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A Rolling Horizon Approach for Stochastic Mixed Complementarity Problems with Endogenous Learning: Application to Natural Gas Markets

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Abstract

In this paper we present a new approach for solving energy market equilibria that is an extension of the classical Nash-Cournot approach. Specifically, besides allowing the market participants to decide on their own decision variables such as production, flows or the like, we allow them to compete in terms of adjusting the data in the problem such as scenario probabilities and costs, consistent with a dynamic, more realistic approach to these markets. Such a problem in its original form is very hard to solve given the product of terms involving decision-dependent data and the variables themselves. Moreover, in its more general form, the players can affect not only each others' objective functions but also the constraint sets of opponents making such a formulation a more complicated instance of generalized Nash problems. This new approach involves solving a sequence of stochastic mixed complementarity (MCP) problems where only partial foresight is used, i.e., a rolling horizon. Each stochastic MCP or roll, involves a look-ahead for a fixed number of time periods with learning on the part of the players to approximate the extended Nash paradigm. Such partial foresight stochastic MCPs also offer a realism advantage over more traditional perfect foresight formulations. Additionally, the rolling-horizon approach offers a computational advantage over scenario-reduction methods as is demonstrated with numerical tests on a natural gas market stochastic MCP. Lastly, we introduce a new concept, the Value of the Rolling Horizon (VoRH) to measure the closeness of different rolling horizon schemes to a perfect foresight bench-

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mark and provide some numerical tests on it using a stylized natural gas market.

Keywords:
Rolling horizon, stochastic mixed complementarity problem, natural gas market, game theory.

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1. Introduction

There are many dynamically changing conditions in today's international energy markets such as increased level of renewable energy supplies that are mandated and the related infrastructural challenges faced by many countries, imperfect competition in international natural gas markets motivated by both economics and politics in some cases, and strongly fluctuating world oil prices to name a few. Consequently, there is a need for energy models that are responsive to these conditions in a way that more realistically depicts the individual players' behavior taking in to account the game theoretic nature of these markets, the role of information related to only partial foresight of future conditions, and in general, the stochastic aspects of demand.

1.1. Rolling Horizon Stochastic MCPs to Approximate Endogenous-Data, Nash-Cournot Problems

In this paper we consider rolling horizon, stochastic mixed complementarity problems (MCPs). As discussed below, there are many reasons why this area should be studied. Perhaps a first main reason though concerns an extension of the traditional Nash-Cournot game in which the variables relate to decisions concerning production, flow, capacity or the like. In the current setting, each player has those variables but also has control of the actual data in the problem such as scenario probabilities that it estimates, costs, or other items that can be adjusted. Considering that just the probabilities can be affected by each player, in essence, player p is solving an optimization problem of the form:

$$\min_{x_{p,s}} \sum_{s} PROB_p^s \left(X^P; X^{-P}\right) f_{ps}\left(X^P; X^{-P}\right) \tag{1a}$$

$$s.t.\ X^P \in S_p\left(X^P; X^{-P}\right) \tag{1b}$$

where s is the index for scenarios (could vary by player) , X^P, is the vector of decisions that player p makes across all time periods, with the other players' decisions, given as X^{-P} fixed. The objective function in the above example is the expected value of a function f_p for player p. The important thing to note in the above example is that both this function as well as the probabilities themselves $PROB_p^s\left(X^P; X^{-P}\right)$ depend on the decision variables decided by both player p and the other ones. This is an especially hard problem to solve given that both $PROB_p^s\left(X^P; X^{-P}\right)$ and $f_{ps}\left(X^P; X^{-P}\right)$ are functions of all players' decision variables and are multiplied by each other. This difficulty is further compounded by the feasible region $S_p\left(X^P; X^{-P}\right)$ also being a function of both player p's decisions as well as the other players' ones (although fixed).

To have a better handle on the importance of allowing such flexibility to this extension of the Nash-Cournot paradigm, various endogenous probability/endogenous data schemes are considered in this paper. More specifically, each player is given a learning algorithm to adjust the data in the problem and we evaluate several such ones to see which ones are better in terms of several important measures such as profits, consumer surplus, and the Value of the Rolling Horizon (VoRH), a new concept we also introduce. These learning algorithms are implemented as part of the rolling horizon methodology we propose wherein each player, after each roll can adjust their data (e.g., scenario probabilities, costs) to further their own ends. Clearly, this is just a heuristic approach to the above much harder problem to solve. We discuss these and other aspects of why the rolling horizon approach for stochastic MCPS is worthy of consideration in what follows but first define MCPs with a specialization later to natural gas markets for illustrative purposes.

In this paper we present a new approach to model such energy markets focusing on an illustrative natural gas market to clarify the concepts. The gas market is described by a mixed complementarity problem (MCP) [5], an often-used format which generalizes single-optimization models via their associated Karush-Kuhn-Tucker (KKT) conditions, multiple optimization problems by market participants, and equilibrium problems not directly traceable to one or more optimization problems [22]. Formally stated, having a function $F : R^n \times R^m \rightarrow R^n \times R^m$, MCP($F$) is to find a vector $(x, y) \in R^n \times R^m$ such that:

$$0 \leq F_x\left(x, y\right) \perp x \geq 0 \tag{2a}$$

$$0 = F_y\left(x, y\right), y \text{ free} \tag{2b}$$

Here x represents the nonnegatively constrained variables with associated nonnegative F components denoted F_x and complementarity between them given by $\perp$ (i.e., $F_x\left(x, y\right)^T x = 0$). Also, y are the free variables with associated components F_y that must equal zero exactly [14], [22].

As described more fully in the next subsections and sections, the current paper combines MCP energy market models, stochasticity, rolling horizon foresight with potential learning by the players, and scenario trees that can change from one roll to the next based on decision made in the previous roll (e.g., endogenous probabilities). While these elements are not new in themselves, the combination of them is novel and leads to a more realistic perspective for modeling energy markets based on MCPs. Moreover, for such a new approach we have also introduced a new concept— the value of the rolling horizon or VoRH which is an attempt to measure how well the stochastic, rolling horizon MCP stacks up against a perfect foresight benchmark. In the subsections that follow, we describe in more detail the advantages of this modeling paradigm.

1.2. Rolling Horizon Stochastic MCP vs. Perfect Foresight MCP, Computational Advantages of Rolling Horizon

To date, the majority of natural gas and electric power market models assume perfect foresight of the time horizon being considered which is less than realistic since market planners don't have perfect information for the entire time horizon. In fact, energy decisions are often made under uncertainty with hedging of worst-case scenarios. With deterministic, perfect-foresight models, such things as costs and other parameters are assumed to be known with 100% certainty. As such, these models while instructive to serve as base cases, are less realistic than ones that allow for stochastic elements and/or some rolling horizon foresight, more in line with the way markets work.

In the latter case, there have been a number of stochastic MCPs. One problem with stochastic MCPs is that an already hard problem to solve (MCP) when large-scale, becomes prohibitively harder when even a small number of scenarios is considered. Two common approaches are to use scenario reduction [51], [39], [29], [23] or some sort of other strategy for stochastic equilibrium problems. For example, Haurie et al. [35] proposed a stochastic dynamic Nash-Cournot model with application in the European gas market. Ventosa et al. [57] used a traditional Stochastic Dynamic Programming (SDP) methodology to address the long term hydrothermal coordination of a generation company operating in a competitive market and at each SDP stage are stated as an MCP for a Cournot market equilibrium to represent the electricity market equilibrium. Dewolf and Smeers [12] built a stochastic Stackelberg game for the European gas market. Luna et al. [47] proposed two models for stochastic equilibrium: one based on the variational equilibrium of a generalized Nash game, and the other on the mixed complementarity formulation. The models differ in how the agents interpret their own actions on the market. Cabero et al. [7] developed a Benders approach for linear complementarity problems (LCP) for risk management in the Spanish power market. Demand, fuel prices and water inflow in the reservoirs are uncertain. Their work considered the master problem that determined output quantities and acceptable risk-levels (CVaR) in an oligopolistic setting among the producers while the subproblem minimized cost for each producer to generate power. Gabriel and Fuller [23] developed a decomposition method for general stochastic MCP and applied it to an electricity market model with stochastic demand with consideration of market power aspects addressed in the subproblems. Lastly, Shanbhag et al. [54] considered a stochastic multi-leader, multi-follower equilibrium problem where the players competed in both the forward and spot markets in successive periods. A stochastic MCP was used to find an appropriate equilibrium and results for a power network were presented.

An alternative approach, and one of the contributions of this paper, is to use a rolling horizon, stochastic MCP in which only a subset of time periods (e.g., four future quarters of a year) are considered at a time. Once roll r has been solved for using H time periods in total including the current one, in the next roll $r+1$, roll r's decision are fixed as parameters. In this way, the associated scenario tree is greatly reduced leading to shorter solution times and also the behavior of the market participants more closely matches how decisions are made in practice and captures unexpected events more accurately. Rolling horizon optimization is not new (see for example, [52], [15], [6], [45], [2], [56]) but not so often used in the energy literature. Several recent examples in energy system optimization are Tuohy et al. [55] for the Irish power sector, Guigues et. al. [33] who proposed a risk-average multistage stochastic program with robust rolling horizon approach to LNG (liquefied natural gas) contracts with a cancelation option and Devine et al. [11] who studied the UK natural gas market. By contrast, rolling horizon MCPs in energy or other disciplines is an unstudied area and thus the current paper offers something new in that domain.

