Quantifying the synchronization of the spikes emitted by coupled lasers

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Synchronization phenomena is ubiquitous in nature, and in spite of having been studied for decades, it still attracts a lot of attention as is still challenging to detect and quantify, directly from the analysis of noisy signals. Semiconductor lasers are ideal for performing experiments because they are stochastic, nonlinear, inexpensive and display different synchronization regimes that can be controlled by tuning the lasers' parameters. Here we analyze experiments done with two mutually optically coupled lasers. Due to the delay in the coupling (due to the finite time the light takes to travel between the lasers) the lasers synchronize with a lag: the intensity time traces show well-defined spikes, and a spike in the intensity of one laser may occur shortly before (or shortly after) a spike in the intensity of the other laser. Measures that quantify the degree of synchronization of the lasers from the analysis of the intensity signals do not fully quantify the synchronicity of the spikes because they also take into account the synchronization of fast irregular fluctuations that occur between spikes. By analyzing only the coincidence of the spike times, we show that event synchronization measures quantify spike synchronization remarkably well. We show that these measures allow to quantify the degree of synchronization and also, to identify the leading laser and the lagging one.

Professor Kurths has done very significant contributions to the fields of synchronization of chaotic systems and complex systems, having studied synchronization phenomena in a wide variety of systems, from biological oscillators to the human brain and the climate system. His work has also allowed a better understanding of the effects of noise in complex systems. In our contribution to this Focus Issue, dedicated to Prof. Kurths' 70th birthday, we analyze experiments on the synchronization of two semiconductor lasers that are mutually optically coupled. Due to the coupling delay (due to the finite time the light takes to travel between the lasers) the lasers synchronize with a lag and the intensity of one laser lags behind the other, or there is alternation. We show that spike coincidence analysis allows to quantify the degree of synchronization and to identify the leading laser and the lagging one.

I. INTRODUCTION

The synchronization of chaotic systems has been studied for decades, and still attracts a great deal of attention due to the many systems that display synchronized behavior, and the different types of synchronization regimes that have been found[1]. Coupled lasers allow to perform real experiments where parameters can be precisely controlled. Therefore, since the first observations of synchronization of two chaotic lasers[2,3], a lot of work has been done to understand the lasers' synchronized behavior[4–27].

Semiconductor lasers are attractive for studying synchronization phenomena because they are inexpensive, emit a chaotic output when they are optically coupled, and their governing equations are well known[28]. Due to the finite speed of light, optical coupling induces a delay (the flight time from one laser to the other), which is usually non-negligible and which can lead to leader-lagger[6,16,18], anticipated[7,10] or isochronal synchronization[11,12,19].

Most of these studies have characterized the lasers' synchronized behavior by using bivariate similarity measures, such as the linear cross-correlation coefficient or the nonlinear mutual information, directly applied to the intensity time traces of the two lasers. However, when the lasers operate near threshold they emit a spiking output (examples of time traces of the output intensities are shown in Fig. 1) and this approach does not fully capture the synchronicity of the spikes because it also takes into account the synchronicity of the fast, irregular fluctuations that occur in between spikes.

A event-based approach was used by Tiana et al.[18] to characterize the synchronization of two lasers, when they operate in the spiking regime. Tiana and collaborators extracted, from the two intensity time series, a single binary time series by assigning 0 or 1 depending on which laser spiked first: if the spike of laser 1 occurs before that of laser 2, the symbol '1' was assigned, otherwise, the symbol '0' was assigned. By analyzing the statistical properties of the binary sequence, Tiana and coworkers characterized the leader-lagger regime transition that occurs, when the pump current of laser 1 increases, from a regime in which laser 2 is the leader because it spikes before laser 1, to a regime in which laser 1 is the leader because it spikes before laser 2.

Here we analyze the experimental data recorded by Tiana et al.[18] using event-coincidence analysis. Specifically, to characterize and quantify the synchronization of the two lasers we use the measures proposed by Quian Quiroga et al.[29], calculated by counting the number of quasi-simultaneous occurrences of spikes, allowing for a short delay in the spike in the signal of one laser with respect to the spike in the signal of the other laser. Event-coincidence analysis has the advantage that it allows for a dynamic, time-varying lag between events, in-

