Multi-Objective Optimisation of a constrained 2000 km Trajectory using Genetic Algorithms

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Abstract

This paper presents a study whereby a typical 2000 km trajectory flown by an aircraft of the Airbus A320 and Boeing 737 family, is imposed operational constraints such as altitude restrictions and operational speeds, and optimised for the reduction of fuel, nitrous oxides (NOx) and time.

The trajectory is divided into two phases, the first encompasses the initial climb following take-off up to 3,000ft, and it is optimised for NOx and fuel, while the second encompasses the en-route climb starting from 6,000ft, the cruise phase, and descent to 8,000ft and it is optimised for fuel and time. The resulting pareto frontiers and the corresponding extremal trajectories are analysed and discussed from an aircraft performance point of view, from which salient conclusions are drawn on the optimal trajectories generated.

Nomenclature

AGL = Above Ground Level
ATM = Air Traffic Management
CAS = Calibrated Airspeed
FL = Flight Level
NOx = oxides of nitrogen
TAS = True Air Speed

1 Introduction

Air transport currently depends entirely on fossil fuels and mainly on gas turbine technology for propulsive means. This implies that every flight contributes to the usage of a finite resource, to the emission of green-house gases and secondary emissions (mainly NOx) and also to noise pollution.

The study in this paper addresses the optimisation of a 2000 km trajectory bound by operational constraints in altitude and speed. Flights of this sector length are common for 150-180 seat category air transports such as the Boeing 737 and Airbus A320 family of aircraft and can be exemplified by routes such as Paris-Athens, Dubai-Islamabad and Los Angeles-Dallas. As part of the problem definition, the altitude throughout the trajectory was constrained to a maximum value of 34,000 feet, which may not be typical of real flights since aircraft are normally capable and allowed to reach altitudes of 36,000 feet and higher. The minimum cruise altitude was selected at 20,000ft to facilitate the optimisation set-up. The aircraft operational speed was constrained (limited) in terms of calibrated air speed below 20,000ft and Mach above. Constraining the cruise altitude to above 20,000ft facilitated the problem setup and avoided potential complications that could result from multiple transitions about this boundary. Consequently, this constraint effectively introduced pseudo-ATM and operational constraints to the problem. Although non-representative of real flights since ATM constraints are normally imposed on relatively short segments of the trajectory, it is, nonetheless, useful in this initial study on a generic trajectory.

A genetic algorithm was used to optimise the trajectory as this class of algorithms is not prone to converge on a local minimum. This is a significant capability in the application of trajectory optimisation with discrete trajectory constraints, where other types of optimisation techniques may be unable to perform the optimisation in a satisfactory or reliable manner. Genetic algorithms, however, require large computational resources, which normally translate to slower processing. This is particularly due to the fact that genetic algorithms are heuristic in nature, requiring the algorithm to cover a relatively large section of the problem space compared to classical techniques. However, genetic algorithms are less sensitive to the initial conditions, although they require careful setup in terms of, amongst others, problem space definition, population size and crossover techniques if they are to converge to a meaningful result.

2 The Problem Setup

The overall trajectory was divided into two separate stages to facilitate different multi-objective assessment. The first stage encompasses the initial climb following take-off, initiating from a screen height of 50 ft at the runway threshold up to 3,000 ft. This trajectory is optimised for NOx and fuel, and it extends only up to 3,000 ft because this is generally the altitude up to which NOx emissions are considered. Normally, this phase of flight involves several configuration changes, allowing the transition from the take-off to the clean configurations. Configuration changes were not incorporated in the assessment in order to limit the complexity of the problem setup and the optimisation time of the genetic algorithm. Whilst this did not afford the analysis of the impact the optimisation criteria have on strategies associated with configuration changes, it provided a preliminary understanding of the impact NOx considerations may be expected to have on the trajectory.

The second stage encompasses the en-route climb starting from 6,000ft, the cruise phase and descent to 8,000 ft. In this case, fuel-optimal trajectories are generated for an aircraft weighing 60 tonnes. The resulting trajectories are compared, analyzed and a number of conclusions are drawn.

