Multipartite entanglement distribution in quantum networks

E. Sutcliffe and <u>A. Beghelli</u>
Optical Networks Group

Thanks to UKRI Project “Quantum Data Centres of the Future” & Delivering Quantum Technologies UCL CDT

The fundamental problem

How to distribute quantum information among several users in the NISQ era?

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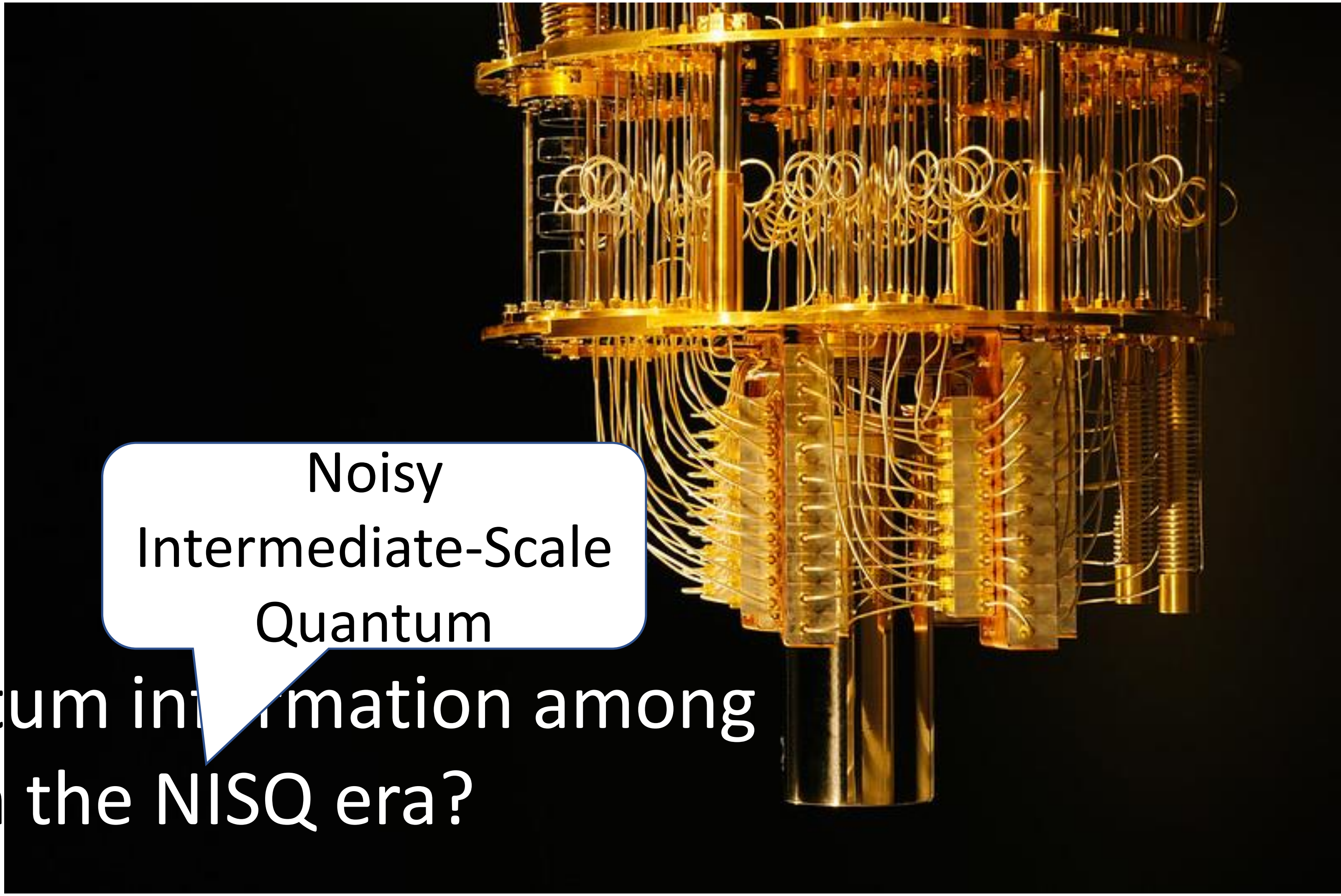
The fundamental problem



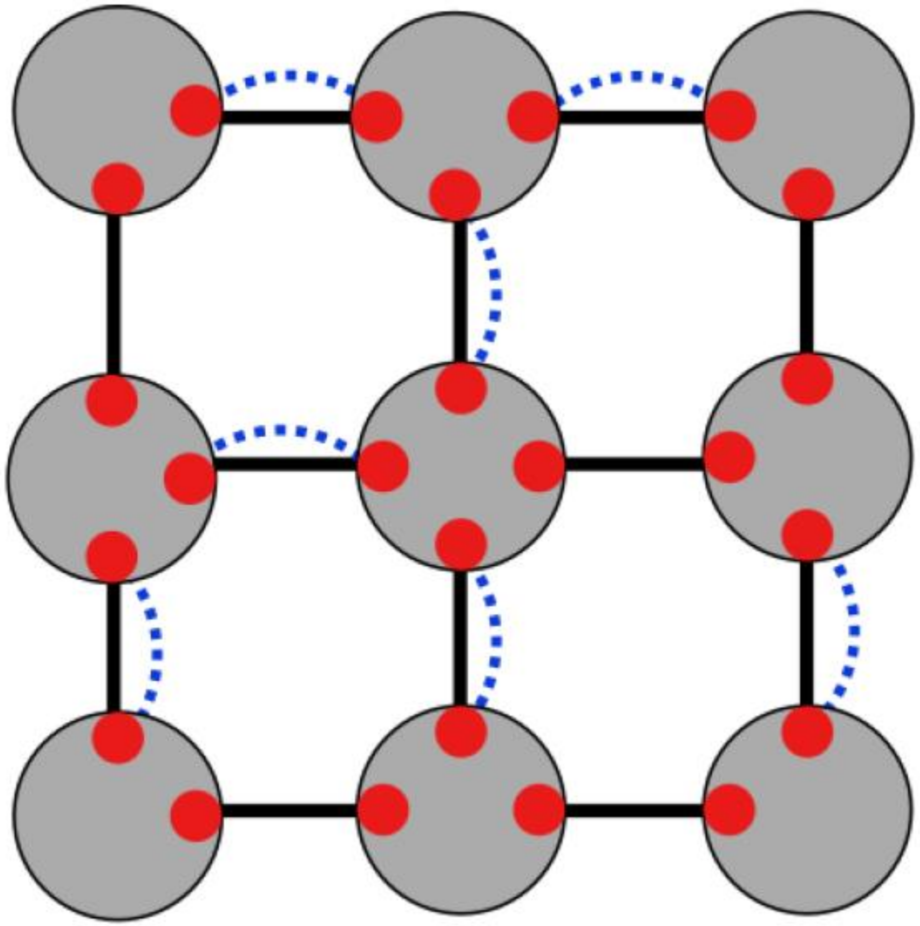
Noisy Intermediate-Scale Quantum

How to distribute quantum information among several users in the NISQ era?

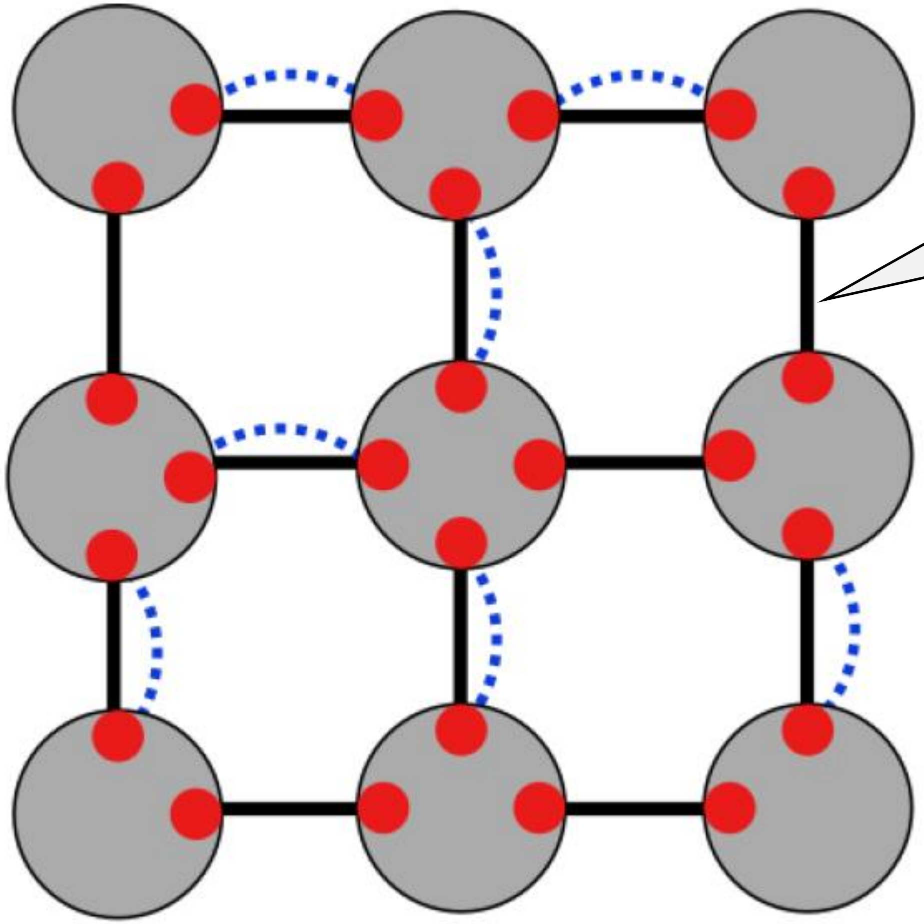
The (potential) applications

- Distributed quantum computing
- Distributed quantum sensing
- Secret sharing

The network model



The network model

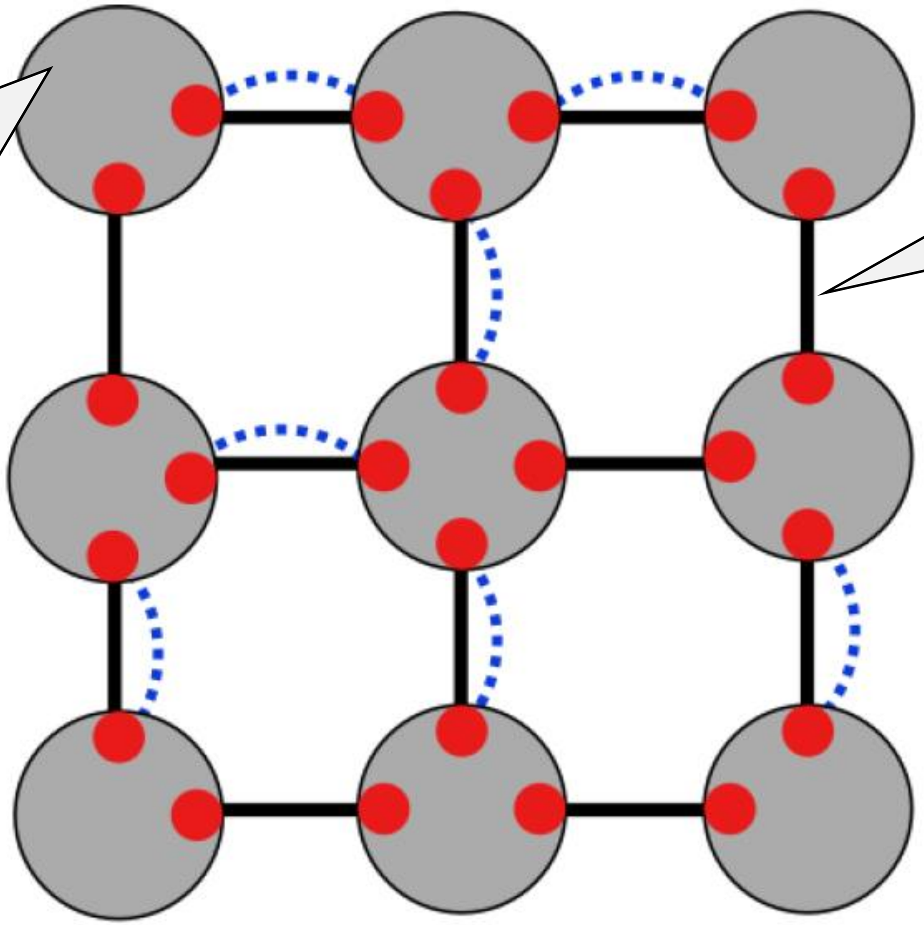


EDGES

- **Optical fibre**

The network model

NODES
- 1 quantum memory per edge (T_c)
- Can act as quantum repeaters
- Can perform entanglement fusion



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The network model

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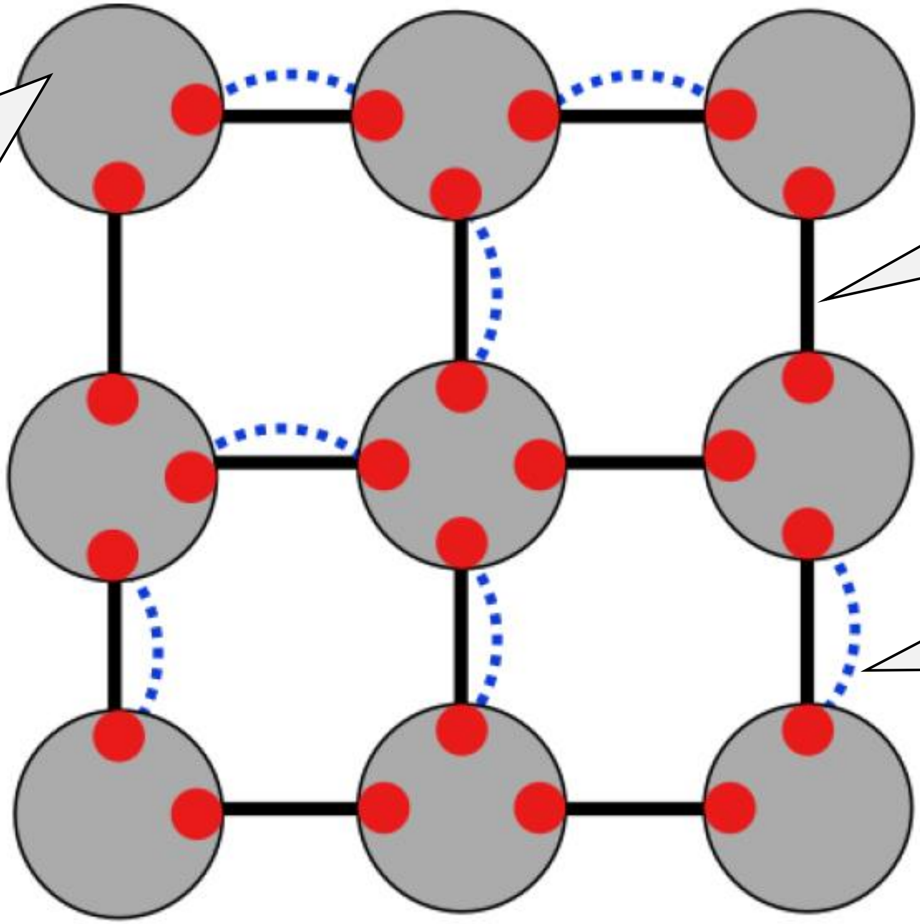