In [46], a natural gas market model is solved recursively, i.e., the model is solved, firstly for the years 2005-2010, then for the years 2010-2015. Although similar, this is different from a rolling horizon. With rolling horizons, typically the same number of periods are used in different rolls of the model. For example, in the first roll, the time horizon is Q1 to Q4 while for the second roll the time horizon is from Q2 to Q5 and so on. Furthermore the variance associated with the demand scenarios for Q2, Q3 and Q4 reduces from roll one to roll two reflecting how, in reality and in contrast to [46], as time moves on information regarding future demand becomes clearer. In addition the model described in [46] does not model any stochastic variables.

It is important to note that the notion of perfect foresight vs. rolling horizons are not the same as including/excluding uncertainty. In perfect foresight (also called "open loop" equilibrium in the economics literature [18], [1]), the decisions for all the time periods are considered at one time by all the players. This is in contrast to the rolling horizon approach in which decisions are taken one time period at a time for all players with some sort of updating (potentially) of the parameters between these time periods. In addition, for a rolling horizon approach (also called "closed loop" in the economics literature [18]), it is understood that all decisions by all the players in prior periods are available. In both these cases the models can be deterministic. By contrast, a stochastic equilibrium involves either of these two notions: perfect foresight or rolling horizon but has some elements of the data uncertain and in the case of this paper, described by finite scenario trees.

Lastly, in Section 4.4 we outline some computational advantages of the rolling-horizon, stochastic MCP approach vs. scenario-reduction methods. As each rolling horizon problem is using a smaller scenario tree than perfect foresight, although solving it a number of times, the end result is that for larger problems the rolling-horizon approach is numerically superior at least on the tests used.

1.3. Endogenous Probabilities and Learning

Besides the above-mentioned computational and realism improvements that come with rolling horizon equilibrium problems (MCPs), there is another advantage that to our knowledge has not been well studied. More specifically, as each roll is a separate solving of an MCP, there is the opportunity to adjust inputs in between these rolls. For example, the new scenario tree for the next roll can be endogenously changed, by one or more players, based on a solution from the previous roll so that the model has endogenous probabilities. In effect, in an MCP setting, the market participant would also have scenario probabilities as decision variables besides more traditional ones like production, flow, or the like. In the case of optimization, decision-dependent uncertainty (endogenous probability) in stochastic programs gives arise to non-convex problems because the recourse model includes a probability multiplied by decision variables. Some relevant work that involves decision-dependent probabilities includes [50], [58], [40], and [41]. Pflug [50] applied a Robbins-Monro procedure for the optimization of simulated Markovian processes.

Viswanath et al. [58] adjusted scenario probabilities based on an MILP and distribution selection with binary variables. Held and Woodruff [40] proposed the network interdiction problem where endogenous uncertainties were attached in the network. Heuristic solution approaches were also shown in this study. Hellemo et al. [41] provided several formulations for modifying probability distributions: 1. Scaling for uniform distribution, 2. Convex combination for discrete distribution, 3. Optimization over parameters of the distribution, and 4. Approximation of distributions.

We believe our approach is a significant improvement in the sense that probabilities and other parameters are potentially updated after each roll. Thus, despite assuming a finite probability mass function (scenario tree), this approach is quite general and allows for any probability distribution and any updating rule. The input data or scenario tree can be changed based on the previous rolls' solution, but it does not make the problem get more complicated to solve since there are not bilinear terms of the form: probability multiplied by decision variable.

This feature of a decision-dependent scenario tree represents a tremendous benefit to the model as it allows for a novel endogenous learning and thus adds even more realism to the model as well as the potential for prescriptive guidance. Besides updating the scenario tree in between rolls, other data elements such as costs can also be modified. Consequently, the rolling horizon equilibrium perspective allows for endogenous spatial-temporal learning on the part of the market participants whose optimization problems make up the equilibrium problem. To be more specific, if for example the market share in profits for a particular gas producer has substantially decreased from one roll to a future one, the rolling horizon format then allows for lowering of costs by the producer involved to try to recover these profits in the next roll. Another example is adjusting in time period $t = r+1$ for over- or under-estimating sales in time $t = r$. The temporal aspects consider how profits, production, etc. change over time in some relative way for just the producer's own decision variables. The spatial aspects for instance, could relate to how well the producer fares with respect to other producers in the same or other nodes.

1.4. Connection to Sequential Games and Online Optimization Problems, the Value of the Rolling Horizon for a Stochastic MCP

From one perspective, this rolling horizon paradigm can be considered as an example of a sequential game ([42], [3]) where the rounds are the rolls themselves. If the time horizon is the full set of time periods then clearly one recovers the perfect foresight approach as a special case. In a market equilibrium characterized by a single-objective optimization problem without endogenous learning (e.g., maximzing social welfare), one would expect that the perfect foresight approach provides a better objective function value than a rolling horizon one and thus the former serves as some sort of benchmark. This is the benchmarking concept behind online optimization [53] for which different strategies to reoptimize after some new information is learned (e.g., in a vehicle routing problem) are compared against the optimization for which all information in the future is known in advance.

Even when no endogenous learning is included, such a straightforward comparison between an online strategy (relating to each roll in the current context), and the perfect-foresight equilibrium is not so obvious. This is because in the case of MCPs, there is no one objective function for all the equilibrium conditions.

A sometimes-used measure for MCPs which can help in comparing the rolling horizon MCP solutions to the perfect foresight ones is the following. The conditions in the MCP can be restated as finding the zero of the following function where $z = (x^T \; y^T)^T$:

$$H_i^{mid}(z) = z_i - mid(l_i, u_i, z_i - F_i(z)), \forall i \tag{3}$$

where $l_i = 0, u_i = +\infty$ for the nonnegative components (x in (2)) and $l_i = -\infty, u_i = +\infty$ for the other components (y in (2)) and $mid(a, b, c)$ is the median operator for the three scalars a, b, c [25], [21]. Consequently, z is a solution to the MCP if and only if $\|H(z)\| = 0$ for any vector norm $\|\cdot\|$ where $H(z) = (H_i(z), \; \forall i)$.

To make the notion of the $H-$function described above more clear, consider the following small example. Find $x \in R_+, y \in R$ so that

$$\begin{array}{l} 0 \leq 10x + 2y - 7 \perp x \geq 0 \\ 0 = 3x + 1y - 2 \qquad y \text{ free} \end{array} \tag{4}$$

where

$$F_1(x, y) = 10x + 2y - 7 \tag{5a}$$

$$F_2(x, y) = 3x + 1y - 2 \tag{5b}$$

The unique solution to (4) is $(x^*, y^*) = \left(\frac{3}{4}, -\frac{1}{4}\right)$ giving $H(x^*, y^*) = 0$ and for the non-solution $(x, y) = (1, -4)$, $\|H(1, -4)\|_1 = 4 > 0$. The H function (3) could be used to test the "efficiency" of the rolling horizon equilibrium solution vs. a perfect foresight approach for MCPs. More specifically, the rolling horizon equilibrium solution z^{rh} for each roll, after concatenating them together could then be evaluated against the vector associated with a perfect foresight solution z^{pf} but evaluated using this H function (or other ones) corresponding to the perfect foresight set of inputs. Such an analysis is done later in the paper using a new concept which we call the Value of the Rolling Horizon (VoRH) based on this $H-$ function. VoRH rather than the more traditional value of the stochastic solution (VSS) [5] is needed here for at least two reasons. First, in stochastic optimization models there is one objective function being optimized so that VSS makes sense. In an MCP setting there is no one objective but rather several possible ones for example profits [61] for each of the players or $||H(x)||$ for a suitable norm $||||$ and function H whose zero matches a solution of the MCP [14] [27] [49] [21] [29]. The second reason is that when learning or more generally having the scenario tree adjusted potentially at each roll, the concept of VSS for fixed probabilities, not dependent on the decision variables makes less sense."

Thus, in summary, the rolling horizon equilibrium problem offers some both computational and modeling realism advantages over more traditional approaches. In the rest of the paper we compare perfect foresight versus rolling horizon perspectives (Section 2), provide a specific natural gas formulation as an example (Section 3), provide selected computational results (Section 4) and conclude the contribution (Section 5).

2. Perfect Foresight vs. Rolling Horizon Perspective With and Without Uncertainty

In this section we provide some observations and counter-examples to more clearly differentiate between perfect foresight and rolling horizon MCPs with and without uncertainty. In Figure 1a the rolling horizon stochastic trees are displayed. In roll 1, the demand for time period 1 is known exactly and is deterministic. For time periods 2-4, demand is stochastic with the variance of demand increasing with time as Figure 1a shows with widening of the branches in the stochastic demand trees.

