade and growth, while preserving tractability in a dynamic stochastic general equilibrium context and being consistent with basic business cycles facts.

This paper contributes to several strands of the economic literature. It contributes to the literature on trade and innovation, which is summarized in Melitz and Redding (2021). To the best of my knowledge, my work is the first to integrate endogenous trade and innovation in an otherwise standard macro-DSGE model.

I contribute also on the literature that introduced endogenous growth through expanding varieties à la Romer (1990b) after the seminal work of Comin and Gertler (2006). Closely related works to mine are Queralto (2020), Kung and Schmid (2015) and Bianchi et al. (2019). They all share endogeneity in growth through expanding varieties but abstract

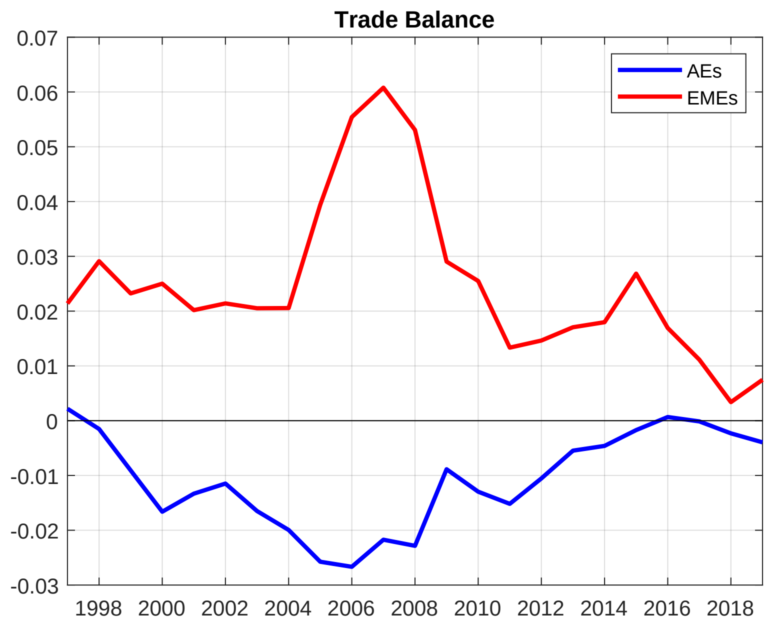

Figure 30: Trade balance of AEs and EMEs, 1997-2019. Correlation is -0.68. See Appendix I.1 for details.

from the international dimension of innovation and trade, which are the focus of my paper.

The model presented is benefiting from the bridge between international macroeconomics and trade built by Ghironi and Melitz (2005). I share with this seminal work the core structure of the model, namely the micro-foundations of trade (Melitz, 2003) and the international bonds trading setting. I expand their framework introducing endogenous growth and international R&D spillovers, which are closely tied with trade and trade balance dynamics.

Of course, this work contributes to the literature on emerging economies. Related works include Senhadji (1998), Aguiar and Gopinath (2007), Arezki and Liu (2020), Benigno et al. (2020) and Giovannini et al. (2019), among the others. My work is more closely related to Giovannini et al. (2019) and Benigno et al. (2020). With the former I share the study of AEs and EMEs in a joint model. They focus on trade balance dynamics after the GFC and put emphasis on the role of commodity prices in guiding it. I instead focus on the developments in productivity, trade and trade balance for AEs and EMEs, thus also proposing a complementary explanation for the observed trade balance reversals. Regarding the latter work, both contributions share international R&D spillovers and endogenous growth but my model is stochastic and focuses on the links between innovation, trade and trade balance while their work focuses more on the effects of a financial integration episode for the current account dynamics.

The paper is structured as follows: Section 2 presents the model and Section 3 the results. Section 4 studies business cycles properties of the data and model. Section 5 concludes.

2 The model

The model developed in this section has a core structure based on Ghironi and Melitz (2005) that is augmented to encompass endogenous growth in product variety à la Romer (1990b). The world economy is populated by two countries (or mass of countries), namely advanced economies (AEs) and emerging and developing economies (EMEs). Variables for the latter are denoted with an asterisk. The two countries are symmetric except for the fact that EMEs will benefit from positive R&D spillovers from AEs (Barro and Sala-i Martin, 1997) and have a lower innovation productivity (to be consistent with being technological followers over the period 1997-2008). Time is discrete, $t \in \mathbb{N}$. I will describe only the home country (AEs); the foreign country (EMEs) country is symmetric except for what specified in the text.

2.1 Household preferences

Following Melitz (2003) and Ghironi and Melitz (2005), I assume there is a continuum of goods indexed by $\omega \in \Omega$ where the measure of the set Ω represents the mass of available goods. The household derives C.R.R.A. lifetime-utility $\mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \frac{C_s^{1-\gamma}}{1-\gamma}$ from consuming a C.E.S. aggregate of these goods, $C_t = \left(\int_{\omega \in \Omega} c_t\left(\omega\right)^{\frac{\theta-1}{\theta}} d\omega \right)^{\frac{\theta}{\theta-1}}$,where $\theta > 1$ is the symmetric elasticity of substitution across goods, $\gamma > 0$ is the inverse of the intertemporal elasticity of substitution and $\beta \in (0,1)$ is the subjective discount factor. Each period t, only a subset of goods $\Omega_t \subset \Omega$ is available. As shown in Dixit and Stiglitz (1977), the associated consumption-based price index is $P_t = \left(\int_{\omega \in \Omega_t} p_t\left(\omega\right)^{1-\theta} d\omega \right)^{\frac{1}{1-\theta}}$ and optimal household consumption is given by $c_t\left(\omega\right) = \left(\frac{p_t(\omega)}{P_t} \right)^{-\theta} C_t$. The household receives a wage rate W_t for supplying inelastically L units of labor in each period.

The EMEs country faces the exact same set-up. Notice that $\Omega_t^* \subset \Omega$ can differ from Ω_t as a result of different optimal choices dictated by differences in the realizations for the shock processes, as it will become clear *infra*.

2.2 Production

In each country there is a continuum of firms, each producing a different variety $\omega \in \Omega$. Firms combine labor $l_t(\omega)$ and capital $k_t(\omega)$ in a Cobb-Douglas fashion and share a common (stationary) aggregate labour productivity Z_t whose process will be described in Section 2.11. As it will be described in Section 2.5, firms are created by an R&D sector using the final good as an input. Upon creation, firms draw their relative productivity level $\varphi > 0$ from a common distribution $G(\varphi)$ with support on $[\varphi_{min}, \infty)$ and remains fixed thereafter. All firms produce in every period until their associated variety becomes obsolete, which happens with probability $\phi \in (0,1)$. Since ϕ is independent of the firm's productivity level, $G(\varphi)$ represents the productivity distribution of all producing firms.

Firms may decide to operate only in the home market or to serve also the foreign market. Exporting involves a melting-iceberg trade cost $\tau_t \geq 1$ and a fixed cost $f_{X,t}$ in units of effective (stationary) labor paid period by period. Therefore, the problem of a firm serving the home market is

$$\begin{aligned}
d_{D,t}(\varphi) = & \max_{p_t(\varphi), y_t(\varphi), l_t(\varphi), k_t(\varphi)} \frac{p_t(\varphi)}{P_t} y_t(\varphi) - w_t l_t(\varphi) - r_t^K k_t(\varphi) \\
& \text{s.t.} \\
& y_t(\varphi) = \varphi (Z_t l_t)^{1-\alpha} k_t^{\alpha} \\
& y_t(\varphi) = \left(\frac{p_t(\varphi)}{P_t} \right)^{-\theta} Y_t
\end{aligned}$$

where w_t is the real wage rate, r_t^K is the real rental rate of capital and $\alpha \in (0,1)$ is the capital share. A firm that exports solves

$$\begin{aligned}
d_{X,t}(\varphi) = & \max_{p_t(\varphi), y_t(\varphi), l_t(\varphi), k_t(\varphi)} \epsilon_t \frac{p_t^*(\varphi)}{P_t^*} y_t^{AE,*}(\varphi) \tau_t^{-1} - w_t l_t(\varphi) - r_t^K k_t(\varphi) - \frac{w_t / N_{D,t}}{Z_t^{1-\alpha}} f_{X,t} \\
& \text{s.t.} \\
& y_t^{AE,*}(\varphi) = \varphi (Z_t l_t)^{1-\alpha} k_t^{\alpha} \\
& y_t^{AE,*}(\varphi) = \left(\frac{p_t^*(\varphi)}{P_t^*} \right)^{-\theta} Y_t^*
\end{aligned}$$

where $y_t^{AE,*}(\varphi)$ is the residual demand faced by a home firm producing variety φ in the foreign market and ϵ_t is the nominal exchange rate defined as units of home currency per unit of foreign. The dependence of the period by period exporting cost on stationary units of labor $\frac{w_t/N_{D,t}}{Z_t^{1-\alpha}} f_{X,t}$, as it will become clear later, ensures that the exporting productivity cut-off is itself stationary, which simplifies exposition without affecting the interpretation of the model[58].

The solution to this problem yields

$$\rho_{D,t}(\varphi) \equiv \frac{p_{D,t}(\varphi)}{P_t} = \frac{\theta}{\theta-1} MC_t(\varphi) \tag{86}$$

$$\rho_{X,t}(\varphi) \equiv \frac{p_{X,t}(\varphi)}{P_t^*} = Q_t^{-1} \tau_t \rho_{D,t}(\varphi) \tag{87}$$

where $Q_t \equiv \frac{\epsilon_t P_t^*}{P_t}$ is the consumption-based real exchange rate and $MC_t(\varphi) = \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \left(\frac{r_t^K}{\alpha}\right)^{\alpha} \left(Z_t^{1-\alpha}\varphi\right)^{-1}$ is the (real) marginal cost faced by the firm. The resulting real profits are

$$d_{D,t}(\varphi) = \frac{1}{\theta} \rho_{D,t}(\varphi)^{1-\theta} Y_t$$

$$d_{X,t}(\varphi) = \begin{cases} \frac{Q_t}{\theta} \rho_{X,t}(\varphi)^{1-\theta} Y_t^* - \frac{w_t/N_{D,t} f_{X,t}}{Z_t^{1-\alpha}} & \text{if firm } \varphi \text{ exports,} \\ 0 & \text{otherwise.} \end{cases}$$

More productive firms (higher φ) charge lower prices and earn higher profits. Since exporting is costly and deterred by a fixed cost, only firms with productivity φ above $\varphi_{X,t} = \inf \{\varphi : d_{X,t}(\varphi) > 0\}$ will decide to export. As in Ghironi and Melitz (2005), assuming a lower bound productivity $\varphi_{min} = \varphi_{min}^*$ enough below $\varphi_{X,t}$ and $\varphi_{X,t}^*$ ensures existence of an endogenously determined non-traded sector that fluctuates over time.