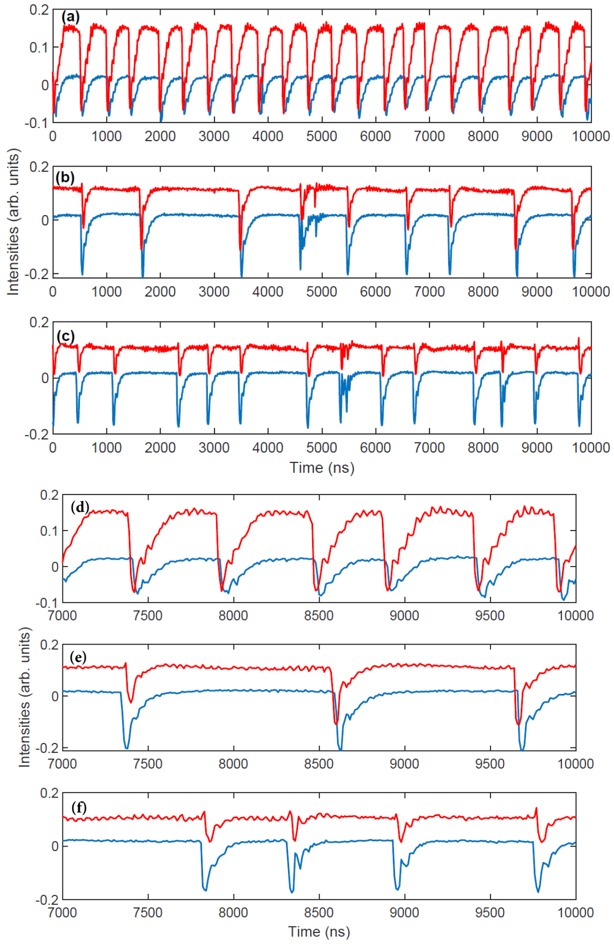

FIG. 1. Examples of time traces of the intensities emitted by the two lasers (the time trace of laser 2, in red, is shifted vertically for clarity). The pump current of laser 1, I_1, is (a,d) 12.30 mA; (b,e) 13.42 mA; (c,f) 13.88 mA; the pump current of laser 2 is $I_2 = 12.25$ mA; panels (d-f) show in detail a few spikes, we note that in (a,d) the spikes of laser 2 anticipate those of laser 1, in (b,e) there is alternation, and in (c,f) the spikes of laser 2 occur after those of laser 1.

stead of a fixed, static lag for the entire time series as used by lagged cross-correlation analysis. This flexibility has made it possible to reveal the synchronicity of climatic extremes that occur in different geographical regions of the Earth[30–33].

By counting the number of spikes emitted by one laser shortly after the other emitted a spike, we show that the measures proposed by Quian Quiroga et al.[29] allow to quantify the degree of synchronization of the spikes emitted by the two lasers, and also, to distinguish three regimes: one in which laser 1 is leader, one in which the lasers alternate, and one in which laser 2 is the leader.

II. EXPERIMENTAL SETUP AND DATASET

The experimental setup was described in[18] and consists of two commercial diode lasers (Mitsubishi ML925B45F) mutually coupled via an optical fiber that gives a coupling delay of 30 ns (the flight time from one laser to the other). The lasers' temperatures and pump currents were adjusted such that the uncoupled lasers emitted optical frequencies as similar as possible. The threshold currents of the uncoupled lasers were $I_{th,1}$=11.10 mA and $I_{th,2}$=11.63 mA. The current of laser 2 was kept fixed at $I_2 = 12.25$ mA and the current of laser 1 was tuned as a control parameter. The intensity signals were simultaneously captured with two 2 GHz photodetectors, amplified by two 2 GHz amplifiers, and recorded with a 1 GHz digital oscilloscope (Agilent DS06104A). The signals were recorded over 5 ms with a sampling time of 10 ns, and 4000-10000 spikes were recorded, depending on the lasers' currents. Time traces examples are displayed in Fig. 1.

III. METHODOLOGY

We use a threshold to define the spike times of the two lasers, $(t_{1,1}, t_{1,2}, \dots, t_{1,i}, \dots)$ with $i \in (1, N_1)$ and $(t_{2,1}, t_{2,2}, \dots, t_{2,j}, \dots)$ with $j \in (1, N_2)$. Following the approach proposed by Quian Quiroga et al.[29], we allow for a time lag, τ, between two synchronous spikes (τ is smaller than the shortest inter-spike interval, to avoid double counting) and count the number of times a spike is emitted by laser 1, shortly after a spike is emitted by laser 2. We denote by $c_{12}(\tau)$ the number of times this occurs (i.e., the number of times $0 \le t_{1,i} - t_{2,i} \le \tau$). We also calculate $c_{21}(\tau)$, which is the number of times a spike is emitted by laser 2, within a time interval τ after laser 1 emits a spike (i.e., the number of times $0 \le t_{2,i} - t_{1,i} \le \tau$).

Then, we calculate the synchronization measures

$$Q(\tau) = \frac{c_{12}(\tau) + c_{21}(\tau)}{\sqrt{N_1 N_2}}, \quad (1)$$

$$q(\tau) = \frac{c_{12}(\tau) - c_{21}(\tau)}{\sqrt{N_1 N_2}}, \quad (2)$$

with N_1 and N_2 being the number of spikes of laser 1 and laser 2, respectively. $Q(\tau)$ and $q(\tau)$ are normalized such that $0 < Q(\tau) \le 1$ and $-1 \le q(\tau) \le 1$. We have that $Q(\tau) = 1$ if and only if for each spike emitted by laser 1 (laser 2), there is a spike emitted by laser 2 (laser 1), within a time interval τ before or after the spike of laser 1 (laser 2). In addition, $q(\tau) = 1$ ($q(\tau) = -1$) if and only if the spikes emitted by laser 2 (laser 1) always precede those emitted by laser 1 (laser 2).

