Chapter 21

Using hypothetical learning trajectories in design research

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Summary

This chapter provides an example of a design research project to substantiate the idea of a hypothetical learning trajectory as discussed in Chapter 3, and to illustrate the type of results that can be expected from analyzing the differences between hypothesized and observed learning. Because hypothetical learning trajectories are often too elaborate to publish in journal articles, we also illustrate how summaries of our findings could be formulated in the form of design principles and conjecture maps.

Background and research question

The key problem addressed by Smit (2013) was that so many multilingual students in mathematics classrooms do not have sufficient linguistic proficiency to participate. An important goal was to give all students, including native speakers with low proficiency in Dutch, access to the language required for mathematical learning. From descriptive research much is known about the problems second-language learners experience in learning subject-specific language. Much less is known, however, about how to remedy the situation, so interventionist research in this domain (Moschkovich, 2010) leading to actionable knowledge is needed. Smit argued that genre-based pedagogy (Gibbons, 2002) and the idea of scaffolding would be useful sources of inspiration, but these ideas had not yet been worked out for mathematics education. The main research question was: *How can teachers in multilingual primary classrooms scaffold students' language required for mathematical learning?*

The mathematical domain that was central in the intervention was the domain of line graphs – a mathematically and linguistically challenging domain.

It is, for example, known that students may be good at pointwise reading of graphs but typically much less so at interpreting segments of graphs that represent periods in time (Leinhardt, Zaslavsky, & Stein, 1990). As explained later in this chapter, the learning goals were summarized in terms of an "interpretative description of line graphs" specified in terms of linguistic and structural features of this text-type (a domain-specific genre). Answering the main question required many stages, including an underpinning of the learning goals, iterative cycles of design, testing, and evaluation (Chapters 1 and 4). Within this design research process we considered it important to address several questions including the following evaluative one: *How did students' proficiency in a genre for interpreting line graphs develop?*

An answer to this question was necessary to check whether students indeed developed the language skills required for interpreting line graphs as planned. Without an at least partially affirmative answer it would be impossible to answer the main question.

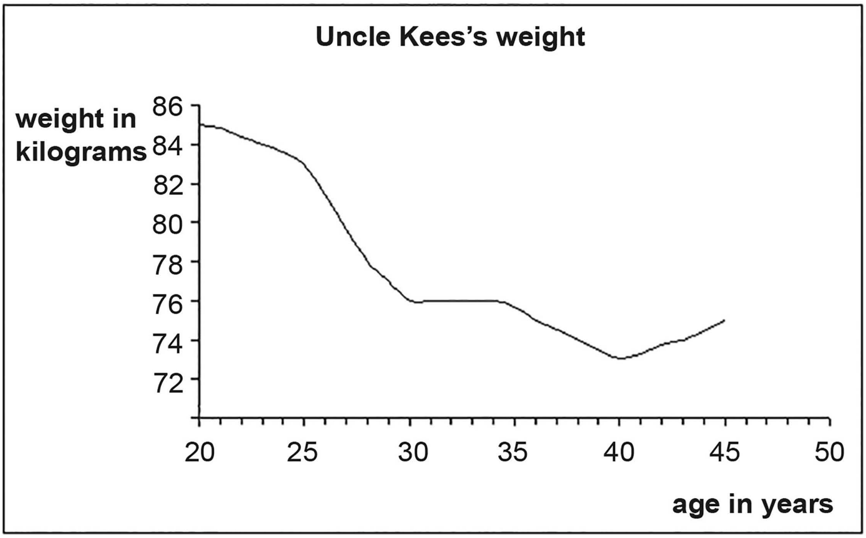
In this chapter we focus on using hypothetical learning trajectories (HLTs, see Chapter 3), in particular how they can be compared with the observed learning of four case study students. Simon (1995) defined these as follows:

> I use the term "hypothetical learning trajectory" to refer to the teacher's prediction as to the path by which learning might proceed. It is hypothetical because the actual learning trajectory is not knowable in advance. It characterizes an expected tendency. Individual students' learning proceeds along idiosyncratic, although often similar, paths.
>
> (p. 135)

> The hypothetical learning trajectory is made up of three components: the learning goal that defines the direction, the learning activities, and the hypothetical learning process – a prediction of how the students' thinking and understanding will evolve in the context of the learning activities.
>
> (p. 136)

Elsewhere we have addressed the question of proficiency development by means of pre- and posttest results of the whole group and a single case study (Smit, Bakker, Van Eerde, & Kuijpers, 2016). The purpose of presenting the analysis is to show how HLTs can be used to gain insight into the process of supporting students to get from A to B.