The optimisations were performed using a multi-objective optimisation algorithm and a number of relevant models, which included an aircraft performance model, an engine model, and an emissions model. A standard atmosphere was used and still air assumed in all optimisations. The aircraft performance model is representative of a generic, single aisle, twin-engine jet airliner of the Airbus A320 / Boeing 737 category. The model determines the aerodynamic state of the aircraft based upon a reduced set of aerodynamic derivatives as well as certain geometric characteristics typical to the class of aircraft being replicated. The model outputs the thrust requirement and flight time for the aircraft when required to travel between initial and final pre-defined states, which are the inputs to the model. The initial and final states are defined in any of a number of ways and include parameters such as altitude, range and velocity vectors. Due to the complexity of the model, which also takes into account factors such as the effects of variations of flight path angle and atmospheric conditions between the initial and final states, the solution is obtained iteratively and is based upon an integration routine.

The engine model is representative of a two spool, high bypass, turbofan engine of the 12 tonne static thrust class such as the CFM56 series engine typically found on A320 and Boeing 737 family aircraft. This model computes the steady-state performance of the engine based upon design-point data, generic component characteristic maps and fundamental thermodynamic relationships. The thrust requirement as well as altitude and Mach number are provided as inputs to the model and the solution is obtained through a number of iterations. The suite of data output by the model includes specific fuel consumption, turbine entry temperature, air mass flow rate and fuel flow rate. The aircraft performance model interfaces with the engine model through the provision of thrust requirement data while fuel flow rate obtained from the engine model is used to update the aircraft weight.

The emissions model is used to determine the fuel burn-related emissions generated by the afore-mentioned engine type. The model is based upon empirical data, reactor models as well as widely reported chemical reaction mechanisms. The model determines the emission indices for Oxides of Nitrogen, Carbon Dioxide and water vapour. The latter two emission types are determined through basic chemical equilibrium while more complex rate chemistry is employed to model oxides of Nitrogen. In order to determine these indices, knowledge of atmospheric conditions and combustor fluid characteristics is required. The former is provided by the atmospheric model, whilst the latter is provided by the engine model.

The model setup, whilst not being numerically precise in terms of specific aircraft performance, afforded the observation of optimal flight trajectory strategies for aircraft typically of the size and configuration referred to in this paper.

2.1 The First Stage – Initial Climb

The initial climb has been defined by 3 segments as shown in Table 1, starting at 50ft up to an altitude of 3000ft. A constant configuration setting (Slats + Flap2) was assumed with gear up. Initial conditions set were:

- weight 60.5 Tonnes
- altitude 50ft above ground level (agl)
- speed 165kts CAS.

Segment	Final Velocity range (CAS)	Altitude range (ft)
1	165 kts	50 - 3000
2	165-250 kts	50 - 3000
3	165-250 kts	3000

Table 1: First stage segment constraints.

2.2 The Second Stage – Climb, Cruise and Descent

The climb, cruise and descent trajectory was defined by 10 segments as shown in Table 2. The initial conditions, not shown in the table, were:

- weight 60 Tonnes
- altitude FL60
- speed 250kts CAS.

Segment	Segment Length	Final Velocity range	Altitude range (FL)
1	63km	270-370kts CAS	100 - 200
2	254km	270-350kts CAS	190 - 290
3	327km	0.75 – 0.82 Mach	200 - 340
4	219km	0.75 – 0.82 Mach	200 - 340
5	123km	0.75 – 0.82 Mach	200 - 340
6	301km	0.75 – 0.82 Mach	200 - 340
7	260km	0.75 – 0.82 Mach	200 - 340
8	241km	0.75 – 0.82 Mach	200 - 290
9	132km	270-370kts CAS	100 - 200
10	14km	256kts CAS	80

Table 2: Second stage segment constraints.

The defined problems were designed such that an aircraft the size and performance of that represented in this paper will have the performance to fly within these boundaries.

3 Results

3.1 The First Stage – Initial Climb

The pareto frontier of the two-objective (fuel-NOx) optimisation is presented in Figure 1. The frontier exhibits moderate smoothness, indicating a good trend line of where the actual boundary can be expected to be. NOx formation in gas turbines is accelerated by higher operating temperatures associated with higher thrust settings (and therefore higher fuel burn) during take-off and initial climb. However, higher thrust settings afford greater efficiency and a quicker climb to 3000ft agl. Consequently, it could be expected that higher thrust levels would be more advantageous in terms of optimising (minimising) fuel burn, whilst for the limitation of NOx emissions, reduced thrust and associated lower engine operating temperatures would be more advantageous.