2.3 Aggregation

I follow Melitz (2003) and Ghironi and Melitz (2005) and define productivity averages to deal with firm heterogeneity. In particular, in AEs there is a mass $N_{D,t}$ of firms producing every period and serving the home market. A fraction of them, $N_{X,t} = [1 - G(\varphi_{X,t})] N_{D,t}$ serves also the foreign market. Moreover, $N_t = N_{D,t} + N_{X,t}$ represents the total mass of varieties available to consumers in any country. Let me denote with $\tilde{\varphi}_D$, $\tilde{\varphi}_{X,t}$ the average productivity levels of, respectively, all firms and exporting only firms and with $\tilde{\varphi}_t$ the weighted productivity average that reflects the combined market share of all firms and the

[58] For the EMEs country, the real wage is made stationary by the AEs endogenous trending variable, $N_{D,t}$. In other words, period by period exporting costs on stationary units of labour in EMEs are $\frac{w_t^*/N_{D,t}}{(Z_t^*)^{1-\alpha}} f_{X,t}$.

output shrinkage linked to exporting:

$$\tilde{\varphi}_D \equiv \left(\int_{\varphi_{min}}^{\infty} \varphi^{\theta-1} g(\varphi)\, d\varphi \right)^{\frac{1}{\theta-1}}$$

$$\tilde{\varphi}_{X,t} \equiv \left(\frac{1}{1-G(\varphi_{X,t})} \int_{\varphi_{X,t}}^{\infty} \varphi^{\theta-1} g(\varphi)\, d\varphi \right)^{\frac{1}{\theta-1}}$$

$$\tilde{\varphi}_t \equiv \left\{ \frac{1}{N_t} \left[N_{D,t} \tilde{\varphi}_D^{\theta-1} + N_{X,t} \left(\tau_t^{-1} \tilde{\varphi}_{X,t} \right)^{\theta-1} \right] \right\}^{\frac{1}{\theta-1}}$$

and, as in Melitz (2003), the latter productivity average $\tilde{\varphi}_t$ plays an important role because it completely summarizes the effects of the distribution of productivity levels $g(\varphi)$ on the aggregate outcome. In other words, the aggregate equilibrium in any country is identical to one with N_t representative firms that all share the same productivity level $\tilde{\varphi}_t$. Or, as it is privileged in Ghironi and Melitz (2005), the economy can be seen as $N_{D,t}$ firms with productivity level $\tilde{\varphi}_D$ serving the home country only and $N_{X,t}$ firms with productivity $\tilde{\varphi}_{X,t}$ exporting.

Thus, the aggregate price index becomes

$$P_t = \left(\int_{\omega \in \Omega_t} p_t(\omega)^{1-\theta} d\omega \right)^{\frac{1}{1-\theta}} = \left(\int_{\varphi_{min}}^{\infty} p_t(\varphi)^{1-\theta} N_t g(\varphi)\, d\varphi \right)^{\frac{1}{1-\varphi}}$$

i.e. $P_t = N_t^{\frac{1}{1-\theta}} p_t(\tilde{\varphi}_t)$ and using $p_t(\tilde{\varphi}_t) = \frac{\theta}{\theta-1} MC_t(\tilde{\varphi}_t) P_t$ one gets $\tilde{MC}_t = N_t^{\frac{1}{\theta-1}} \frac{\theta-1}{\theta}$ and average total profits $\tilde{d}_t = \frac{Y_t}{N_t}\frac{1}{\theta}$, where variables with a *tilde* are computed at $\tilde{\varphi}_t$, and represent, thus, average values. I will use this notation henceforth.

2.4 Parameterization of productivity draws

As in Ghironi and Melitz (2005), I assume that productivity φ is distributed Pareto with lower bound φ_{min} and shape parameter $\kappa > \theta - 1 : G(\varphi) = 1 - \left(\frac{\varphi_{min}}{\varphi}\right)^{\kappa}$. Thus, $\tilde{\varphi} = \nu \varphi_{min}$ and $\tilde{\varphi}_{X,t} = \nu \varphi_{X,t}$, where $\nu = \left(\frac{\kappa}{\kappa-(\theta-1)}\right)^{\frac{1}{\theta-1}}$ and the share of exporting firms in AEs is $\frac{N_{X,t}}{N_{D,t}} = 1 - G(\varphi_{X,t}) = \left(\frac{\nu\varphi_{min}}{\tilde{\varphi}_{X,t}}\right)^{\kappa}$. The threshold export productivity must satisfy $d_t(\varphi_{X,t}) = 0$ which allows to compute the average export profit as $\tilde{d}_{X,t} \equiv d_t(\tilde{\varphi}_{X,t}) = d_t(\nu\varphi_{X,t}) = (\theta-1)\left(\frac{\nu^{\theta-1}}{\kappa}\right)\frac{w_t/N_{D,t}}{Z_t^{1-\alpha}} f_{X,t}$.

2.5 Innovation in AEs

Each period, there is a single entrepreneur that conducts R&D using the final good S_t in order to produce a new variety. The productivity of the innovation sector is ϑ_t and is taken as exogenous by the innovator. Following Kung and Schmid (2015) and Comin and Gertler (2006), I assume that R&D technology is of the type

$$\vartheta_t = \frac{\chi N_{D,t}}{S_t^{1-\eta} N_{D,t}^{\eta}}$$

where $\chi > 0$ is a scale parameter and $\eta \in [0,1]$ is the elasticity of new varieties with respect to R&D. This technology features positive spillovers from the aggregate stock of varieties (innovations) $\partial\vartheta/\partial N_D > 0$, as in Romer (1990b), and a congestion externality $\partial\vartheta/\partial S > 0$ that raises the cost of developing new varieties as the aggregate level of R&D raises. To ensure that the growth rate of new varieties is stationary, the congestion effect depends positively on $N_{D,t}$, meaning that the marginal gain from R&D declines the more varieties are produced.

An innovator cannot direct her research effort toward a specific variety $\omega \in \Omega_t$. Instead, the R&D effort is undirected and the innovator cares about the expected discounted firm value $\tilde{V}_t$ generated by the new variety, which is the present discounted value of average profits. Therefore, the innovator solves

$$\begin{aligned}
&\max_{S_t} \vartheta_t S_t \left(\mathbb{E}_t M_{t+1} \tilde{V}_{t+1}\right) - S_t \\
&\text{s.t.} \\
&\tilde{V}_t = \tilde{d}_t + (1-\phi)\, \mathbb{E}_t M_{t+1} \tilde{V}_{t+1}
\end{aligned}$$

where, I recall, $\phi \in (0,1)$ is the probability of variety obsolescence and $\vartheta_t S_t$ represent newly created varieties. The optimality condition reads

$$\frac{1}{\vartheta_t} = \mathbb{E}_t \left[M_{t+1} \tilde{V}_{t+1}\right] \tag{88}$$

which imposes that, at the optimum, the marginal cost of conducting research (the LHS) equals its marginal benefit (the RHS)[59].

[59]One could also think of this set up as one in which there is an unbounded mass of perfectly competitive innovators. Then the solution to the problem would still be given by equation (88) which would in that case be interpreted as a free-entry condition, with the same economic meaning in the main text.

The aggregate stock of varieties evolves according to

$$N_{D,t+1} = \vartheta_t S_t + (1 - \phi) N_{D,t}. \tag{89}$$

Notice that, exactly because the innovator is not able to direct her research effort toward a specific variety, the aggregate stock that she directly affects is $N_{D,t+1}$, the total mass of firms producing in home, and not N_t which is the total mass of varieties available to consumers in any country[60]. Thus she affects N_t only indirectly.

Using the law of motion of varieties (89), the innovator optimality condition can be rewritten as

$$S_t = \mathbb{E}_t \left[M_{t+1} \tilde{V}_{t+1} \right] (N_{D,t+1} - (1 - \phi) N_{D,t}) \tag{90}$$

where M_{t+1} is the stochastic discount factor and will be defined in Section 2.8. Equation (90) pins down the equilibrium amount of R&D investment, S_t.

The role of the R&D sector is key to the goal of the paper: it introduces non-stationarity in the model and allows shocks to have long-run consequences on trade. The trend of the mass of firms producing every period $N_{D,t}$ will transfer to the trend of the total mass of varieties available to consumers in any country N_t thus inducing long-run growth in the economy.

2.6 Innovation in EMEs

The R&D sector in EMEs is symmetric to that in AEs with the only difference that they experience R&D spillovers from the stock of knowledge of AEs. Thus, their R&D technology is of the type

$$\vartheta_t^* = \frac{\chi^* \left(N_{D,t}^* \right)^{1-\psi} (N_{D,t})^{\psi}}{(S_t^*)^{1-\eta} \left[\left(N_{D,t}^* \right)^{1-\psi} (N_{D,t})^{\psi} \right]^{\eta}} \tag{91}$$

where $\psi \in (0, 1)$ captures the extent to which EMEs benefit from the stock of knowledge of AEs. This set up is consistent with Barro and Sala-i Martin (1997) and empirical evidence on R&D spillovers in Coe and Helpman (1995) and in Coe et al. (2009). Consistently with evidence of EMEs being technological laggards over the period 1997-2008 (Figure 29, left), I focus on the case $N_{D,0}^* < N_{D,0}$. As it is shown in the context of a technological diffusion

[60]Because the innovator is not guaranteed to create a new variety with productivity φ above the export cutoff level $\varphi_{X,t}$, which indeed happens with probability $1 - G(\varphi_{X,t})$.

model in Barro and Sala-i Martin (1997), the assumption of R&D spillovers (from the leader to the follower) and of EMEs being technologically inferior at the beginning of time, is enough to allow the existence of parameters calibration such that, on a BGP, the ratio of the endogenous trending variables, $n_t \equiv N^*_{D,t}/N_{D,t}$ lies between zero and one, and all variables (domestic and foreign) grow at the same rate of the technological leader, $g_t \equiv N_{D,t+1}/N_{D,t}$. This is crucial in avoiding explosive behaviours in an international set-up and makes possible to apply conventional perturbation techniques to solve the model once it has been detrended.

2.7 Aggregate production

In order to derive an expression for aggregate production, let me follow the same steps for computing the aggregate price index:

$$Y_t = \left(\int_{\omega \in \Omega} y_t\left(\omega\right)^{\frac{\theta-1}{\theta}} d\omega \right)^{\frac{\theta}{\theta-1}} = \left(\int_{\varphi_{min}}^{\infty} y_t\left(\varphi\right)^{\frac{\theta-1}{\theta}} N_t g\left(\varphi\right) d\varphi \right)^{\frac{\theta}{\theta-1}} = N_t^{\frac{\theta}{\theta-1}} \tilde{y}_t$$

where $\tilde{y}_t \equiv y_t\left(\tilde{\varphi}_t\right) = \tilde{\varphi}_t \left(Z_t l_t\left(\tilde{\varphi}_t\right)\right)^{1-\alpha} k_t\left(\tilde{\varphi}_t\right)^{\alpha}$. It is then possible to write the aggregate production function as

$$Y_t = N_t^{\frac{\theta}{\theta-1}} \tilde{\varphi}_t \left(Z_t l_t\left(\tilde{\varphi}_t\right)\right)^{1-\alpha} k_t\left(\tilde{\varphi}_t\right)^{\alpha} \tag{92}$$

Using the input market clearing conditions (in the production sector)

$$K_t \equiv \int_{\omega \in \Omega_t} k_t\left(\omega\right) d\omega = N_t \tilde{k}_t$$

$$L_t \equiv \int_{\omega \in \Omega_t} l_t\left(\omega\right) d\omega = N_t \tilde{l}_t$$

in equation (92), the production function becomes

$$Y_t = \tilde{\varphi}_t N_t^{\frac{1}{\theta-1}} K_t^{\alpha} L_t^{1-\alpha}.$$

In order to obtain balanced growth, it is necessary that the aggregate production function is homogeneous of degree one in the accumulating factors K_t and N_t which amounts at requiring $\frac{1}{\theta-1} = 1-\alpha$[61]. This implies that there exists a balanced growth path where aggregate output is proportional to N_t and average profits $\tilde{d}_t$ are stationary[62]. With this restriction it is possible to rewrite the aggregate production function in a more familiar way as

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{93}$$

where $A_t \equiv (Z_t N_t)^{1-\alpha} \tilde{\varphi}_t$ is the Solow residual, or labour productivity, which has an endogenous component N_t driven by optimal R&D effort chosen by the innovator and an exogenous one, Z_t, driven by i.i.d. shocks[63].