IV. RESULTS

Figure 2 displays the variation of the synchronization indicators, Q and q, as a function of the time interval τ in which a spike is counted. We fix the current of laser 2 and vary the current of laser 1, I_1 (time traces examples were shown in Fig. 1). For low I_1, in panel (a) we see that if the time interval is too short (shorter than the flight-time) no synchronized spikes are found, but if τ is long enough, $Q = 1$ and $q = -1$, which means that the spikes of laser 1 always precede those of laser 2. We also notice that Q and q display well-defined plateaus whose duration is about 10 ns; these plateaus are due to the

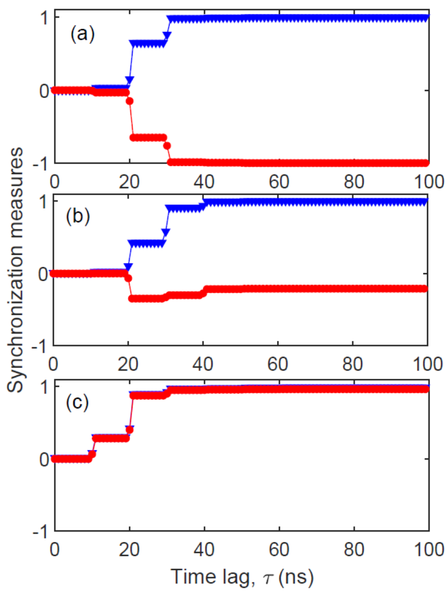

FIG. 2. Synchronization indicators (Q in triangles, q in circles) as a function of the lag time. The pump currents are as in Fig. 1: (a) $I_1 = 12.30$ mA; (b) $I_1 = 13.42$ mA; (c) $I_1 = 13.88$ mA. The sharp steps are due to the sampling time, 10 ns.

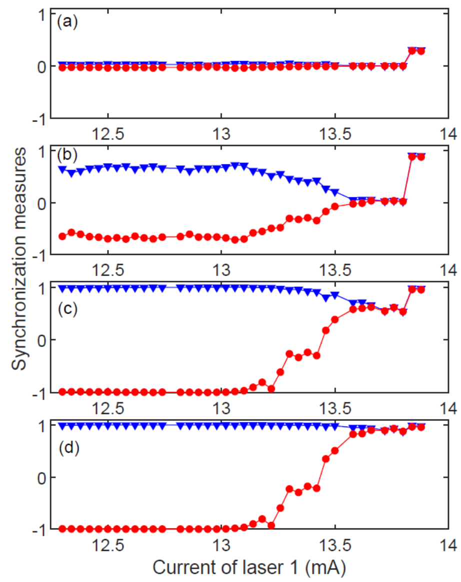

FIG. 3. Synchronization indicators (Q in triangles, q in circles) as a function of the current of laser 1. The lag time, τ, is (a) 15 ns; (b) 25 ns; (c) 35 ns; (d) 45 ns.

fact that the sampling time of the intensity time series is 10 ns (i.e., the spike times are detected with 10 ns resolution).

For a higher current, Fig. 2(b), we see that for τ long enough the spikes are fully synchronized ($Q = 1$) but there is no clear leader, as q takes a small negative value. For an even higher current, in Fig. 2(c) we see that for τ long enough the spikes are fully synchronized ($Q = 1$), but now laser 2 is the leader ($q = 1$, i.e., the spikes of laser 2 always precede those of laser 1).

We remark that in the three panels, the plateaus that occur as the time lag increases are artifacts due to the discretization of the intensity dynamics; a smaller sampling time would result in smoother variations. However, the memory of the oscilloscope limited the number of data points that could be acquired, and in order to record the output intensities of the two lasers over a sufficiently long time interval, a relatively long sampling time (10 ns) was used.

Figure 3 displays the synchronization indicators, Q and q, as a function of the current of laser 1, for different values of τ. For small τ, Fig. 3(a), no synchronized spikes occur, except for the highest current values, for which there are few synchronized spikes. For longer τ, Figs. 3(b)-(d), the degree of synchronization increases and a smooth transition from a regime where laser 1 is the leader ($q = -1$) to a regime where laser 2 is the leader ($q = 1$) occurs.

Figures 4(a), (b) summarize these results by plotting in gray color code Q (panel a) and q (panel b) vs. the current of laser 1 and the lag time. Here we see that, as expected, if τ is too small, there are no synchronized spikes ($Q \sim 0$). For τ large enough $Q \sim 1$ for all pump currents, but we note that lowest degree of synchronization ($Q \sim 0.6 - 0.8$ depending on τ) does not occur when there is leader-lagger alternation (i.e., where laser 1 and laser 2 alternate to spike first, and $q \sim 0$), but occurs at a slightly higher current value, for which laser 1 tends to spike before laser 2 ($q > 0$).

Next we inspect the spiking dynamics as a function of time, by computing Q and q over non-overlapping segments of the time series. Figure 5 shows results obtained by dividing the time series in 25 segments and calculating, in each segment, Q and q with $\tau = 35$ ns (panels (a) and (b) respectively). Panels (c) and (d) show the percentage of spikes of laser 1 that are emitted after spikes of laser 2, and the percentage of spikes of laser 2 that are emitted after spikes of laser 1, respectively. We see that for low currents values, laser 1 spikes after laser 2; for higher current values Q gradually decreases and the percentage of spikes of laser 2 that occur after the spikes in laser 1 grows. For the highest current values almost all spikes of laser 2 are emitted after those of laser 1 and Q and q increase sharply, which is consistent with the values seen in Fig. 3(c) that were calculated over the whole the time series.