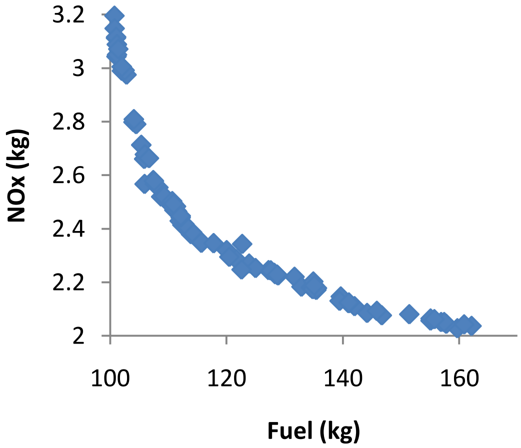


Figure 1: The Pareto Frontier for NOx and fuel burn for the first stage

Figures 2, 3 and 4 present the profiles of the two extreme operating conditions, namely those minimising NOx and fuel burn respectively. As can be observed, the strategy for minimum fuel burn to 3000ft requires a more rapid climb than that for minimum NOx emission, and this is achieved through the application of higher thrust and greater fuel flow (burn) rate, attaining 3000ft within 5km as compared to just under 15km for the minimum NOx trajectory. The rate of climb in the third segment is approximately 2800ft/min for the minimum fuel burn case and 800ft/min for the minimum NOx trajectory. The latter strategy, which exhibits a very low climb rate, is currently not normally adopted operationally as aircraft tend to maintain high rates of climb in the initial climb. It is worthwhile noting, however, that the optimisation strategy did not take into account factors such as noise abatement procedures, engine out performance limitations and obstacle clearance.

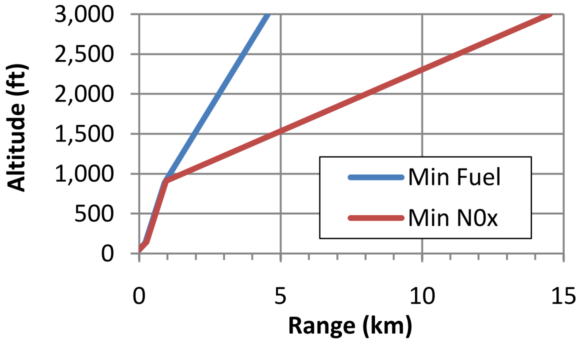
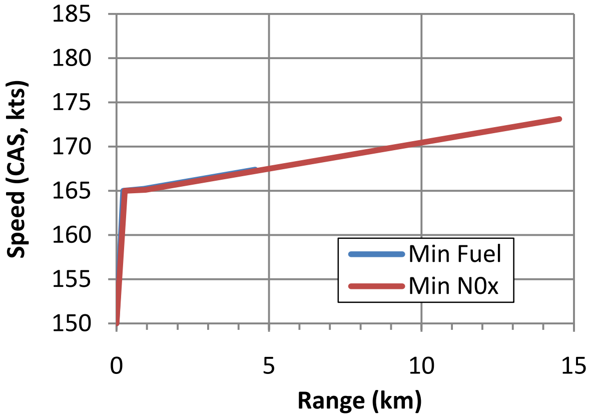
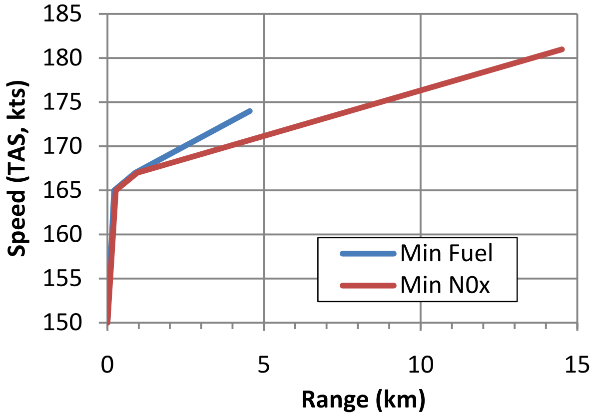


Figure 2: Trajectory profiles for minimum fuel burn and minimum NOx emissions during initial climb