This emphasis on process rather than just two points of measurements (pre- and posttest) may sound laudable, because this emphasis on students' learning processes is not often seen in experimental studies such as randomized controlled trials. However, we hasten to add that this process analysis did not end up in a journal article because the analysis procedure and results were rather tedious and wordy. This consideration is the reason why we also explore in this chapter what summaries of our findings in terms of a design principle or a



At the age of 20 Uncle Kees weighs 85 kilograms. Between his 20th and his 25th birthday, he slowly loses weight. The graph descends gradually. Between his 25th and his 30th birthday his weight decreases quickly. You can tell as the graph shows a steep fall. From his 30th to his 35th birthday his weight remains more or less the same. The graph is constant. Between his 35th and his 40th birthday he slowly loses weight; the graph gradually descends. When Uncle Kees is 40 his weight reaches its minimum: about 74 kilograms. From the age of 40 on his weight increases slightly. In this part, the graph gradually rises.

Figure 21.1 Line graph and exemplary text from a genre for interpreting line graphs

conjecture map look like. This chapter thus also tries to address the question of how insights from design research can be presented in a succinct form.

Summary of the design

To answer the main question, Smit (2013) conducted three cycles of design research in schools with a high percentage of multilingual students, mostly first- and second-generation immigrants. After two cycles of co-design with an experienced teacher (Smit & Van Eerde, 2011), the design consisted of nine lessons accompanied by:

1 A specification of the *learning goals*: mathematical understanding of line graphs but also awareness of the features of the genre in which Smit and the teacher wanted students to talk about line graphs. For an exemplary text in the genre, see Figure 21.1. The structure and linguistic features of

Table 21.1 Structure and linguistic features of the genre

	Structure features	*Examples*
	A student proficient in this genre . . .	
S1	describes each segment in terms of what happens in reality	*Between his 25th and his 30th birthday his weight quickly diminishes*
S2	describes each segment in terms of the course of the graph	*The graph descends gradually*
S3	describes the starting point of the line graph	*At the age of 20 Uncle Kees weighs 85 kilograms*
S4	describes peaks and troughs when present in the graph	*When Uncle Kees is 40, his weight reaches its minimum: about 74 kilograms*
	Linguistic features	*Examples*
	A student proficient in this genre . . .	
L1	includes general academic language in the interpretation of reality	*. . . his weight decreases quickly*
L2	includes topic-specific mathematical language in the description of the course of the graph	*descends gradually*
L3	distinguishes between gradations (e.g., of steepness) to express mathematical precision	*The graph shows a steep fall* *The graph descends gradually* *He slowly loses weight*
L4	uses words such as *as*, *at*, *in*, and *when* to refer to moments in time (i.e., points in the graphs)	*At the age of 20* *In 2010*
L5	uses word combinations such as *from . . . to*, *between . . . and*, and *from . . . onward* to refer to periods in time (i.e., segments of the graph)	*Between his 20th and his 25th birthday* *From 2010 to 2012*

the genre were based on the literature (Table 21.1) and informed by interviews with mathematics educators on what ways of talking they would find desirable at this level of education (Grades 5–6, when students are about 9–11 years old).

2 A series of lessons including instructional activities. The overall series was informed by the idea of scaffolding: temporary adaptive support, which we characterized in terms of diagnosis, responsiveness, and handing over independence (Smit, Van Eerde, & Bakker, 2013). The idea of scaffolding in turn informed the structure of the genre-based lesson series in four phases: building the field, modeling the genre, joint construction, and independent writing (the teaching and learning cycle as Gibbons, 2002, calls it).