V. CONCLUSIONS

We have used event-coincidence analysis to characterize the synchronicity of the optical spikes emitted by two mutually coupled lasers. The methodology is based on obtaining, from the two time trances of the lasers' intensities, two sequences of spike times. We have found that counting the spikes that occur almost simultaneously (within a short time interval between them) allows a precise quantification of the degree of synchronization of the spikes and also, allows to quantify the leading-lagging dynamics. With the asymmetric measure q, we have found a gradual transition from a regime in which laser 2 spikes before laser 1, to a regime in which

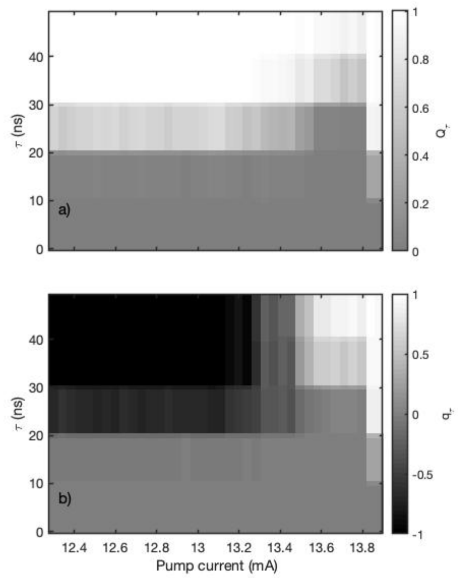

FIG. 4. Synchronization indicators in gray color scale (a: Q, b: q) as a function of the lag time and the current of laser 1.

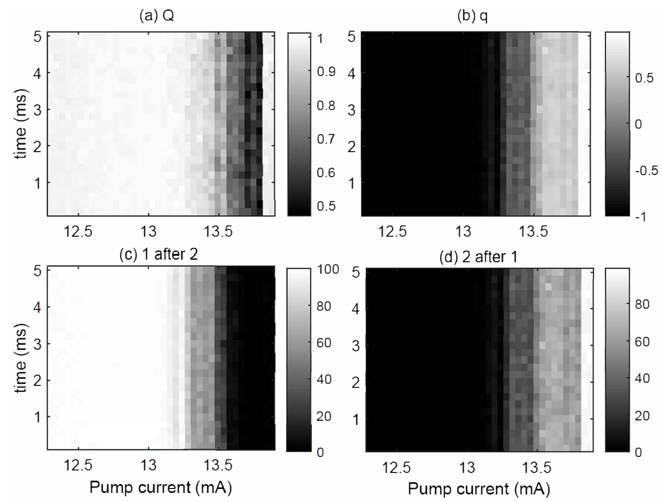

FIG. 5. (a,b) Synchronization indicators in gray color scale (a: Q, b: q) as a function of time and the current of laser 1. (c,d) Percentage, in gray color scale, of spikes of (c) laser 1 emitted after the spikes of laser 2; (d) of laser 2 emitted after the spikes of laser 1. In all panels $\tau = 35$ ns.

laser 2 spikes after laser 1. The symmetric measure Q reveals that the synchronization level is in general high (provided a long enough interval is allowed to count the spikes as synchronized) and decreases gradually when laser 2 looses its leadership role. This gradual decrease is followed by a sharp increase when laser 1 becomes the leading laser.

It will be interesting, for future work, to apply this methodology to several coupled lasers, and also, two lasers that are driven by the same master laser. In addition, our approach can be useful to better understand the spiking dynamics of coupled nano- and micro-lasers, whose output signals are often noisy, and which are expected to be building blocks of the next generation of energy-efficient, spike-based photonic neural networks.