2.8 Household budget constraint and intertemporal choices

Households consume the aggregate consumption good C_t, invest in the stock market x_t and in physical capital I_t. Moreover, the household can also issue bonds in domestic currency, B_{t+1} or invest in foreign bonds (denominated in the foreign currency) $Q_t B_{*,t+1}$. I follow Ghironi and Melitz (2005) and, to induce stationarity and determinacy, assume households have to pay fees to financial intermediaries to adjust their bonds position[64]. The fees are then rebated to domestic households with transfers.

Therefore, in each period t, the household receives interest income on bond holdings, dividend income and potential capital gain on stock holdings, physical capital income and labour income. As a result, the period budget constraint in units of the consumption good is

$$\begin{aligned} C_t + B_{t+1} + Q_t B_{*,t+1} + \frac{\mu}{2} B_{t+1}^2 + \frac{\mu}{2} Q_t B_{*,t+1}^2 + \xi_t x_{t+1} + I_t = w_t L + (1 + r_{t-1}) B_t \\ + Q_t \left(1 + r_{t-1}^*\right) B_{*,t} + (\xi_t + D_t) x_t + r_t^K K_t + T_t \end{aligned} \tag{94}$$

where r_{t-1} and r_{t-1}^* are, respectively, the consumption based interest rate on home and foreign bond holdings, $\mu > 0$ is a scale parameter, L are the units of labour available to the household, ξ_t is the stock price and D_t is the aggregate dividend (or aggregate firms' profits net of the R&D cost) defined as $D_t \equiv \int_{\omega\in\Omega_t} d_t(\omega)\, d\omega - S_t = N_t \tilde{d}_t - S_t$. The transfers from financial intermediaries are denoted with T_t and in equilibrium amount to $\frac{\mu}{2} B_{t+1}^2 +$

[61]Notice that, for a steady state to be feasible, permanent technical change must be expressible in a labor augmenting form, as stated in Swan (1964) and Phelps (1967) and stressed in King et al. (1988).

[62]This restriction is imposed also in Kung and Schmid (2015) and Queralto (2020).

[63]Now it is also possible to notice that, denoting with g_x the growth rate of a generic variable x, $g_Y = (1-\alpha) g_{N_D} + \alpha g_K$ and since $g_K = g_{N_D}$ then $g_Y = g_{N_D}$.

[64]Fees are a convex function of households bonds holdings

$\frac{\mu}{2}Q_tB^2_{*,t+1}$. Capital evolves according to the standard law of motion $K_{t+1} + \Upsilon(K_{t+1}, K_t) = I_t + (1-\delta)K_t$, where $\Upsilon()$ is a convex adjustment cost function as in Kollmann (1998)[65]. Households maximize their lifetime utility subject to the budget constraint (94) yielding standard first order conditions:

$$C_t^{-\gamma}(1+\mu B_{t+1}) = \beta(1+r_t)\mathbb{E}_t C_{t+1}^{-\gamma} \tag{95}$$

$$C_t^{-\gamma}(1+\mu B_{*,t+1}) = \beta(1+r_t^*)\mathbb{E}_t \frac{Q_{t+1}}{Q_t} C_{t+1}^{-\gamma} \tag{96}$$

$$\xi_t = \mathbb{E}_t\left[M_{t+1}(\xi_t + D_{t+1})\right] \tag{97}$$

$$\left[1+\Upsilon_{K_{t+1}}(K_{t+1},K_t)\right] = \mathbb{E}_t\left[M_{t+1}\left(r^K_{t+1} + (1-\delta) - \Upsilon_{K_{t+1}}(K_{t+2},K_{t+1})\right)\right] \tag{98}$$

where (95) and (96) are, respectively, the Euler equation for home and foreign bonds, (97) is the one for stocks and (98) the one for capital and $M_{t+1} \equiv \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}$ is the stochastic discount factor.

2.9 Equilibrium

In equilibrium $x_t = x_{t+1} = 1$, the labour market clears $L_t = L$, and bonds market clearing requires

$$B_{t+1} + B^*_{t+1} = 0$$

$$B_{*,t+1} + B^*_{*,t+1} = 0$$

where B^*_{t+1} denotes EMEs holdings of the AEs bond and $B^*_{*,t+1}$ EMEs holdings of their bonds. Using the fact that in equilibrium transfers are equal to the fees, $T_t = \frac{\mu}{2}B^2_{t+1} + \frac{\mu}{2}Q_tB^2_{*,t+1}$, in the budget constraint (94) yields aggregate accounting

$$C_t + I_t + S_t + NX_t = r_{t-1}B_t + Q_t r^*_{t-1}B_{*,t} + w_tL + N_t\tilde{d}_t + r^K_t K_t$$

which tells that consumption C_t plus investment in physical capital I_t and in the creation of new varieties S_t plus net exports NX_t, is equal to labour income w_tL and investment income (from stocks, physical capital and bonds) $N_t\tilde{d}_t + r^K_tK_t + r_{t-1}B_t + Q_tr^*_{t-1}B_{*,t}$[66]. Net

[65] $\Upsilon(K_{t+1}, K_t) = \frac{\Phi}{2}\frac{(K_{t+1}-\varsigma K_t)^2}{K_t}$, with $\Phi, \varsigma > 0$.

[66] Notice that for a household working in the R&D sector the labour income she receives is then detracted from firms' profits as it also represent the cost for creating new varieties. Therefore the household, *de facto*, works in the production sector to earn the labour income and in the R&D sector to create new varieties and

exports are defined as

$$NX_t = Q_t N_{X,t} \left(\tilde{\rho}_{X,t}\right)^{1-\theta} C_t^* - N_{X,t}^* \left(\tilde{\rho}_{X,t}^*\right)^{1-\theta} C_t$$

where $\tilde{\rho}_{X,t} \equiv \tilde{\rho}_t \left(\tilde{\varphi}_{X,t}\right)$ and $\tilde{\rho}_{X,t}^* \equiv \tilde{\rho}_t^* \left(\tilde{\varphi}_{X,t}^*\right)$ are average export prices of AEs and EMEs firms, respectively. Net exports are equal to the current account, $CA_t \equiv B_{t+1} + Q_t B_{*,t+1} - B_t - Q_t B_{*,t}$.

The complete set of non-linear equations describing the non-stationary system as well as the definition of equilibrium is left in the Appendix I.4. In order to solve the model with perturbation methods, all the AEs and EMEs trending variables are stationarized dividing them by $N_{D,t}$. The resulting system of equations is stationary and can be linearly approximated about the deterministic balanced growth path. I do so using Dynare (Adjemian et al., 2011).

2.10 Balanced growth path

The balanced growth path of this world economy is simple. In the very long-run, all trending variables grow at the same rate of growth of the stock of domestic varieties in AEs, both in AEs and in EMEs[67]. This implies that the trade share does not grow on a BGP. This is genuine, as it cannot reasonably grow for ever (it cannot exceed 1). Nonetheless, its behaviour in the short/medium-run is very interesting and closely tied to innovation activities in both countries groups. Defining the proximity to the technological frontier as $n_t \equiv N_{D,t}^*/N_{D,t}$ and detrended variables as $\underline{x} \equiv x/N_D$ for a generic variable x and x^*, the trade share can be rewritten as

$$TS_t = \frac{Q_t^\theta \left(1 - G\left(\varphi_{X,t}\right)\right) \left[\tau_t \frac{\theta}{\theta-1} \tilde{MC}_{X,t}\right]^{1-\theta} \underline{C_t^*} + \left(1 - G\left(\varphi_{X,t}^*\right)\right) n_t \left[Q_t \tau_t \frac{\theta}{\theta-1} \tilde{MC}_{X,t}^*\right]^{1-\theta} \underline{C_t}}{\underline{Y_t} + Q_t \underline{Y_t^*}} \tag{99}$$

which conveys very neat intuition: keeping world output constant, the trade share increases when (i) trade costs τ_t and/or average export marginal costs decrease $\left(\tilde{MC}_{X,t}, \tilde{MC}_{X,t}^*\right)$, (ii) when demand increases (C_t^*, C_t) and (iii) in case of a decline in the export cut-off productivity $\left(\varphi_{X,t}, \varphi_{X,t}^*\right)$ as all these changes (*cæteris paribus*) makes prices smaller and thus

enjoy a greater dividend income.

[67] In particular, $g_\tau = g_Q = g_{\varphi_X} = g_{\varphi_X^*} 0$, $g_Y = g_{Y^*} = g_C = g_{C^*} = g_{N_D^*} = g_{N_D}$ and $g_{\tilde{MC}_X} = g_{\tilde{MC}_X^*} = \frac{1}{\theta-1} g_{N_D}$, where the fact that $g_Q = 0$ comes from the fact that $g_{N_D^*} = g_{N_D}$ while $g_\tau = 0$ is by assumption. See Appendix I.3 for details.

exports more appealing; most importantly, (iv) the trade share increases when the proximity of EMEs to the technological frontier, n_t, increases. The intuition is that, a closer proximity to the frontier is associated with an increase in both the mass of domestic varieties in EMEs and in a decline in the export cut-off productivity in both countries groups[68]; therefore, ex-post there is a greater mass of varieties and a larger fraction of them is exported, increasing the trade share. It is true that, in general equilibrium, the increase in n_t affects all other variables in equation (99) but these effects are of second order while the one on the export volume is of first order and hence is dominant. As it will become clearer looking at impulse responses, the proximity to the frontier n_t plays a crucial role in inducing persistence in the effects of innovation on trade.

2.11 Calibration and forcing processes

I follow the international real business cycle literature (Backus et al., 1992) and start estimating a bivariate process for the cyclical component of TFP of the form $\ln(\boldsymbol{Z}_{t+1}) = \boldsymbol{A}\ln(\boldsymbol{Z}_t) + \boldsymbol{\epsilon}^z_{t+1}$ where $\boldsymbol{\epsilon}^z$ is the vector of zero-mean i.i.d. normal over time shocks with variance-covariance matrix $\boldsymbol{V}^z$ and $\boldsymbol{Z}_t = \begin{bmatrix} Z_t & Z_t^* \end{bmatrix}'$ is the vector of exogenous TFP components. The matrices $\boldsymbol{A}$ and $\boldsymbol{V}^Z$ are estimated on the TFP series used in Figure 29 (left, blue line) detrended following Hamilton (2018)[69]. This yields

$$\boldsymbol{A} = \begin{bmatrix} .62 & .851 \\ -.012 & .389 \end{bmatrix}$$

where the spillover coefficients (the off-diagonal elements of $\boldsymbol{A}$) are not statistically significant[70]. Hence I estimate the same specification without spillovers and obtain

$$\boldsymbol{A} = \begin{bmatrix} .580 & 0 \\ 0 & .633 \end{bmatrix}$$

and the standard deviation for AEs and EMEs shocks is .043 and .097 respectively, with correlation .414. Differently from the approach of Backus et al. (1992), I do not symmetrize

[68] As the increase in n_t is accompanied by a rise in innovation activities in both countries that then boost overall demand and thus export profits.