3 HLTs for each lesson (see Table 21.3 for an example). These HLTs were adaptations of HLTs used in earlier design cycles, which in turn were informed by information gathered on students' starting points (pretest), specifications of the learning goals, and progressive insight from the design cycles along with continuing to read the relevant literature and discussing intermediate findings with researchers and the co-designing teacher. Furthermore, the HLTs functioned as the teacher's guidelines for what to focus on in her teaching and what to look out for in student contributions so she could be responsive to diagnoses she made of student linguistic proficiency levels.

4 A repertoire of *scaffolding strategies* that the teacher was asked to use during the lessons, in particular during whole-class discussion (Table 21.2). This list was the result of reading the literature on scaffolding but also filtering out the most effective ones in earlier design cycles. This repertoire (along with the design of the instructional sequence of nine lessons) is an important element in the answer to the research question, namely how teachers can scaffold students' language required for mathematical learning.

Table 21.2 Strategies for scaffolding language and examples for each strategy

	Strategies	*Examples*
1	Reformulating or extending students' spoken or written utterances	[In response to *the graph goes higher and higher up*:] *Yes, the graph does rise steeply.*
2	Explicitly referring to or reminding of linguistic features (e.g., topic-specific words or temporal prepositions), or doing so implicitly by referring to or pointing at the word list, or by referring explicitly to supportive gestures	*Look, the word you are looking for is written down here.*
3	Explicitly referring to or reminding of structure features (e.g., the use of a specific type of language such as topic-specific language)	*Into how many segments can we split the graph?*
4	Asking students to improve language (e.g., asking for more precise language) or to elaborate their utterance	*How can we rewrite this in more mathematical language?*
5	Repeating correct student utterances	*Yes, the graph does descend slowly.*
6	Asking for or explicitly encouraging students to independently produce spoken or written language	*And now try to formulate a sentence yourself.*

Table 21.3 HLT as formulated for lesson seven

Mathematical and linguistic goals
• Students view a line graph as a representation involving different segments (stages) and can determine these segments (stages) independently. • Students develop a richer understanding of what points and segments of a line graph represent. • Students describe each segment in terms of reality and in terms of the course of the graph, using previously made agreements written on the whiteboard. • Students describe reality by deploying general academic language (e.g., his *weight decreases slowly*) and they describe the graph's course by deploying topic-specific language (e.g., the *graph descends gradually*); in both cases they make use of the growing word list on the classroom wall. • Students can relate words belonging to general academic language to words belonging to topic-specific language in terms of meaning (e.g., the relation between *decrease* and *descend*). • Students can correctly use temporal prepositions (*from . . . to*; *between . . . and*; *at*; *in*) for referring to points of the graph (moments in time) or segments of the graph (periods in time).
Starting points
• Students have been introduced to topic-specific words (constant, axis, etc.). • Students are familiar with reading off information from line graphs and tables. • Students have collaboratively constructed graphs themselves. • Students have difficulty interpreting changing direction in a graph. • Students have been introduced to the idea that a graph can be divided into segments and that each segment can be described in terms of reality and in terms of its direction. • In most cases, students include the description of important points (peaks, troughs, starting point) when describing and interpreting line graphs. • Some students still have difficulty using temporal prepositions to refer to periods or moments in time: *from . . . to*, *between . . . and*, *at*, *in*, etc.
Instructional activities
1 Teachers and students collectively divide a line graph representing Uncle Kees's weight into segments. 2 Students match sentences about Uncle Kees's weight (reality) and about the course of the graph with the different segments of the graph, followed by a whole-class discussion. 3 Teacher explains temporal prepositions to the whole class, visually supported by using timelines; whole-class discussion of examples. 4 Students conduct a writing activity in which they fill in temporal prepositions in a writing frame containing a text about Uncle Kees's weight in the targeted genre.

Table 21.3 (Continued)