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Entanglement link

$p=p_{op}*(1-p_{loss})$

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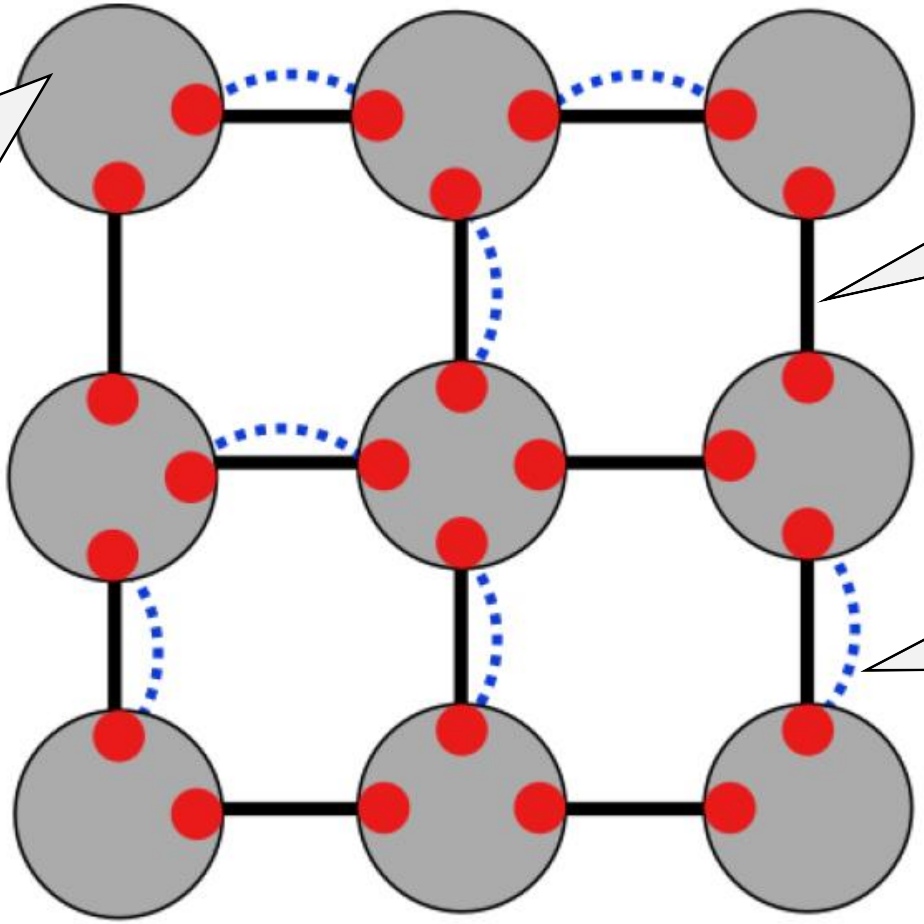
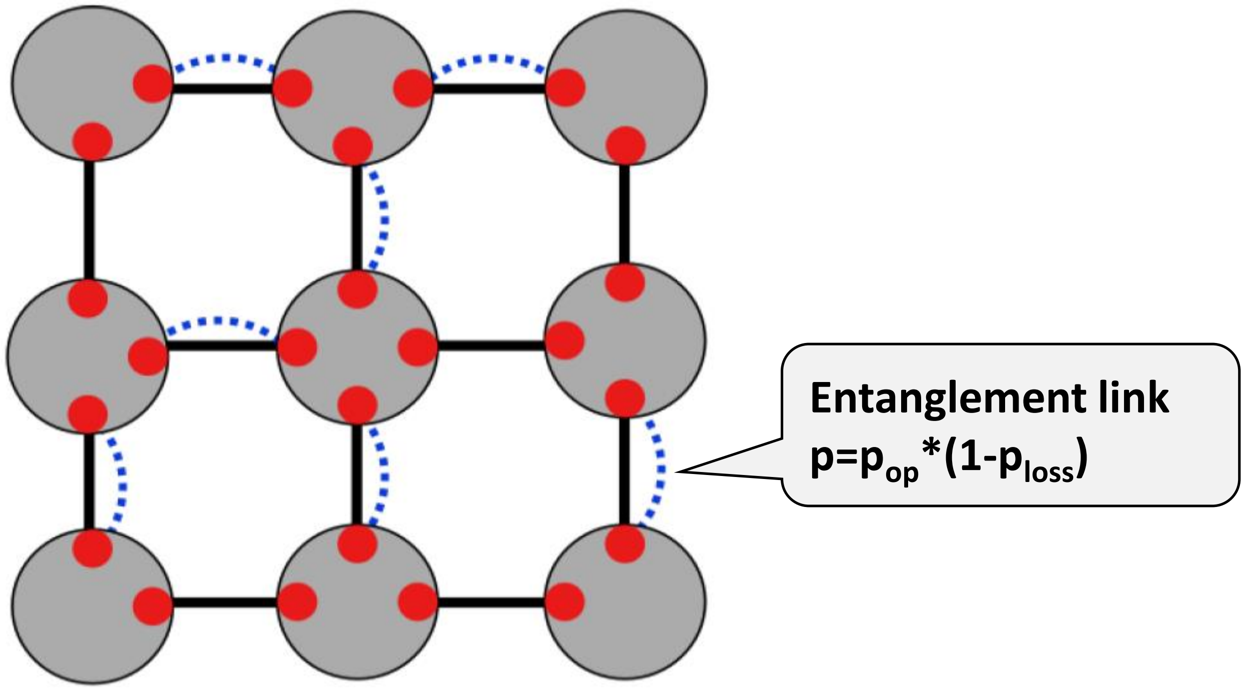


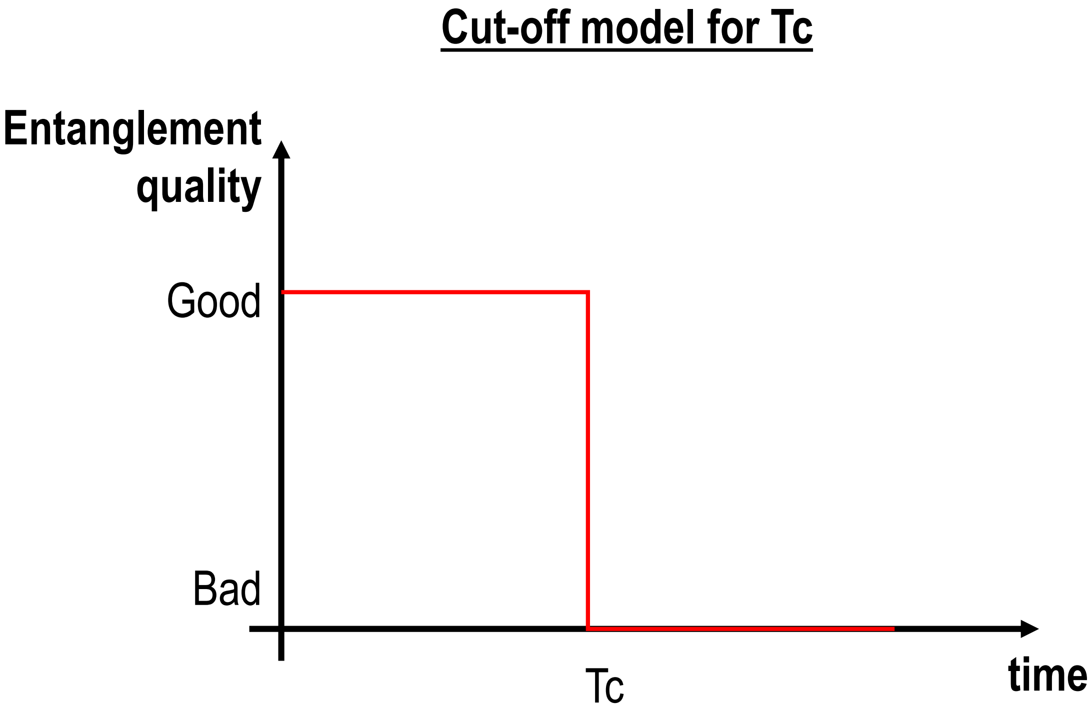
EDGES
- Optical fibre

Entanglement link
$p=p_{op}*(1-p_{loss})$

Tc: Quantum memory decoherence time



T_c: Time a qubit stored in a quantum memory can maintain entanglement [1]



Examples of decoherence times in P. E. Black et al. “Quantum Computing and Communication”, Advances in Computers, vol. 56, pp.189-244, 2002

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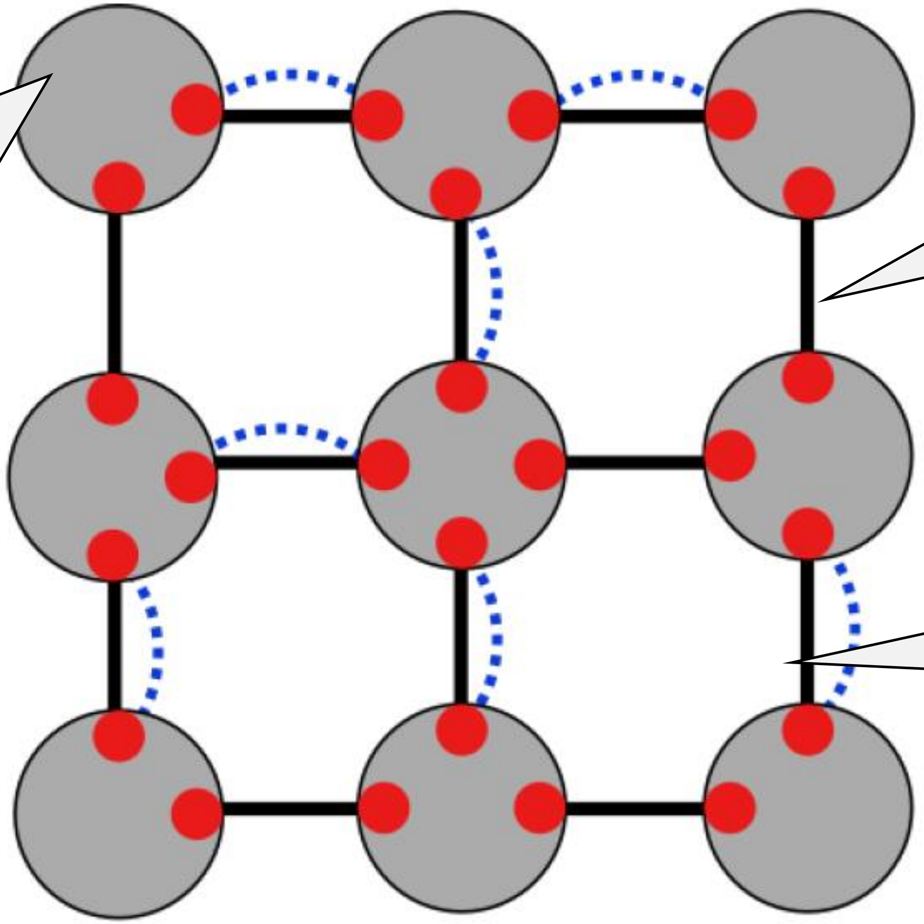
The network model

NODES
- **1 quantum memory per edge (T_c)**
- **Can act as quantum repeaters**
- **Can perform fusion**



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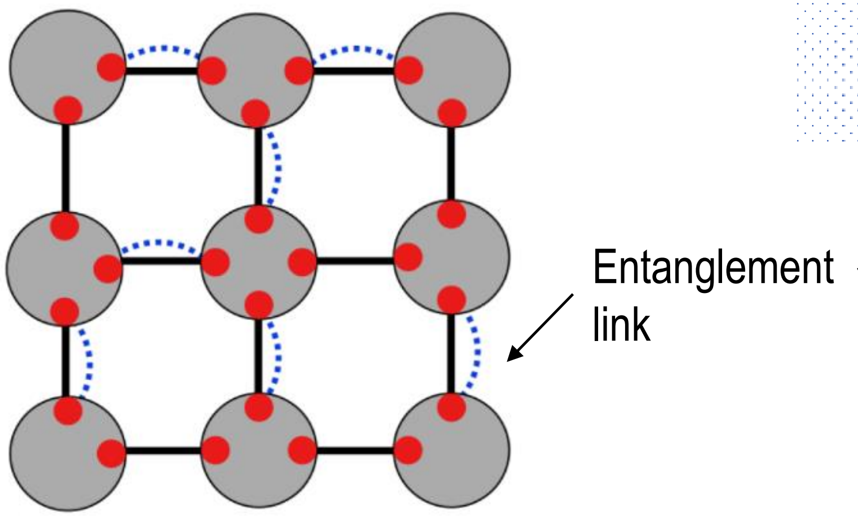
Quantum repeaters