In roll 2, the demand for time period 2 becomes exactly known and deterministic. For time periods 3 and 4 demand remains stochastic but with less variance from the previous roll. Figure 1a also illustrates how information regarding time period 5 is only fed into the model in roll 2. A similar pattern can also be seen for rolls 3-5 in Figure 1a.

Figure 1b displays the stochastic demand tree associated with the perfect foresight version of the model. This graph shows that only demand at time period 1 is deterministic while demand at all other time periods is stochastic. This is in contrast to Figure 1a. In addition, Figure 1b also show how information regarding demand for all time periods is known at the start of the model which is again in contrast to Figure 1a.

Now consider an MCP of the form (6) with specific time periods $t = 1, \ldots, T$ and associated decision variables $(x^t, y^t), t = 1, \ldots, T$. This assumes that each variable has a t subscript which in many instances is not overly restrictive. The first question is whether or not if a deterministic rolling horizon approach is applied to solve (6), one MCP for each time period, will the solution set of that set of problems be the same as if the perfect foresight approach is used. The answer depends on the separability of the MCP function F as shown in the next result.

Essentially this result is saying if there is no linkage between the time periods either in constraints or data updating (or otherwise), then the two modeling paradigms of perfect foresight and rolling horizon match up. For most models including the gas market one considered in this paper this no-linkage assumption is not present. For instance, in the numerical examples presented in Section 4, there is linkage between the time periods as, after each MCP, the amount of gas in storage is updated using the amount of gas injected/extracted in the previous roll. In addition, the stochastic demand tree in these examples is also updated between rolls as time new periods are added to the tree and the variance associated with demand for gas at time t reduces. Clearly the answer is yes if the MCP function F is separable in t.

2.1. Value of Rolling Horizon

Besides comparing a rolling horizon vs. a perfect foresight solution to a deterministic MCP as discussed above, it is instructive to consider such a comparison when some of the data are stochastic. In that case, there are two effects to consider. First, the rolling horizon vs. perfect foresight aspects assuming nonseparability of the MCP function F in light of Theorem 1. Second, there are the stochastic aspects similar to the notion of value of the stochastic solution (VSS) for optimization problems as described in [5]. Different from an optimization problem, there is not one objective function to use but rather a number of merit functions such as the H function described earlier. Additionally, also different from optimization is the fact that due to the rolling horizon nature of the problem, the scenario tree itself may be endogenous changing due to equilibrium solutions at each roll. We introduce a new measure, the value of the rolling horizon (VoRH), to understand the importance of rolling horizon taking into account the two points just mentioned.

In order to explain VoRH, we first set up appropriate notation. First, we consider an MCP of the following form: find $x^t \in R^{t,n_x}, y^t \in R^{t,n_x}$

such that [1]

$$\begin{array}{l} 0 \le F_x^t\left(x,y\right) \perp x^t \ge 0 \\ 0 = F_y^t\left(x,y\right),\ y^t \text{ free} \end{array} \text{for } t = 1,\ldots,T \tag{6}$$

where

$$\begin{aligned} (x,y) &= \left\{\left(x^t,y^t\right), t=1,\ldots,T\right\} \\ F_x\left(x,y\right) &= F_x^t\left(x,y\right), t=1,\ldots,T \\ F_y\left(x,y\right) &= F_y^t\left(x,y\right), t=1,\ldots,T \\ F_x^t\left(x,y\right) &: R^{t,n_x} \to R^{t,n_x}, F_y^t\left(x,y\right) : R^{t,n_y} \to R^{t,n_y} \end{aligned}$$

Let SOL^{pf} be the solution set to (6) solving for all time periods at the same time, i.e., the perfect foresight perspective. For such a solution to (6), the H function defined earlier must necessarily satisfy $\left\|H_{\left(\overline{x}^{pf},\overline{y}^{pf}\right)}\left(\overline{x}^{pf},\overline{y}^{pf}\right)\right\| = 0$ for any vector norm $\|\cdot\|$. Note that the H function now is made dependent on a particular perfect foresight solution $\left(\overline{x}^{pf},\overline{y}^{pf}\right) \in SOL^{pf}$.

The VoRH concept is based on the notion of seeing how a solution to the rolling horizon set of problems compares with the perfect foresight one using the norm of this H function. More specifically, suppose that SOL^{rh} is the solution set to (6) but solving using a rolling horizon approach for each roll or time period. If $\left|SOL^{pf}\right| = \left|SOL^{rh}\right| = 1$, then a natural question is how far off from zero is using the rolling horizon solution in place of the perfect foresight one via the norm of the H function? Consequently, in the case of singleton solution sets, we have the following first definition of the VoRH.

Definition: *Assume that* $\left|SOL^{pf}\right| = \left|SOL^{rh}\right| = 1$. *Then, the value of the rolling horizon (VoRH) for problem (6) is defined as*

$$\left\|H_{\left(\overline{x}^{pf},\overline{y}^{pf}\right)}\left(\overline{x}^{rh},\overline{y}^{rh}\right)\right\| \tag{8}$$

and is always non-negative where $\left(\overline{x}^{pf},\overline{y}^{pf}\right) \in SOL^{pf}$ so that $\left\|H_{\left(\overline{x}^{pf},\overline{y}^{pf}\right)}\left(\overline{x}^{pf},\overline{y}^{pf}\right)\right\| = 0$.

Since the solution sets SOL^{pf} and SOL^{rh} may in fact not be singletons, a more generalized version of the above definition of VoRH is needed and is as follows.

[1] This notation implicitly assumes that there is the same set of variables for x and y, repeated for each time period t. This is not restrictive and can be relaxed by allowing n_x to vary by time period.

Definition: The value of the rolling horizon (VoRH) for problem (6) is defined as

$$\inf_{\left(\overline{x}^{rh},\overline{y}^{rh}\right)\in SOL^{rh}} \left\{ \sup_{\left(\overline{x}^{pf},\overline{y}^{pf}\right)\in SOL^{pf}} \left\| H_{\left(\overline{x}^{pf},\overline{y}^{pf}\right)}\left(\overline{x}^{rh},\overline{y}^{rh}\right)\right\| \right\} \tag{9}$$

where $\left\| H_{\left(\overline{x}^{pf},\overline{y}^{pf}\right)}\left(\overline{x}^{pf},\overline{y}^{pf}\right)\right\| = 0$.

3. Formulation

In this section, we describe the rolling horizon formulation for a stochastic natural gas market MCP. This formulation is then applied to a three-node system to provide insights into the rolling horizon approach.

3.1. Overview of Rolling Horizon and Market Participants

As stated before, the procedure a sequence stochastic MCPs each with partial foresight. Each of the stochastic MCPs corresponds to a roll of the rolling-horizon. At each roll r the model looks at $\bar{H}$ time steps ahead: The first timestep is $t = r$ and the last one is $t = r + \bar{H} - 1$. The set of timesteps for roll r is $T(r) = \{r, .., r + \bar{H} - 1\}$. For example, if $\bar{H} = 4$ then the time set for the first roll would be $T(1) = \{1, 2, 3, 4\}$. For the second roll it would be $T(2) = \{2, 3, 4, 5\}$ and so on. For each roll r, demand now (i.e., demand at time $t = r$) is known exactly and is scenario-independent. The first-stage decisions from the previous roll plus adjustments are used to meet this exactly known demand. For each roll r, first-stage decisions (gas production, injection, etc.) are made on how to meet the, at this point, uncertain demand for the next timestep ahead. These first-stage decisions are used in the next roll $(r + 1)$ to meet the exactly known demand in that roll. For each roll r, hypothetical decisions are made on how to meet the uncertain demand for all time steps greater than one time step ahead. In total, over all rolls, there are $TT = |R| + \bar{H} - 1$ timesteps where R is the set of rolls. Once the MCP for a given roll is solved, the model steps forward to the next MCP problem where the uncertain demand for the first time step ahead in the previous roll becomes known exactly. In this way, the model updates itself with decisions made in the previous roll. The above approach should be contrasted with a single roll optimization where all the future time periods are considered which we have called perfect foresight.

In the gas market model to be presented, only two sets of players are modeled: producers and a transportation system operator (TSO). Clearly many more market players/functions are available but to keep the model illustrative yet computationally manageable, these two sets of players were chosen. The producer is endowed with decisions on production, storage, as well as exports so in some sense it is a generalized version of actual ones. As the market being considered is small (roughly the size of the market operated by PJM[2]), we do not include liquefied natural gas (LNG) aspects as are considered in more global gas models such as the World Gas Model [26], Columbus [38], etc.

Tables 1-6, shown in the Appendix, describe the variables in the model, the data, and the functions involved. The following conventions are used: lower-case Roman letters indicate indices or variables, upper-case Roman letters represent parameters (i.e., data, functions), Greek letters indicate endogenous or exogenous prices while thousand cubic meters and million cubic meters are represented by kcm and mcm respectively.

3.2. Producer p's problem for roll r

Producer p maximizes profit by deciding how much gas to sell, to produce, inject to storage, extract from storage and how much to flow to other nodes/markets. Note that the expected value ($E_{s(r)}$) for the producer and other players means a weighted summation. That is

$$E_s\left(x^s\right) = \sum_s PROB_r^s x^s$$

where $PROB_{pr}^s$ is the probability of scenario s for roll r and x^s is the particular variable whose expectation is being taken. Thus, when the KKT conditions are taken relative to this variable x^s, there will be a factor of $PROB_{pr}^s$ in front of the variable. The producer's convex programming problem is given below with the associated KKT conditions shown in the Supplementary Appendix. This assumes the cost functions are convex.