[69] I set the lags to 1 $(p = 1)$ and the forecast is for 2-steps ahead $(h = 2)$, the yearly equivalents of Hamilton (2018)'s recommendation for quarterly data. This isolates cyclical movements of 2 years.

[70] .851 is significant at the 8% level while -.012 at the 90% one. Spillover coefficients are not significant neither detrending the original series linearly nor with the Hodrick and Prescott (1997) filter.

the matrix $\boldsymbol{A}$ because of the deep structural differences between AEs and EMEs.

Turning to the parameter values, I calibrate the model so that, on the BGP, it is consistent with 1997-2008 first moments for AEs and EMEs (on annual basis). I set the subjective discount factors $\beta = \beta^*$ to a standard value of 0.99^4. The inverse of the elasticity of the intertemporal substitution γ is set to a standard value of 2. Regarding R&D, the productivity parameter (which is a pure scale parameter) χ is set to 6.6% for AEs so that the BGP growth rate of new varieties is 0.99% which is close to the true average (0.90%) one over the period 1997-2008. In EMEs instead the same parameter χ^* is set to 97% of the AEs one, *i.e.* to 6.52% so that the BGP distance from the frontier n is 0.85 which is close to the average value (0.89) over the period 1997-2008 according to PWT data. The elasticity of new varieties with respect to R&D η is set to 0.3, in line with empirical findings of Hall et al. (1986). The obsolescence rate ϕ is set to 10% in line with Ghironi and Melitz (2005) whom set it to match the US empirical level of 10% job destruction per year. I follow Ghironi and Melitz (2005) and set $\tau = 1.44$ in line with Obstfeld and Rogoff (2000) and calibrate the fixed export cost f_X to roughly match the number of US exporting plants (21%, Bernard et al., 2003) and set it to 0.095 which yields 20% of exporting plants in AEs and 16% in EMEs. This ensures a trade share on the BGP of 22%, in line with the average for the period 1997-2008. Following Ghironi and Melitz (2005) and Bernard et al. (2003) I set κ equal to 1.56 ensuring that the standard deviation of log US plant sales is 1.67*4 on annual basis[71]. The capital share α is set to a standard value of 30% while capital depreciates at a 4.3% annual rate, the mean between the GDP-weighted average capital share for AEs and EMEs over the period 1997-2019 using PWT labour share data. As in Ghironi and Melitz (2005) I set $\varphi_{min} = 1$ while the R&D spillover parameter, ψ, is set to 0.5 to get close to the conditional correlation between trade and proximity (0.95) observed in the data. Regarding the parameters governing capital adjustment costs, I set ς to 1 so that in the steady state adjustment costs are zero, and Φ to 8 to match the investment to output volatility ratio over the period 1997-2019. Parameter values are reported in Table 10.

3 Results

The clearest way to see the model's implications regarding productivity and trade is looking at impulse response functions. Given the asymmetry in the shock process, IRFs are different for the two countries, depending on which shock one wishes to analyze.

[71] Thus $\kappa = \frac{1}{1.67} - 1 + \theta = 1.56$.

Table 10: Parameter values

Parameter	Description	Value	Source/Target
$\beta = \beta^*$	Subjective discount factor	0.99^4	standard
γ	Inverse of elasticity of int. substitution	2	standard
χ	R&D scale parameter	0.066	$g_{N_D} = 1.009$
χ^*	R&D scale parameter	0.0647	$n = 0.85$
η	Elasticity of new varieties to R&D	0.3	Hall et al. (1986)
ϕ	Obsolescence rate	1%	Ghironi and Melitz (2005)
ψ	R&D spillover	0.5	$\text{corr}\left(TS, n \mid \epsilon^*\right) = 0.95$
f_X	Fixed export cost	0.095	20% exporting plants, Bernard et al. (2003)
τ	Melting-iceberg trade cost	1.44	$\tilde{TS} = 21\%$,Obstfeld and Rogoff (2000)
κ	Pareto shape parameter	1.56	Bernard et al. (2003)
α	Capital share	0.3	standard
δ	Depreciation rate	4.3%	sample weighted average
φ_{min}	Lower bound of Pareto distribution	1	Ghironi and Melitz (2005)
ς	Capital adj. cost	1	zero adj. cost in BGP
Φ	Capital adj. cost	8	I/Y volatility
a_{11}	Autocorrelation of exogenous AEs TFP	.580	estimate
a_{22}	Autocorrelation of exogenous EMEs TFP	.633	estimate
σ_ϵ	Volatility of exogenous AEs TFP	4.3%	estimate
σ_{ϵ^*}	Volatility of exogenous EMEs TFP	9.7%	estimate
$\rho_{\epsilon,\epsilon^*}$	Correlation of TFP innovations	.414	estimate

Nonetheless, it seems more appropriate to focus on shocks to the exogenous component of EMEs TFP, ϵ_t^*, as the standard deviation of the latter is twice as large of that in AEs.

Figure 31 reports IRFs to a one standard deviation increase in the exogenous component of EMEs TFP. Following a one standard deviation shock in EMEs, on impact, the threshold productivity for becoming an exporter $\varphi_{X,t}^*$ decreases, leading to an increase in firms' average profits, $\tilde{d}_t^*$, and thus in their average value, $\tilde{V}_t^*$. This stimulates investment in R&D, S_t^*, which then translates in an increase in the total number of varieties in EMEs, N_t^*, and in the proximity to the technological frontier, n_t. The decrease in the cut-off productivity $\varphi_{X,t}^*$ expands the mass of exporting firms. Hence, there are more firms (due to the increase in R&D spending) and a larger fraction of them is an exporter (due to the decrease in the cut-off productivity) . This increases export volume in EMEs by more than the increase in world GDP. The final effect is a very persistent increase in the trade share[72].

Turning to AEs, since there are no spill-overs in the exogenous component of the TFP process, a ϵ_t^* shock does not rise AEs GDP, Y_t, on impact. Indeed, larger demand from EMEs lowers the cut-off productivity $\varphi_{X,t}$, increasing firms' average value $\tilde{V}_t$ and thus R&D spending S_t. This has positive effects on the AEs growth rate $g_{N_{D,t}}$. AEs' desire to increase R&D and investment in physical capital exceeds the increase in GDP and thus AEs run trade deficits in the short/medium run. But as the effect of increased R&D and physical investment translates into higher GDP, the need to borrow gradually fades away, and AEs start to run trade surpluses.

The increased innovation activity in AEs has beneficial effects in EMEs thanks to the R&D spillover assumption; it enforces the Romer (1990b)'s effect and initiates a virtuous technology-improvement cycle which keeps bringing EMEs more and more closer to the frontier, contributing to the permanent rise in trade. The persistence of the proximity (n_t) increase, and thus of trade, is controlled by the parameter ψ,which governs the extent to which EMEs benefit from AEs technological improvements (see equation 91)[73]. The higher the ψ, the less scope there is in the domestic varieties stock evolution in EMEs for a role of EMEs innovation. Everything is guided by AEs and EMEs act as pure followers. This makes less persistent the increase in the proximity and in trade.

The model is thus successful in replicating the positive co-movement between trade

[72]On impact this positive effect on the trade share is dominated by the increase in world GDP and as a result the trade share decreases below its BGP value. Already from the second year, the export-volume-effect starts to dominate and trade increases. After 5 years trade increases persistently above its BGP value.

[73]Notice that, using (91) in the (detrended) law of motion of $N_{D,t}^*$, one gets the law of motion of the proximity to the frontier: $n_{t+1} = \left[\chi^* n_t^{(1-\psi)(1-\eta)} (S_t^*)^\eta + (1-\phi) n_t\right] / g_{t+1}$and it is immediate to see that, as $\psi \to 1$, the autocorrelation of n decreases.

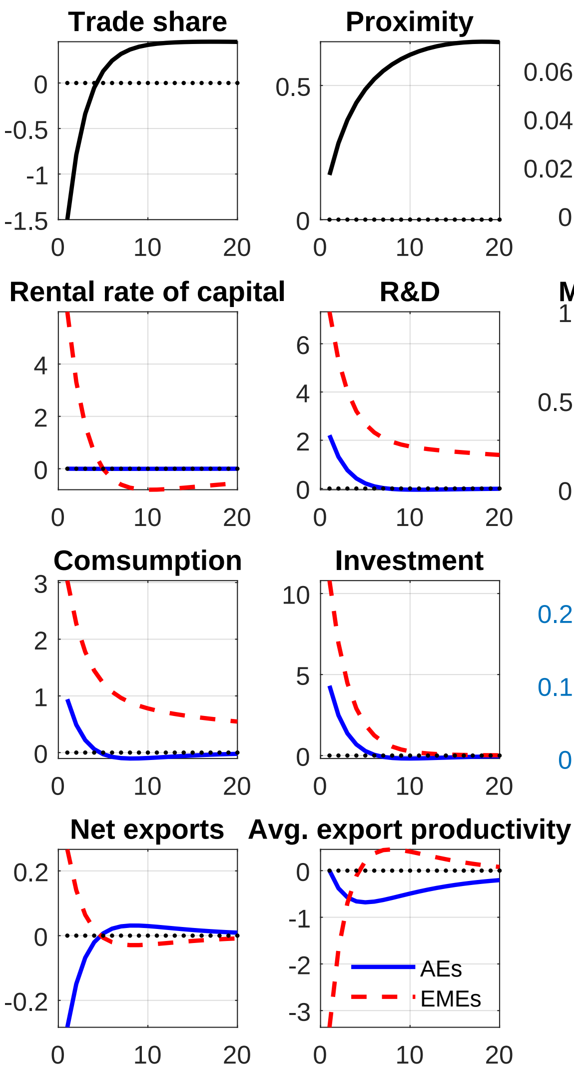

Figure 31: Impulse responses to a 1 standard deviation increase to EMEs exogenous TFP (ϵ_t^*). Time is in years. Percentage deviations from the balanced growth path. All variables share the same y-axis except for GDP, Firms value and wages where AEs are on the left and EMEs on the right.

and the proximity to the technological frontier (the conditional correlation is 0.96) and trade balance reversals observed in the data. Moreover, it also offers an explanation for the observed trade stagnation in the post-GFC period. According to the model, the latter is a consequence of shocks to the exogenous component of TFP becoming more symmetric, milder and negatively correlated. Indeed, after the GFC, the standard deviation of the shocks dropped by, respectively, 32% and 38% in AEs and EMEs, their sum of squared deviations dropped by 60% and their correlation went negative (from 0.51 to -0.08). The contribution of this paper is to link innovations to the exogenous component of TFP to the long-run behaviour of itself and trade through R&D dynamics. Such idiosyncratic, country-wide, shocks, when strong and asymmetric enough, shape countries desire to engage in innovation activities. During the 1997-2008 period, shocks in EMEs were relatively larger compared to that in AEs and shared a common sign. This boosted innovation activities both in the AEs and in EMEs thanks to the increase in firms' valuations and R&D spillovers. This virtuous cycle induced a long-lasting increase in trade as more goods were developed and a growing fraction of them was exported. This pattern came to an halt after the GFC because of milder, more similar and negatively correlated shocks. Innovation incentives were muted over that period (Benigno and Fornaro, 2018) and as a result trade tracked almost one to one GDP and trade balances were stable.