(a)



(b)

Figure 3: Speed profiles for minimum fuel burn and minimum NOx emissions during initial climb (a) CAS, (b) TAS

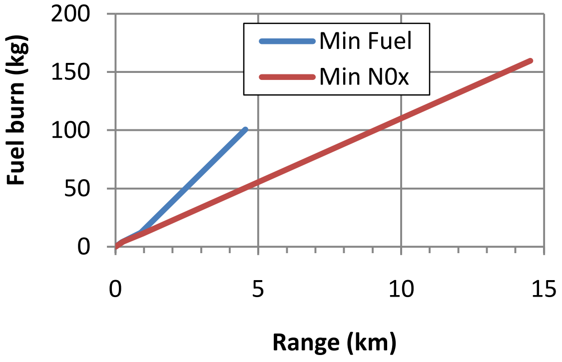


Figure 4: Fuel burn for minimum fuel burn and minimum NOx emissions during initial climb

3.2 The Second Stage – Climb, Cruise and Descent

The two-objective (fuel-time) optimisation of the trajectory profile presented resulted in the pareto frontier presented in Figure 5. The frontier exhibits good convergence with a smooth characteristic throughout the operational regime. As expected, the frontier demonstrates how reduced flight time is achieved at the cost of higher fuel burn. The relative weighting given to fuel burn and flight time will define the optimal operating point on the frontier. This operating point is, as for the case of the NOx/Fuel Burn front consideration, identified by finding the point at which the slope of the tangent defines this relative weighting. For example, if flight time and fuel burn are given an equal weighting, the optimal operating point would be the point where the slope is equal to -1, which is in the region of 7,200kg fuel burn and a flight time of 2 hours and 13 minutes (8,000 seconds) for the trajectory constraints defined in this work.

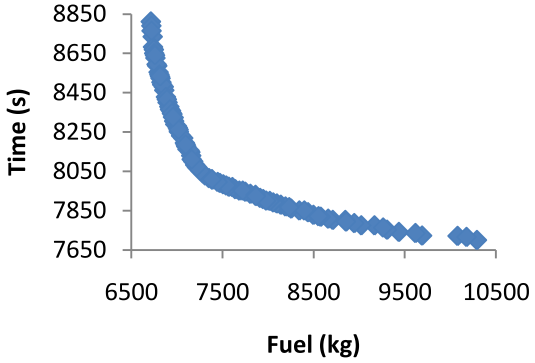


Figure 5: The Pareto Frontier for Time and fuel burn for the second stage

Although the models are generic in nature, the pareto frontier suggests good representation of aircraft of the class, as a similar trajectory profile for a 60 tonne aeroplane would involve a fuel burn of about 6.6 tonnes over 2½ hours and a cruise altitude of 38,000ft. Fuel burn at 38,000ft can be expected to be about 10% less than that at 34,000ft at Mach 0.78, which, due to the higher true airspeed at lower altitude, will translate to about a 5% greater fuel efficiency at the higher altitude over a 2000km sector.

It is interesting to analyse the trajectory strategies determined by the optimisation algorithm for minimum fuel burn and minimum time of flight. Figures 6 and 7 present the flight profiles in terms of altitude vs. range and speed at the start and end of each segment (which represent pseudo-waypoints).

3.2.1 Minimum Fuel Burn Trajectory

For the case of minimum fuel burn, the strategy adopted is the expected, namely climbing to cruise at maximum altitude for fuel efficiency. However, the results show that the climb is constrained, as the aircraft reaches the maximum altitude in all the first eight segments (Table 2). Indeed, an unconstrained aircraft of the class represented in this work should achieve FL340 within about 200km at a weight of 60 tonnes. This indicates that greater efficiency would be achieved if the aircraft were allowed to climb quicker, which it is capable of doing. It is interesting to note that the selected climb speed is higher than the cruise speed, as shown in Figure 6. Whilst this results in a greater fuel burn in the initial part of the climb, it affords the aircraft to achieve higher altitudes earlier and thus save fuel through prolonged flight there. In essence, this implies a trade-off in fuel burn between segments of the trajectory for global optimisation. This strategy is also evident in Figure 8, which illustrates a high fuel flow (burn) rate in the first segment (close to that for minimum flight time, which disregards fuel burn considerations) in order to climb away from the lower altitudes as quickly as possible. This is also understandable in the context that, for a given CAS, TAS increases disproportionately with altitude. In still air, TAS is what defines the ground speed and hence the flight time. As a result, it is expected that it will be advantageous to expedite climb beyond that providing minimum fuel burn during climb in order to reach higher altitudes quicker. It will be interesting, in future work, to analyse the effect of the altitude constraint during climb on the optimal climb speed.