[2]PJM is a regional transmission organization that coordinates the movement of wholesale electricity in 13 states in the Eastern half of the United States.

$$
\begin{aligned}
\max_{sales^*_{pmtr}, prod^*_{pmtr}, flows^{*,prod}_{patr}, inj^*_{pmtr}, xtr^*_{pmtr}} \sum_m \sum_{t=r}^{r+\bar{H}-1} \quad & D_t DAYS_t \Bigg\{ E_{s(r)} \Bigg[\pi^s_{mtr} sales^s_{pmtr} \\
& - C^{production}_{pmtr}(prod^s_{pmtr}) \\
& - \sum_{a \in A(p)} (\tau^{REG}_{at} + \tau^s_{atr}) flows^{s,prod}_{patr} - C^{storage}_{pmtr}(inj^s_{pmtr}, xtr^s_{pmtr}) \Bigg] \Bigg\} \\
& - D_{t=r} DAYS_{t=r} \Bigg(RU^{prod}_{pmr} prod^{adj+}_{pm(t=r)r} + RO^{prod}_{pmr} prod^{adj-}_{pm(t=r)r} \\
& + RU^{sales}_{pmr} sales^{adj+}_{pm(t=r)r} + RO^{sales}_{pmr} sales^{adj-}_{pm(t=r)r} \\
& + RU^{inj}_{pmr} inj^{adj+}_{pm(t=r)r} + RO^{inj}_{pmr} inj^{adj-}_{pm(t=r)r} \\
& + RU^{xtr}_{pmr} xtr^{adj+}_{pm(t=r)r} + RO^{xtr}_{pmr} xtr^{adj-}_{pm(t=r)r} \\
& + \sum_{a \in A(p)} (RU^{flows}_{par} flows^{adj+,prod}_{pa(t=r)r} + RO^{flows}_{par} flows^{adj-,prod}_{pa(t=r)r}) \Bigg) \\
& - D_{t=r+1} DAYS_{t=r+1} E_{s(r)} \Bigg[RU^{prod}_{pmr} prod^{SS+,s}_{pm(t=r+1)r} \\
& + RO^{prod}_{pmr} prod^{SS-,s}_{pm(t=r+1)r} \\
& + RU^{sales}_{pmr} sales^{SS+,s}_{pm(t=r+1)r} + RO^{sales}_{pmr} sales^{SS-,s}_{pm(t=r+1)r} \\
& + RU^{inj}_{pmr} inj^{SS+,s}_{pm(t=r+1)r} + RO^{inj}_{pmr} inj^{SS-,s}_{pm(t=r+1)r} \\
& + RU^{xtr}_{pmr} xtr^{SS+,s}_{pm(t=r+1)r} + RO^{xtr}_{pmr} xtr^{SS-,s}_{pm(t=r+1)r} \\
& + \sum_{a \in A(p)} (RU^{flows}_{par} flows^{SS+,s,prod}_{pa(t=r+1)r} + RO^{flows}_{par} flows^{SS-,s,prod}_{pa(t=r+1)r}) \Bigg]
\end{aligned} \tag{10}
$$

subject to:

$$
\begin{aligned}
& prod^s_{pmtr} + \sum_{a \in a^{in}(m)} (1 - LOSS_a) flows^{s,prod}_{patr} + xtr^s_{pmtr} \\
& = sales^s_{pmtr} + \sum_{a \in a^{out}(m)} flows^{s,prod}_{patr} + inj^s_{pmtr} \;\; \forall t, s, m \;\; (\lambda^{s,p1}_{pmtr})
\end{aligned} \tag{11a}
$$

$$
\begin{aligned}
MINSTOR_{pm} \le\, & INITSTOR_{pmr} + \sum_{e=r}^{t} DAYS_e[(1 - LOSS_m) inj^s_{pmer} - xtr^s_{pmer}] \\
& \forall t, s, m \;\; (\lambda^{s,p2}_{pmtr})
\end{aligned} \tag{11b}
$$

$$
INITSTOR_{pmr} \quad + \sum_{e=r}^{t} DAYS_e[(1 - LOSS_m) inj^s_{pmer} - xtr^s_{pmer}] \le MAXSTOR_{pm}
$$

$$\forall t, s, m \ (\lambda^{s,p3}_{pmtr}) \tag{11c}$$

$$flows^{s,prod}_{patr} \geq CONTRACTS_{pat} \quad \forall pats \ (\lambda^{s,p4}_{patr}) \tag{11d}$$

$$prod^{s}_{pmtr} \leq DP^{\max}_{pm} \forall s, t, m \ (\lambda^{s,p5}_{pmtr}) \tag{11e}$$

$$prod^{FS}_{pmtr} \leq DP^{\max}_{pm}, \ \ t = r+1, \forall m \ (\lambda^{p6}_{pmtr}) \tag{11f}$$

$$\sum_{t=r}^{r+\bar{H}-1} DAYS_t prod^{s}_{pmtr} \leq TP^{\max}_{pmr} \quad \forall s, m \ (\lambda^{s,p7}_{pmr}) \tag{11g}$$

$$inj^{s}_{pmtr} \leq DI^{\max}_{pm} \ \forall s, t, m \ (\lambda^{s,p8}_{pmtr}) \tag{11h}$$

$$inj^{FS}_{pmtr} \leq DI^{\max}_{pm}, t = r+1, \forall m, \ (\lambda^{p9}_{pmtr}) \tag{11i}$$

$$xtr^{s}_{pmtr} \leq DX^{\max}_{pm} \ \forall s, t, m \ (\lambda^{s,p10}_{pmtr}) \tag{11j}$$

$$xtr^{FS}_{pmtr} \leq DX^{\max}_{pm}, t = r+1, \forall m, \ (\lambda^{p11}_{pmtr}) \tag{11k}$$

$$sales^{s}_{pmtr} = SALES^{previous}_{pmt(r-1)} + sales^{adj+}_{pmtr} - sales^{adj-}_{pmtr}$$
$$t = r, \forall s, m \ (\lambda^{s,p12}_{pmtr}) \tag{11l}$$

$$sales^{s}_{pmtr} = sales^{FS}_{pmtr} + sales^{SS+,s}_{pmtr} - sales^{SS-,s}_{pmtr},$$
$$t = r+1, \forall s, m \ (\lambda^{s,p13}_{pmtr}) \tag{11m}$$

$$inj^{s}_{pmtr} = INJ^{previous}_{pmt(r-1)} + inj^{adj+}_{pmtr} - inj^{adj-}_{pmtr} \ \ t = r, \forall s, m \ (\lambda^{s,p14}_{pmtr}) \tag{11n}$$

$$inj^{s}_{pmtr} = inj^{FS}_{pmtr} + inj^{SS+,s}_{pmtr} - inj^{SS-,s}_{pmtr} \ \ , t = r+1, \forall s, m \ (\lambda^{s,p15}_{pmtr}) \tag{11o}$$

$$xtr^{s}_{pmtr} = XTR^{previous}_{pmt(r-1)} + xtr^{adj+}_{pmtr} - xtr^{adj-}_{pmtr} \ \ t = r, \forall s, m \ (\lambda^{s,p16}_{pmtr}) \tag{11p}$$

$$xtr^{s}_{pmtr} = xtr^{FS}_{pmtr} + xtr^{SS+,s}_{pmtr} - xtr^{SS-,s}_{pmtr} \ \ , t = r+1, \forall s, m \ (\lambda^{s,p17}_{pmtr}) \tag{11q}$$

$$flows^{s,prod}_{patr} = FLOWS^{previous,prod}_{pat(r-1)} +$$
$$flows^{adj+,prod}_{patr} - flows^{adj-,prod}_{patr} \ \ t = r, \forall s, a \ (\lambda^{s,p18}_{patr}) \tag{11r}$$

$$flows^{s,prod}_{patr} = flows^{FS,prod}_{patr} +$$
$$flows^{SS+,s,prod}_{patr} - flows^{SS-,s,prod}_{patr} \ \ , t = r+1, \forall s, a(\lambda^{s,p19}_{patr}) \tag{11s}$$

$$prod^{s}_{pmtr} = PROD^{previous}_{pmt(r-1)} + prod^{adj+}_{pmtr} - prod^{adj-}_{pmtr}$$
$$t = r, \forall s, m \ (\lambda^{s,p20}_{pmtr}) \tag{11t}$$

$$prod^{s}_{pmtr} = prod^{FS}_{pmtr} + prod^{SS+,s}_{pmtr} - prod^{SS-,s}_{pmtr} \ \ t = r+1, \forall s, m(\lambda^{s,p21}_{pmtr}) \tag{11u}$$

Producer p's objective function (10) maximizes their expected profit in all time periods less the recourse cost of making adjustments to first-stage decisions from the previous roll (e.g., $SALES^{previous}_{pmt(r-1)}$ for sales) in $t = r$ and second-stage decisions in $t = r + 1$, across all markets. The expected profit of producers is the money they receive from sales less the cost of production, less the cost associated with flowing gas through pipelines and less the cost of injections and extractions to and from storage.