Entanglement swapping: To achieve remote entanglement between two nodes



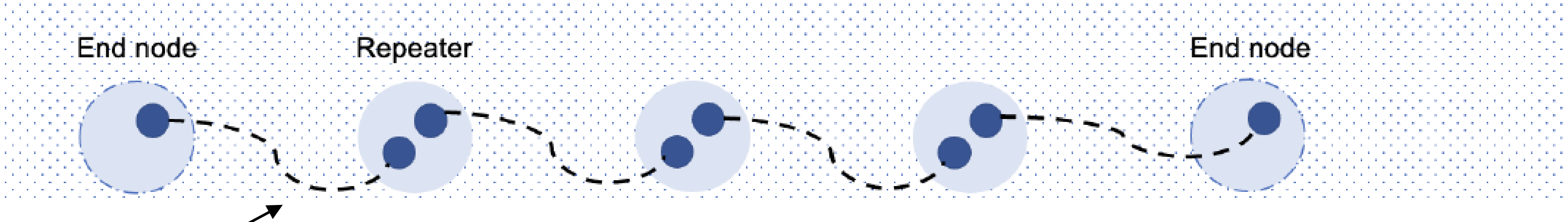
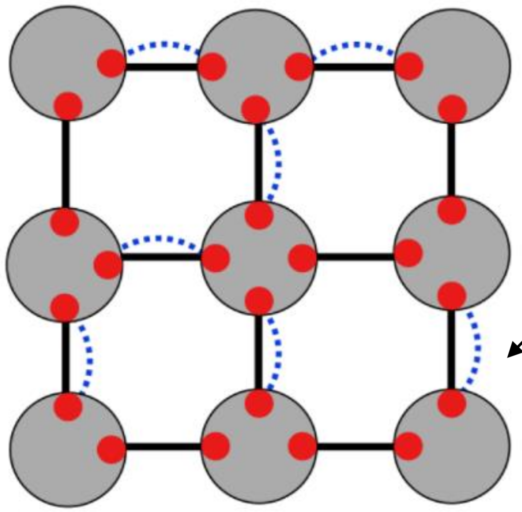
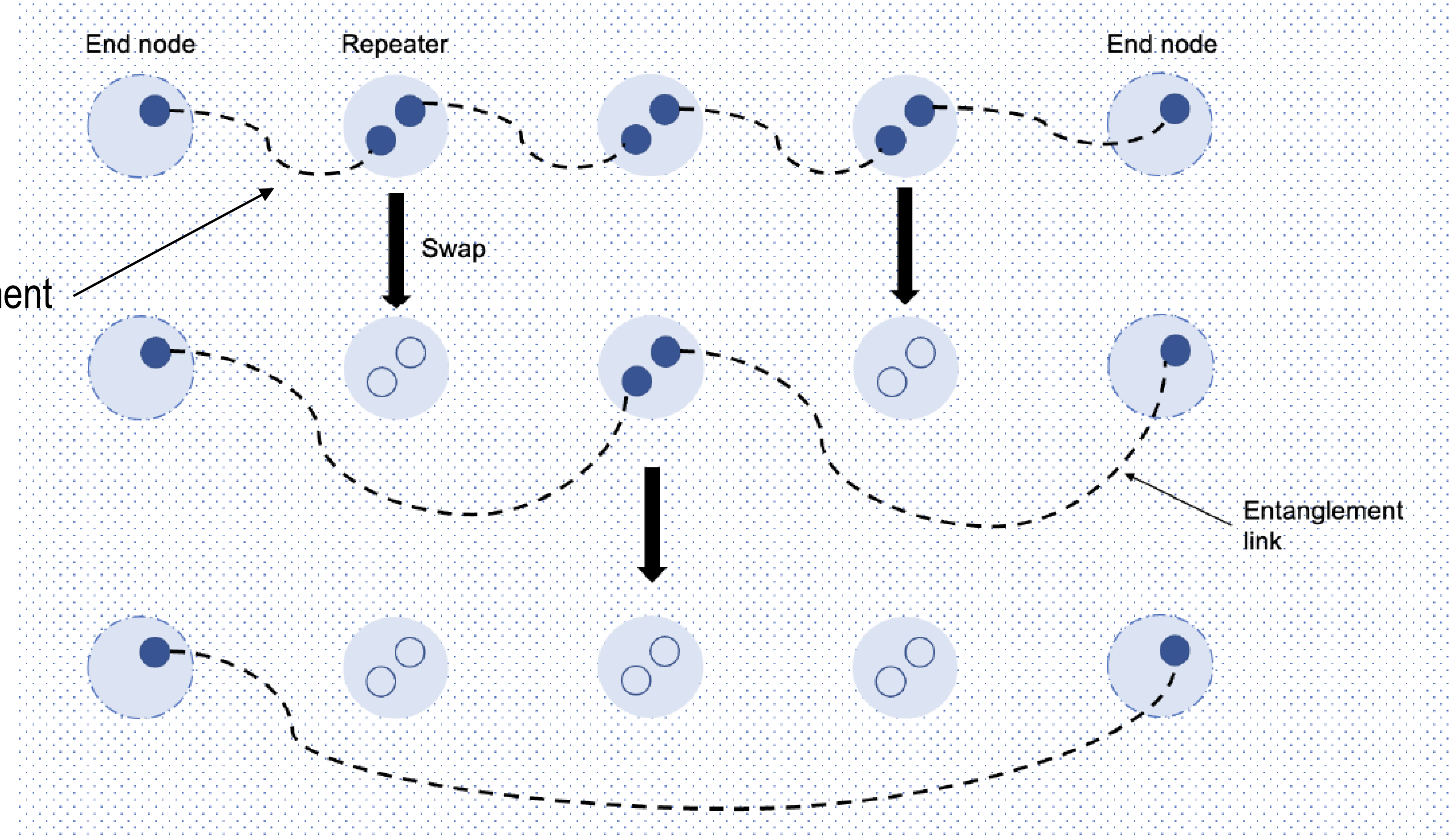


Image: Natasha Siow, MSc student from Quantum Technologies @UCL

Quantum repeaters

Entanglement swapping: To achieve remote entanglement between two nodes





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The network model

NODES
- 1 quantum memory per edge (T_c)
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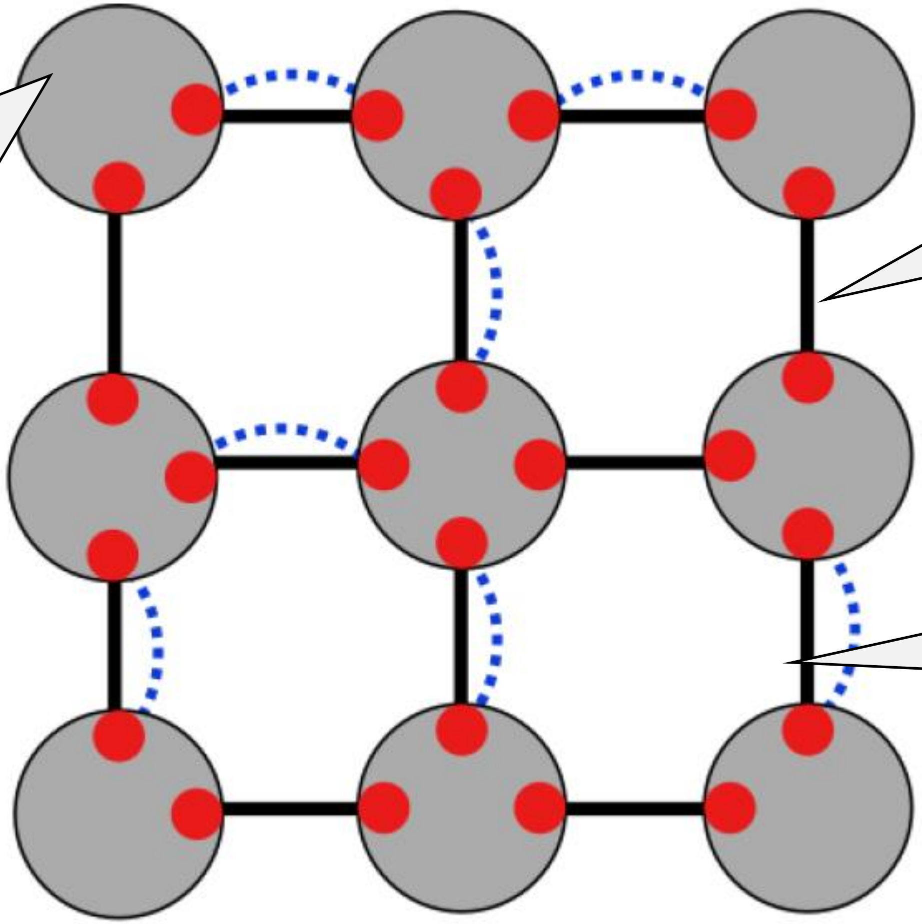
- Can perform entanglement fusion

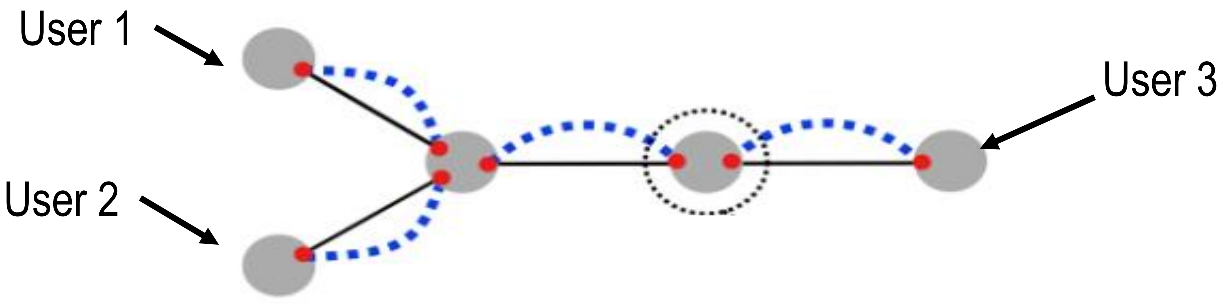


EDGES
- Optical fibre

Entanglement link
$p=p_{op}*(1-p_{loss})$

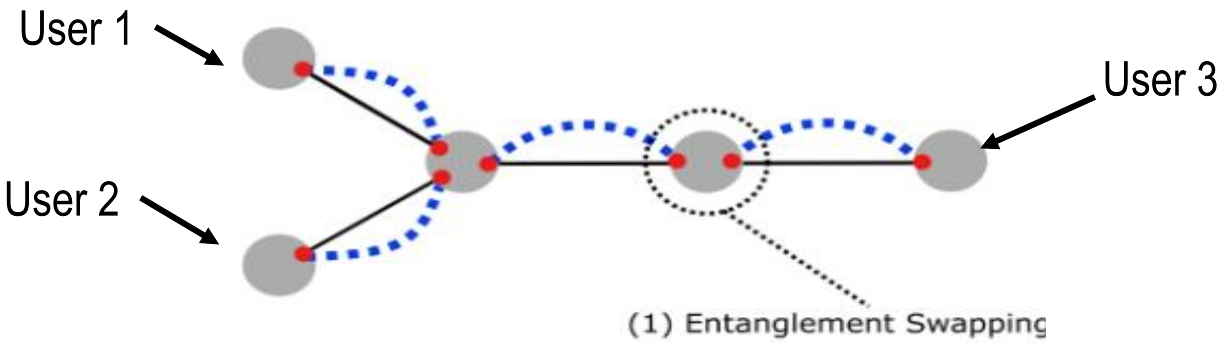
Entanglement fusion

To generate a multipartite entanglement state



Entanglement fusion

To generate a multipartite entanglement state



Entanglement fusion

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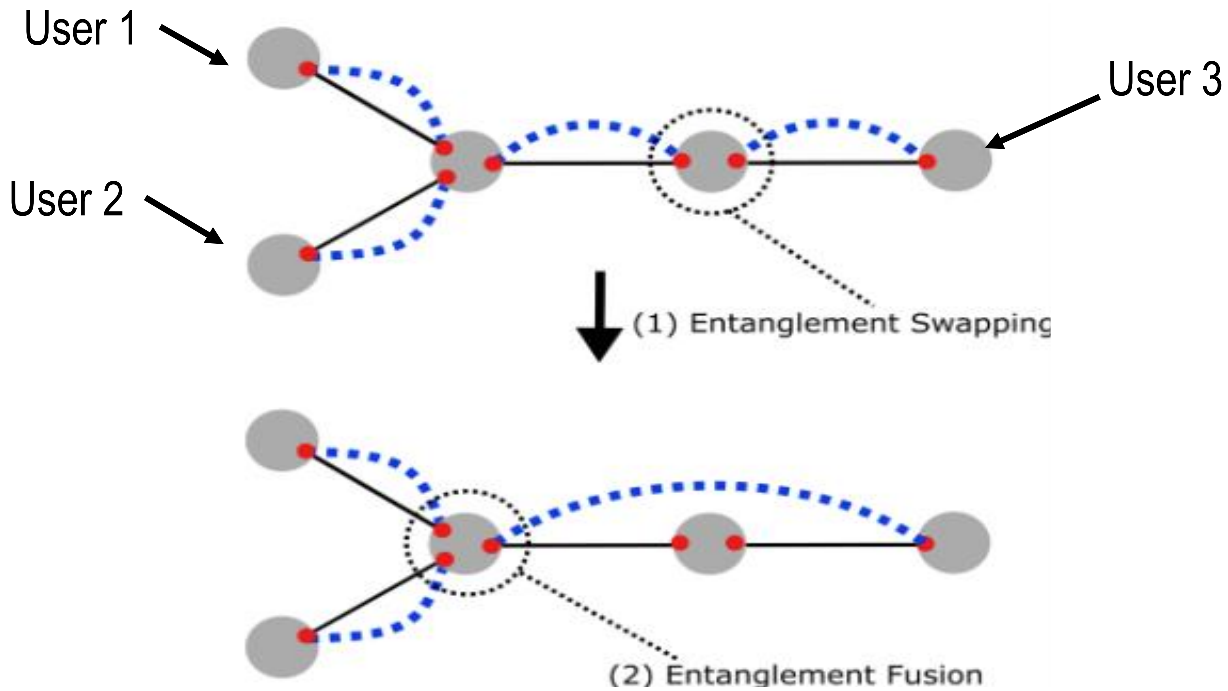

