

Editorial

Maximizing the Quality of Learning Opportunities for Every Student

Jinfa Cai, Anne Morris, Charles Hohensee, Stephen Hwang, Victoria Robison,
Michelle Cirillo, Steven L. Kramer, and James Hiebert
University of Delaware

Arthur Bakker
Utrecht University

For five decades, *JRME* has sought to publish high-quality mathematics education research that advances the field's knowledge and has a positive impact on the teaching and learning of mathematics in the classroom. The journal's 50th anniversary represents an opportune time for the research community to take a step back, assess what progress has been made on the major problems of the field, and consider the most important problems that could orient research in the future. As we look across educational scholarship, we find that among the most robust findings from research on teaching and learning is that students' learning is "ultimately determined and constrained by the opportunities they have had to learn" (National Research Council, 2001, p. 31). As the field begins the next five decades of mathematics education research, this familiar, long-standing statement offers a simple but powerful lens that could refocus how researchers address the most important problems facing the mathematics education community. In this year's editorials, we will use this lens as we discuss five overarching problems and associated research questions that we believe the field must address if it is to add important new knowledge and have a more substantive impact on practice over the next 50 years. In each editorial, we will elaborate on one of these overarching problems by identifying a number of more specific research questions related to that problem. In this editorial, we discuss the first of the five overarching problems: defining and measuring learning opportunities precisely enough to study how to maximize the quality of the opportunities experienced by every student.

Refocusing on Learning Opportunities

Maximizing *learning* for every student has long been and continues to be a critical and challenging goal for teachers and researchers. We propose that *learning opportunities* can serve as a productive construct that researchers can use to help the field achieve this goal. Although it may seem obvious that learning opportunities play a significant role in students' learning, this has serious implications for the directions

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of future research (Henningsen & Stein, 1997; Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013; Walkowiak, Pinter, & Berry, 2017). It means that a (if not *the*) significant and enduring problem in mathematics education is understanding how to maximize the quality of learning opportunities for every student.

The construct of learning opportunities (or opportunity to learn) has long been recognized as a mediator between teaching and learning. International assessments have included measures of opportunity to learn as indicators of students' classroom experiences (Floden, 2002; Husén, 1967). Direct assessments of these experiences have documented differences in opportunities as seen through students' eyes (Clarke, Keitel, & Shimizu, 2006). Theories of mathematics learning have used classroom learning opportunities as the site for describing the variations in experiences that explain differences in learning outcomes (Kullberg, Kempe, & Marton, 2017; Marton, 2015).

Despite playing a central role in many research programs and despite the widely cited claim that learning opportunity remains the best predictor of student learning, the construct itself is underdefined. This may be because researchers assume that the concept is clear and does not require a comprehensive and precise definition. Instead, proxies are often used to measure opportunity to learn. Time spent on a topic (Berliner, 1979), whether a topic was taught (Husén, 1967), and characteristics of the tasks used in classrooms (Henningsen & Stein, 1997) have all been used to assess opportunity to learn. The problem is that when learning opportunity is used with vaguely defined criteria and assessed with a wide range of measures, what counts as a learning opportunity loses its meaning. The category is too big and imprecise to have clear research value. As a result, data about learning opportunities cannot be shared across studies, and results cannot be compared or built upon by other researchers. We believe that this helps explain why the field still knows relatively little about what, exactly, counts as a high-quality learning opportunity; what counts as a learning opportunity for a particular student; how to create high-quality learning opportunities for each student; and even how to recognize and measure learning opportunities with precision.

In this editorial, we propose the first overarching problem facing the field of mathematics education to be defining and measuring learning opportunities precisely enough to study how to maximize the quality of the opportunities experienced by every student. Because we believe that the construct of learning opportunities can help researchers and teachers understand the relationships between teaching and learning and, in turn, work toward maximizing the quality of opportunities for every student, we believe that the field should refocus its attention on this construct. In the sections below, we describe challenges in conducting research that directly addresses the measurement of learning opportunities, and we explore what must be done to make such research possible.