Constraint (11a) ensures that the amount of gas producer p has entering market m equals the amount of gas they have exiting that market. Lower and upper bounds for the amount of gas producer p can have in storage at time t is provided by constraints (11b) and (11c) respectively. Constraint (11d) ensures that producer p must meet any contract that requires them to have a (pre-defined) fixed amount of gas flowing through pipeline a. An upper bound on the daily amount of gas producer p can produce in market m is provided in constraints (11e) and (11f) whilst constraint (11g) ensures the total amount of gas produced by producer p in market m, across all time steps, is capped. Similar constraints for the daily injection and extractions rates to/from storage are given by constraints (11h) - (11k).

Constraint (11l) allows $SALES^{previous}_{pmt(r-1)}$ (a first-stage decision from the previous MCP) to be adjusted. Note: the right-hand-side of equation (11l) contains no superscript s ensuring $sales^{s}_{pmtr}$ is the same across all scenarios for $t = r$. In contrast, constraint (11m) allows $sales^{FS}_{pmtr}$ to be adjusted, by second-stage decisions, for each scenario s. Equations (11n) - (11u) provide similar constraints for injections variables, extractions variables, flows through pipelines variables as well as production variables. The variable in the parentheses, alongside constraints (11a) - (11u), represent the Lagrange multipliers associated with that constraint.

Finally, all primal variables in producer p's problem are constrained to be non-negative. The superscripts $adj+$ and $adj-$ are associated with variables in time period $t = r$ only while the superscripts FS, $SS+$ and $SS-$ are associated with variables in time period $t = r + 1$ only.

3.3. Transportation system operator (TSO) problem for roll r

The next player is the transportation system operator who provides an economic mechanism to efficiently allocate transport capacity to producers. They are modeled as profit maximizers but in practice this objective could also be social welfare maximization. The TSO's convex programming problem is the following with the associated KKT conditions shown in the Supplementary Appendix. This assumes the cost function is convex.

$$\max_{flow^{*,tso}_{atr}} \sum_{a} \Bigg\{ \sum_{t=r}^{r+\bar{H}-1} D_t DAYS_t E_{s(r)} \Big[(\tau^{s}_{atr} + \tau^{REG}_{at}) flows^{s,tso}_{atr} - C^{a}(flows^{s,tso}_{atr}) \Big]$$
$$- D_{t=r} DAYS_{t=r} (RU^{flows}_{ar} flows^{adj+,tso}_{a(t=r)r} + RO^{flows}_{ar} flows^{adj-,tso}_{a(t=r)r})$$
$$- D_{t=r+1} DAYS_{t=r+1} E_{s(r)} \Big[RU^{flows}_{ar} flows^{SS+,s,tso}_{a(t=r+1)r} + RO^{flows}_{ar} flows^{SS-,s,tso}_{a(t=r+1)r} \Big] \Bigg\} \quad (12)$$

subject to:

$$flows^{s,tso}_{atr} = FLOWS^{previous,tso}_{at(r-1)} + flows^{adj+,tso}_{atr} - flows^{adj-,tso}_{atr},\quad t = r, \forall s, a \ (\lambda^{s,tso1}_{atr}) \quad (13a)$$

$$flows^{s,tso}_{atr} = flows^{FS,tso}_{atr} + flows^{SS+,s,tso}_{atr} - flows^{SS-,s,tso}_{atr},\quad t = r+1, \forall s, a \ (\lambda^{s,tso2}_{atr}) \quad (13b)$$

$$flows^{s,tso}_{atr} \leq DA^{\max}_{a} \ \ \forall s, t, a \ (\lambda^{s,tso3}_{atr}) \quad (13c)$$

$$flows^{FS,tso}_{atr} \leq DA^{\max}_{a}, t = r+1, \ \forall a \ (\lambda^{tso4}_{atr}) \quad (13d)$$

In a similar manner to the producer's problem their objective function is to maximize their expected profits less the recourse cost of making adjustments to first-stage decisions from the previous roll in $t = r$ and second-stage decisions in $t = r + 1$. The TSO's expected profit is the money they receive from producers less the cost associated with flowing gas through each pipeline a. The TSO receives two payments for each unit of gas flwon through one of their pipelines; a pre-defined regulated charge (τ^{REG}_{at}) and a market price (τ^{s}_{atr}).

Constraint (13a) allows $FLOWS^{previous,tso}_{at(r-1)}$ (a first-stage decision from the previous MCP) to be adjusted. In a comparison to similar constraints presented in producer p's problem, note that the right-hand-side of equation (13a) contains no superscript s ensuring $flows^{s,tso}_{atr}$ is the same across all scenarios for $t = r$. In contrast, constraint (13b) allows $flows^{FS,tso}_{atr}$ to be adjusted, by second-stage decisions, for each scenario s. Constraints (13c) and (13d) provide an upper bound on the daily amount of gas that can flow through each pipeline a. The variable in the parentheses, alongside constraints (13a) - (13d), represent the Lagrange multipliers associated with that constraint. In addition all primal variables in the TSO's problem are constrained to be non-negative.

3.4. Market-Clearing Conditions for roll r

Besides optimization problems for each of the producers and the TSO, there are market-clearing conditions. The market-clearing conditions for the producers balance supply and demand for gas by market and are shown in (14a). This equation ensures that the price of gas in each scenario s, market m and time period t (π^s_{mtr}) is equal to an inverse demand curve. Similarly for the TSO, the flows requested by the producers and allowed on the pipelines by the TSO are balanced in (14b). The Lagrange multipliers for these two sets of market-clearing conditions, respectively, π^s_{mtr} and τ^s_{atr}, are the marginal supply and marginal transportation prices.

$$\pi^s_{mtr} = Z^s_{mr} - B^s_{mr} \sum_p DAYS_t sales^s_{pmtr} \forall mtrs, \ \pi^s_{mtr} \text{ free} \tag{14a}$$

$$flows^{s,tso}_{atr} = \sum_p flows^{s,prod}_{patr} \ , \forall atrs, \ \tau^s_{atr} \text{ free} \tag{14b}$$

3.5. The Complete MCP at Roll r

The Karush-Kuhn-Tucker (KKT) conditions for both producers and the TSO are presented in the Supplementary Appendix. Assuming that all cost functions are convex these conditions are both necessary and sufficient for optimally for both players. The MCP for roll r consists of these KKT conditions in addition to the market clearing conditions (14).

3.6. Update rules after each MCP (or roll)

In natural gas markets many market parameters parameters update over time, for example, information regarding demand. In this section we show now the rolling horizon MCP captures this behavior. After each roll of the model, the following parameter changes take place:[3]

- All first-stage decisions in the previous MCP become the *previous* parameters for the next MCP, e.g., $flows^{FS}_{at(r-1)} \to FLOWS^{previous}_{atr}$
- The initial amount of gas in storage is updated based on the actual decisions (i.e., decisions made in time period $t = r$) made in the previous roll for injections and extractions for storage:
- Similarly, the total production capacity for producer p in market m ($TP^{\max}_{pmr}$) reduces by the actual amount of gas produced for $t = r$ in

[3]When a learning algorithm is used in the model, further model parameters (e.g., probabilities, production costs or recourse costs) are also updated. How this is implemented is described in detail in Section 4.3. The update rules presented in this section are in place in the model regardless of whether or not a learning algorithm is used or not.

the previous roll ensuring that production capacities deplete over time.

- The parameters associated with the stochastic demand tree update reflecting how information regarding demand updates throughout time, i.e., demand becomes less uncertain and eventually known actually as time moves forward. The stochastic demand for $t = r+1$ now becomes deterministic and known exactly. The stochastic demands for $t = r+1,..,r+\bar{H}-1$ remain uncertain but with reduced variance while information regarding stochastic demand for time period $t = r+\bar{H}$ now becomes available to the model.

- The index for roll increases by one: $r \to r+1$

4. Numerical Results for Three-Node Model

4.1. Data

In this section numerical examples of the rolling horizon MCP are presented. The model is formulated with $|P| = 3$ producers, $|M| = 3$ markets, $|S| = 3$ demand scenarios and with $|R| = 8$ rolls. The three markets are connected via $|A| = 4$ pipelines as shown in Figure 2 and roughly represent the following states in the United States (including the District of Columbia):

- $M = 1$: New Jersey, New York and Pennsylvania
- $M = 2$: Illinois, Indiana, Michigan, Ohio, Wisconsin
- $M = 3$: Delaware, District of Columbia, Florida, Georgia, Maryland, North Carolina, South Carolina, Virginia, West Virginia

For each roll/MCP solved, the time horizon is $\bar{H} = 4$ which means that the total number of time steps is $TT = |R|+\bar{H}-1 = 11$. Each time step represents a season, i.e., spring, summer, fall or winter. As a result, $DAYS_t = 89$ for $t = 1,5,9$ and $DAYS_t = 92$ for $t = 2,3,4,6,7,8,10,11$. As the model is being solved over a relatively short timescale, no discount factor is considered, i.e., $D_t = 1\ \forall t$. Each of the producers plus the TSO assign equal probabilities to the demand scenarios. Hence, $PROB^s_{pr} = \frac{1}{|S|} = \frac{1}{3}\ \forall p,s$ and $PROB^s_r = \frac{1}{|S|} = \frac{1}{3}\ \forall s$. Note: in some of the example presented in Section 4.3, these probabilities change between rolls.