In cruise, the aircraft would be expected to climb or to slow down gradually as the aircraft becomes lighter through progressive fuel burn. However, in this test case, the aircraft flies at the constraints of maximum altitude and minimum mach number, thus denying the optimisation process from adopting one of these strategies. Consequently, the fuel flow rate will be rising gradually during cruise (Segments 3-8) as a result of the aircraft progressively flying further away from its maximum range operating point.

The descent of the minimum fuel burn trajectory commences towards the end of the 8th segment and this is only because the trajectory constraint required the aircraft to be at or below FL280 by the end of this segment. Consequently, the majority of the descent, that from FL280 down to FL80, was performed in the 146km of the last two segments. The average rates of descent in these two sectors are computed to be just under 2000 ft/min and 1500 ft/min respectively. The descent in segment 9 is also constrained in that the initial altitude is at FL290, the highest possible to maintain efficient cruise, whilst the final altitude attained is FL10. Considering that segment 9 extends 132km, the average glide ratio will be just under 23:1, which will need thrust above idle, since a glide ratio of about 20:1 is achieved with idle thrust. This implies that, due to limitations in the problem setup, the optimal trajectory is limited by the constraint of the initial and final conditions of the descent segment, thus not affording the aircraft to fly longer (or higher) in segment 8, which then results in the higher than idle thrust setting during the descent in segment 9. It is interesting to observe that the end speed of segment 9 rises to a CAS of 337kts and then the excess kinetic energy is traded off into potential energy, resulting in a relatively low rate of descent during segment 10, as the aircraft decelerates to the final CAS of 250kts at idle thrust. This strategy is probably the same as that adopted during climb, where the aircraft flies faster at lower altitudes to reduce the time spent at lower altitudes, where fuel burn is higher.

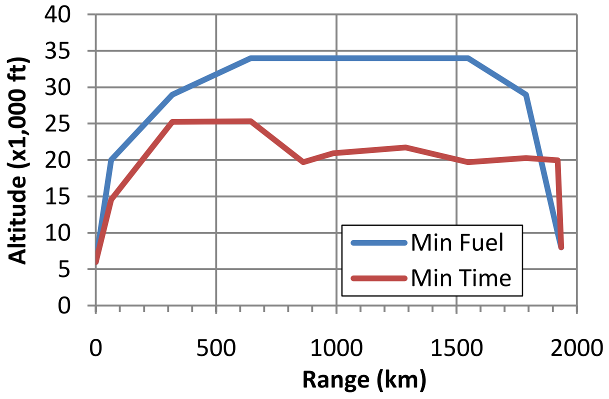
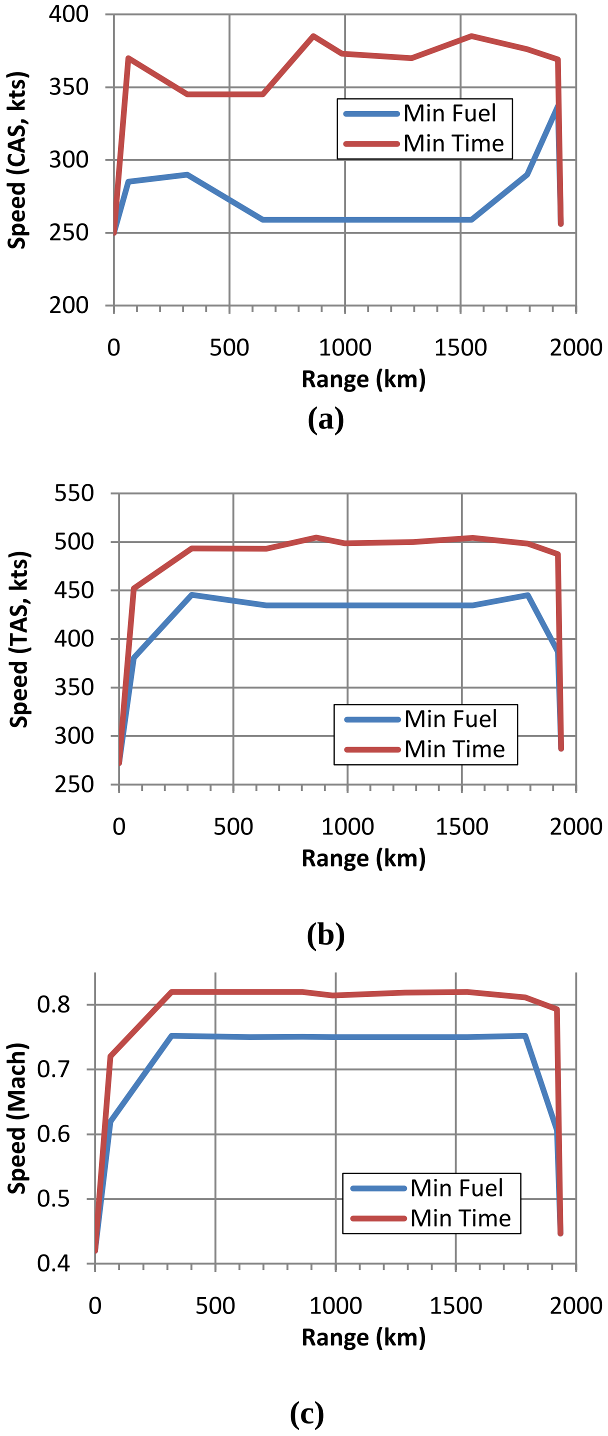