The urgency of extending and refining the research on learning opportunities comes, in part, from the fact that high-quality learning opportunities are unequally distributed. There is compelling evidence that some students receive much higher quality opportunities than others (Jackson & Wilson, 2012). There is also overwhelming evidence that the unequal access to high-quality learning opportunities is correlated with a number of interrelated social issues, such as ethnicity, race, culture, language, socioeconomic status, geographic location, and financial

resources, as well as teacher training, retention, and capacity (e.g., Barwell, Moschkovich, & Setati Phakeng, 2017; Darling-Hammond, 2007; Diversity in Mathematics Education Center for Learning and Teaching, 2007; Hedges & Nowell, 1999; Martin, 2000; Martin, Rousseau Anderson, & Shah, 2017; Tate, 2008). As this body of work clearly indicates, researchers in mathematics education are already studying the quality of learning opportunities from a variety of perspectives and have provided important insights upon which future research can build. Nevertheless, much more needs to be done.

Given the nature of our collective perspective as an editorial team, we examine this multifaceted problem from a view that foregrounds the learning opportunities that arise when students participate in classroom activities and engage with instructional tasks. We acknowledge that ours is only one perspective and that we are not directly addressing important opportunity-to-learn factors outside of the classroom. We also acknowledge the importance of mathematics learning that happens as students participate in activities outside of classrooms, such as at home, in museums, and through computer and mobile games (Jackson, 2011; Nemirovsky, Kelton, & Civil, 2017). However, we believe that addressing the research questions we pose here will provide insights that could benefit work conducted from many perspectives, including those that focus on the urgent issues of equity, diversity, inclusion, and learning outside of classrooms that are connected to the problems we discuss.

Before proposing specific research questions for the future, we note that considering learning opportunities entails considering specific learning goals. Particular experiences might constitute a learning opportunity relative to one goal but not another. There are many legitimate goals, and we see the selection of goals as ultimately dependent on value judgments (Hiebert, 1999), although such judgments may be informed by research (van den Heuvel-Panhuizen, 2005). Teachers might select different goals over the course of the school year, and they might pursue multiple goals simultaneously (Lampert, 2001). Goals might be strands of mathematical proficiency, such as procedural fluency, conceptual understanding, or adaptive reasoning (National Research Council, 2001); standards described in policy documents (e.g., College voor Toetsen en Examens, 2019; Department for Education, 2014; Ministry of Education of China, 2011; National Council of Teachers of Mathematics [NCTM], 2000; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010); mastery of specific content or skills; development of specific beliefs about mathematics or identities as students of mathematics; and so forth. Consistent with our perspective, the learning goals we have in mind are mostly focused on the context in classrooms where students are working to develop understanding, skills, beliefs, and dispositions in specific content domains (Cai et al., 2017a). However, we believe that the approach we take could be useful for other kinds of learning goals as well.

What Kinds of Measures and Research Designs Will Reveal the Nature of Learning Opportunities?

Given that the overarching problem for this editorial is defining and measuring learning opportunities precisely enough to study how to maximize the quality of

the opportunities experienced by every student, the first research question we propose is about the research process itself: What kinds of measures and research designs will reveal the nature of learning opportunities? We believe that researchers could study and develop research measures and research designs that will be especially sensitive to identifying and describing the nature of learning opportunities. As noted earlier, the construct of learning opportunities is under-defined. How do researchers and teachers know when an opportunity to learn actually exists? How do they know when a student is experiencing or taking advantage of a high-quality learning opportunity? Without answers to these questions, there is no way to know whether teachers are maximizing the quality of learning opportunities for every student in their classrooms. The classroom activities and instructional tasks that maximize one student's opportunity may be rather different from those that maximize another's. Despite previous work, it is not easy for us to specify what researchers could do to begin addressing these questions with the precision and depth required to answer them.

We propose that a first step toward studying what counts as a high-quality learning opportunity in the classroom is to figure out what this phenomenon is and how it can be studied. Researchers must develop complete and precise descriptions of what counts as a learning opportunity that helps particular students achieve a particular learning goal. Without such descriptions—that is, without specific operationalizable definitions of learning opportunities—it would not be productive to conduct research on the presence of learning opportunities in classrooms, on the effects of particular learning opportunities on students' learning, or on interventions that aim to improve the quality of learning opportunities in classrooms. Designing measures goes hand in hand with developing definitions. In addition, we believe it crucial that researchers study the nature of research designs that are best suited to studying the nature of learning opportunities.