Assumptions about how the instructional activities support mental activities that lead to the mathematical goals
(Ad 1) In discussing how to divide a line graph in different segments, students realize that changes in the course of the graph (direction) represent changes in reality. (Ad 2) In matching sentences with segments of the graph, students' understanding of gradations of steepness in the graph is promoted. (Ad 3) By using a timeline in visualizing the use of temporal prepositions, which is related to the horizontal axis in the representation, students' mathematical understanding of moments and periods in time is promoted. (Ad 4) In consciously employing temporal prepositions in the activity of interpreting and describing a line graph, students develop conceptual understanding of points versus segments of the graph as well as of changes in the course of the graph.
Assumptions about how the instructional activities support mental activities that lead to the linguistic goals
(Ad 1) In discussing how to divide a line graph in different segments, students prepare the activity of interpreting and describing a line graph. (Ad 2) In matching pre-formulated sentences with segments of a line graph, students are provided with genre sentences that will foster their genre proficiency. Furthermore, by providing students with given formulations, they can focus on and develop their understanding of attributing *both* an interpretation *and* a description to each segment of the graph. (Ad 3) Students reinforce their knowledge and use of temporal prepositions. (Ad 4) By actively using temporal prepositions in a meaningful context, students' (second) language development concerning this aspect of the genre is promoted.

Data analysis: comparison of hypothetical and observed learning

Here we only address the aforementioned research question about the development of student proficiency in the genre of interpreting line graphs. Data collection consisted of audio recordings of interviews held with four case-study students of mixed ability after each lesson, as well as before and after the teaching experiment. Their pseudonyms were Abdul, Moad, Rabia, and Youness (for selection criteria see Smit, 2013). Each interview lasted 10 to 15 minutes and was carried out by two members of the research team following an interview scheme that was aligned with the HLT of that lesson. In line with the overall phasing of the lessons, the students increasingly were asked to write about graphs. All interviews were transcribed verbatim.

For each lesson, Smit (2013) compared hypothesized learning with observed learning. Table 21.4 provides an example of how she approached this for students that she interviewed right after every lesson. The first column summarizes the activity conducted in class, with hypothesized learning in the second column. Then the third column shows relevant quotes from the interviews and the fourth column the conclusion of the comparison. Again, this is not necessarily how the analysis would be presented in a journal article, but it shows how it was conducted.

We focused the cross-case analysis of students' genre proficiency as evidenced in the interviews on the following three topics – each including two particular features of the genre in which we intended students to interpret line graphs (see Table 21.1):

1 The organization of the graph in segments and the description of each segment both in terms of reality and in terms of the course of the graph (structure features 1 and 2: S1 and S2).
2 The use of general academic language for describing reality and topic-specific language for describing the course of the graph (linguistic features L1 and L2).
3 The use of temporal prepositions to refer to either moments or periods in time (L4 and L5).

Table 21.4 Example of the comparison of hypothesized learning and students' utterances during interviews (Lesson 7)

Activity	*Hypothesized learning*	*Quotes from interviews*	*Conclusion*
Whole-class explanation of temporal prepositions, visually supported by using timelines; whole-class discussion of examples. Writing activity in which students fill in temporal prepositions in a writing frame containing a text about Uncle Kees's weight in the targeted genre.	Students reinforce their knowledge and use of temporal prepositions and gain a better conceptual understanding of points versus segments in the line graph. By actively using temporal prepositions in a given meaningful context, students' (second) language development concerning this aspect of the genre is promoted.	His weight? His weight was first 85 kilograms. Then, yes then he descended very much. Then it was, from his, from his 20th to his 25th he was, he went descending little. Descended gradually. And then from his 25th to his 30th he descended quickly. Then he stays from 30 up and to including 35 he stays gradually, then he stays 76 kilograms. [. . .]	Youness has started to actively use temporal prepositions. When he does, it is mostly done correctly (confirming hypothesized learning). However, it also occurs incorrectly (up and to including), and mixed with his previous style of "then . . . " (rejecting hypothesized learning).

To trace the four case-study students' progress concerning these three topics, we carried out one analysis for each topic. Table 21.4 provides an example, concerning the interview held after lesson 7, of how the HLT was employed (see also Table 21.3 for the HLT for lesson 7 and Figure 21.1 for the line graph discussed). From the interviews we identified all students' quotes in which genre features were used in a correct, partly correct, or incorrect way. As we conducted separate analyses for the three aforementioned topics, all transcripts of interviews were analyzed through three different lenses (structure features, linguistic features, and temporal propositions). In the conclusions based on students' quotes, one researcher summarized the extent to which hypothesized learning was confirmed. The same researcher formulated a few key aspects of each case-study student's development in the particular topic that seemed most typical. These key aspects can be seen as a summary of a student's learning process that is based on all separate conclusions drawn for one student for each topic. As a cross-case analysis (Borman, Clarke, Cotner, & Lee, 2006; Miles & Huberman, 1994) conclusions across students were summarized. Another researcher from the team judged both the conclusions and the summaries. The rare instances of disagreement were discussed until agreement was reached.