Entanglement fusion

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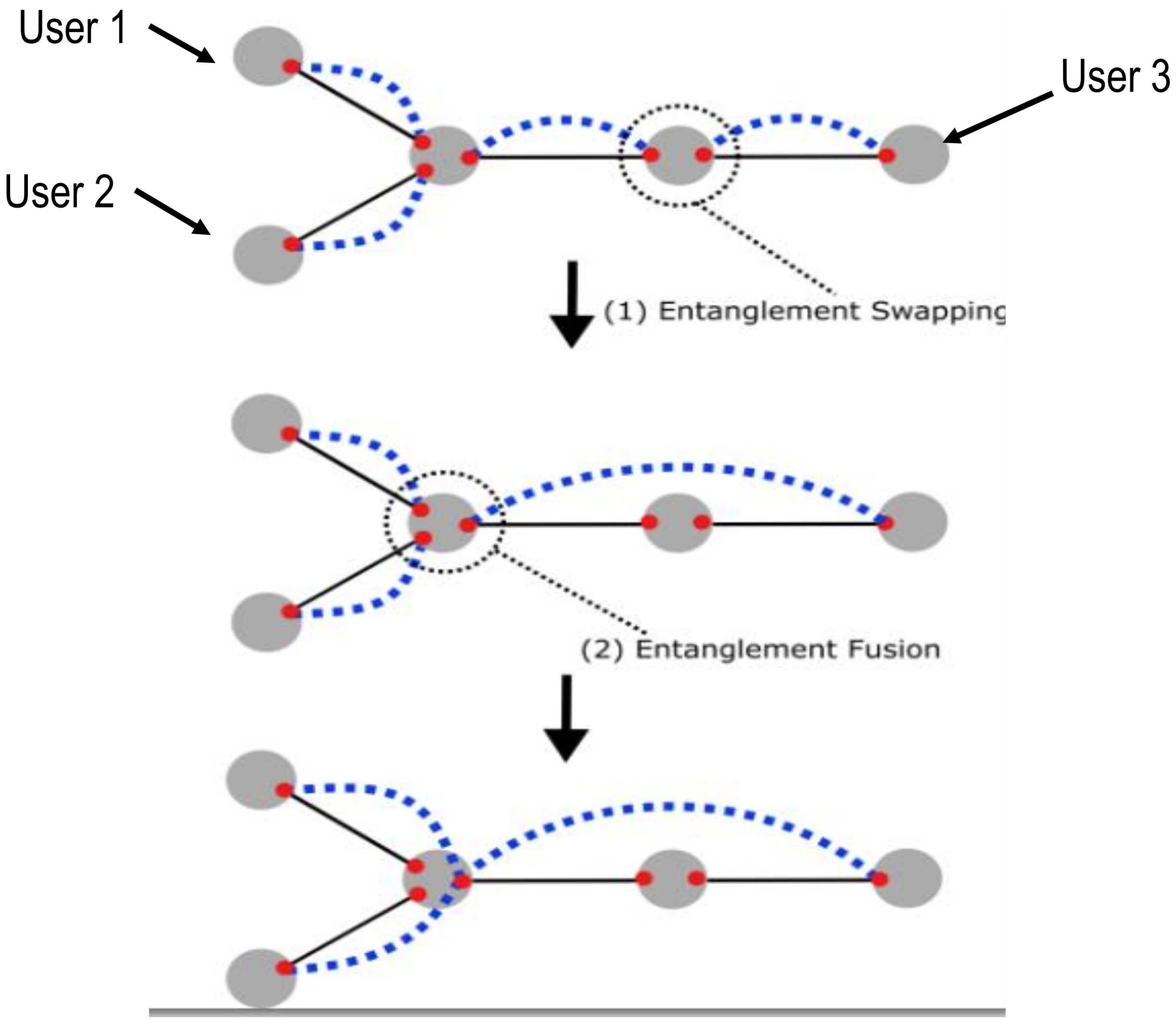
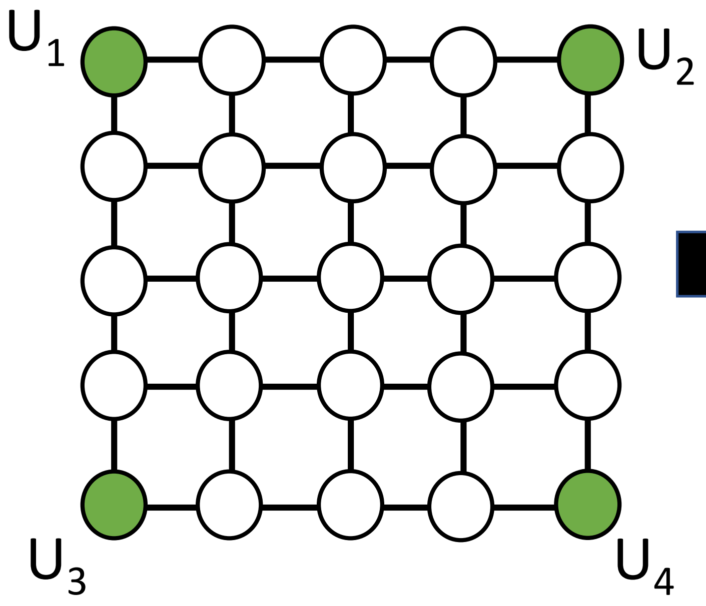
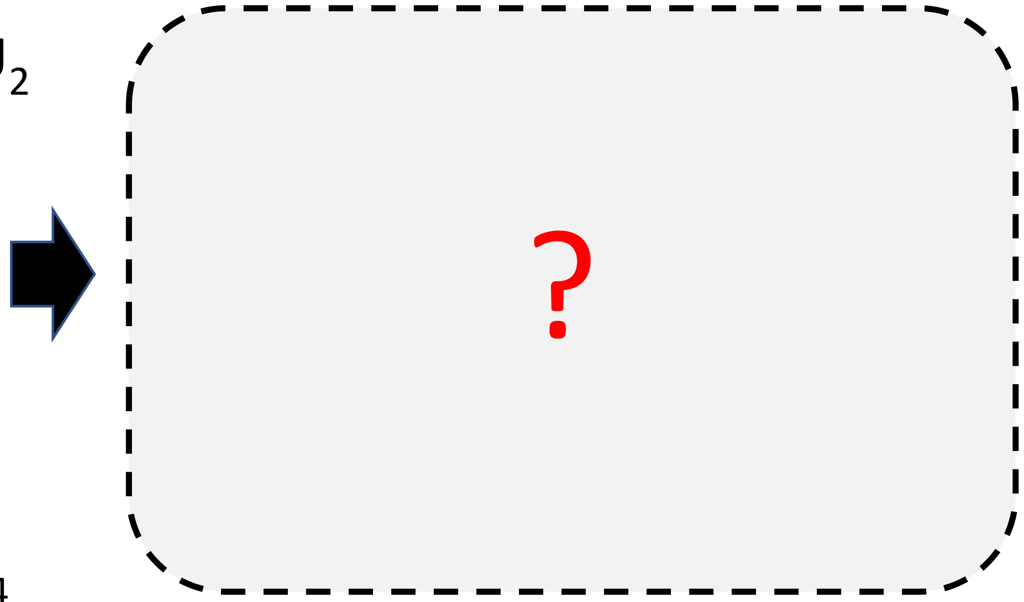


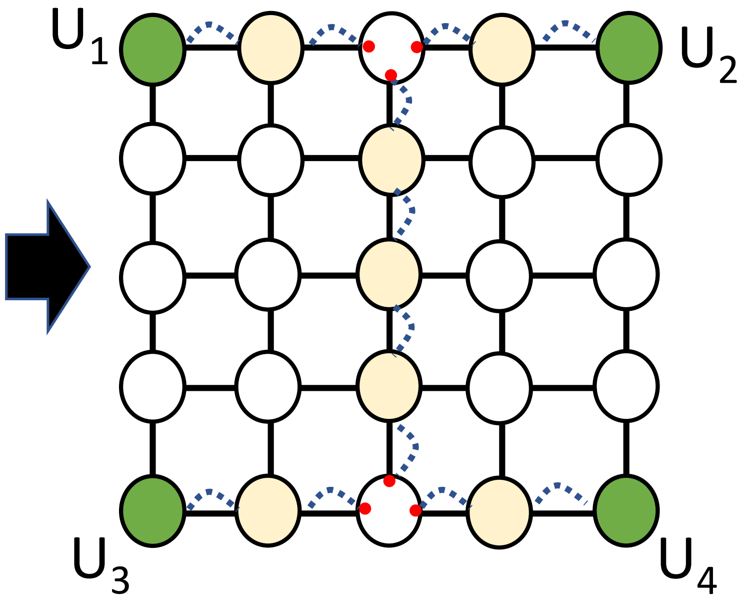
The problem

INPUT:
1. Network topology
2. Set of users U

OUTPUT:
- Routing solution
- Swap & fusion points



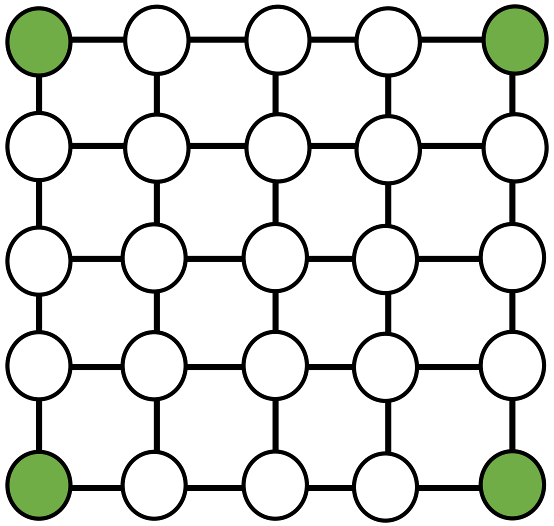




ENTANGLEMENT DISTRIBUTION PROTOCOL

Previous work (SP)

INPUT:
1. Network topology
2. Set of users U



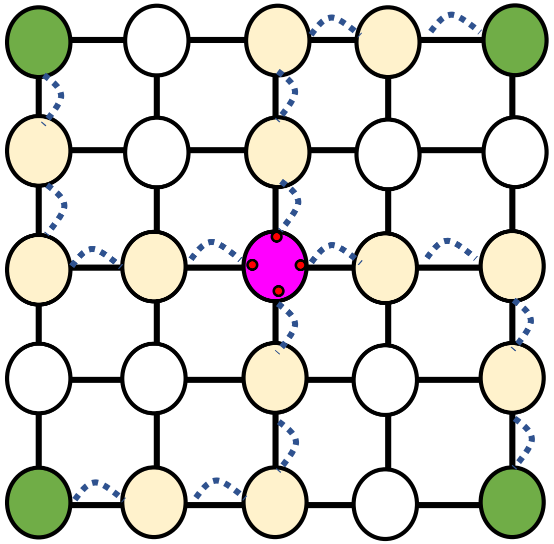
1. Establish a central node

2. Attempt remote entanglement - on **precalculated** edge-disjoint routes

3. Perform fusion at the central node

ENTANGLEMENT DISTRIBUTION PROTOCOL

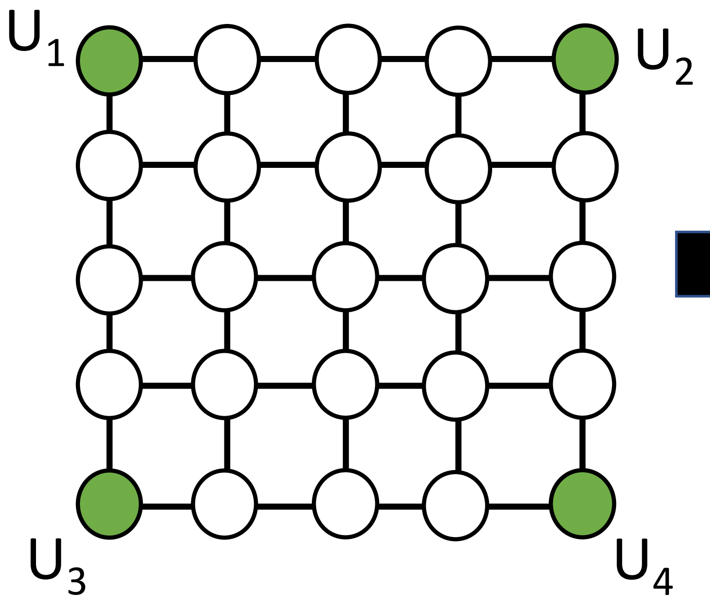
OUTPUT:
- Routing solution
- Swap & fusion points



Proposal 1: MP-G

INPUT:

1. Network topology
2. Set of users U

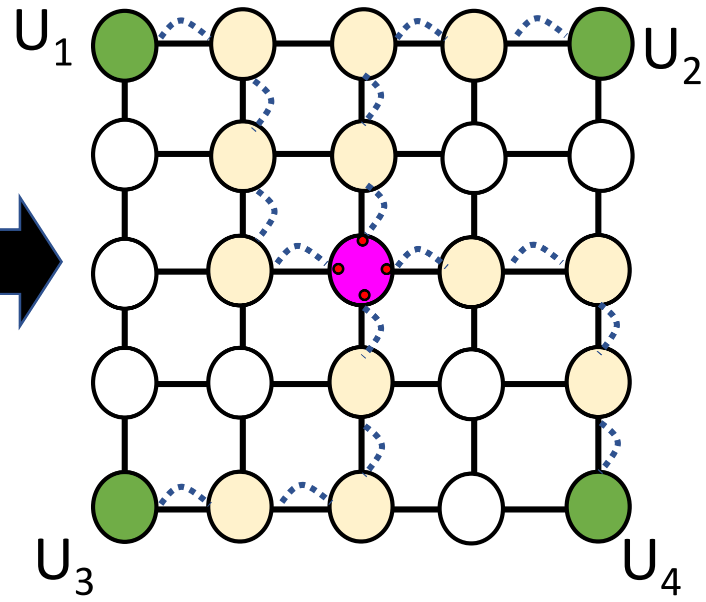


1. Establish a central node
2. Create entanglement links
3. Attempt remote entanglement - on edge-disjoint routes calculated **on the fly**
4. Perform fusion at the central node

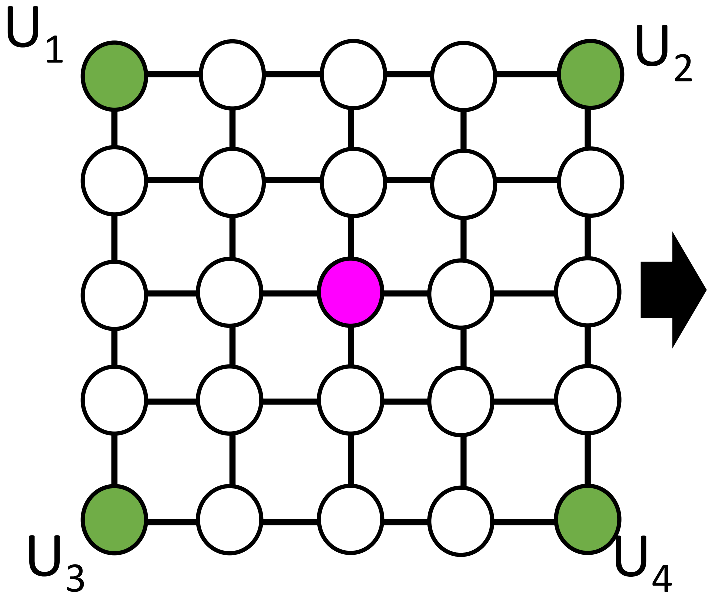
ENTANGLEMENT DISTRIBUTION PROTOCOL

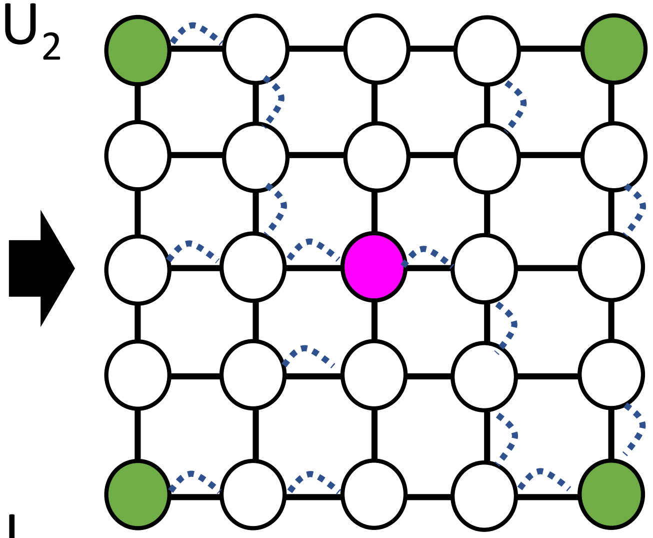
OUTPUT:

- Routing solution
- Swap & fusion points



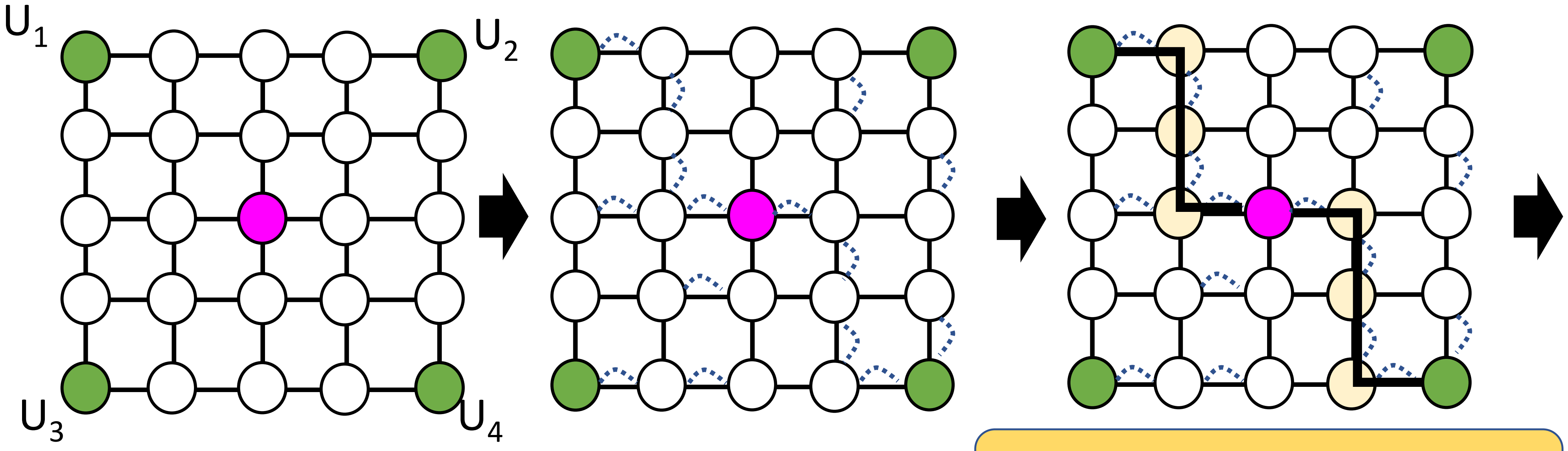
Execution example (T_c=2 time slots)





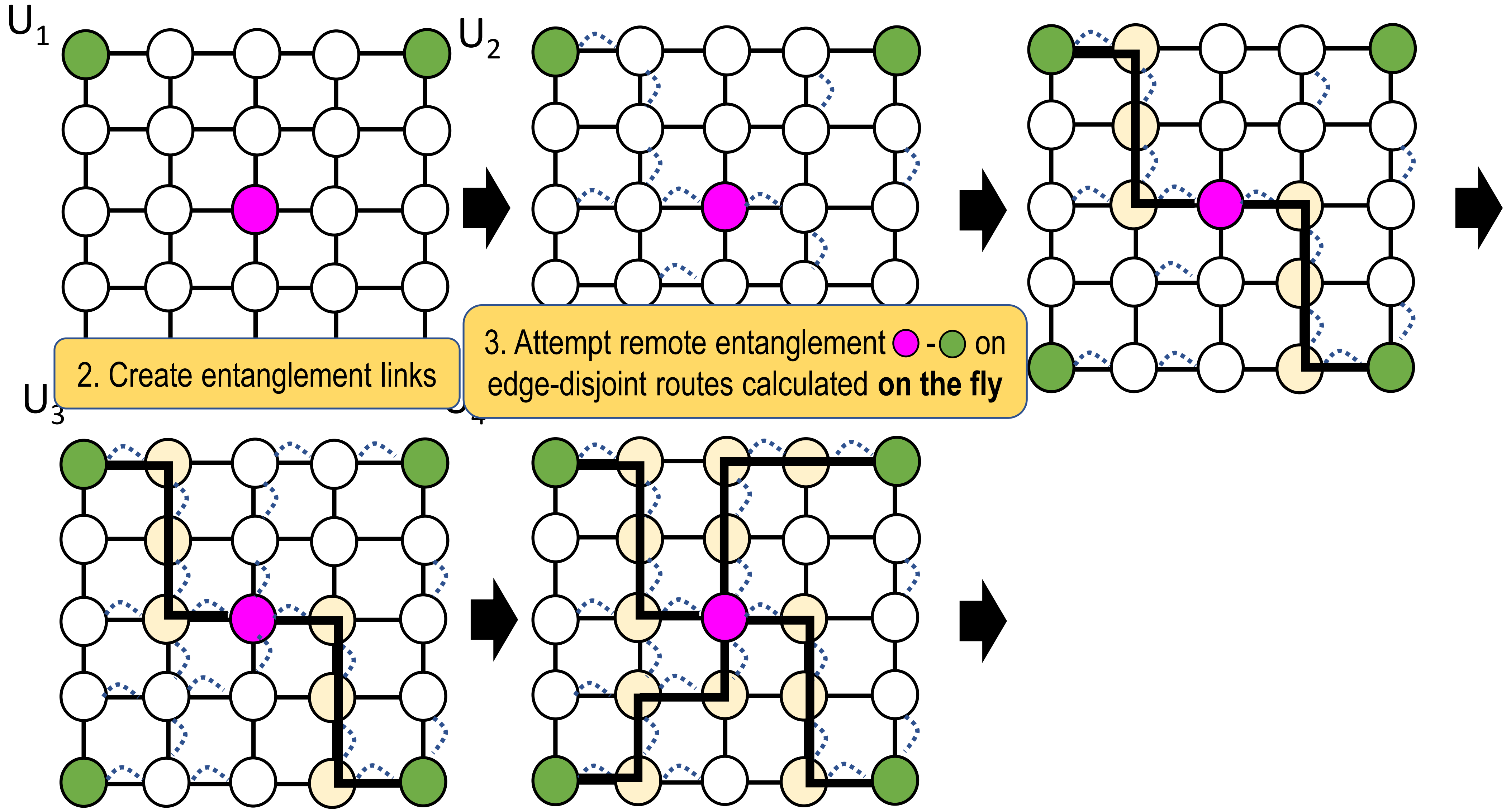
2. Create entanglement links

Execution example (T_c=2 time slots)



3. Attempt remote entanglement ● - ● on edge-disjoint routes calculated **on the fly**

Execution example (T_c=2 time slots)



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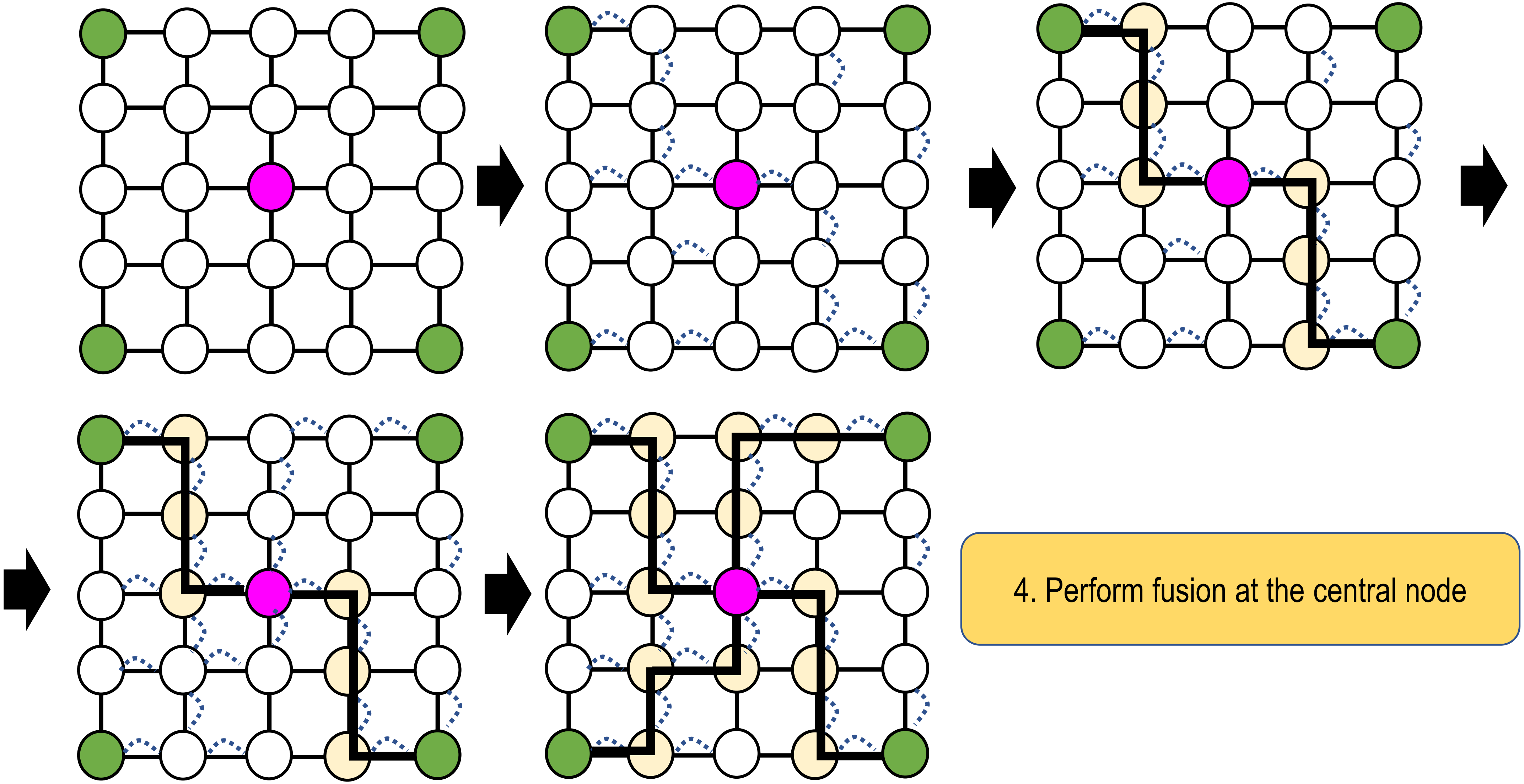
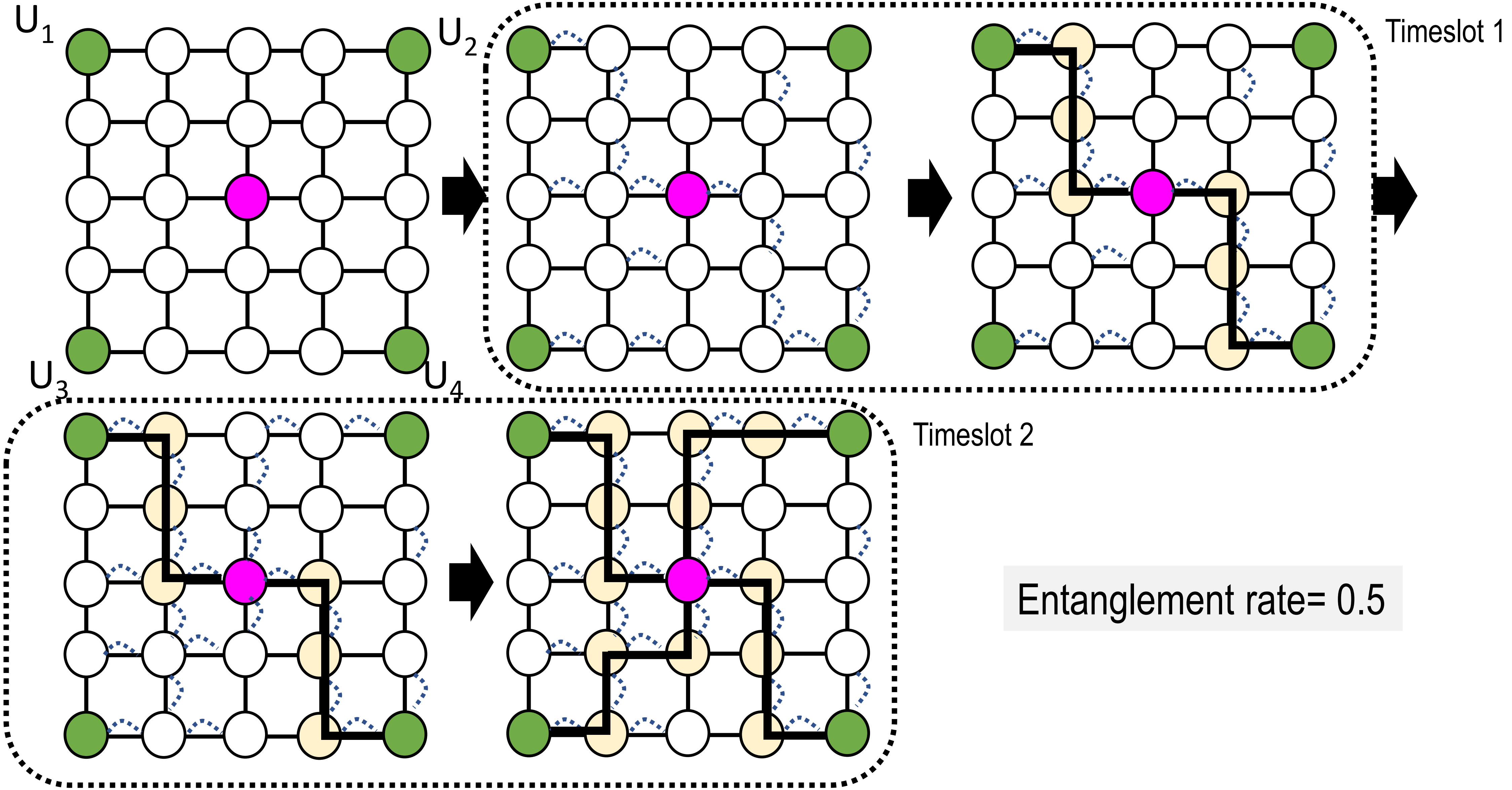


4. Perform fusion at the central node

Execution example (T_c=2 time slots)

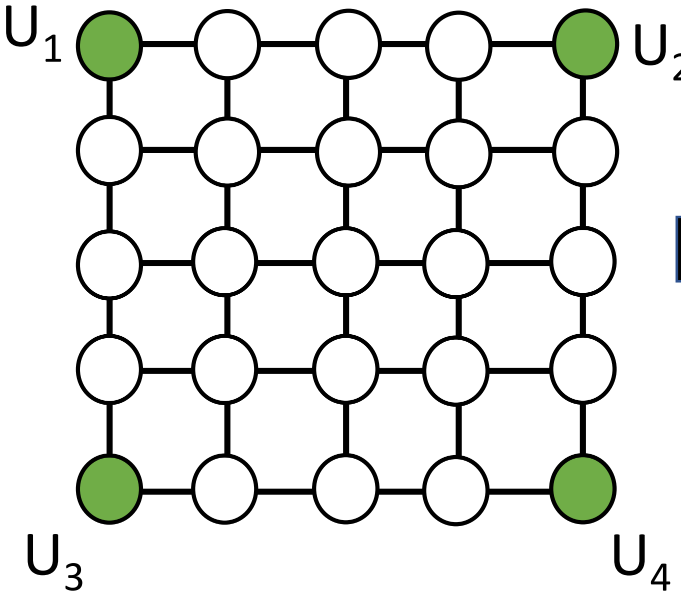


Entanglement rate= 0.5

Proposal 1: MP-C

INPUT:

1. Network topology
2. Set of users U

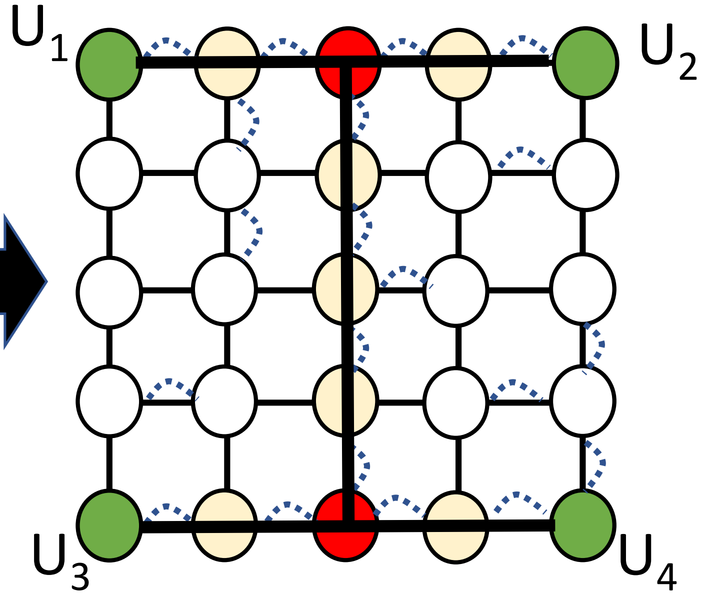


1. Create entanglement links
2. Calculate the Steiner tree covering all users
3. When tree ready, swapping & fusion (fork nodes)

ENTANGLEMENT DISTRIBUTION PROTOCOL

OUTPUT:

- Routing solution
- Swap & fusion points



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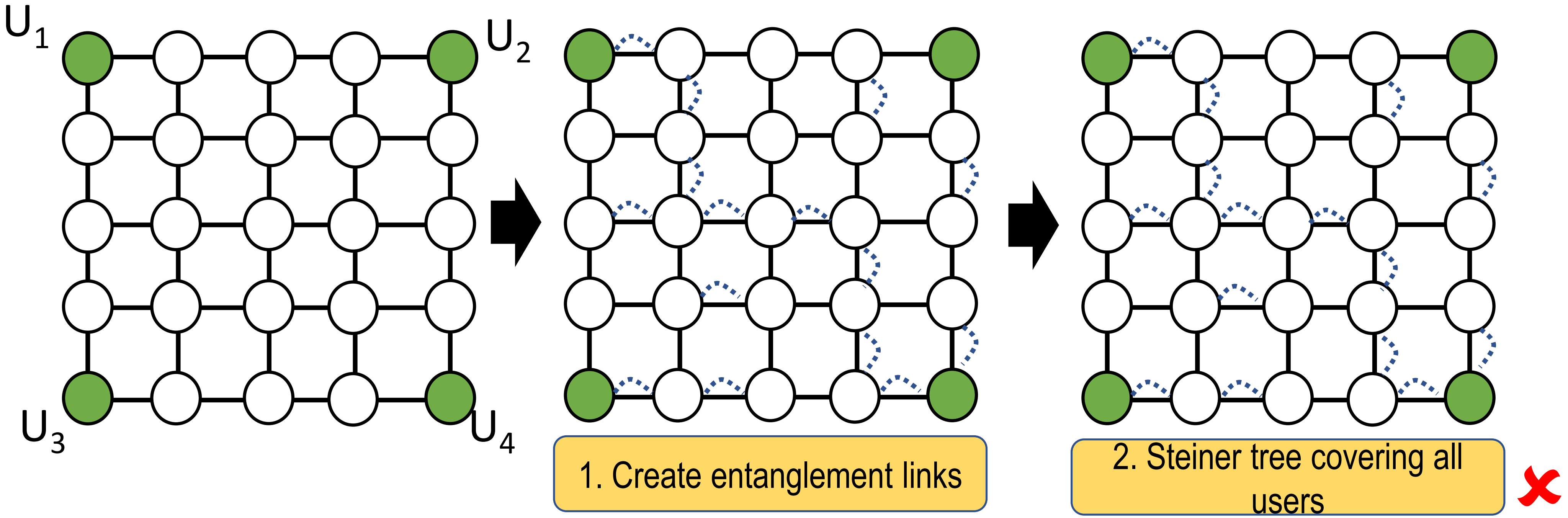

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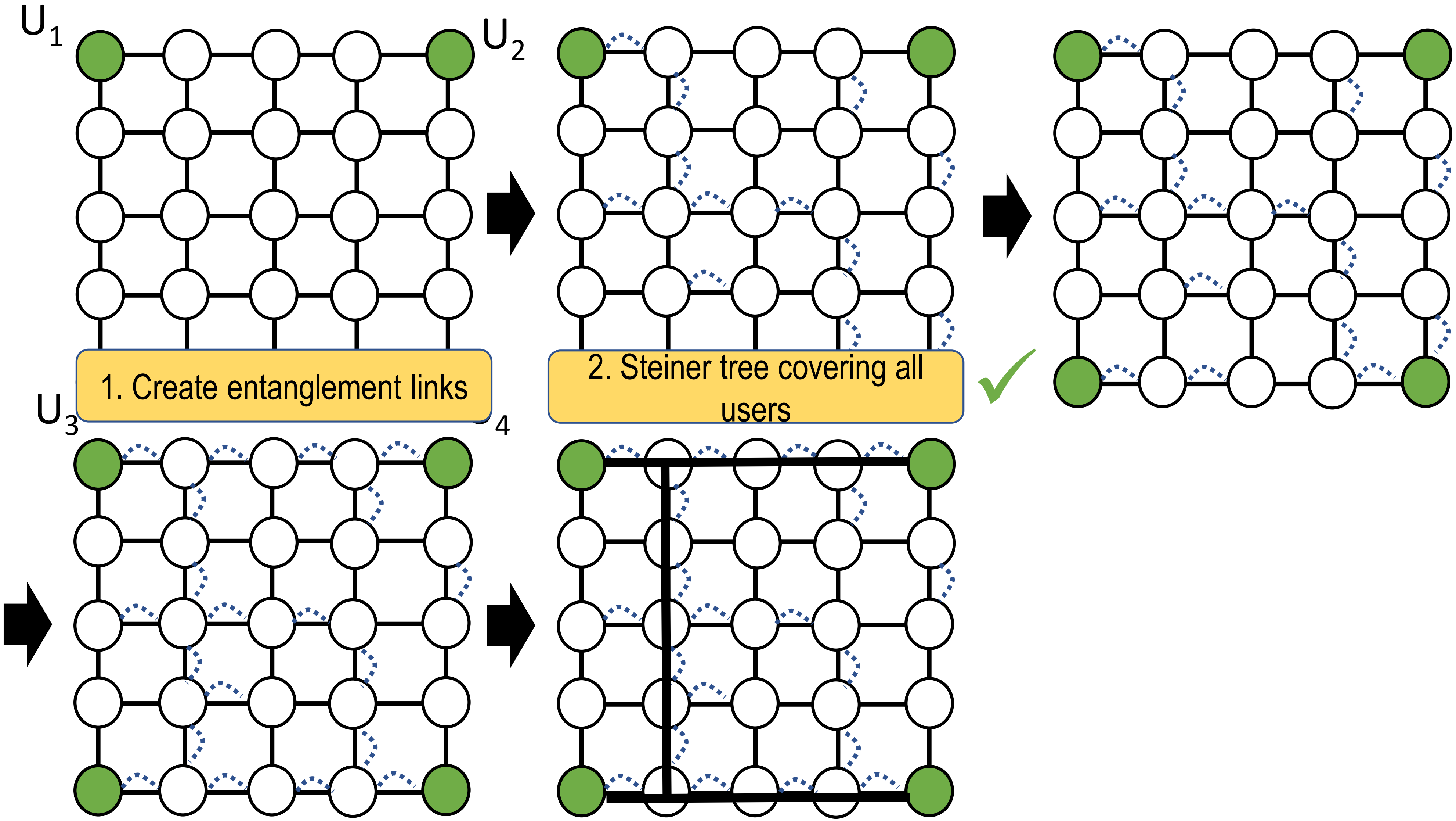

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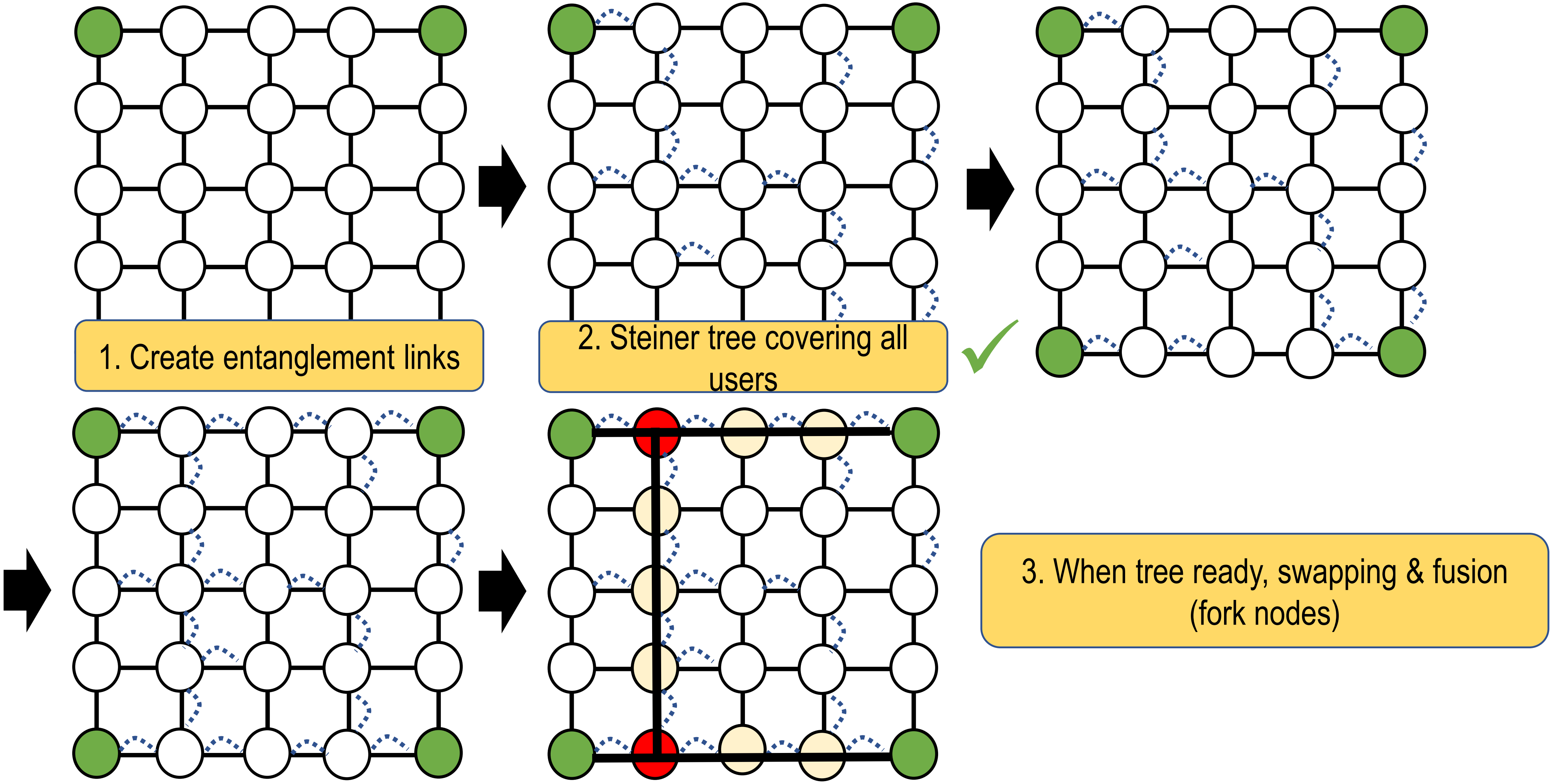