In the subsections below, we unpack further the larger research question posed in the section heading (i.e., “What Kinds of Measures and Research Designs Will Reveal the Nature of Learning Opportunities?”) into subquestions. By posing these subquestions as research questions, we are claiming that there are critical aspects of these questions that can and should be answered empirically.

Developing Innovative Measures That Assess Features of Learning Opportunities

What can, and should, be measured? As we noted earlier, the selection of learning goals is ultimately based on value judgments. But once the learning goals are specified, researchers can ask what counts as learning opportunities that help students achieve the goals. We begin our discussion of measuring learning opportunities by reiterating the need for defining precisely the construct of learning opportunity. In our view, any definition of a classroom-based learning opportunity is likely to focus on the interactions among three factors—mathematical tasks, teaching, and students (Cohen, Raudenbush, & Ball, 2003). Interactions among the factors in this instructional triangle are likely to define the major elements of a classroom learning opportunity. What is not clear is how aspects of each of these factors matter for creating learning opportunities or for maximizing their quality.

How does a particular constellation of tasks, instructional moves, and engagement of students create a high-quality learning opportunity? We believe that it will be impossible to isolate the effects of any one of these factors because the nature of their interaction will determine whether a classroom activity or experience becomes a learning opportunity for particular students relative to a specified goal. So, the empirical question we pose here is *What kinds of interactions among tasks, teaching, and students create learning opportunities for a specific learning goal?*

Although the field is far from developing a consensus and operational definition of learning opportunity, some researchers have begun this work. As noted earlier, it has been included in international assessments and figures prominently in some mathematics learning theories. In addition, researchers who have investigated inequities in learning opportunities within and across classrooms have necessarily assessed the nature of learning opportunities available to students in their samples (e.g., Boaler, 1998; Boaler & Selling, 2017; Gutiérrez, 1996, 2000; Jackson et al., 2013; Martin, 2000). This work provides a good starting point for developing comprehensive and precise definitions of learning opportunities.

How can researchers develop these measures? Developing measures of any construct depends on having a clear operational definition of the construct. In fact, if a valid measure can be created, researchers often take this as evidence that the operational definition of the construct is robust. However, as measures are being developed, it can be the case that difficulties in designing the measure signal that the construct needs further definition. It can also be the case that a measure must be adjusted in response to changes in the definition. Thus, defining a construct and developing a measure for the construct are two sides of the same coin, and they can inform each other.

Because learning opportunities are created through the interactions of (at least) tasks, teaching, and students, measures of learning opportunities will need to develop through analyzing interactions among these factors and describing how some interactions help students achieve a specified learning goal more than others. The interactions among the three factors create complexities that will likely be managed only by multiple iterations of studies that carefully build on successive findings. Multiple researchers will need to collaborate by designing preliminary measures, sharing them, trying them out in multiple settings, comparing findings, revising the measures, and repeating the cycle multiple times. As we have stated above, one way to reduce complexity is to clarify, from the beginning, the learning goal against which the potential learning opportunities will be evaluated. This will constrain to some extent the number of scenarios that must be anticipated. For example, assessing the contributions of tasks, teaching, and students will likely look different if the learning goal is the memorization of basic facts versus understanding the role of zero in computation procedures versus developing perseverance in solving challenging mathematical problems.

Although we do not underestimate the daunting challenge of designing and developing these measures, we believe this work worth doing because it is likely to yield considerable benefits. One benefit of making progress on this research agenda will be a set of measures that can be used by all researchers engaged in assessing the quality of learning opportunities for a variety of learning goals.

Shared measures would then allow researchers from different perspectives to fill in pieces of the puzzle that actually fit together. Sharing data across research programs will enable the field to make clear and cumulative progress in studying learning opportunities and how to maximize their quality.

A second benefit of developing measures of learning opportunities is that they will provide well-specified targets for classroom interventions. If researchers use the instructional triangle as a starting point for defining learning opportunities, the measures they create will assess much of what researchers often call the *conditions* of classroom learning. We have argued in earlier editorials that the effects of classroom interventions depend on such conditions (Cai et al., 2017b, 2017c, 2018). Developing measures of learning opportunities thus means describing and accounting for these conditions. Because learning opportunities can be considered mediators between teaching and learning (i.e., teaching helps students achieve learning goals if it creates productive opportunities for students to learn), classroom interventions can be focused on creating the conditions—the interaction patterns—that the measures capture as contributing to high-quality learning opportunities.