Summary of results of the cross-case analysis

In this section, we present the development of case-study students' proficiency in the genre throughout the lesson series. As mentioned before, it may be part of a dissertation (monograph), but it may not be the form of analysis presented in a journal article because journals typically prefer more concise and general results.

Students increasingly included all segments of the line graph in their graph descriptions. In early lessons, as anticipated in the HLTs, students showed difficulty with inclusion of all segments in their interpretative description of a line graph. This particularly held for Rabia (who described no segments after lesson 3, so she only gave a description in general terms) and for Abdul (two out of four segments included after lesson 3). Over time, however, the case-study students provided increasingly complete interpretative descriptions. An exception is Moad: He did not show a clear improvement over time in terms of segments included in his interpretative descriptions.

Concerning the genre features of describing each segment in terms of reality (structure feature one, abbreviated as S1) and in terms of the course of the graph (S2) we conclude that hypothesized learning corresponded with observed learning only to a limited extent: Youness only once, and Abdul as well as Rabia only twice (out of eight interviews) included interpretation of reality *and* description of the course of the graph for a segment of the graph. Moad did not describe graph segments in a dual sense during the interviews. The median lesson number from which onward the other three students included descriptions of reality was eight (S1). The median lesson number from which onward all four students included a description of all segments was six (S2).

Concerning students' use of general academic language, we concluded that both Rabia and Youness included little general academic language in line graph descriptions. This can be related to their tendency to focus on the course of the graph rather than on interpreting reality. Furthermore, the fact that general academic language was context-dependent for each activity (e.g., *losing weight*), whereas topic-specific language (*rise*, *descend*, *constant*, and *gradually*) was used for each of these contexts cannot be ignored. Observed development thus did not fully correspond to anticipated development.

When interpreting reality, students tended to draw unjustified causal conclusions (e.g., "here he does sports"), a phenomenon also observed by Leinhardt et al. (1990). Confirmations of hypothesized learning were found in occasional (adequate) use of general academic language in later interviews, for instance Abdul's use of *increase*, as well as students' capability to provide correct word meanings for *increase* and *decrease* in later interviews (both Abdul and Moad). In brief (L1), Rabia did not include much general academic language in her line graph descriptions. The other three did so toward the end of the series. For example, words such as *increase* and *decrease* were correctly used and defined later on.

Students increasingly used topic-specific language to describe the course of the line graph. This corresponded with hypothesized learning. They did this in increasingly differentiated and adequate ways. However, all case-study students occasionally used topic-specific language to interpret reality in unconventional ways (as in "Uncle Kees descends with his weight") as well as occasionally used topic-specific language informally or ungrammatically.

When developing proficiency in a new genre, students need time and space to explore such new ways of using language. Imperfections in deploying the genre should in our view be interpreted as manifesting language development rather than as deficient employment of the genre. We further remark that topic-specific language, although increasingly used by case-study students, proved conceptually difficult in several instances in interviews: for example, the difference between *constant* and *gradually* (cf. analysis of Abdul's learning). Despite the repeated attention to these words and underlying mathematical conceptions, all students kept struggling with their meanings. In brief, all four students increasingly used words such as *descend*, *rise*, *constant*, and *gradually*; and they did so with increasing correctness (L2).

Corresponding with hypothesized learning, students began using temporal prepositions more to describe points (L4) or segments (L5) on the graph. This implies that they improved their ability to distinguish between moments (represented by points) and periods (represented by segments) in time. Thus, by adequately using temporal prepositions, students improved in mathematical precision concerning line graph interpretation. Occasional self-corrections related to the use of temporal prepositions indicate in our view that students developed a heightened awareness of the need to interpret and describe line graphs more precisely, implying that students became more independent.