The maximum daily production rates ($DP^{\max}_{pm}$) for producer p in market m are given in Table 7 while, for the first roll, the total production capacity ($TP^{\max}_{pmr}$) is given by the following equation:

$$TP^{\max}_{pmr} = (365000)(DP^{\max}_{pm}). \tag{15}$$

Note: as Section 3.6 describes, $TP^{\max}_{pmr}$, reduces between rolls when there has been production in that field. The values for $DP^{\max}_{pm}$ are based on information provided by the United States' Energy Information Authority [4] while the values for $TP^{\max}_{pmr}$ are arbitrarily chosen. The cost function $C^{production}_{pmtr}(x)$ for producer p in market m is a Golombek cost function [26] and is given by the following equation:

$$C^{production}_{pmtr}(x) = (\alpha_{p,m} - \gamma_{p,m})x + \frac{1}{2}\beta_{p,m}x^2 + \gamma_{p,m}(DP^{\max}_{pm} - x)\ln\left(\frac{DP^{\max}_{pm} - x}{DP^{\max}_{pm}}\right) \forall t, r, \tag{16}$$

where the parameters $\alpha_{p,m}$, $\beta_{p,m}$ and $\gamma_{p,m}$ are given in Tables 8 - 10 respectively.

The maximum daily injection to and extraction from storage rates are displayed in Tables 11 and 12 respectively. The minimum ($MINSTOR_{pm}$) and maximum ($MAXSTOR_{pm}$) amount of gas allowed in storage for each producer p in market m is zero and 10^6, mcm respectively. For the first roll of the model the initial amount of gas in storage ($INTSTOR_{pm}$) is set to zero for each producer p in market m. For subsequent rolls, the initial amount is determined by the update rules described in Section 3.6. The storage cost function is assumed linear as follows:

$$C^{storage}_{pmtr}(inj^s_{pmtr}, xtr^s_{pmtr}) = 1.7(inj^s_{pmtr} + xtr^s_{pmtr}), \quad \forall p, m, t, r, s. \tag{17}$$

The value of 1.7 is arbitrary but illustrative. The maximum daily capacity for each pipeline a ($DA^{\max}_a$) is given in Table 13 while the TSO's pipeline cost function is also assumed to be linear as follows:

$$C^a(flows^{s,tso}_{atr}) = (C^{MARG}_a)(flows^{s,tso}_{atr}), \quad \forall t, r, s, \tag{18}$$

where C^{MARG}_a is given in Table 14. The regulated pipeline tariff that the TSO receives is set equal to the marginal pipeline cost, i.e., $\tau^{REG}_{at} = C^{MARG}_a$, $\forall t$. Again, these costs are both arbitrary and illustrative.

The loss factors associated with injections to storage and flows through pipelines are both set to 5%, i.e, $LOSS_a = 0.05, \forall a$ and $LOSS_m = 0.05, \forall m$. In the numerical examples described in this section, no contract flows are assumed meaning $CONTRACTS_{pat} = 0, \forall p, a, t$ while all recourse costs are given a value of 0.2.

The slope values associated with the inverse demand function (see equation (14a)), are given by the following equation:

$$B^s_{mtr} = \bar{B}_{mtr}, \quad \forall m, t, s, \tag{19}$$

[4] http://www.eia.gov

where $\bar{B}_{mtr}$, is the deterministic demand slope for market m and time t with values displayed in Table 15. The corresponding demand intercepts are given as follows:

$$Z^s_{mtr} = VAR^s_t \bar{Z}_{mtr}, \quad \forall m, t, s, \tag{20}$$

where $\bar{Z}_{mtr}$ are the deterministic demand intercepts for market m at time t and VAR^s_t are the multipliers applied to the deterministic demand giving increased/decearsed demand to scenario s in time t. The values for $\bar{Z}_{mtr}$ are 723.481, 643.743 and 483.49 for markets $m = 1, 2, 3$, respectively, $\forall t, r$. Table 16 displays the values for VAR^s_t. In this numerical example, demand scenario $s = 3$ is deemed to be the *high* demand scenario, scenario $s = 2$ the *low* demand scenario and $s = 1$ the *middle* demand scenario.

4.2. Perfect Foresight vs. Rolling Horizon Foresight

In this example, we show how the rolling horizon MCP format captures unexpected events more realistically than a perfect foresight approach. In particular we show how and why the price spike, as a result of the unexpected event, reduces as information regarding the event becomes known earlier. The unexpected event in this example is increased demand for $t = 7$. Increased demand is obtained by increasing the deterministic demand intercept ($\bar{Z}_{mtr}$) levels, described in Section 4.1, by 50%. In the *Base Case*, the rolling horizon MCP is run without any increased demand and with all parameters as described in Section 4.1. In the *No Foresight* Case, none of the players have information about the increased demand until roll 7; see Figure 3b for the increase in demand for $t = 7$ in roll $r = 7$ only, compared with the *Base Case* in Figure 3a.

In the *1 Period Ahead Foresight* Case, each player can see the increased demand one period (roll) ahead, i.e., in roll 6. See Figure 3c for the increase in demand for $t = 7$ in rolls $r = 6$ and $r = 7$ only, compared with the base case in Figure 3a. Note: while the values for the stochastic demand increase for $t = 7$ in roll 6 for the *1 Period Ahead Foresight* case, the probabilities associated with the high, low and medium demand scenarios remain the same.

In the *3 Period Ahead Foresight* Case, each player can see the increase in demand three periods (rolls) ahead, i.e., in roll 4. See Figure 3d for the increase in demand (compared with the base case in Figure 3a) for $t = 7$ for all rolls from $r = 4$ to $r = 7$. Again, while the values for the stochastic demand increase for $t = 7$ in rolls 4-6 for the *3 Period Ahead Foresight* case, the probabilities associated with the high, low and medium demand scenarios remain the same.

In the *Perfect Foresight* Case, there is only one roll of the model and each player can see all time periods ahead at the start of the model. As a result, each player can see the increased stochastic demand from the start of the model; again, see Figure 4 for the increase in demand for $t = 7$ compared with Figure 1b.

Figure 5 shows how increased information allows for smaller price spikes for the prices in market $m = 1$ ($\pi_{m=1,t=1,r}$) while Figure 6 displays the net injections of gas storage across all markets, i.e, the sum of all injections to storage less the sum of all extractions. In the *No Foresight* Case, Figure 6 explains how only a small amount of gas is extracted from storage in roll 7 because, not knowing about the increased demand in previous rolls, a relatively small amount of gas is in storage prior to roll 7. As a result, Figure 5 shows how this case has the largest spike in price for roll 7.

In the *One Period Foresight* and *Three Period Foresight* Cases, there is prior knowledge of the stressed demand. As a result, Figure 6 shows increases in the amount of gas injected to storage in rolls 5 and 6 which in turn allows for an increased amount of gas to be withdrawn from storage in roll 7. Hence, as Figure 5 shows, the price spikes in roll 7 for these two cases is not as big as in the *No Foresight* Case. However, because of the increase in the amount of gas injected into storage, there is increased prices seen in the rolls prior to roll 7 for these two cases (relative to the *Base Case* and *No Foresight* Case).

As Figure 6 shows, the *Perfect Foresight* Case has the largest amount of injections (cumulatively) to storage before roll 7, leading to the largest amount of extractions from storage in roll 7. This allows the price spike in roll 7 to be the smallest but there are increased prices (relative to the Base Case) seen from roll 1.

In the *One Period Foresight*, *Three Period Foresight* and *Perfect Foresight* Cases, the market is able to prepare for the increased demand by injecting gas into storage prior to roll 7. The more gas that is injected to storage prior roll 7 ensures smaller prices spikes in roll 7.

Withdrawals from storage in roll 8 can only be seen in the *Base Case* and in the *Perfect Foresight* Case. This is because, in all other cases, storage facilities are emptied to meet the increased demand in roll 7.

Similar gas price trends and spikes were found for markets $m = 2$ and $m = 3$ as those presented for $m = 1$ in Figure 5, i.e., the sooner the market knows about the increased demand, the smaller the price spike in roll 7 is relative to the *Base Case*.

4.2.1. VoRH results for increased demand example

Table 17 displays the VoRH results associated with the examples described above in Section 4.2. Here the solutions from the *No Foresight*, *One Period Foresight* and *Three Period Foresight* cases are compared with the *Perfect Foresight* solutions.[5,6] The second column of Table 17 displays the VoRH results as calculated by equation (8) while in the third column these VoRH figures relative to the *Base Case* VoRH are presented.