4 International business cycles

The study of links and international spillovers across countries through the lens of joint models dates as back as to Backus et al. (1992) and Backus et al. (1994) whom works gave rise to the international business cycle literature. As my paper is among the first to study advanced and emerging economies in a joint model, it seems appropriate and useful to analyze the business cycle properties of AEs and EMEs in the data and test the model's behaviour under this respect, in the spirit of Backus et al. (1992). This section digs into this exercise.

4.1 Data properties

Business cycle properties of the data are reported in Table 11. As one would have expected given the fact that emerging economies are still undergoing massive developments, the standard deviations are larger almost for each variable for EMEs when compared to AEs.

Table 11: Data Properties of AEs and EMEs over the period 1997-2019

	Standard deviation (percent)		Relative std.		Correlation with output		Cross-country relative Std	Cross-country correlation
	AEs	EMEs	AEs	EMEs	AEs	EMEs	AEs to EMEs	
y	.93	1.24	1	1	1	1	.75	.36
c	.75	1.25	.80	1.01	.97	.78	.59	.49
n	.68	.64	.73	.52	.76	.23	1.05	-.41
i	2.77	2.53	2.98	2.03	.82	.60	1.10	-.46
tb	.88	1.46	.94	1.18	-.13	.11	.60	-.68
s	2.58	3.86	2.77	3.10	.72	.36	.67	.20
tot	2.28	2.28	2.44	1.83	.14	.37	1	-1

Notes: Variables are: Real Output (y), Private final consumption (c), Gross fixed capital formation (i), Civilians employed (n), net exports over GDP (tb), Terms of trade (tot) and business enterprise expenditure on R&D (s). The terms of trade for EMEs is simplified to be the inverse of that of AEs. Data are annual and from OECD and BLS. Details are in the Appendix I.1. All entries refer to variables in natural logarithms (except tb) and Hodrick and Prescott (1997) filtered with a smoothing parameter of 6.25, as suggested in Ravn and Uhlig (2002). Sample period is 1997-2019 for all variables but p, for which it is 1998-2019, due to data availability.

The only exceptions are employment and investment, which are more volatile in AEs[74].

Turning to contemporaneous correlations with output, these are in line with the classic statistics for the US and other advanced economies for the AEs group; for EMEs instead, almost any variable is much less procyclical than in AEs. It is interesting to notice that employment is not much procyclical in EMEs (0.23) and that the trade balance comoves positively with output, differently from what happens in AEs. That is very peculiar as in two-country models such as Backus et al. (1992) and Backus et al. (1994) effort was made to explain the countercyclicality of the trade balance. Moreover, such an asymmetry will prove difficult to be generated by a standard symmetric model.

Looking at cross country correlations, outputs have a correlation of .36 which is close to the correlation the US have with Australia, Austria, Germany, Italy, Japan, Switzerland and UK[75], in terms of output. Regarding the cross country correlation of consumption, generally for advanced economies it is less stronger than that of output; looking instead at AEs and EMEs the picture turns upside down. The cross country correlation for consumption is .49, higher than that of output and close to that the US have with the UK (.43) and Europe (.46). Investment in R&D is also positively correlated across countries

[74]This may be due to the GFC which affected more AEs than EMEs.

[75]Table 2 in Backus et al. (1992).

groups (.21) while employment and investment are negatively correlated.

Regarding within country correlations, output and the terms of trade comove positively while the trade balance and the terms of trade comove negatively, as it is the case for the median advanced country[76].

To summarize, the statistics for the AEs group are broadly in line with the classic statistics regarding advanced economies. The EMEs macroeconomic variables instead are more volatile, except for employment and investment, and less procyclical. The trade balance for EMEs displays a peculiar positive comovement with output and cross country correlations are high, if compared to cross country correlations of the US with advanced economies. This evidence points toward the importance to start studying and digging into the link between advanced and emerging economies as their business cycles are closely tied.

4.2 Model's properties

Model's business cycle statistics are reported in Table 12. As this is the first attempt to study AEs and EMEs in a joint model, it seems more appropriate to focus on cross country relative standard deviations instead of looking at point estimates of the standard deviation for each variable. The objective is thus to check whether the model is able to capture the main cross country differences between AEs and EMEs rather than being fully consistent with within countries groups statistics[77]. In that respect the model does a good job in getting close to relative standard deviations for output, consumption, the trade balance and R&D investment while it is unable to generate more volatile investment in AEs relatively to EMEs. The model is also good in generating the right amount of prociclicality in AEs but it fails to do so for EMEs as the implied contemporaneous correlation of consumption, investment and R&D with output are too high. Moreover the model is not able to capture the asymmetric behaviour of trade balances in AEs and EMEs as it predicts a positive but too high correlation for the EMEs one (.87 in the model vs .11 in the data) and a counterfactual positive one for AEs (.05 in the model vs -.13 in the data). The model predicts a counterfactual positive comovement between the trade balance and the terms of trade for both countries groups while it predicts a positive correlation of output with the terms of trade for the EMEs (.86 in the model vs .37 in the data) and a negative

[76] Table 1 of Backus et al. (1994).

[77] Of course, as a second step it would be very interesting to dig into also the within countries groups statistics. This is left to future research.

Table 12: Model's theoretical moments

	Standard deviation (percent)		Relative std.		Correlation with output		Cross-country relative Std	Cross-country correlation
	AEs	EMEs	AEs	EMEs	AEs	EMEs	AEs to EMEs	
y	1.98	4.31	1	1	1	1	.46	.42
c	1.38	1.95	.70	.45	.85	.97	.71	.91
i	6.03	8.99	3.05	2.09	.85	.98	.67	.91
tb	1.12	1.21	.57	.28	.05	.87	.93	-1
s	3.21	5.35	1.62	1.24	.86	.99	.60	.87
tot	2.14	2.14	1.08	.39	-.08	.86	1	-1

Notes: Variables are deflated by the average price index as in Ghironi and Melitz (2005), *i.e.*, for any variable X, its real counterpart is $x \equiv {}^{PX}/_{\tilde{P}}$. The terms of trade is defined as $tot \equiv {}^{\tilde{MC}_X}/_{Q\tilde{MC}^*_X}$ and the trade balance is $tb \equiv {}^{nx}/_{y}$. All entries are averages of 1000 simulations of 23 periods long samples (as in the real data) and Hodrick and Prescott (1997) filtered with a smoothing parameter of 6.25, as suggested in Ravn and Uhlig (2002).

counterfactual one for AEs (-.08 in the model vs .14 in the data).

Standard deviations of investment relative to that of output are consistent with the data for both countries groups while that of consumption is consistent for AEs but for EMEs is half of the observed one. Relative standard deviations for the terms of trade are of the correct ordering (higher in AEs than in EMEs) but too small. For the trade balance instead the ordering is incorrect and numbers are far from the data counterpart.

Cross country correlations are of the correct sign for all variables but for investment. The model predicts a cross country correlation for the output of .42 while in the data it is .36 and that for consumption is greater than that for output consistently with the data even if too high (.91 in the model vs .49 in the data). The artificial economy predicts a correlation between R&D investments of .87, too high if compared to the data, .20.

Overall, the model gets the big picture of international business cycles between advanced and emerging economies but it faces apparent limitations. Since this is the first paper to analyze AEs and EMEs in a joint model, in order to shed light on the model's performance regarding business cycle statistics matching, it is instructive doing the same exercise with a plain vanilla Backus et al. (1994) model. This provides a useful benchmark to which compare the baseline model to understand whether the model's limitations are a byproduct of the additional micro-foundations introduced relatively to trade and innovation or if they are embedded in the international joint structure of two-country models and/or missing mechanism peculiar to the AEs-EMEs tie.

Table 13: BKK model's theoretical moments

	Standard deviation (percent)		Relative std.		Correlation with output		Cross-country relative Std	Cross-country correlation
	AEs	EMEs	AEs	EMEs	AEs	EMEs	AEs to EMEs	
y	3.27	7.62	1	1	1	1	0.43	.34
c	2.98	5.20	.91	.68	.88	.99	.57	.80
n	.48	1.10	.15	.14	.86	.99	.43	-.24
i	8.87	17.12	2.72	2.25	.89	.98	.52	.78
tb	1.58	1.58	.48	.21	.09	.89	1	-1
tot	6.37	6.37	1.95	.84	.09	.88	1	-1

Notes: Variables are described in Appendix I.5 where tb corresponds to nx in the model while tot to p and i to x. All entries are averages of 1000 simulations of 23 periods long samples (as in the real data) and Hodrick and Prescott (1997) filtered with a smoothing parameter of 6.25, as suggested in Ravn and Uhlig (2002).

Details regarding the model and its calibration are left in the Appendix 14 while its statistics are reported in Table 13. Results are pretty similar to those of the baseline model. The BKK model suffers almost the same quantitative gaps of the baseline model: it is not able to generate more volatile investment in AEs relatively to EMEs, predicts a counterfactual positive comovement between the trade balance and the terms of trade and a positive cross country correlation of investment while in the data is positive. Moreover, it overpredicts procyclicality in EMEs. The BKK model is instead successful in generating a positive comovement between output and the terms of trade (.09 in the model vs .14 in the data). Hence, even though the standard Backus et al. (1994) was a good starting point for matching basic business cycle statistics for advanced economies, and thus a good benchmark to which compare my baseline model, it does as well as the latter.

These results point toward the need of a more suitable structure to study the very asymmetric but highly intertwined business cycles between AEs and EMEs. The baseline model I am proposing captures business cycles stylized facts as well as a standard BKK model with the advantage of being able to explain the observed patterns over the period 1997-2019 and having a micro-founded structure for trade and innovation. I thus believe it is a good starting point for the analysis of AEs and EMEs links.

5 Conclusion

This paper presented interesting developments regarding advanced and emerging economies over the last three/four decades. I built a two-country model with firms heterogeneous in productivities à la Melitz (2003), growth occurring endogenously via expanding varieties (Romer, 1990b) and international R&D spillovers (Barro and Sala-i Martin, 1997), able to explain these developments. Namely, following a shock, firms' profits increase, decreasing the export productivity cut-off. Forward looking entrepreneurs engage in innovation activities, with the aim of acquiring the increased profits through the development of new varieties. Since historical shocks had been larger in EMEs than in AEs, the larger demand coming from EMEs triggers investment in R&D also in AEs and via international R&D spillovers this reinforces the innovation activity in EMEs. The stock of varieties grows faster in EMEs than in AEs, due to the larger shock size and EMEs get closer to the technological frontier. As there is a larger mass of varieties and a greater fraction of them is exported, trade increases faster than GDP, closely tracking the dynamic behaviour of the EMEs proximity to the technological frontier. The whole dynamics is accompanied by an initial trade balance deterioration in AEs, as borrowing finances the increased R&D spending which, on impact, exceeds the increased output. As time goes by, trade balances dynamics reverse as the initial desire to invest in R&D in AEs becomes sustained by the increased output; the opposite occurs in EMEs, where the shock propagates more persistently, and their international position deteriorates, consistently with the observed pattern.