Despite all case-study students' progress concerning their use of temporal prepositions, they showed differences in the way they developed proficiency. Abdul and Rabia predominantly used moment-related temporal prepositions (e.g., *at*) for interpreting both points and segments in early interviews. Only in later interviews did they start to adequately use temporal prepositions for describing periods in time. Although Youness initially focused on periods (across-time reading) more than on moments (pointwise reading), he continued to use moment-related temporal prepositions for describing periods until interview six. Remarkably, it is only in the last interview (nine) that he correctly referred to a moment in time. Moad stuck to the use of moment-related words, even when referring to periods in time (i.e., when conducting across-time reading). In brief, in the first interviews, all students predominantly use a limited repertoire of temporal prepositions (e.g., *first*, *then*). They all expand their repertoire over time, for example with *from . . . to*, *after*, *at*. Their capability to distinguish between moments and periods increases throughout interviews. There are some instances of relapses but also of correcting initially incorrect usage. In the beginning they make many mistakes (e.g., *from* without *to*) but in the later interviews they mostly use these temporal prepositions correctly.

In summary, the cross-case analysis of the variability between the four students showed gradual progress with some minor falling back over the course of the lessons. By and large their development was in line with what we had intended or predicted in the HLTs. For instance, they all showed progressive capability of using topic-specific language (*rise*, *descend*, *gradually*, *constant*) to describe the course of the line graph. For none of the four case-study students did it become a habit to provide graph descriptions in terms of reality (e.g., "slowly loses weight") and in terms of the course of the graph (e.g., "descends gradually"). In such cases, where hypothetical and observed learning differed, the differences indicate that even more attention and time is needed to support multilingual and language-weak students' learning processes in the required genre.

Dear reader, did you manage to get through this section? We doubt it. Reviewers of our manuscripts found such writing to be tedious and, indeed, the link to broader, more interesting theory is not easy to make. In our own view, only sharing the results in this way stays "too close to the data," as many researchers would call it.

Reflection on the analysis and results

As mentioned previously, results of such evaluative analysis can be boring to some readers unless they are very experienced in this subject. Yet such evaluation is necessary to check whether the intervention led to increased proficiency in what was defined as the learning goals. Without some evidence of this it would not make sense to make claims about how teachers could scaffold their students in the intended genre for interpreting line graphs.

But an answer to our evaluative research question is not enough. As one of the reviewers commented, in the results from the analysis the relation between teaching and learning was out of sight. In the end we decided to focus a new analysis on one case-study student's development (Abdul) over the nine lessons in relation to the main learning activities of each lesson. This allowed us to embed one student's learning in a story about the design of the instructional activities and the teacher's employment of scaffolding strategies (Smit et al., 2016). Thus we lost detailed information about the variation among students' progress but gained the opportunity to give the reader insight into the question of how teachers can scaffold students' mathematical language – the type of actionable knowledge that we were after in the first place.

So what is worth sharing with which audience? In our experience, most readers prefer brief results that are easy to digest and remember. Perhaps this is why numbers are so popular as ways to report aspects of interventions. To explore how our findings could be shared more concisely we now turn to conjecture maps (Sandoval, 2014) for a research-oriented audience, and design principles (Van den Akker, 1999) for a more practice-oriented audience (see Chapter 3).

Conjecture map of the scaffolding case

To present analysis and results in a concise form, we now explore whether a conjecture map would be useful for communicating the main insights from Smit's (2013) design research project. In the next section we do the same for design principles.

The *high-level conjecture* that we formulated in retrospect is: Using instructional materials inspired by genre pedagogy and idea of scaffolding, teachers can scaffold strategies to help their students develop proficiency in the language needed for interpreting line graphs. This conjecture was *embodied* in a design that consisted of different elements:

1 Following heuristics from genre pedagogy, the instructional materials aimed to support students' long-term build-up and progress toward learning goals. Hence we designed a series of nine lessons consisting of learning activities that were modeled after the teaching and learning cycle, inspired by the idea of scaffolding (high support in the beginning, handing over independence toward the end). This cycle consists of a series of four stages in which a particular text-type needed at school is introduced, modeled, jointly practiced, and eventually individually performed by the students. It is to be used in content classrooms (e.g., history, science, or mathematics). Underlying this cycle is the idea that students need to gradually develop language skills along a mode continuum (Gibbons, 2002) from spoken-like everyday language into written-like academic language, bridged by literate spoken language, also referred to as "bridging discourses" (as in

Gibbons's book title, 2006). The written-like academic language includes those aspects of the second language that are most relevant to curriculum learning.