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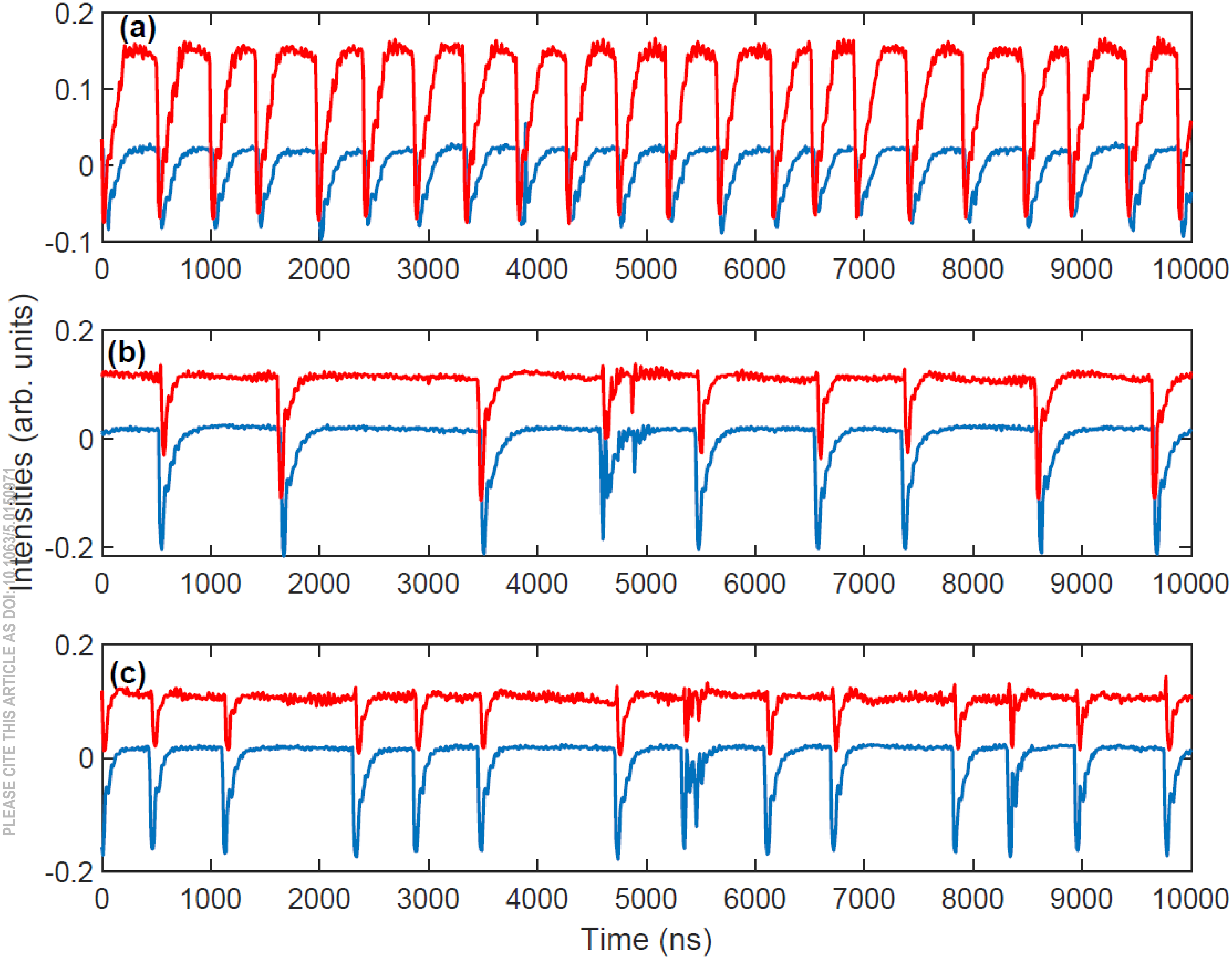
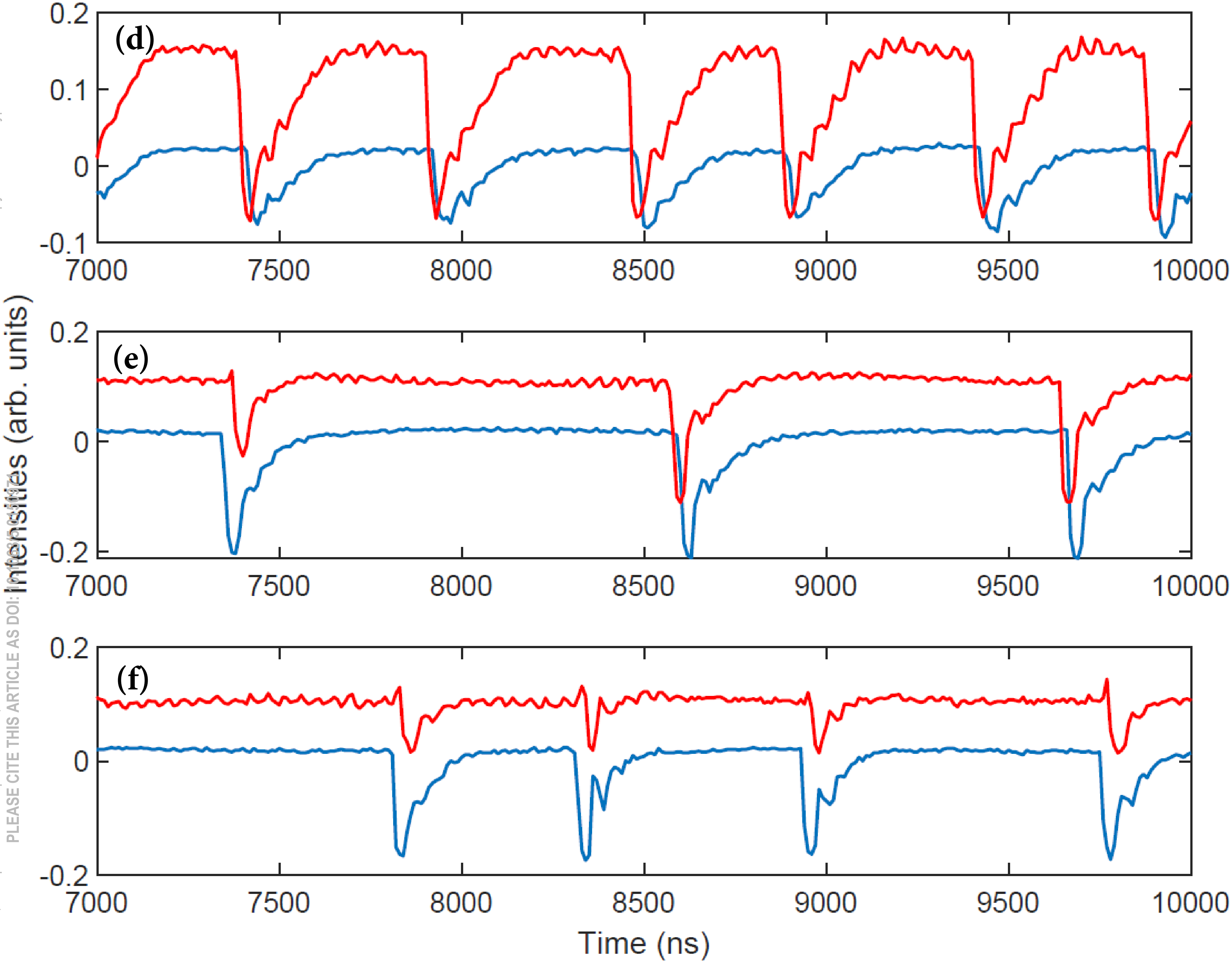