The model has also been tested on a quantitative ground, checking its ability to match basic business cycles facts regarding AEs and EMEs. The model is roughly consistent with basic facts even though the asymmetric nature of those statistics make it difficult for a joint model with few heterogeneity across countries to account for them. The paper thus also calls for more research on the joint relationship between advanced and emerging economies, a task made urgent by the close tie their business cycles show. The model I presented in this work should serve as a good starting point for this task, being it able to explain recent developments in the international links between AEs and EMEs, together with business cycles facts, while preserving tractability.

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I Appendix

I.1 Data description

I.1.1 Dataset for business cycles statistics

In this section I describe the dataset built to produce Figures 29 and 30 and Table 11.

Data regarding Emerging Economies are not present at an exhaustive length at quarterly frequency for very important EMEs representatives such as China, India and Indonesia. I thus opted for annual data.

In the main text I sometimes refer to IMF and/or OECD classification of countries. The IMF classification embodies:

- AEs: Australia, Canada, Czech Republic, Denmark, Iceland, Israel, Japan, Korea, New Zealand, Norway, Sweden, Switzerland, United Kingdom, United States, and Euro area (19 countries).
- EMEs: Chile, Colombia, Costa Rica, Hungary, Mexico, Poland, Turkey, Bulgaria, China (People's Republic of), Croatia, India, Indonesia, Romania, Russia and South Africa.

while the OECD one:

- AEs: Australia, Austria, Belgium, Canada, Chile, Colombia, Costa Rica, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, Korea, Lithuania, Latvia, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Slovak Republic, Slovenia, Spain, Sweden, Switzerland, Turkey, United Kingdom and United States.
- EMEs: Bulgaria, China (People's Republic of), Croatia, India, Indonesia, Romania, Russia and South Africa.

The following time series are from the OECD. GDP, consumption (households and non-profit institutions serving households), gross fixed capital formation (investment in physical capital), exports and imports of goods and services and business enterprise expenditure on R&D (BERD are in current USD, current PPPs and deflated by Consumer Price Index for All Urban Consumers, All Items in U.S. City Average, Percent Change from Year Ago, Annual, Seasonally Adjusted, taken from the FRED database. Employment is also taken from the OECD and is number of persons employed, measured in thousands.

The AEs and EMEs aggregates are built taking a weighted average of each country time series (according to IMF or OECD classification), weighted with time varying real GDP weights.

Regarding the terms of trade, computations are a little bit more convoluted. I compute it as the ratio between the import price index and the export price index between these two aggregates[78]. As a proxy for the Advanced Countries aggregate I chose the USA due to data availability. The Bureau of Labor Statistics provides data on import price index by locality of origin. I therefore picked Mexico and China as locality of origin as representatives of the Emerging Countries aggregate because are the second and third, respectively, trade partners of USA, with a share in total USA imports of 15.94% and 7.21% in 2018 (Source: World Integrated Trade Solution)[79]. This data are available from 2004. Thus, from 1998Q1 to 2003Q4 I used as a proxy for Emerging Countries origin of USA imports the aggregate *Latin America* which is composed by Mexico, Central America, South America and Caribbean. From 2004Q1 until 2019Q4 I instead used a weighted average of the Mexico and China import price index with weights given by import shares from World Integrated Trade Solution.

With respect to the export price index instead, data by locality of destination in the BLS are available from 2017M12. An ideal substitute for it could be an average of the import price indexes of EMEs that trade mostly with the USA (*i.e.* that import mostly from the USA and the USA export mostly to them). Top USA EMEs export destinations are Mexico (15.94%), China (7.21%), Korea (3.39%), Brazil (2.38%) and India (2.01%). China, India and Brazil are of no help as for the first two there are no date while for the latter only starting from 2006Q1. Hence, I used as a synthetic export price index for the USA toward Emerging Countries a weighted average of Mexico and Korea import price indexes, retrieved from IMF International Financial Statistics database. The Mexican index is available from 1998Q1 to 2015Q2 while the Korean one 1998Q1-2019Q4. Mexico is and ideal proxy for the USA to EMEs export price index because 76.49% of its exports are toward the USA. Korea's exports toward the USA instead are only 12.08% of the total (USA are the second Korean export partner. The first is China with 26.81% and the third is Vietnam with 8.04%). The

[78] I am following Backus et al. (1994) in defining the terms of trade as the inverse of trade theorists definition but in so doing it corresponds to the real exchange rate convention applied in international macroeconomics.

[79] The other localities of origin present in the BLS database are Canada, EU, Germany, Latin America (Mexico, Central America, South America, and the Caribbean), Pacific Rim (China, Japan, Australia, Brunei, Indonesia, Macao, Malaysia, New Zealand, Papua New Guinea, Philippines, and the Asian Newly Industrialized Countries) and Japan. Therefore there are no Emerging Countries available alone but Mexico, China and Latin America.

weights are given by USA export shares from World Integrated Trade Solution. Since these data are quarterly I then annualize them by simple averaging.

These series are then logged and filtered with Hodrick and Prescott (1997) using a smoothing parameter of 6.25 as suggested in Ravn and Uhlig (2002) and used to compute business cycles statistics of Table 11.

I.1.2 TFP estimation (Figure 29, left)

Regarding the TFP series labelled "BKK estimation" in the main text, they are obtained estimating the TFP for each country as in Backus et al. (1992), namely $\ln \lambda = \ln y - (1-\theta) \ln n$, where λ is TFP, y is real output, n is total employment and θ the capital share. The series for the labour share $(1-\theta)$ are from the Penn World Tables (Feenstra et al., 2015). The aggregate TFP measure is then obtained as a weighted average (according to IMF or OECD classification), with time varying real GDP weights.

The TFP series labelled "PWT" in the main text is the TFP at constant national prices (2017=1) from PWT (Feenstra et al., 2015). The aggregate TFP measure is then obtained as a weighted average (according to IMF or OECD classification), with time varying real GDP weights (which this time were computed on real GDP at current USD, constant PPPs, from the OECD, deflated by Consumer Price Index for All Urban Consumers, All Items in U.S. City Average, Percent Change from Year Ago, Annual, Seasonally Adjusted, taken from the FRED database).

I.2 Other figures

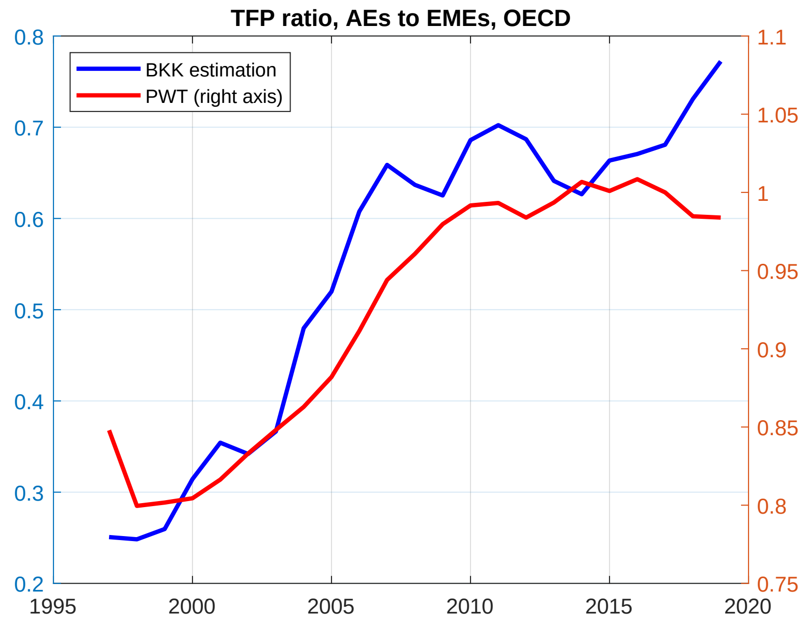

Figure 32: EMEs TFP relative to that of AEs, 1997-2019. Countries are classified according to OECD. See Appendix I.1.

I.3 Balanced growth path details

Let me define $\underline{x} \equiv \frac{x}{N_D}$ and $\underline{x}^* \equiv \frac{x^*}{N_D}$ as, respectively, de-trended home and foreign variables. Let me start from the non-stochastic balanced growth path (NS-BGP) growth rate of the real exchange rate Q_t. Notice that the cost of adjusting bond holdings implies that in equilibrium bonds holdings are zero. As a consequence, along the BGP there is balanced trade which implies that home and foreign exports must be the same, *i.e.*,

$$Q_t N_{X,t} \left(\tilde{\rho}_{X,t}\right)^{1-\theta} C_t^* = N_{X,t}^* \left(\tilde{\rho}_{X,t}^*\right)^{1-\theta} C_t \tag{100}$$

where $N_{X,t} = \left(1 - G\left(\varphi_{X,t}\right)\right) N_{D,t}$ and $N_{X,t}^* = \left(1 - G\left(\varphi_{X,t}^*\right)\right) N_{D,t}^*$ so equation (100) can be rewritten as

$$Q_t \left(1 - G\left(\varphi_{X,t}\right)\right) \left(\tilde{\rho}_{X,t}\right)^{1-\theta} \underline{C}_t^* = \left(1 - G\left(\varphi_{X,t}^*\right)\right) \left(\tilde{\rho}_{X,t}^*\right)^{1-\theta} \underline{C}_t. \tag{101}$$

Notice that

$$\left(\tilde{\rho}_{X,t}\right)^{1-\theta} = \left[Q_t^{-1}\tau_t\rho_{D,t}\left(\tilde{\varphi}_{X,t}\right)\right]^{1-\theta} = Q_t^{\theta-1}\left[\tau_t\frac{\theta}{\theta-1}\tilde{MC}_{X,t}\right]^{1-\theta} \tag{102}$$

$$\left(\tilde{\rho}^*_{X,t}\right)^{1-\theta} = Q_t^{1-\theta}\left[\tau_t\frac{\theta}{\theta-1}\tilde{MC}^*_{X,t}\right]^{1-\theta} \tag{103}$$

and that the average marginal cost for exporting firms

$$\tilde{MC}_{X,t} = \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{r_t^K}{\alpha}\right)^{\alpha}\left(Z_t^{1-\alpha}\tilde{\varphi}_{X,t}\right)^{-1}\left(\frac{N_{D,t}}{N_{D,t}}\right)^{1-\alpha} = \left(\frac{w_t/N_{D,t}}{1-\alpha}\right)^{1-\alpha}\left(\frac{r_t^K}{\alpha}\right)^{\alpha}\left(Z_t^{1-\alpha}\tilde{\varphi}_{X,t}\right)^{-1}N_{D,t}^{1-\alpha}$$

grows at the rate $g_{\tilde{MC}_X} = (1-\alpha)\, g_{N_D}$ and since I restricted $\frac{1}{\theta-1} = 1-\alpha$ then $g_{\tilde{MC}_X} = \left(\frac{1}{\theta-1}\right) g_{N_D}$ which implies $g_{\tilde{MC}_X^{1-\theta}} = -g_{N_D}$[80]. The same reasoning applies to $\tilde{MC}^*_{X,t}$ which grows at the rate $g_{\tilde{MC}^*_X} = \left(\frac{1}{\theta-1}\right) g_{N_D^*}$. Going back to (101), substituting (102) and (103) gives

$$Q_t^{2\theta-1}\left(1-G\left(\varphi_{X,t}\right)\right)\left[\tau_t\frac{\theta}{\theta-1}\right]\underline{\tilde{MC}}_{X,t}^{1-\theta}N_{D,t}^{-1}\underline{C}_t^* = \left(1-G\left(\varphi^*_{X,t}\right)\right)\left[\tau_t\frac{\theta}{\theta-1}\right]\left(\underline{\tilde{MC}}^*_{X,t}\right)^{1-\theta}N_{D,t}^{*-1}\underline{C}_t$$

which gives the equilibrium value of the real exchange rate on a BGP

$$Q_t = \left[\frac{\left(1-G\left(\varphi^*_{X,t}\right)\right)}{\left(1-G\left(\varphi_{X,t}\right)\right)}\left(\frac{\underline{\tilde{MC}}^*_{X,t}}{\underline{\tilde{MC}}_{X,t}}\right)^{1-\theta}\frac{N_{D,t}}{N^*_{D,t}}\right]^{\frac{1}{2\theta-1}}$$

and since $G\left(\varphi^*_{X,t}\right)$ and $G\left(\varphi_{X,t}\right)$ are stationary[81], the exchange rate grows at

$$g_Q = \frac{1}{2\theta-1}\left(g_{N_D} - g_{N_D^*}\right)$$

but since on a BGP it must be that $g_{N_D} = g_{N_D^*}$ then $g_Q = 0$.