2 Accompanying HLTs that functioned as guidelines for the teacher of what to focus on (here to co-design and be used as teacher guidelines rather than research instruments).
3 An overview of the domain-specific genre features for interpreting line graphs that captured key aspects of what the teacher could focus on (Table 21.1).
4 A repertoire of scaffolding strategies that the teacher could use whenever she thought suitable during class or group discussion (Table 21.2).

Design conjectures that specify how design characteristics are expected to lead to mediating processes or mechanisms might be summarized as follows:

1 The tasks and teacher elicit mediating processes: student reasoning about graphs, contributions to small-group and whole-class discussion, language production, and classroom interaction. The opportunity to produce language, to highlight one example, is considered a key mechanism or mediating process in second-language learning (Gibbons, 2002).
2 If teachers know on the basis of HLTs the detailed learning goals and hypotheses about how students will reason, and what language they will use, they will better diagnose mathematical and linguistic levels of student reasoning and respond adaptively (where diagnosing and responding adaptively are examples of intended scaffolding processes).
3 If teachers are aware of features of the genre, it helps them and their students focus on what matters most in developing student proficiency in this genre. The mediating process here is teachers' and students' increasing awareness of linguistic and structure genre features.
4 The scaffolding strategies will make students aware of what are correct or desired ways of talking and writing about line graphs, and invite them to start reasoning about line graphs in the intended genre. For example, asking students to say something more precisely is also a push toward independence, so another mediating process here is handing over independence, one of the key characteristics of scaffolding.

Theoretical conjectures restricted to some of the scaffolding strategies (Table 21.2) might be formulated as follows:

1 If teachers repeat correct linguistic expressions from students, the rest will hear them better and implicitly learn that these are approved.
2 If teachers make explicit which student utterances are considered linguistically correct and mathematically adequate, their peers will hear what are considered better ways of talking in the genre. The establishment and

internalization of relevant mathematical and linguistic norms is assumed to contribute to proficiency development.

3 Through producing language within the genre and receiving responsive feedback from the teacher, students become more independent users of the genre of interpreting line graphs.

Outcomes: the learning goal was that students would become much more proficient in the predefined genre of interpreting line graphs. Our analysis shows to what extent this was the case for four case-study students. A single case study and a pre-posttest comparison with an effect size estimate can be found in Smit et al. (2016).

Note that much of what is presented here as a conjecture map could also be part of an overall HLT for the whole intervention. Note that HLTs per lesson functioned as more detailed guidelines.

We think that Sandoval's idea of conjecture mapping is very useful in various stages of a design research project. In retrospect, however, one limitation that we face when using it for this particular design study is that conjecture maps are rather linear: As formulated by Sandoval, conjecture maps assume a one-way influence of design characteristics via mediating processes to outcomes. Yet when embodying the ideas of genre pedagogy and scaffolding in a design, we inevitably need feedback loops. For example, if teachers or design researchers observe during or in between lessons that adjustments in the design are needed, they make adjustments. Their on-the-fly inferences then form the basis for redesign (arrow back from mediating processes or intermediate outcomes into the embodiment of design characteristics). The concept of scaffolding being enacted here even requires a continuous adaptivity to what happens in the classroom. Some of these adaptive responses are purely how a teacher interacts with students, but some may lead to changes in the instructional activities. More generally, conjecture maps are useful for macro-cycles rather than for micro-cycles of design research.

Another limitation of conjecture maps is they have no time dimension that captures how particular processes are expected to follow each other. To some extent this limitation can be overcome, as illustrated in Chapters 18 and 19, in the vertical dimension of the various boxes of conjecture maps. If hypothesized development over time is important in a design study, HLTs seem better geared to explicate how intermediate activities assist in getting closer to the learning goals.