To repeat, the research question we pose here is *How can researchers develop measures of learning opportunities?* We note that this question is one that entails conducting research on a key process of doing research. We believe that there are more and less productive ways for researchers to develop measures of learning opportunities, but the field has little or no evidence about what these ways are. This leaves an opening for empirical research.

What Kind of Research Designs Will Be Useful?

Thus far, we have argued that until the field has better descriptions of learning opportunities and better ways to measure them, researchers will have a great deal of trouble studying the impact of improvement efforts on changes in students' learning. Earlier, we also suggested that one likely aspect of research designs that address this problem will be iteration. In other words, to make progress on the previous questions, researchers can expect to conduct multiple (often small) studies that move incrementally toward more comprehensive and precise answers. We conjecture that many research methods commonly used to generate descriptive and evaluative data in educational settings will be useful for conducting research on learning opportunities. For example, observations of classroom interactions, analysis of lesson videos, examination of student work, evaluation of mathematical tasks, assessment of students' entry knowledge and their level of engagement, and documentation of instructional moves by the teacher could all be useful approaches to examining learning opportunities. In addition, gathering information on each student's perspective will likely play an important role (Clarke, 1997; Clarke et al., 2006).

However, we expect that new approaches will also need to be developed to conduct this research. Our field has a history of using new and different approaches as researchers have addressed new questions (Inglis & Foster, 2018). Teaching experiments, action research, discourse analysis, multilevel statistical models, and design research are examples of research designs, methodologies, and tools that researchers have developed to study the phenomena and contextual factors involved in the learning and teaching of mathematics. Innovation in mathematics

education research often involves using creative methods that go beyond the usual research paradigms. In fact, this issue of *JRME* includes an article using social network analysis to better understand students' individual and collective study habits (Alcock, Hernandez-Martinez, Godwin Patel, & Sirl, 2020) as well as an article that uses the statistical technique of propensity weighting to examine the complex connections between students' attitudes and mathematical preparation and their mathematics achievement (Sonnert, Barnett, & Sadler, 2020).

Given how challenging it may be to conduct consequential research on learning opportunities, it is likely that new research designs, methods, and instruments will be needed. Answering questions about the learning opportunities that students experience might require immensely complex analyses of potentially overwhelming amounts of data (see, e.g., Nuthall's [2004] approach, which involved a comprehensive collection of audiovisual recordings of every student throughout a lesson). Our field has not yet figured out how to do this kind of work without radically paring down our outcomes of interest. Thus, we offer the following research question: *What research designs and methodological tools will enable us to investigate questions about the quality of learning opportunities that students experience?*

How Can Researchers Study the Quality of Learning Opportunities for Students in Classroom Settings?

Armed with operational definitions of learning opportunities, measures that can identify and describe them, and research designs that enable productive work analyzing learning opportunities in classrooms, researchers can begin studying the quality of the opportunities. The questions now move beyond deciding whether a classroom experience provides a learning opportunity for a particular goal to assessing the degree to which an opportunity helps students achieve that goal. The quality of a learning opportunity is likely determined by the same three classroom factors and their interactions that we have already discussed: tasks and their potential for supporting learning, the teaching moves that are used to implement the task, and the entry knowledge and dispositions of the students that might affect their inclination to engage in and access the learning opportunity. As one example of different interactions among these factors that could create learning opportunities of different quality, consider the QUASAR project and the changes that were induced by the nature of the teaching (Stein, Smith, Henningsen, & Silver, 2000). Although a mathematical task was the same, the teacher could affect the experience that students had with the task by working on it with students in different ways.

How might researchers study the contribution of these factors as they interact to create learning opportunities of different levels of quality? In our first set of editorials running through the 2017 and 2018 issues of *JRME*, we proposed a vision of future research in mathematics education that followed a radically different pathway than most research today. In this vision, teachers and researchers work as partners to identify instructional problems and engage in a process, similar to iterations of design research, that would involve repeating cycles of making and testing small changes across multiple, contextually similar classrooms that share

an instructional problem. We suspect that these teacher–researcher partnerships could produce the kind of fine-grained research needed to study the quality of learning opportunities. How might this play out in research that addresses each of the three parts of the instructional triangle? In the subsections below, we unpack the larger research question posed earlier in the section heading (i.e., “How Can Researchers Study the Quality of Learning Opportunities for Students in Classroom Settings?”) into subquestions that could be investigated through such partnerships.