Going back to the definition of trade share, exploiting balanced trade, it can be rewritten

[80] The de-trended real wage $w_t/N_{D,t}$ is stationary.

[81] Recall that profits $\tilde{d}_t = \tilde{d}_{D,t} + \left(1-G\left(\varphi_{X,t}\right)\right)\tilde{d}_{X,t} = \frac{1}{\theta}\frac{Y_t}{N_t}$ and $\tilde{d}^*_t = \tilde{d}^*_{D,t} + \left(1-G\left(\varphi^*_{X,t}\right)\right)\tilde{d}^*_{X,t} = \frac{1}{\theta}\frac{Y^*_t}{N^*_t}$ are stationary which implies that their components are also stationary. This means that the export thresholds $\varphi_{X,t}$ and $\varphi^*_{X,t}$ are stationary.

as

$$TS = \frac{Q^{\theta}\left(1-G\left(\varphi_X\right)\right)N_D\left[\tau\frac{\theta}{\theta-1}\tilde{MC}_X\right]^{1-\theta}C^*}{Y+QY^*}$$

and it is now clear that, since the growth rate of foreign demand is simply

$$g_{C^*} = g_{N_D^*} = g_{N_D} \tag{104}$$

the growth rate of TS on a NS-BGP is

$$g_{TS} = g_Q + g_{(1-G(\varphi_X))} + g_{N_D} + (1-\theta)\left(g_\tau + g_{\tilde{MC}_X}\right) + g_{C^*} - \zeta g_Y - (1-\zeta)\left(g_Q + g_{Y^*}\right)$$

where ζ is the share of AEs GDP in world GDP (which is constant on the BGP). Using (104) and $g_Q = 0$ together with $g_{N_D} = g_{N_D^*}$ one gets $g_{TS} = 0$[82].

I.4 Non-stationary equilibrium system of equations

Laws of motion for the mass of firms serving the domestic market

$$N_{D,t+1} = \vartheta_t S_t + (1-\phi)N_{D,t} \tag{105}$$

$$N^*_{D,t+1} = \vartheta^*_t S^*_t + (1-\phi)N^*_{D,t} \tag{106}$$

Innovation FOCs

$$\frac{1}{\vartheta_t} = \mathbb{E}_t\left[M_{t+1}\tilde{V}_{t+1}\right] \tag{107}$$

$$\frac{1}{\vartheta^*_t} = \mathbb{E}_t\left[M^*_{t+1}\tilde{V}^*_{t+1}\right] \tag{108}$$

R&D productivity

$$\vartheta_t = \frac{\chi N_{D,t}}{S_t^{1-\eta}N_{D,t}^{\eta}} \tag{109}$$

$$\vartheta^*_t = \frac{\chi^*\left(N^*_{D,t}\right)^{1-\psi}\left(N_{D,t}\right)^{\psi}}{S_t^{1-\eta}\left[\left(N^*_{D,t}\right)^{1-\psi}\left(N_{D,t}\right)^{\psi}\right]^{\eta}} \tag{110}$$

Firms values

$$\tilde{V}_t = \tilde{d}_t + (1-\phi)\,\mathbb{E}_t M_{t+1}\tilde{V}_{t+1} \tag{111}$$

[82] I am assuming $g_\tau = 0$ on a BGP.

$$\tilde{V}_t^* = \tilde{d}_t^* + (1-\phi)\,\mathbb{E}_t M_{t+1}^* \tilde{V}_{t+1}^* \tag{112}$$

Average total profits

$$\tilde{d}_t = \frac{Y_t}{N_t}\frac{1}{\theta} \tag{113}$$

$$\tilde{d}_t^* = \frac{Y_t^*}{N_t^*}\frac{1}{\theta} \tag{114}$$

Discount factors

$$M_{t+1} = \mathbb{E}_t \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \tag{115}$$

$$M_{t+1}^* = \mathbb{E}_t \beta \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\gamma} \tag{116}$$

Average profits for exporting firms

$$\tilde{d}_{X,t} = (\theta-1)\left(\frac{\nu^{\theta-1}}{\kappa}\right)\frac{w_t/N_{D,t}}{Z_t^{1-\alpha}} f_{X,t} \tag{117}$$

$$\tilde{d}_{X,t}^* = (\theta-1)\left(\frac{\nu^{\theta-1}}{\kappa}\right)\frac{w_t^*/N_{D,t}^*}{\left(Z_t^*\right)^{1-\alpha}} f_{X,t} \tag{118}$$

Average profits for non-exporting firms

$$\tilde{d}_{D,t} = \frac{1}{\theta}\left(\frac{\theta}{\theta-1}\right)^{1-\theta} \tilde{MC}_{D,t}^{1-\theta} Y_t \tag{119}$$

$$\tilde{d}_{D,t}^* = \frac{1}{\theta}\left(\frac{\theta}{\theta-1}\right)^{1-\theta} \left(\tilde{MC}_{D,t}^*\right)^{1-\theta} Y_t^* \tag{120}$$

Total profits accounting

$$\tilde{d}_t = N_{D,t}\tilde{d}_{D,t} + N_{X,t}\tilde{d}_{X,t} \tag{121}$$

$$\tilde{d}_t^* = N_{D,t}^*\tilde{d}_{D,t}^* + N_{X,t}^*\tilde{d}_{X,t}^* \tag{122}$$

Average prices

$$\tilde{\rho}_t = \frac{\theta}{\theta-1}\tilde{MC}_t \tag{123}$$

$$\tilde{\rho}_t^* = \frac{\theta}{\theta-1}\tilde{MC}_t^* \tag{124}$$

Average marginal costs

$$\tilde{MC}_t = \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{r_t^K}{\alpha}\right)^{\alpha}\left(Z_t^{1-\alpha}\tilde{\varphi}_t\right)^{-1} \tag{125}$$

$$\tilde{MC^*}_t = \left(\frac{w_t^*}{1-\alpha}\right)^{1-\alpha}\left(\frac{r_t^{K*}}{\alpha}\right)^{\alpha}\left(\left(Z_t^*\right)^{1-\alpha}\tilde{\varphi}_t^*\right)^{-1} \tag{126}$$

Average marginal cost for non-exporting firms

$$\tilde{MC}_{D,t} = \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{r_t^K}{\alpha}\right)^{\alpha}\left(Z_t^{1-\alpha}\tilde{\varphi}_D\right)^{-1} \tag{127}$$

$$\tilde{MC^*}_{D,t} = \left(\frac{w_t^*}{1-\alpha}\right)^{1-\alpha}\left(\frac{r_t^{K*}}{\alpha}\right)^{\alpha}\left(\left(Z_t^*\right)^{1-\alpha}\tilde{\varphi}_D^*\right)^{-1} \tag{128}$$

Average marginal cost for exporting firms

$$\tilde{MC}_{X,t} = \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{r_t^K}{\alpha}\right)^{\alpha}\left(Z_t^{1-\alpha}\tilde{\varphi}_{X,t}\right)^{-1} \tag{129}$$

$$\tilde{MC^*}_{X,t} = \left(\frac{w_t^*}{1-\alpha}\right)^{1-\alpha}\left(\frac{r_t^{K*}}{\alpha}\right)^{\alpha}\left(\left(Z_t^*\right)^{1-\alpha}\tilde{\varphi}_{X,t}^*\right)^{-1}$$

Aggregate firms FOC

$$\frac{w_t}{r_t^K} = \frac{1-\alpha}{\alpha}\frac{K_t}{L} \tag{130}$$

$$\frac{w_t^*}{r_t^{K*}} = \frac{1-\alpha}{\alpha}\frac{{K_t}^*}{L^*} \tag{131}$$

Total mass of available varieties

$$N_t = N_{D,t} + N_{X,t} \tag{132}$$

$$N_t^* = N_{D,t}^* + N_{X,t}^* \tag{133}$$

Fraction of exporting firms

$$\frac{N_{X,t}}{N_{D,t}} = 1 - G\left(\varphi_{X,t}^*\right) = \left(\frac{\nu\varphi_{min}}{\tilde{\varphi}_{X,t}^*}\right)^{\kappa} \tag{134}$$

$$\frac{N^*_{X,t}}{N^*_{D,t}} = 1 - G\left(\varphi^*_{X,t}\right) = \left(\frac{\nu\varphi_{min}}{\tilde{\varphi}^*_{X,t}}\right)^{\kappa} \tag{135}$$

Euler equation for bonds

$$C_t^{-\gamma}\left(1 + \mu B_{t+1}\right) = \beta\left(1 + r_t\right)\mathbb{E}_t C_{t+1}^{-\gamma} \tag{136}$$

$$C_t^{-\gamma}\left(1 + \mu B_{*,t+1}\right) = \beta\left(1 + r_t^*\right)\mathbb{E}_t \frac{Q_{t+1}}{Q_t} C_{t+1}^{-\gamma} \tag{137}$$

$$\left(C_t^*\right)^{-\gamma}\left(1 + \mu B^*_{t+1}\right) = \beta\left(1 + r_t\right)\mathbb{E}_t \frac{Q_t}{Q_{t+1}}\left(C^*_{t+1}\right)^{-\gamma} \tag{138}$$

$$\left(C_t^*\right)^{-\gamma}\left(1 + \mu B^*_{*,t+1}\right) = \beta\left(1 + r_t^*\right)\mathbb{E}_t\left(C^*_{t+1}\right)^{-\gamma} \tag{139}$$

Euler equation for stocks

$$\xi_t = \mathbb{E}_t\left[M_{t+1}\left(\xi_t + D_{t+1}\right)\right] \tag{140}$$

$$\xi_t^* = \mathbb{E}_t\left[M^*_{t+1}\left(\xi_t^* + D^*_{t+1}\right)\right] \tag{141}$$