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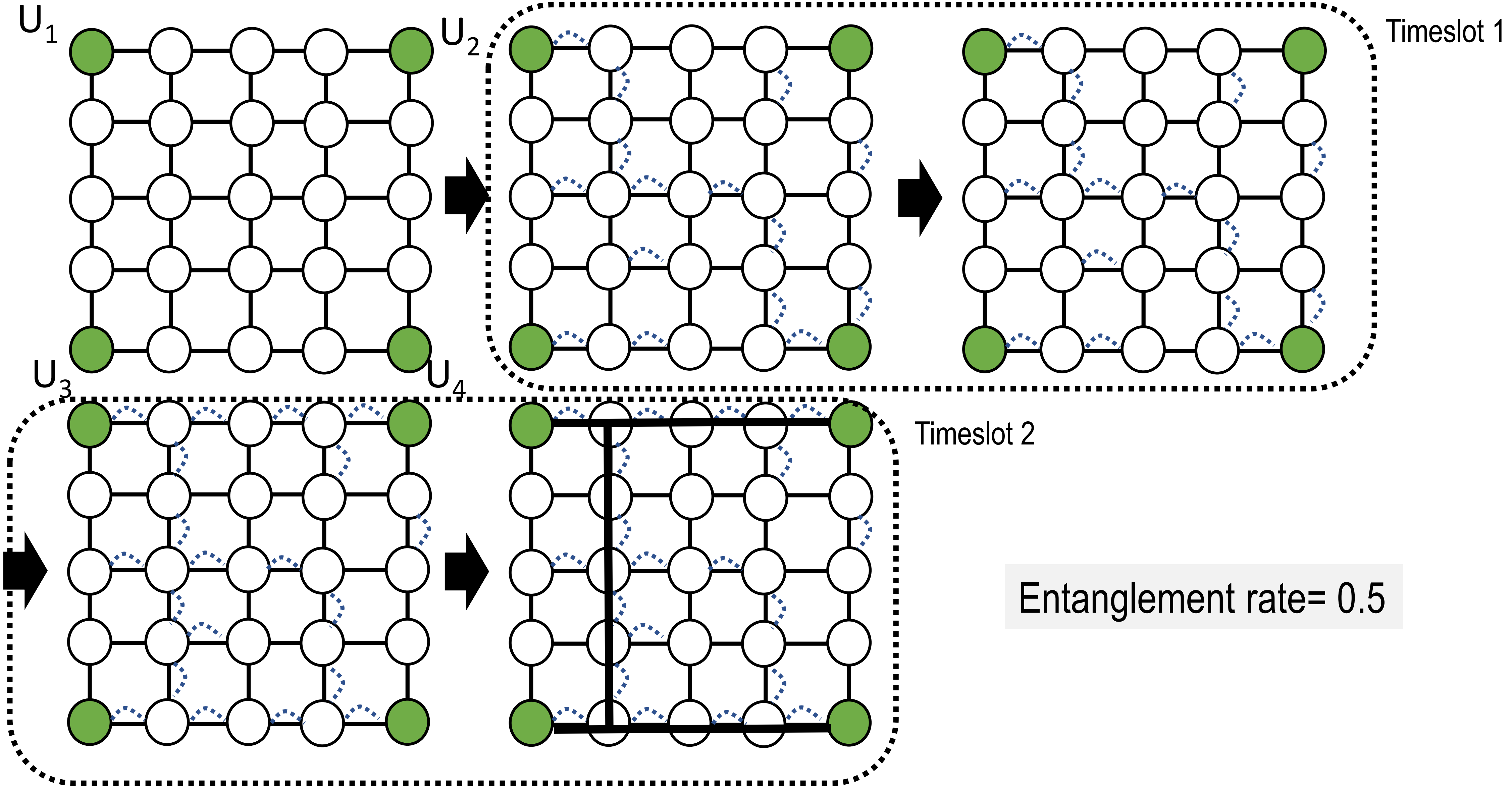
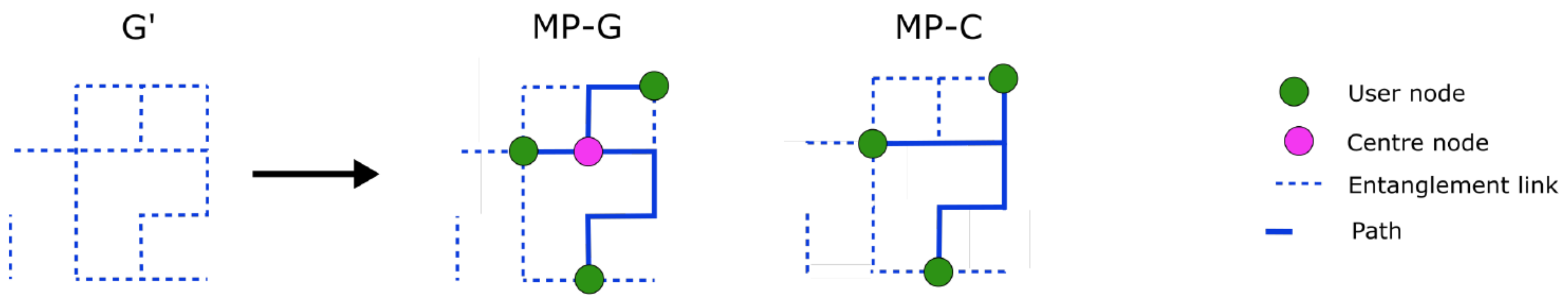


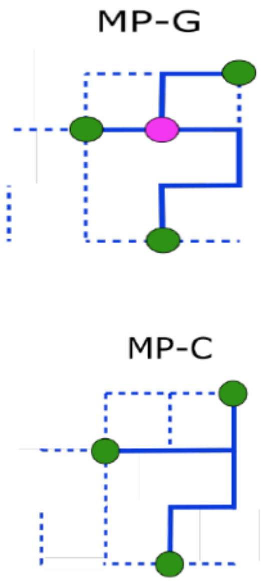
Entanglement rate= 0.5

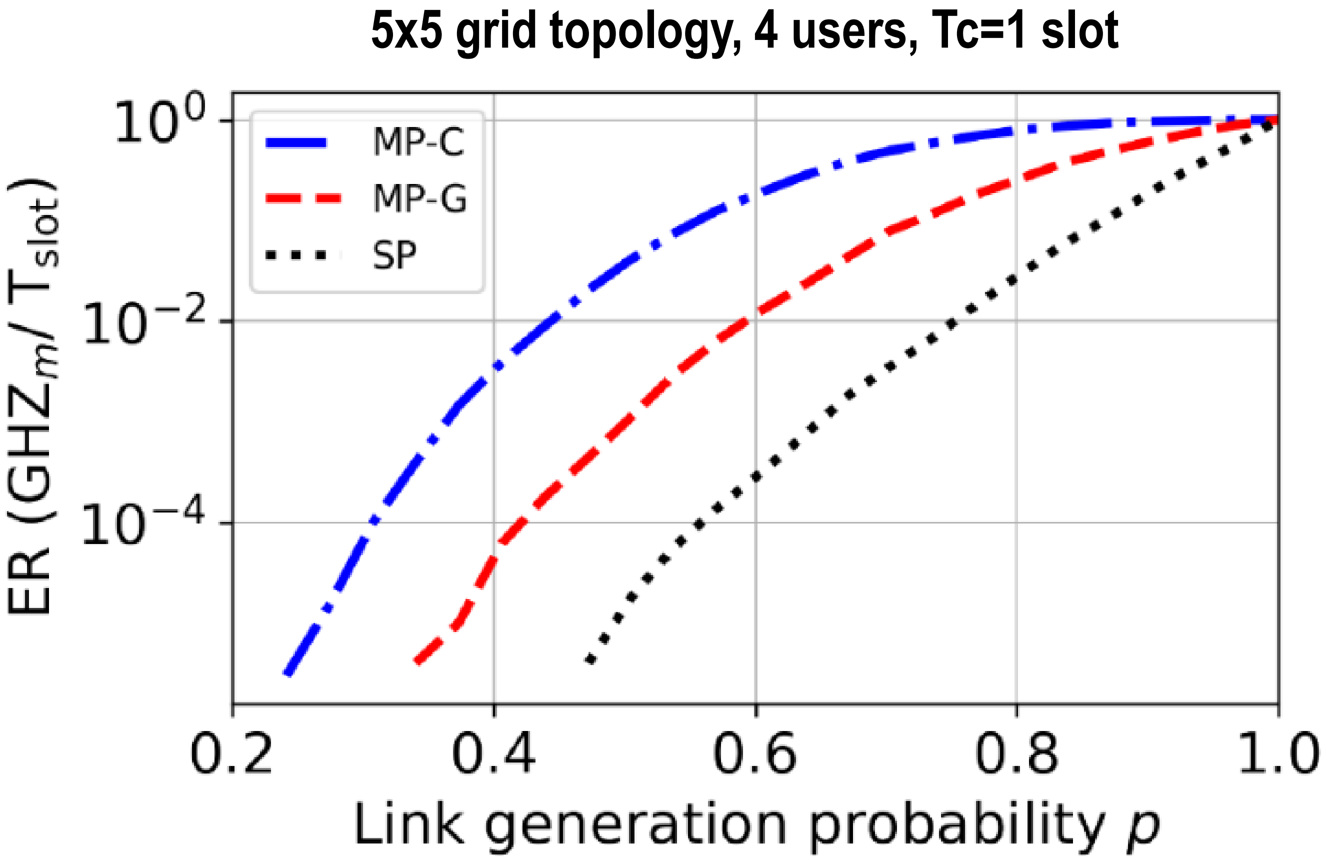
Your guess

Which of the 2 proposed protocols achieve a higher entanglement rate?



Simulation results: Grids I

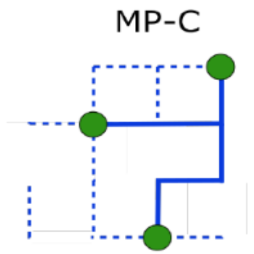


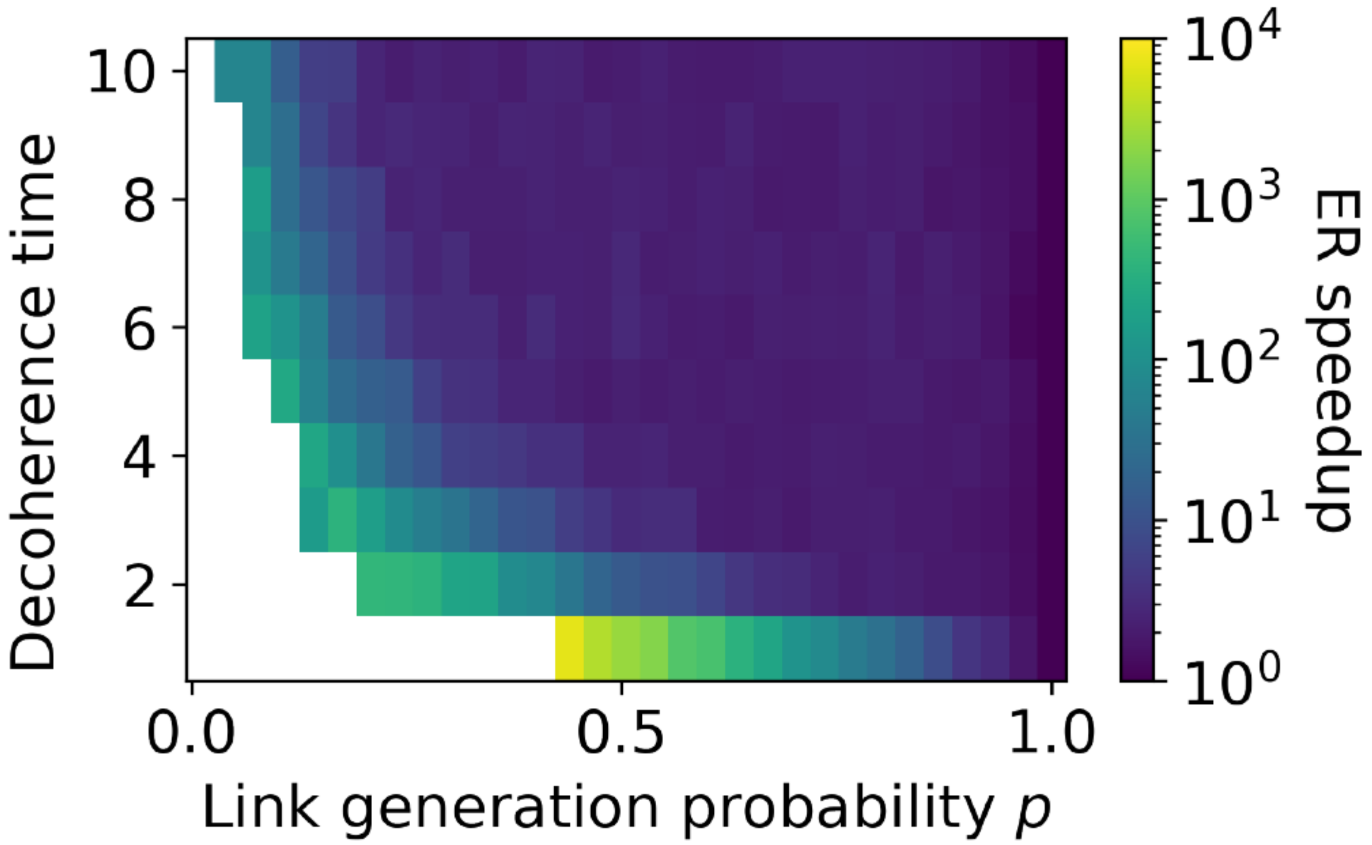


Significant entanglement rate improvement using multipath routing

Simulation results: Grids II

Speed up of MP-C over SP

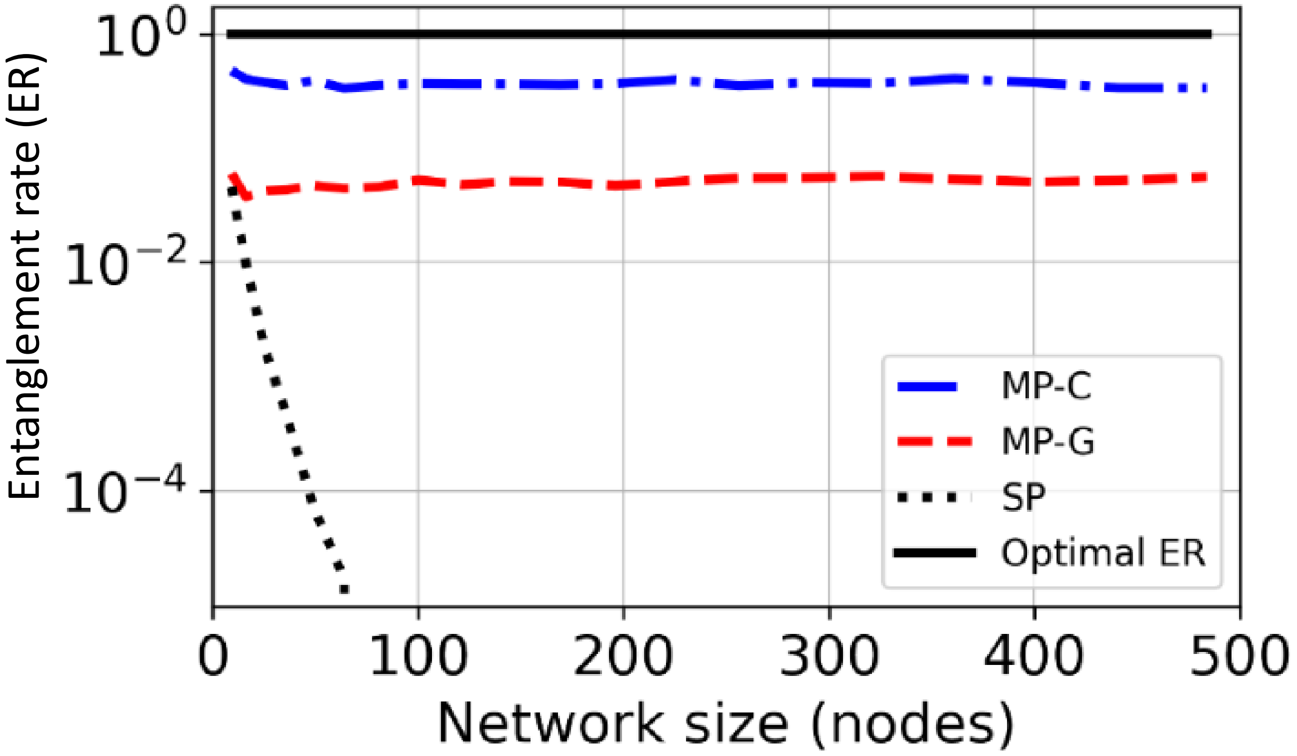




5x5 grid topology, Tc=1 -10 slots

Higher gains where most needed

Simulation results: Grids III

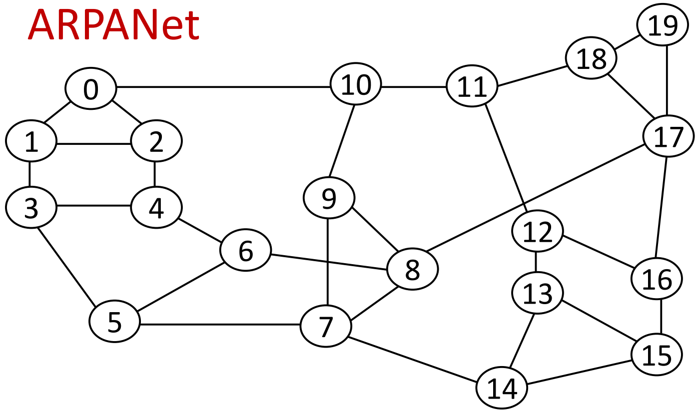


p=0.675, Tc=1 slot

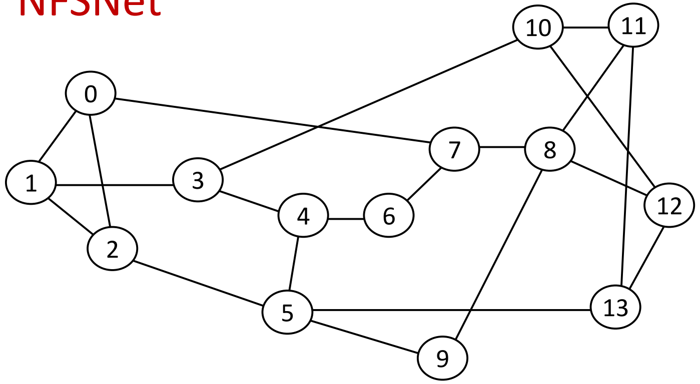
- Multipath routing entanglement rate **independent of network size**
- ER for shortest path routing decays exponentially