(a)
(b)
(c)
Intensities (arb. units)
Time (ns)
0.2
0.1
0
-0.1
-0.2
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000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Intensities (arb. units)
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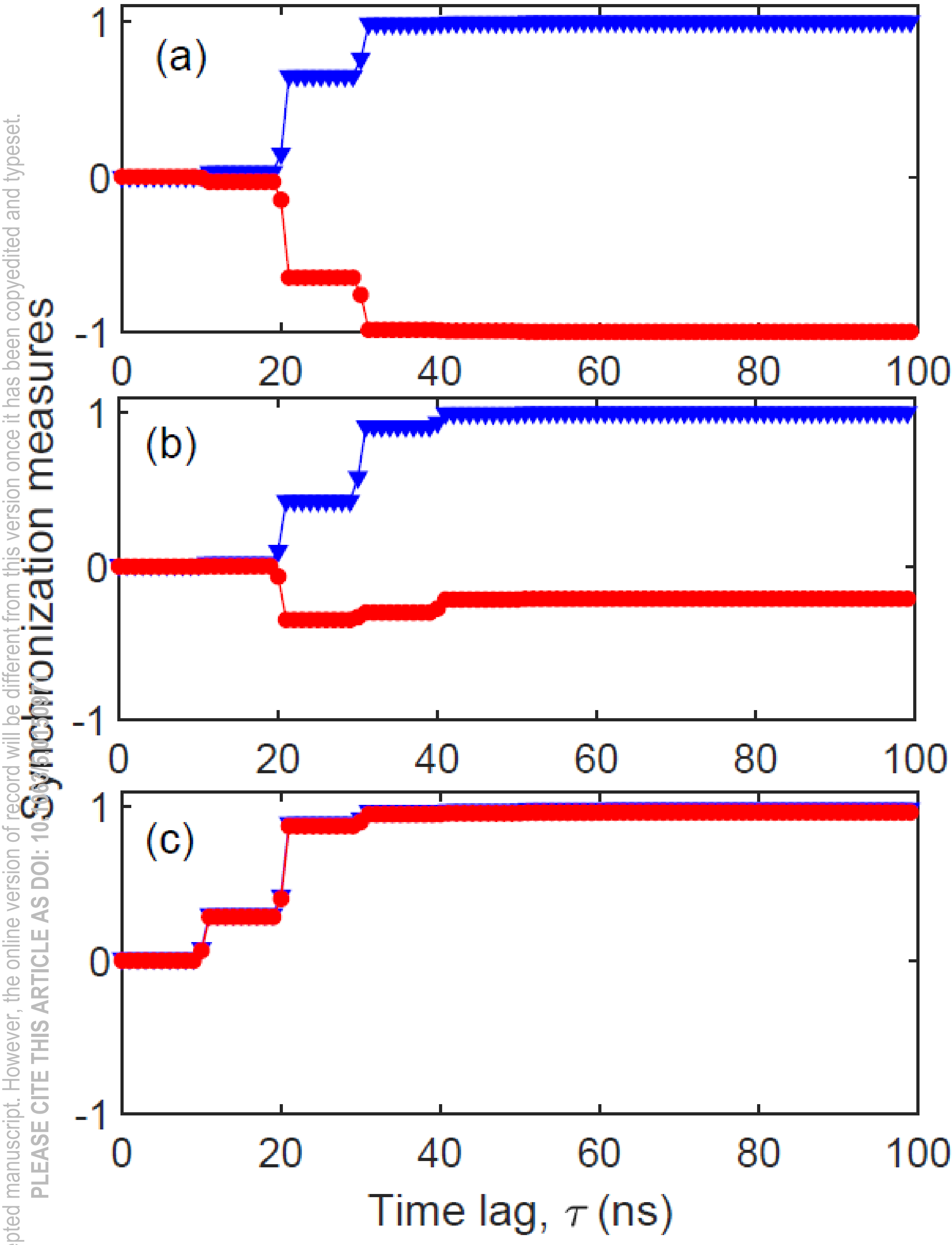