Figure 6: Trajectory profiles for minimum fuel burn and minimum time of flight



(a)

(b)

(c)

Figure 7: Speed profiles for minimum fuel burn and minimum time of flight (a) CAS, (b) TAS, (c) Mach.

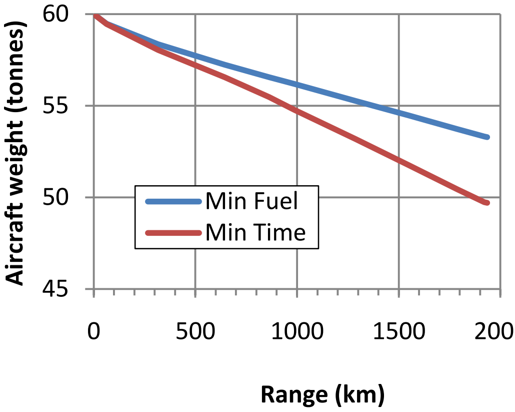


Figure 8: Weight profiles resulting from fuel burn during flight for minimum fuel burn and minimum time of flight

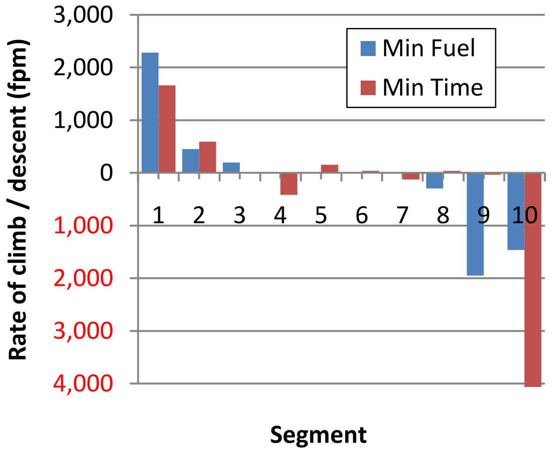


Figure 9: The average rate of climb and descent in each segment for minimum fuel burn and minimum time of flight

3.2.2 Minimum Flight Time Trajectory

The minimum flight time trajectory exhibits a lower climb gradient (Segments 1 and 2) than that in the trajectory for minimum fuel burn, in order to afford a quicker TAS during the climb. The aircraft reaches FL250 at about the same time as when the minimum fuel burn trajectory is flown, but in this case it will have covered a greater range. This implies that, for minimum flying time, the climb gradient is traded for faster climb speeds. In this case, the aircraft accelerates to the maximum constraint speed of 370kts CAS in the first segment and this affords the aircraft to climb to FL145 by the end of the segment. The result suggests that it may be more advantageous for the aircraft to climb faster than 370kts CAS with a lower climb gradient if it were not constrained to fly at this speed.