Simulation results: Mesh topologies

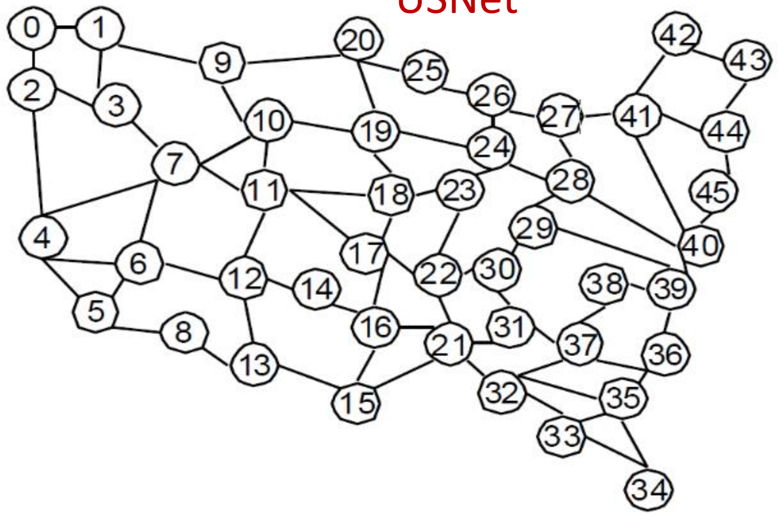
ARPANet



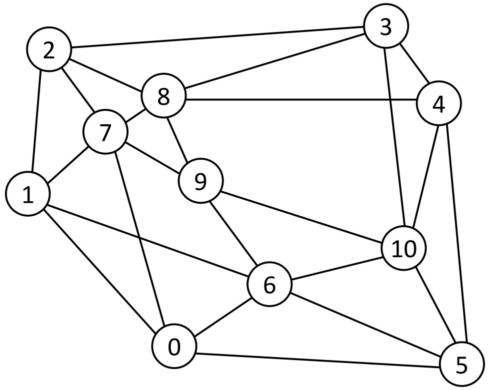
NFSNet



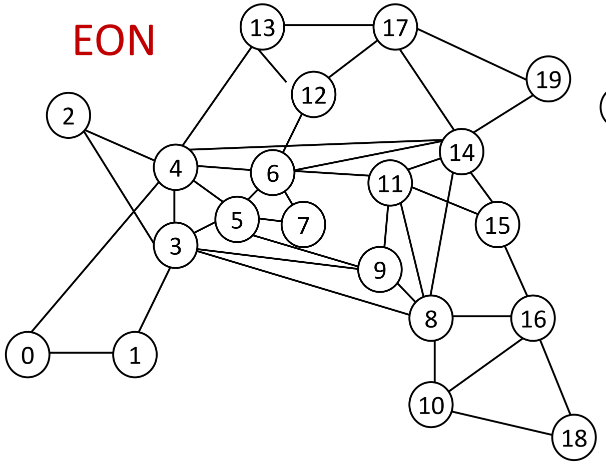
USNet



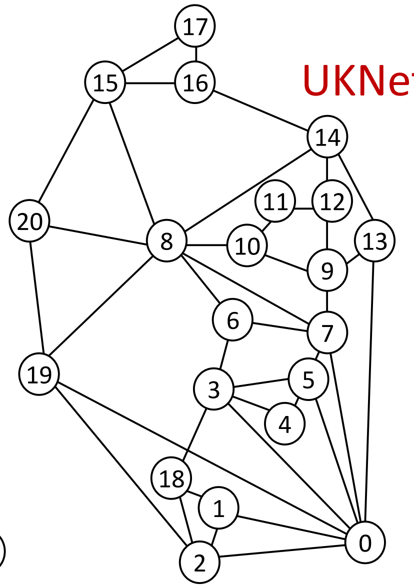
Eurocore



EON

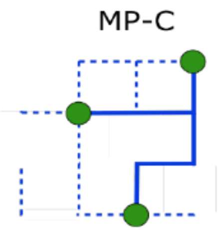


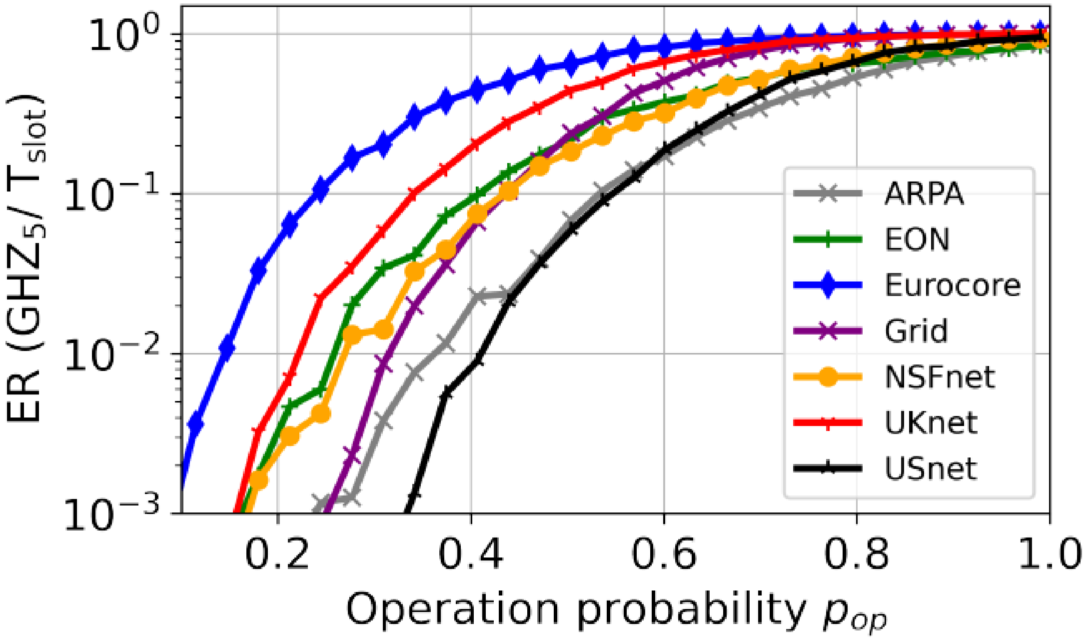
UKNet



Network Name	Nodes	Edges	Average edge length (km)	Average nodal degree
ARPA	20	31	6.09	3.1
EON	20	39	7.24	3.9
Eurocore	11	25	4.26	4.55
NSFnet	14	21	5.09	3.0
UKnet	21	39	1.38	3.71
USnet	46	76	4.34	3.3
Grid (Fig. 2)	36	60	1.0	3.33

Simulation results: Mesh I





10^4 different 5-user combinations, each attempted for up to 5000 timeslots, Tc=1 slot

- High average node degrees (Eurocore, EON and UKNet) networks achieved higher ERs.

Network Name	Nodes	Edges	Average edge length (km)	Average nodal degree
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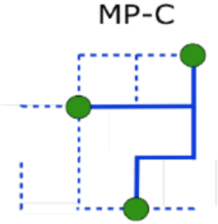
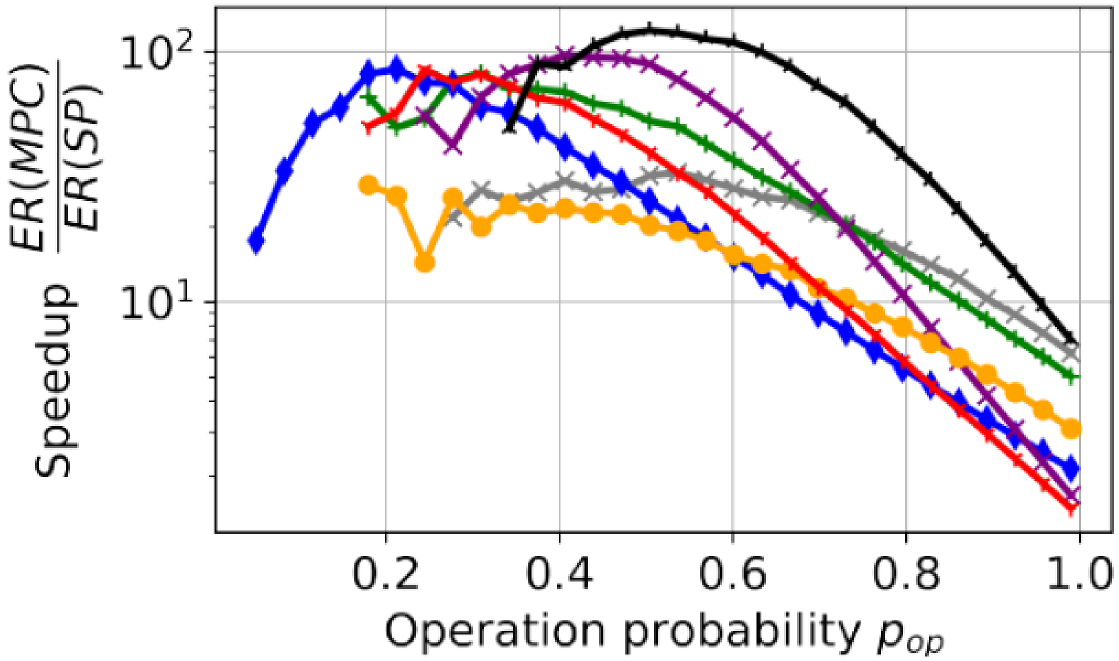
Simulation results: Mesh II



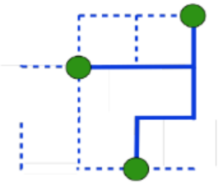


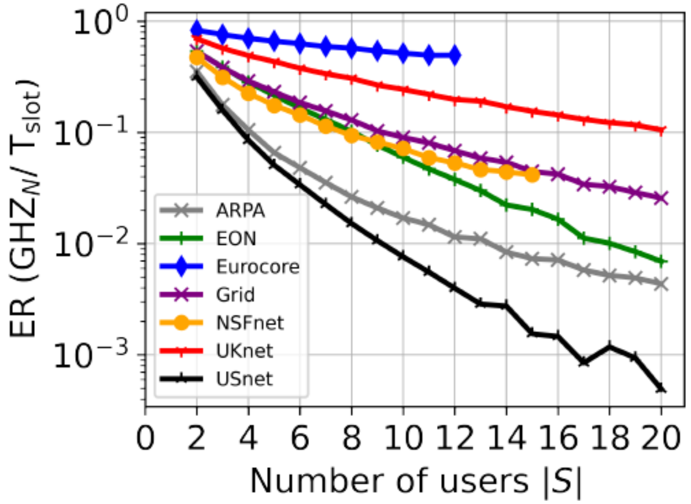
10^4 different 5-user combinations, each attempted for up to 5000 timeslots, Tc=1 slot

Maximum speedup at intermediate values of p_{op} (varied for different topologies)

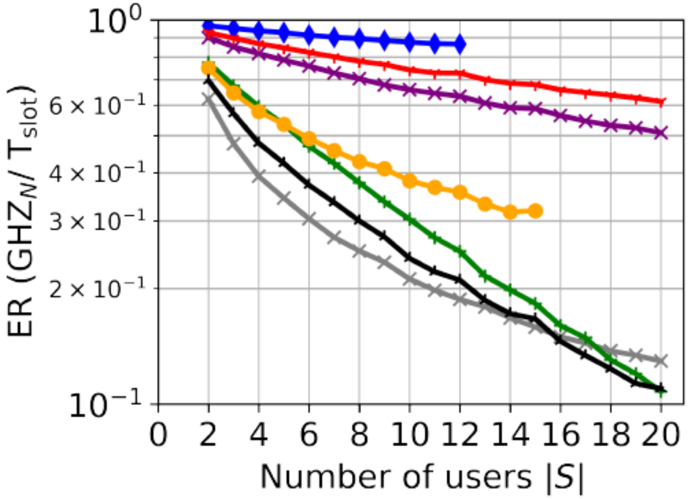
Simulation results: Mesh III

MP-C

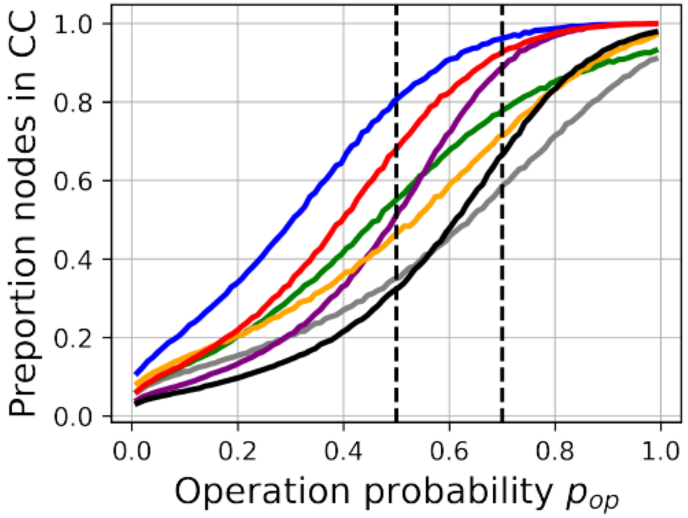




(a) p_{op}=0.5



(b) p_{op}=0.7



(c)

- There is a **critical value of p** at which a high percentage of nodes are in a single connected component.
- This value is **topology-dependant**

Summary

- Multipath **increases ER** several orders of magnitude (NISQ p & T_c)
- Multipath makes **ER size-independent** (for threshold p)
- Multipath **ER is topology-dependant** (nodal degree)