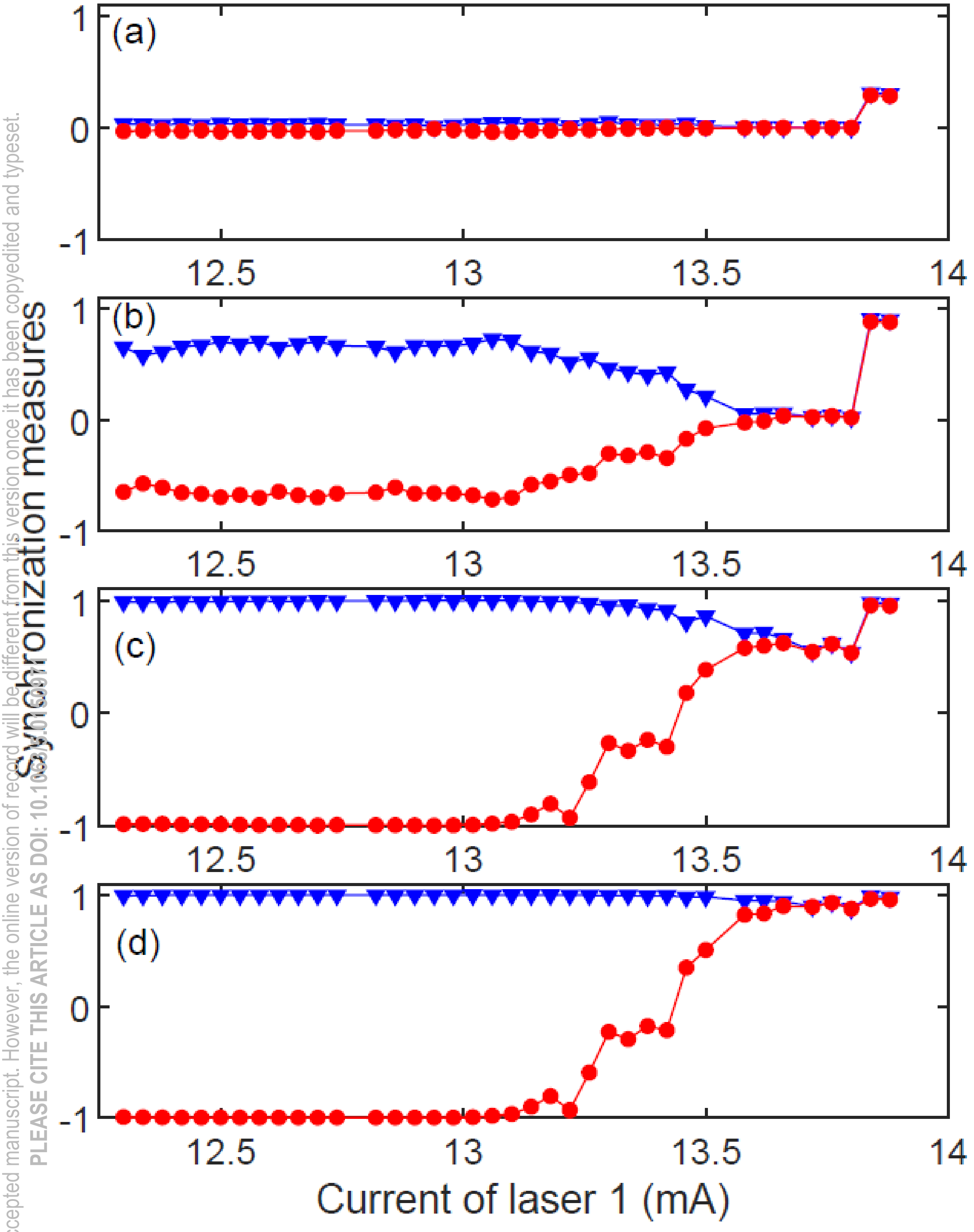
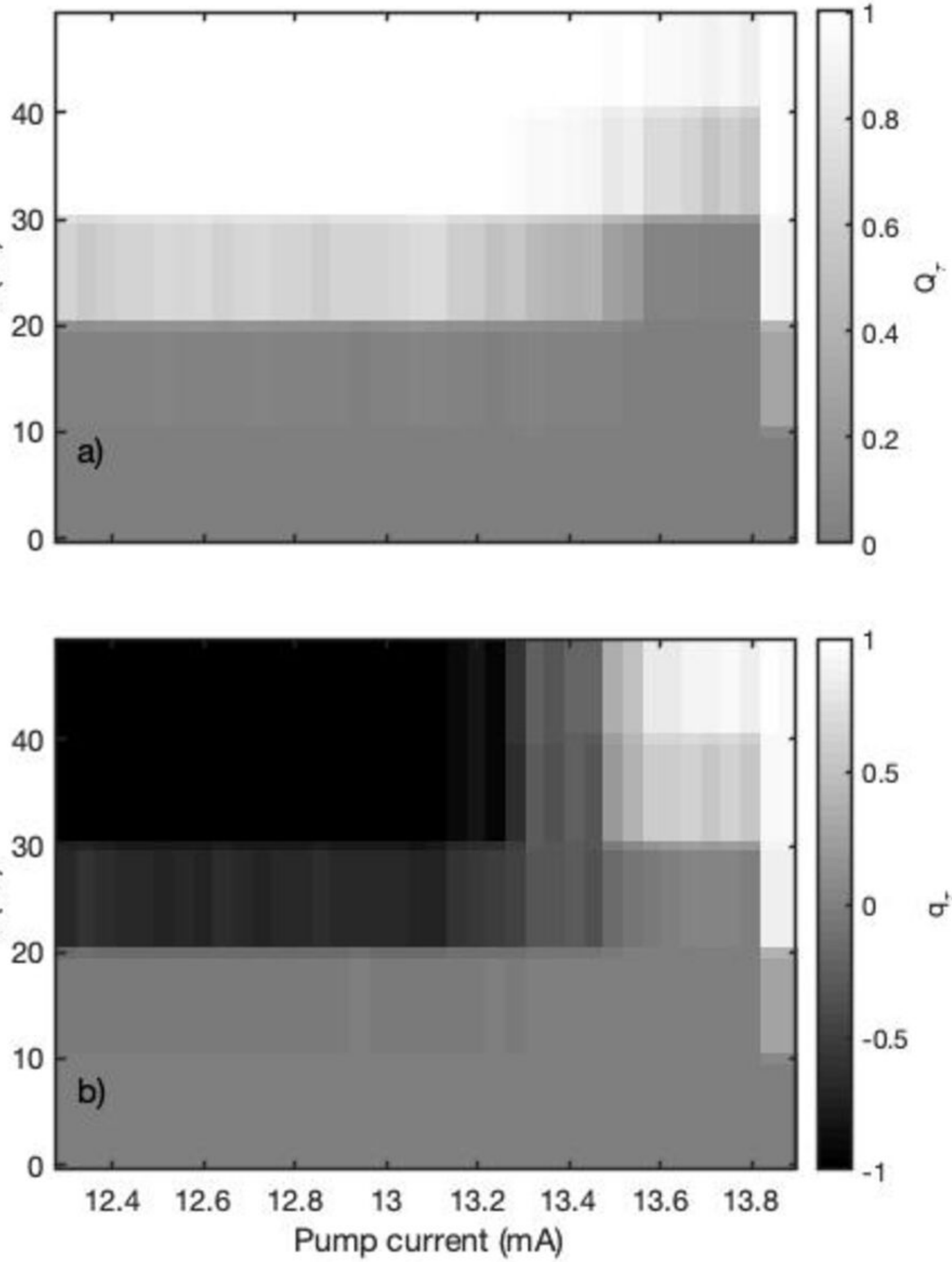
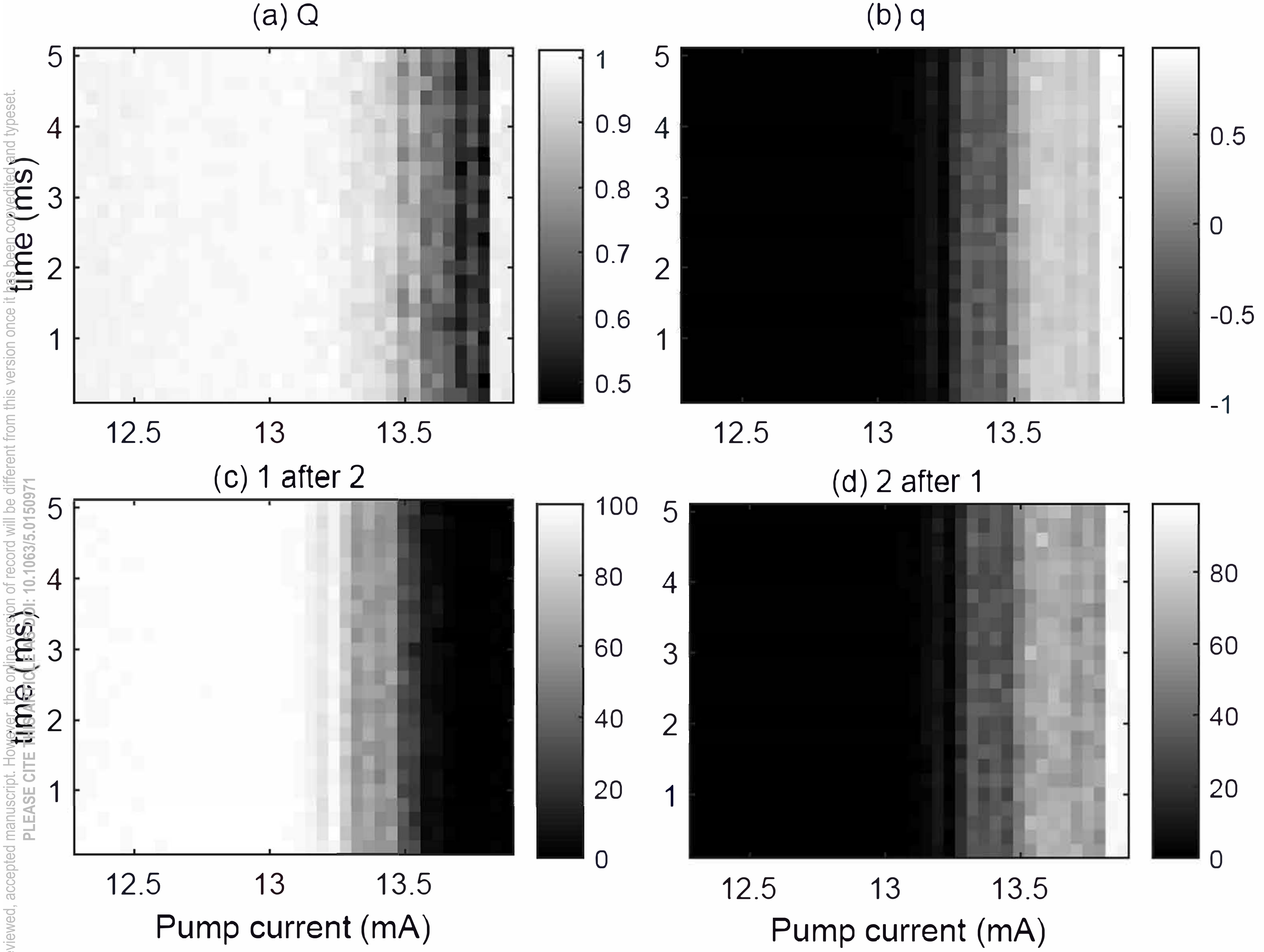

(a)
(b)
(c)
Synchronization measures
Time lag, τ (ns)
1
0
-1
0
20
40
60
80
100


(a)
(b)
(c)
(d)
1
0
-1
12.5
13
13.5
14
Synchronization measures
Current of laser 1 (mA)


a)
b)
τ (ns)
0
10
20
30
40
Q_τ
0
0.2
0.4
0.6
0.8
1
q_τ
-1
-0.5
0
0.5
1
12.4
12.6
12.8
13
13.2
13.4
13.6
13.8
Pump current (mA)



(a) Q
(b) q
(c) 1 after 2
(d) 2 after 1
time (ms)
Pump current (mA)
12.5
13
13.5