Clearly, however, a balance between climb rate and speed needs to be found. A higher climb rate affords the aircraft to fly a higher TAS (and thus a higher ground speed) quicker and achieves higher altitudes earlier, until the flight becomes mach limited above FL200.

Since the intended cruise segments (3 to 8) were constrained in terms of mach number (M0.75-0.78), the trajectory understandably settled at the lowest altitude allowed (FL200), which results in the highest possible ground speed.

The climb to FL250, essentially overshooting the optimal altitude of FL200 may not be expected theoretically. This strategy may be due to the relatively large length of segment 2 (254km), the fact that FL200 is traversed (which leads to the discrete change over from CAS to Mach constraints) and the limitation of the models used in the optimisation, where the trajectory parameters change only gradually between the initial and final states of the segment. In such circumstances, it is probable that the preferred (faster flight) strategy was to climb above FL200 to enable the aircraft to fly at higher TAS. This is plausible, given that at FL200, M0.82 results in a CAS of 383kts, which is higher than the CAS limit defined in the constraint below FL200. This means that, if the flight in the segment remained below FL200, the aircraft would have flown at the CAS limit of 370kts, which would result in a TAS below 490kts, which is lower than that achieved by the selected strategy, although only marginally so (approximately 1 to 2%).

The strategy of varying altitude in cruise is not evident to the authors and indeed, they are of the opinion that this may be due to the optimisation process not fully converging to the actual minimum point of the mathematical function. Indeed, with the aircraft at FL250 at the end of segment 2, the trajectory would have been expected to descend to FL2000, maintaining that altitude until the end of the cruise, flying at Mach 0.82. This would have resulted in a constant TAS of 503.7kts, affording an improvement of the order of 0.3% in the overall flight time.

The graphs indicate that descent strategy adopted for the minimum flight time trajectory is to maintain FL200 until the end of segment 9, prior to rapid descent and deceleration to 256kts CAS in segment 10. This allowed the aircraft to fly at the highest TAS during cruise and to then descend rapidly, again flying at the highest CAS of 370kts. Thus, a rapid descent from FL200 down to FL10 was achieved with an average rate of descent of 9300ft/min, which, although mathematically possible from a performance point of view with the models used in this work, would not be practical in practice for several operational reasons. Nevertheless, the result clearly illustrates the strategy adopted for minimum flight time. It is also relevant to observe that the aircraft is constrained to slow down during segment 8 because of the speed constraint of 370kts CAS, which is Mach 0.79 at FL200. Being constrained by CAS rather than Mach justifies the strategy of maintaining the highest allowed altitude, which is again constrained at FL200.

4 Concluding Remarks

This paper has presented the results of an optimisation of a 2000km trajectory that is bound by operational constraints in terms of altitude and speed. The results have illustrated the strategies adopted by the optimisation process with such constraints and has highlighted the limitations introduced in the problem definition and by the limited number of segments in the trajectories. It has formed the basis of a preliminary study against which more complex problems can be compared in future.

5 Future Work

It will be interesting to assess the effect a number of characteristics of the problem definition may have on the output of the optimization process. The performance of genetic algorithms is affected by factors such as population size, number of generations and cross-over algorithms and these may have an impact on convergence as well as processing time. Given the heuristic nature of the optimisation technique, it is also interesting to assess the repeatability of such results. Another point of significant interest is the consideration of the impact the number of segments into which the trajectory is partitioned for the optimisation process has on the accuracy of the results. Whilst it is obvious that a larger number of segments will generate higher accuracy results, it is interesting to study the nature of the errors (or approximations) brought about by the segmentation process. This is particularly of interest in the light that the performance model has been designed and implemented to cater for dynamic and non-linear effects within a single segment, including changes in local atmospheric conditions (which are particularly relevant during climb and descent). The complexity of the model has been designed to offset some of the limitations introduced by the large segments and it is therefore of interest to assess how the overall process performs in this respect.

Finally, it is also interesting to assess how the strategy will vary with changing operational conditions such as dispatch weight and altitude constraints.