How Do Students’ Entry Competencies and Dispositions Contribute to Creating and Realizing Learning Opportunities?

A range of entry competencies and dispositions may influence whether an individual student actualizes the full potential of a learning opportunity. These competencies and dispositions include affective factors, such as mathematics anxiety or self-efficacy, and cognitive factors, such as how well the student understood the previous day’s lesson or whether the student already understands a concept to which he or she is able to connect concepts introduced in the current lesson. Our primary point here is that an opportunity presented by the teacher could be taken up by one student but not another because of individual differences. Indeed, the Sonnert, Barnett, and Sadler (2020) article in this issue attempts to disentangle two such characteristics: attitude toward mathematics and mathematical preparation. Research is needed to identify what individual competencies and dispositions are salient to learning opportunities and in what ways they interact with teaching and tasks.

How Does the Task Contribute to Creating and Realizing Learning Opportunities?

A substantial body of research has investigated how characteristics of tasks and their implementation are associated with different kinds of student thinking and learning (Sullivan, Knott, & Yang, 2015). Doyle (1988) argued that the work that students do creates their learning opportunities and that tasks that students are given set the parameters for the work they do. Stein and Lane (1996) found a relationship between the level of cognitive demand of mathematical tasks and the level of mathematical understanding that students develop. We believe that the mathematical task plays a central role in the kind of interactions that are possible and the nature of learning opportunities created. But the specific influence that task characteristics have on learning opportunities has not been thoroughly studied in ways that would support causal claims about the impact of certain characteristics of tasks on students’ learning (Otten, Webel, & de Araujo, 2017). There remains much to learn about precisely how the task interacts with teaching and students to create learning opportunities of particular quality.

How Does Teaching Contribute to Creating and Realizing Learning Opportunities?

Even when tasks are of high quality and intended to be cognitively demanding, the implementation of the task has a powerful influence on whether the desired learning opportunities are realized. Several studies have documented ways in

which teaching has a critical effect on the potential of a task. For example, the TIMSS video study (Hiebert et al., 2003, 2005) provided clear evidence of differences in teaching across countries—differences that meant tasks with similar potential for high cognitive demand apparently resulted in very different kinds of opportunities to learn. Thus, a goal of research on the quality of learning opportunities should be to help teachers focus their teaching on creating the conditions for high-quality learning opportunities.

NCTM (2014) has highlighted a set of effective mathematics teaching practices that research suggests could support high-quality mathematics learning, presumably by creating high-quality learning opportunities. For example, the teaching practices advocated in *Principles to Actions: Ensuring Mathematical Success for All* include “use and connect mathematical representations” and “facilitate meaningful mathematical discourse” (NCTM, 2014, p. 10). Indeed, as Jackson, Garrison, Wilson, Gibbons, and Shahan (2013) found, the ways that teachers engage in such practices to introduce a task appear to influence students’ opportunities to learn in the ensuing whole-class discussions. Again, however, research is needed to study, at a fine-grained level, how particular teaching practices (and even specific teaching moves) can contribute to the quality of learning opportunities. This research can then inform teacher education and professional development efforts.

Which Research Approaches Are Best Suited to Studying Differential Instruction as a Way to Maximize the Quality of Learning Opportunities for Every Student?

In this section, we focus on the last part of the question posed as the title for this editorial, namely how to maximize the quality of learning opportunities *for every student*. In other words, we are joining the idea of creating high-quality learning opportunities with the idea of differentiating instruction. Specifically, we are focusing on how instruction can be differentiated to maximize the quality of the opportunity for each student. Although differentiating instruction is not a new concept and maximizing each student’s learning is in danger of becoming an overused slogan, we believe that a learning-opportunities lens can offer a useful perspective for making progress on this problem.

There will be substantive differences among the individuals in any group of learners as to whether a specified instructional task provides them with a learning opportunity. As such, different students might need different opportunities to maximize their learning. Or, from a different perspective, instruction might need to be varied to assist every student to engage with an equally high-quality learning opportunity. Clearly, this problem is incredibly complex. We simplify the problem somewhat by asking subquestions about tailoring instruction for groups of students rather than for each individual student.