Euler equation for capital

$$\left[1 + \Upsilon_{K_{t+1}}\left(K_{t+1}, K_t\right)\right] = \mathbb{E}_t\left[M_{t+1}\left(r^K_{t+1} + (1-\delta) - \Upsilon_{K_{t+1}}\left(K_{t+2}, K_{t+1}\right)\right)\right] \tag{142}$$

$$\left[1 + \Upsilon_{K_{t+1}}\left(K^*_{t+1}, K^*_t\right)\right] = \mathbb{E}_t\left[M^*_{t+1}\left(r^{K*}_{t+1} + (1-\delta) - \Upsilon_{K_{t+1}}\left(K^*_{t+2}, K^*_{t+1}\right)\right)\right] \tag{143}$$

Budget constraint

$$C_t + B_{t+1} + Q_t B_{*,t+1} + I_t + S_t = w_t L + N_t\tilde{d}_t + r_t^K K_t + \left(1 + r_{t-1}\right)B_t + Q_t\left(1 + r^*_{t-1}\right)B_{*,t} \tag{144}$$

$$C_t^* + B^*_{*,t+1} + \frac{1}{Q_t}B^*_{t+1} + I_t^* + S_t^* = w_t^* L^* + N_t^*\tilde{d}_t^* + r_t^{K*}K_t^* + \left(1 + r_{t-1}\right)\frac{1}{Q_t}B_t^* + \left(1 + r^*_{t-1}\right)B^*_{*,t} \tag{145}$$

Law of motion for capital

$$K_{t+1} + \Upsilon\left(K_{t+1}, K_t\right) = I_t + (1-\delta)K_t \tag{146}$$

$$K^*_{t+1} + \Upsilon\left(K^*_{t+1}, K^*_t\right) = I^*_t + (1-\delta)K^*_t \tag{147}$$

Current account - net exports identity

$$B_{t+1} + Q_t B_{*,t+1} - B_t - Q_t B_{*,t} = NX_t \tag{148}$$

Net exports

$$NX_t = Q_t N_{X,t} \left(\tilde{\rho}_{X,t}\right)^{1-\theta} C_t^* - N_{X,t}^* \left(\tilde{\rho}_{X,t}^*\right)^{1-\theta} C_t \tag{149}$$

$$NX_t^* = \frac{1}{Q_t} N_{X,t}^* \left(\tilde{\rho}_{X,t}^*\right)^{1-\theta} C_t - N_{X,t} \left(\tilde{\rho}_{X,t}\right)^{1-\theta} C_t^* \tag{150}$$

Bonds market clearing

$$B_{t+1} + B_{t+1}^* = 0 \tag{151}$$

$$B_{*,t+1} + B_{*,t+1}^* = 0 \tag{152}$$

Average domestic firms productivity

$$\tilde{\varphi}_D = \nu \varphi_{min} \tag{153}$$

$$\tilde{\varphi}_D^* = \nu \varphi_{min} \tag{154}$$

Weighted average productivity

$$\tilde{\varphi}_t \equiv \left\{ \frac{1}{N_t} \left[N_{D,t} \tilde{\varphi}_D^{\theta-1} + N_{X,t} \left(\tau_t^{-1} \tilde{\varphi}_{X,t} \right)^{\theta-1} \right] \right\}^{\frac{1}{\theta-1}} \tag{155}$$

Resource constraint

$$Y_t = C_t + I_t + S_t + NX_t \tag{156}$$

$$Y_t^* = C_t^* + I_t^* + S_t^* + NX_t^* \tag{157}$$

Production function

$$Y_t = \tilde{\varphi}_t N_t^{\frac{1}{\theta-1}} K_t^{\alpha} L_t^{1-\alpha} \tag{158}$$

$$Y_t^* = \tilde{\varphi}_t^* N_t^{*\frac{1}{\theta-1}} K_t^{*\alpha} L_t^{*1-\alpha}. \tag{159}$$

A non-stationary equilibrium in this economy is defined as a sequence of prices, quantities and of exogenous processes such that the system of equations (105)-(159) is satisfied.

I.5 BKK model used for Table 13 in Section 4.2

The model used to perform the comparison exercise in Section 4.2 is from Backus et al. (1994) with the addition of capital adjustment costs as in Kollmann (1998). I report its structure here for convenience.

There are two countries, each inhabited by a representative household. Each country is specialised in the production of a single good, using domestic labor (that is immobile across countries) and its own technology. Absorption, *i.e.* consumption and investment, is a mixture of domestic and foreign goods aggregated through an Armington Aggregator (Armington, 1969) into a non-tradable good.

I.5.1 Technology

Each country is fully specialised in the production of a single good, labeled a and b for country 1 and 2, respectively. Production occurs in a Cobb-Douglas fashion, implying the following resource constraints

$$z_{1t}k_{1t}^{\theta}n_{1t}^{1-\theta} = y_{1t} = a_{1t} + a_{2t}$$

$$z_{2t}k_{2t}^{\theta}n_{2t}^{1-\theta} = y_{2t} = b_{1t} + b_{2t}$$

in countries 1 and 2, respectively, where θ is the capital share parameter, τ_t the symmetric transport cost and y_{it} the GDP in country i. For country 1, a_{1t} is domestic consumption of domestically produced good a while a_{2t} are exports from country 1 to country 2 of the domestically produced good. *Mutatis mutandis* definitions of b_{it} are derived for each i. The vector $\boldsymbol{z}_t = [z_{1t}, z_{2t}]$ describes TFP and follows the bi-variate autoregression $\ln(\boldsymbol{z}_{t+1}) = \boldsymbol{A}\ln(\boldsymbol{z}_t) + \boldsymbol{\epsilon}_{t+1}^{z}$ where $\boldsymbol{\epsilon}^{z}$ is zero-mean i.i.d. normal over time with variance $\boldsymbol{V}^{z}$ that will be described shortly.

I.5.2 Households

Preferences are described, for each country $i = 1, 2$ by

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[c_{it}^{\mu}(1-n_{it})^{1-\mu}\right]^{\gamma}}{\gamma}$$

where c_{it} and n_{it} are consumption and hours worked while $\mu \in (0,1)$ is the consumption share parameter and $\gamma > 1$ measures household's relative risk aversion and intertemporal elasticity of substitution. Consumption c_{it} and investment in capital x_{it} are a composite of domestic and foreign goods aggregated through an Armington aggregator (Armington, 1969) homogeneous of degree one:

$$c_{1t} + x_{1t} = G_1(a_{1t}, b_{1t}) = \left[\omega_1 a_{1t}^{-\rho} + \omega_2 b_{1t}^{-\rho}\right]^{-1/\rho}$$

$$c_{2t} + x_{2t} = G_2\left(b_{2t}, a_{2t}\right) = \left[\omega_1 b_{2t}^{-\rho} + \omega_2 a_{2t}^{-\rho}\right]^{-1/\rho}$$

and $\sigma \equiv 1/(1+\rho)$, with $\rho \geq -1$, is the elasticity of substitution between foreign and domestic goods. Notice that both countries absorb the same proportion of domestic and foreign goods.

Following Kollmann (1998), changing the level of capital from one period to another is subject to convex adjustment costs and its law of motion is thus

$$k_{it+1} + \frac{\Phi}{2}\frac{\left(K_{it+1} - \vartheta K_{it}\right)^2}{K_{it}} = (1-\delta)\, k_{it} + x_{it}$$

for each country i, where $\delta \in [0,1]$ is the depreciation rate and $\Phi > 0, \vartheta > 0$.

By homogeneity of degree one of the Armington aggregator, first order conditions with respect to Armington aggregator's arguments a_{1t} and b_{1t} imply that the (symmetric) prices of the two goods for country 1 in period t in units of the composite good are

$$q_{2t} \equiv G^1_{b_{1t}}$$

$$q_{1t} \equiv G^1_{a_{1t}}$$

and the terms of trade for country 1 can be defined as

$$p_t \equiv \frac{q_{2t}}{q_{1t}}.$$

Hence, in equilibrium absorption in country 1 is $c_{1t} + x_{1t} = q_{1t}a_{1t} + q_{2t}b_{1t}$ and, using the resource constraint, output is $y_{1t} = \frac{(c_{1t}+x_{1t})}{q_{1t}} + (a_{2t} - pb_{1t})$, *i.e.* the sum of absorption and net exports[83]. The trade balance is measured as the ratio of net exports and output, and in country 1 reads

$$nx_{1t} = \frac{a_{2t} - pb_{1t}}{y_{1t}}.$$

I.5.3 Parameter values, forcing processes and solution

The forcing process used is the same of the main model[84].

[83] Since the Armington aggregator is defined with symmetric weights, it can be shown that, for country 2, $q^2_{2t} \equiv G^2_{a_{2t}} = q_{2t}$ and $q^2_{1t} \equiv G^2_{b_{2t}} = q_{1t}$ and therefore the terms of trade of country 2 is simply the inverse of the terms of trade of country 1: $p^2_t = (p_t)^{-1}$.

[84] See section 2.11 for details.

Table 14: Parameter values for the BKK model

Parameter	Description	Value	Source/Target
β	Discount factor	0.99^4	sample weighted average
μ	Consumption share	0.34	Backus et al. (1994)
γ	Elasticity of int. substitution	-1	Backus et al. (1994)
θ	Capital share	30%	standard
δ	Depreciation rate	4.3%	sample weighted average
ω_1	Armington (1969)'s weight	0.74	import share 0.17
ω_2	Armington (1969)'s weight	0.26	import share 0.17
Φ	Capital adj. cost	9.1	y_i/x_i volatility
ϑ	Capital adj. cost	1	zero SS adjustment costs

Regarding parameter values, I set the same value for the corresponding BKK parameter. I treat the real data and the model-generated data in the same way so I use an annual calibration. Parameter values are reported in Table 14. I set the discount factor β at 0.99^4 which is a conventional value in annual international macroeconomics and is close to the implied average discount factor (0.94) computed over GDP-weighted interest rates for AEs and EMEs over the period 1997-2019[85]. The consumption share parameter μ and household's IES γ are set as in Backus et al. (1994) respectively at 0.34 and -1. The capital share parameter θ is set to 30%, a standard value while the capital depreciation rate δ is set to 4.3%, the mean between the GDP-weighted average capital share for AEs and EMEs over the period 1997-2019 using PWT labour share data.. The elasticity of substitution between foreign and domestic goods, σ is set to 1.5, as in Backus et al. (1994) while the Argminton's weights to, respectively 0.74 ad 0.26 for $\omega_1$and ω_2 so that the import share of AEs is 0.17 in steady state (slightly below the true one in 1997, 0.18). The convex adjustment costs parameter ϑ is set to 1 so that in steady state no adjustment costs are paid while Φ to 9.1 to get close to the true standard deviations of investments to that of output for both AEs and EMEs, as in the baseline model.

Given parameter values, an equilibrium is computed solving numerically a linear approximation of the social planner's problem who weights equally household's utilities in the two countries.

[85]The time series I am using for this computation is the long-term interest rate from OECD. The sample is constituted by the same countries already listed according to the IMF classification, except that Bulgaria, Croatia and Romania are missing due to lack of data. GDP-weights are computed over different sub-samples because some EMEs present missing data-points.