The *No Foresight* case has the largest VoRH indicating that the solutions arising form this case are furthest away from the *Perfect Foresight* solutions. This is not surprising because, as explained previously for Figures 5 and 6, the *No Foresight* case has the largest price spike for $t = 7$ and market participants only adjust the storage levels when they become aware of the increased demand in roll $r = 7$, which is in contrast to the *Perfect Foresight* case.

However, the *One Period Foresight* case has a smaller VoRH value compared with the *Three Period Foresight* case suggesting that the solutions from the *One Period Foresight* case are closer to the *Perfect Foresight* solutions that those from the *Three Period Foresight* case. This is a surprising result because one would expect, as more foresight is added to the model, that the solutions would become closer to those from the *Perfect Foresight* case, all things being equal.

When these results were analyzed more closely it was found that the median value for $|H_{i,\left(\overline{x}^{pf},\overline{y}^{pf}\right)}\left(\overline{x}^{rh},\overline{y}^{rh}\right)|$ (see equation (8)), for each of the cases, was zero suggesting that there are a few $|H_{i,\left(\overline{x}^{pf},\overline{y}^{pf}\right)}\left(\overline{x}^{rh},\overline{y}^{rh}\right)|$ values that are skewing the VoRH results. As a result, the VoRH figures were calculated again with $|H_{i,\left(\overline{x}^{pf},\overline{y}^{pf}\right)}\left(\overline{x}^{rh},\overline{y}^{rh}\right)|$ values greater than two excluded; see column four of Table 17. As before, column five of Table 17 displays these VoRH figures relative to the *Base Case* VoRH. The results in these columns show that as more foresight of the increased demand is added to the model, the VoRH figures decrease informing us that the solutions to the rolling horizon MCP get closer to those from the *Perfect Foresight* case as more information regarding demand is added to the model. Thus, these re-calculated results show the benefit of using VoRH in comparing solutions from perfect foresight with those from rolling horizon. Finally, it must noted that this analysis is an initial study of the VoRH and future work will examine the metric further.

[5]The *Base Case* VoRH is excluded from this analysis as this case does not contain any increased demand and is thus not comparable with any of the other cases.

[6]This analysis assumes locally unique solutions in each case.

4.3. Learning algorithm and endogenous probabilities

In this section we examine the effects of several different learning algorithms relative to approximating the extended Nash-Cournot problem discussed in the introduction. Clearly, for the original form of the problem getting solutions even for the three-node network would be challenging given the non-convex nature of each optimization problem when players can adjust data. Consequently, the KKT conditions would not be valid (in general) so that the resulting MCP could not be formed. As an alternative, we compare the rolling horizon results in terms of producer profits, consumer surplus, and VoRH to gain insight using four learning algorithms as discussed next.

4.3.1. The Learning Algorithms

All four of the learning algorithms are based on making adjustments on either scenario probabilities and/or costs relative to a shift in market share calculated from the producer profits. The idea is that the producer will take some sort of action either in adjusting these scenario probabilities or its costs (e.g., lower them) to be more competitive and as such, this activity will approximate the more complicated, original approach discussed in the introduction where both probabilities and decision variables are under the control of the players.

Each producer is given the following threshold levels of market share:

- Producer 1: 35%
- Producer 2: 5%
- Producer 3: 57%

Any deviation below these shares results in the appropriate learning algorithm activating. The learning algorithms are also not proportional to the amount of deviation below these thresholds.

For learning algorithm 1, each of the three producers only takes action if their own market share drops below the threshold value stated above. If this occurs, then the producer increases the probability of the high demand scenario by one-sixth and decreases the probabilities for the other two scenarios each by one-twelfth. Note that initially, each of the three demand scenarios have a equal probability of one-third for each scenario. The idea is that the producer is trying to increase its own market share by making the higher demand scenario more likely to adjust its decisions accordingly for larger profits.

For learning algorithm 2, a similar line of reasoning to the previous one is employed except that the producer decreases its α parameter by 50%. Recall that α represents the coefficient of the linear term in the Golombek production cost function.

For learning algorithm 3, both the producer's own scenario probabilities and costs are activated when its market share goes below the threshold. In that event, the probability for the high demand scenario is increased, the probability for the other two scenarios are decreased, and the α parameter is decreased by 50% as stated above.

Lastly, while the first three algorithms only considered the producer's own market share as a trigger for adjusting scenario probabilities or its costs, by contrast, learning algorithm 4 also considers the actions of another producer. The rule for this algorithm from the perspective of producer $p = 1$ or 2, is if its market share goes below its threshold or if the α parameter for producer 3 was decreased in the previous roll, signaling producer 3 getting more competitive by reducing its costs, then the action by producer $p = 1, 2$ follows that of learning algorithm 3. Producer 3 in this situation follows the logic of learning algorithm 3. This algorithm all things being equal should result in more adjustments by the players.

Table 18 describes the results of these numerical experiments for each of the learning algorithms compared with the case when there is no learning algorithm. This table displays profits for each of the producers, consumer surplus[7] and VoRH expressed as percentage deviations from the no-learning case. The most dramatic changes occur in comparing no learning vs. learning algorithm 4 when the other producers' behavior is taken into account. Indeed, the profits for producer 1 are improved by over 16% and for producer 3 they are increased by about 13%. This comes at a definitive loss for producer 2 whose profit tumbles about 36%. This occurs perhaps because for producer 2, their high demand scenario probability increases to 1 given that they frequently update given their market share dips below their threshold. This leads to overproducing and hence they have a high recourse cost and consequently reduced profits. In fact, for three of the four learning algorithms, producer 2 does worse than with no learning and it is only learning algorithm 2 where they actually make more profits than no learning. This is due to producer 2 not adjusting their scenario probabilities at all. Also, it is interesting to note that producer 3 always does better by any learning algorithm related perhaps to being the player with the largest market share and hence "weathering the storm" of all the adjustments by the

[7]For further details on we calculated consumer surplus see [22].

other two players. It is only learning algorithm 2, related to each producer potentially lowering their costs, when all three of the producers do better than no learning. This is perhaps related to the fact that cost-cutting measures, independent of market share always benefit the bottom line and would thus potentially be a more attractive strategy (at least in this example), than adjusting scenario probabilities as attempted in the numerical tests. In terms of consumer surplus the two learning algorithms that involve cost reductions do best as would be expected since the consumers are not having to pay as much to the producers.

Figure 7 shows the prices in market $m = 1$ by roll considering each of the learning algorithms. Perfect foresight has the lowest prices given the least changes occurring to the system and hence the least uncertainty reflected in the prices. Learning algorithm 4 is the 2nd lowest set of prices indicating perhaps that, relative to prices at least, this algorithm is best to eliminate some uncertainties and data adjustment by the producers. Lastly, no learning or having each producer just adjusting it's own probabilities (learning algorithm 1) result in the highest prices perhaps reflecting disadvantages to the consumer as the producers are not receiving appropriate market signals to adjust their decisions accordingly. Similar results were found for markets $m = 2$ and $m = 3$.

From Table 18 we see that the perfect foresight approach always does slightly better in terms of profit and consumer surplus than the no-learning case potentially due to the increased value stemming from more information about future demand. Also, the perfect foresight case has much less variation in the producer profits than learning algorithms 1-4 since the data are not adjusted for each roll. The Value of the Rolling Horizon shows dramatically higher values than perfect foresight (which would have a value of zero) and no learning which is small and almost zero. These large VoRH values indicate that relative to the perfect foresight cases, the learning algorithms 1-4 are dramatically different in their solutions due to all the data adjustments and the resulting decisions made by the players. By contrast, a low VoRH for the no-learning case means that it does not differ so much from having perfect foresight which is also evidenced in the the profit figures for perfect foresight close to those for the no-learning case.

4.4. Computational efficiency

To test the computational efficiency of the rolling-horizon approach described above, several tests were performed using the three-node network outlined earlier. For simplicity, all experiments assumed linear costs. Specifically, a perfect foresight model with 10,000 demand scenarios was the base from which we started. These 10,000 scenarios differed by random demand obtained by assigning random values to VAR_t^s using a uniform distribution between the maximum and minimum values in Table 16 for each time period. Then, a fast forward scenario-reduction algorithm [39] was applied to produce a much smaller number of scenarios: 3, 10, 50, and 100 as indicated in Table 19.

As shown in Table 20, with just 3 scenarios, there were 2386 variables in the rolling-horizon MCP which was solved for 4 time periods and 8 rolls as outlined above. By contrast, the perfect-foresight approach from which 3 scenarios were selected had 5095 variables due to a larger number of time periods considered at the same time. 10 replications using these 3 scenarios were then performed on a 3.3GHz i5-4590 quad-core processor with 8GB of RAM and with a convergence tolerance used of 10^{-6}, the default for the PATH solver. Since the same 3 scenarios were not selected for each of these 10 tests, the CPU times varied as shown in these results. The minimum, median, and maximum CPU times for both the perfect foresight /scenario reduction and rolling horizon (summed over 8 rolls) approaches are shown in Tables 19 and 20 respectively. Using the median CPU the rolling-horizon approach is slower but this is expected for small problems where the repeated rolls add too much time compared to just solving the perfect foresight version with only 3 scenarios.