As indicated above, every classroom teacher must deal with significant differences among students. The differences will fall along multiple dimensions—gender, race, ethnicity, language proficiency, socioeconomic status, level of entry knowledge and skills, dispositions toward mathematics, and so on. Therefore, multiple complexities exist. How, then, do teachers maximize the learning opportunities for groups of students who differ on one or more of these dimensions? What

kind of research would contribute to building useful information for teachers, who often struggle with finding answers to this question? Moreover, given that efforts to differentiate instruction for groups of students can produce unintended consequences (e.g., labeling certain students or groups of students as less able to learn complex mathematics and fostering negative mathematics identities for some students), how might educational research in this area avoid such pitfalls?

Once again, we propose a research question that is a metaquestion about the research process itself: *What research questions could be posed about differentiating instruction for groups of students that can be addressed empirically with answers that are useful to teachers?* In other words, what might researchers ask about the learning opportunities created by using tasks or teaching implementations that differ across groups of students?

We can offer three observations for researchers contemplating this work. First, there is much work to build on. For example, researchers have done extensive work on gender differences in mathematics and what those differences might mean for instruction (Hanna, 2002; Leder, 2019; Lubienski & Ganley, 2017). A good deal of work has also explored productive differentiation in instruction for African American students (Jackson & Wilson, 2012; Martin et al., 2017). In addition, researchers have considered the implications of other dimensions, such as language proficiency (Barwell et al., 2017; Halai & Clarkson, 2016) and dispositions toward mathematics, including phenomena such as mathematics anxiety (Mammarella, Caviola, & Dowker, 2019; Middleton, Jansen, & Goldin, 2017). Researchers would be wise to read extensively in these and similar areas, both within and outside of mathematics education. A second observation is that it makes sense to study groups determined by dimensions that are relevant to mathematics learning. A clear rationale or hypothesis should be established that demonstrates the likelihood that differences along a particular dimension could help determine the kinds of learning opportunities that will maximize the learning for these students. A final observation is that single dimensions do not define students. Any group defined along a single dimension will include subgroups at the intersection of other dimensions (e.g., historically marginalized groups). Thus, students within any group will have significant individual and subgroup differences, many of which might affect what counts as a high-quality learning opportunity for them. Thus, research on learning opportunities will need to account for different intersections of group membership and the ways that these intersections may afford or constrain the learning opportunities available for specific subgroups of students.

Conclusions

Ultimately, the goal of educational research is to improve students' learning. In order to learn, students must have opportunities to learn. However, the field of educational research, in general, and mathematics education research, in particular, has thus far only begun to understand what learning opportunities are and how to maximize their quality for every student. In this editorial, we chose to turn our attention to the nature of the research agenda that focuses on this overarching problem. Among the empirical questions we have raised, we proposed three research questions that, in essence, are questions about research itself: (1) What

kinds of measures and research designs will reveal the nature of learning opportunities? (2) How can researchers study the quality of learning opportunities for students in classroom settings? and (3) What research approaches are best suited to studying differential instruction as a way to maximize the quality of learning opportunities for every student? We do not claim that these and their associated subquestions are the only research questions that need to be answered to maximize the quality of learning opportunities for every student nor do we claim that our focus on learning opportunities is new. Rather, we pose these questions because they are enduring, pressing questions that have yet to be adequately addressed. Moreover, we believe that taking learning opportunities seriously as a theoretical and methodological lens can allow the field to make real progress on persistent, challenging problems in the teaching and learning of mathematics. We intend this editorial to serve as a springboard for the field of mathematics education to collectively and systematically address the overarching problem in education research of studying and maximizing the quality of learning opportunities for every student, and we believe the outcomes of this work can have a significant impact on instructional practice in classrooms.

In our March 2020 editorial, we will turn our attention to another overarching problem: In many countries, new efforts to improve classroom teaching and learning must always start over. Indeed, most teachers must also start over when they begin teaching, unable to take advantage of what veteran teachers have already learned. In educational systems around the globe, there is not yet a robust infrastructure for capturing, recording, and sharing knowledge about what works in actual classrooms to improve the learning opportunities that students experience. Rather than accumulating knowledge in artifacts that could be shared and improved over time, the best ideas are lost when those who remember them leave the profession. We see this as an overarching problem in mathematics education. Thus, in the next editorial we will discuss key research questions that we believe could help the field make progress on this problem.

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