Reformulation as a design principle

As a second way of presenting Smit's (2013) findings more succinctly, we now make an attempt to summarize them in the format of a design principle. We then explore to what extent conjecture maps and design principles are equivalent. When we rework the conjecture map of the previous section to be a design principle, following Van den Akker's (1999) format, we get something like this:

If you want to scaffold the language that students in multilingual mathematics classrooms (context) need to participate in learning about a mathematical topic (purpose), you are advised to use instructional materials inspired by genre pedagogy and the general idea of scaffolding (see appendices of Smit, 2013) and ask the teacher to use scaffolding strategies from the repertoire in Table 21.2 (procedures) because

- The teaching and learning cycle from genre pedagogy has already been deployed successfully in several domains such as science (Gibbons, 2002);
- Using scaffolding strategies and employing them responsively to students' linguistic levels invites mediating processes that are known to support (second-)language development (Gibbons, 2002);
- Students in Smit's (2013) research indeed made considerable progress in the genre of interpreting a mathematical topic (line graphs).

Note that we have used ideas from the conjecture map to strengthen the arguments that are called for in Van den Akker's (1999) format (Chapter 3):

1 Arguments about design experiences and prior success with similar designs
2 Arguments about what is known from the literature about mediating process or mechanisms in relation to outcomes
3 Arguments based on evaluation from the underlying research.

This exercise shows a few things: Design principles and conjecture maps are different things but can capture similar ideas. A major difference is that design principles in Van den Akker's format do not specify mediating processes (unless incorporated via arguments as we have done above), whereas they are explicit in conjecture maps. Furthermore, while design principles are formulated in practical advisory terms (do this, do that), conjecture maps are more research oriented (conjectures about relations between design, mechanism, and outcomes that can be empirically tested). The choice for one or the other therefore depends on the focus (design or research) and hence the audience one is addressing.

Reflections on HLTs, conjecture maps, and design principles

Last, we reflect on the strengths of each of these research instruments. All three together function as an interface between theory and practice. They are informed by both theoretical insights and practical experience. They can all be useful at different phases in design cycles. For example, in the preparation and design phase, they can help to make explicit what matters theoretically and help to specify what the design is supposed to accomplish (see Chapters 18 and 19). All three instruments can also assist in the communication with other stakeholders such as teachers. For example, Bakker (2004) and Smit and Van Eerde (2011) shared their HLTs with the collaborating teachers who implemented the designs.

HLTs and conjecture maps are more research oriented than design principles (in the Van den Akker, 1999 or 2013, format), because they are configurations of testable conjectures. In the data analysis phase we therefore see a clearer role for HLTs and conjecture maps than for design principles (also see Chapter 3). Yet in the communication phase to potential users, design principles sound easier to digest for nonresearchers. For example, in his work on school development, Mintrop (2016) formulates design principles to summarize the advice coming from the research he does in collaboration with schools.

One strength of HLTs is that they incorporate a developmental dimension that hypothesizes students' progression through learning activities. They are about how to get learners from A to B. The starting point, for example students' prior knowledge or attitudes, is explicitly in the picture – in contrast to conjecture maps or design principles. Yet, conjecture maps may be enhanced with a time dimension (as indicated earlier in Chapters 18 and 19). One could also use multiple conjecture maps to show progression. Sometimes people who read about HLTs initially consider them to be linear, but they are neither meant to be (Simon, 1995), nor need to be (see Bakker, 2004, on the branching of trajectories).

All three instruments are intended to stay hypothetical: It is acknowledged that each new setting may require local adjustment. Table 21.5 summarizes the advantages and disadvantages of the three instruments. We do not want to suggest that design researchers have to choose between these three research

Table 21.5 Overview of the nature and advantages as well as disadvantages of HLTs, conjecture maps, and design principles

	General nature	*Advantages*	*Disadvantages*
HLT	Research oriented, but more local than conjecture maps	Time dimension Developmental Testable conjectures Explicit attention to prior knowledge or expertise	Tedious to report Difficult to summarize Trajectory sounds linear but need not be
Conjecture map	Research oriented but more general than HLTs	Compact graphical representation Mediating processes in focus Testable conjectures	Linear No time dimension No feedback loops
Design principle	Advice directed to users	Formulated as actionable knowledge or advice	Mediating processes not necessarily in focus

instruments. First of all, other options may also work: Some design researchers use none of these three. Second, as this chapter shows, it is often possible to formulate a conjecture map or design principle on the basis of HLTs, or transform a conjecture map into a design principle. Whether this makes sense depends on the audience to which one wants to communicate as well as the time and space one has available to do so.

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