For the case of 10 scenarios, both approaches are approximately the same in terms of CPU time. However, when a larger number of scenarios are considered, specifically 50 or 100 scenarios, the rolling-horizon approach is significantly faster. From Tables 19 and 20 we see that the median CPU for 50 scenarios using rolling horizon is about only 9% of the perfect foresight version of the problem. This large speed-up when using a rolling horizon is no doubt due to the significantly smaller number of variables (36,837 vs. 81,987) that is needed when only partial foresight is considered. This computational advantage of the rolling-horizon stochastic MCP approach also holds up for the 100-scenario case shown in Tables 19 and 20. The results here are even more dramatic. Specifically, the scenario-reduction/perfect foresight approach fails to solve the problem in 21,600 seconds (6 hours) which was when the GAMS program was stopped. However, the rolling-horizon approach solves all the instances with a median time of about 3.39 hours. As a further test we allowed one of the scenario-reduction/perfect foresight experiments with 100 scenarios to be run to completion and found it took over 39 hours (CPU time) to obtain an optimal solution. This further highlights the benefits of the rolling horizon approach.

5. Conclusions

In this paper we have introduced and developed the concept of a stochastic mixed complementarity problem (MCP) for a rolling horizon with a natural gas market application. This format, while more closely matching how energy other markets function, also allows for decision-dependent scenario-tree probabilities and endogenous learning by the market participants. Several theoretical concepts (e.g., the Value of the Rolling Horizon) and numerical results were developed and shown to validate the rolling horizon approach.

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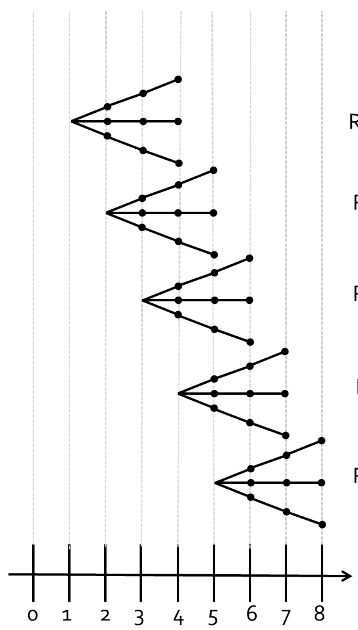
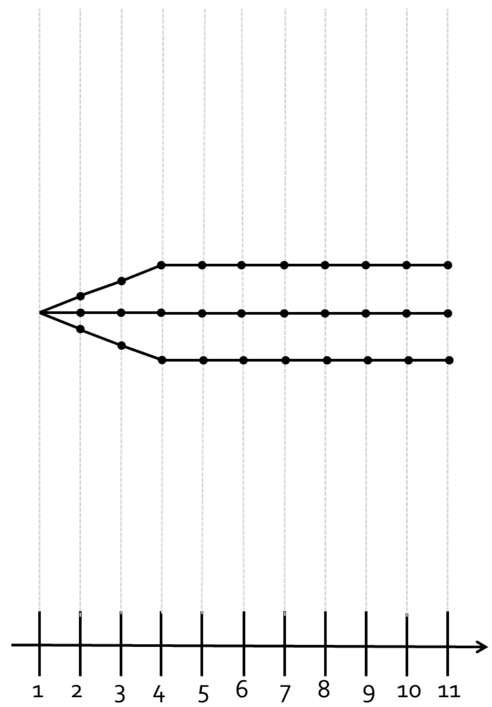
6. Appendix

6.1. Figures



(a) Rolling horizon of stochastic demand scenarios tree for rolls $r = 1$ to $r = 5$.



(b) Perfect foresight of stochastic demand tree.

Figure 1

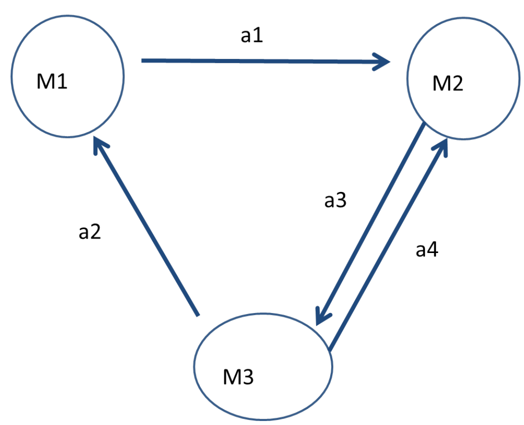
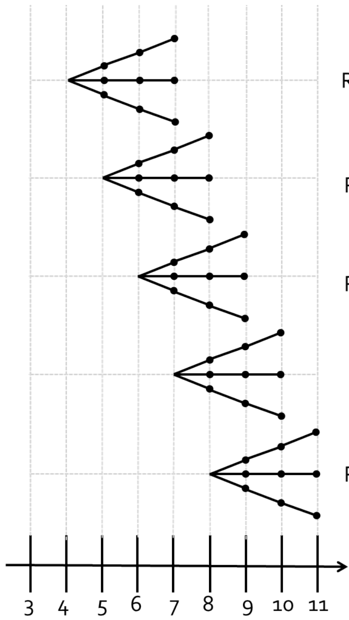
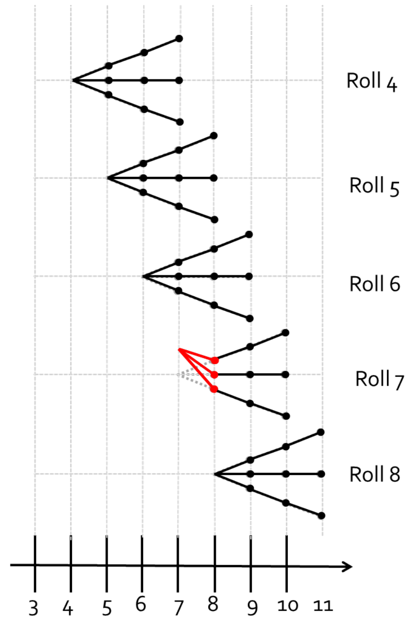


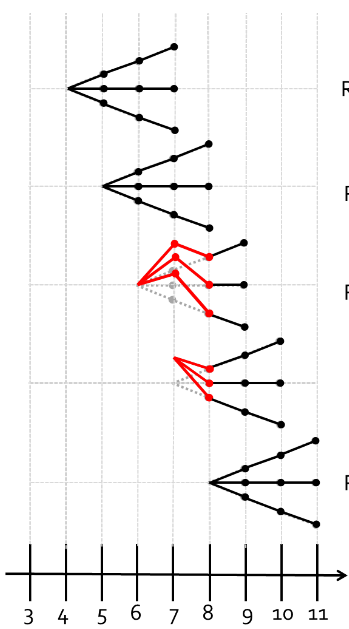
Figure 2: Pipelines connecting three markets.



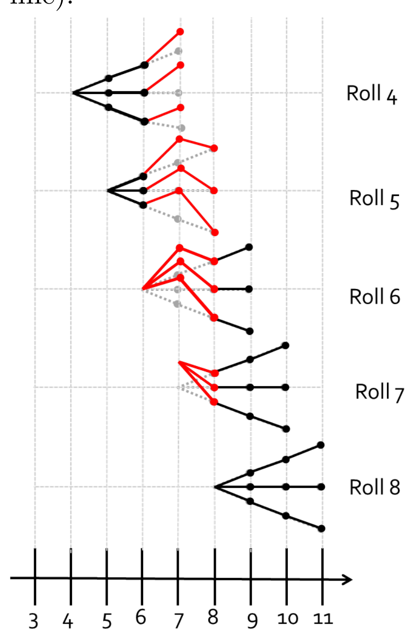
(a) Base case (no increase in demand).



(b) No foresight of increased demand for time $t = 7$ (continuous line) with Base Case (dotted line).



(c) One period ahead foresight of increased demand for time $t = 7$ (continuous line) with Base Case (dotted line).



(d) Three period ahead foresight of increased demand for time $t = 7$ (continuous line) with Base Case (dotted line).

Figure 3: Stochastic demand scenario trees for rolls $r = 4$ to $r = 8$ for increased demand example.

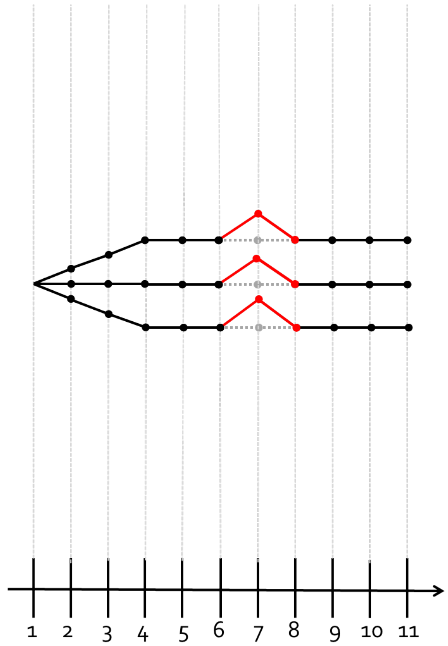


Figure 4: Perfect foresight of increased stochastic demand for $t = 7$ (continuous line) with Base Case (dotted line).

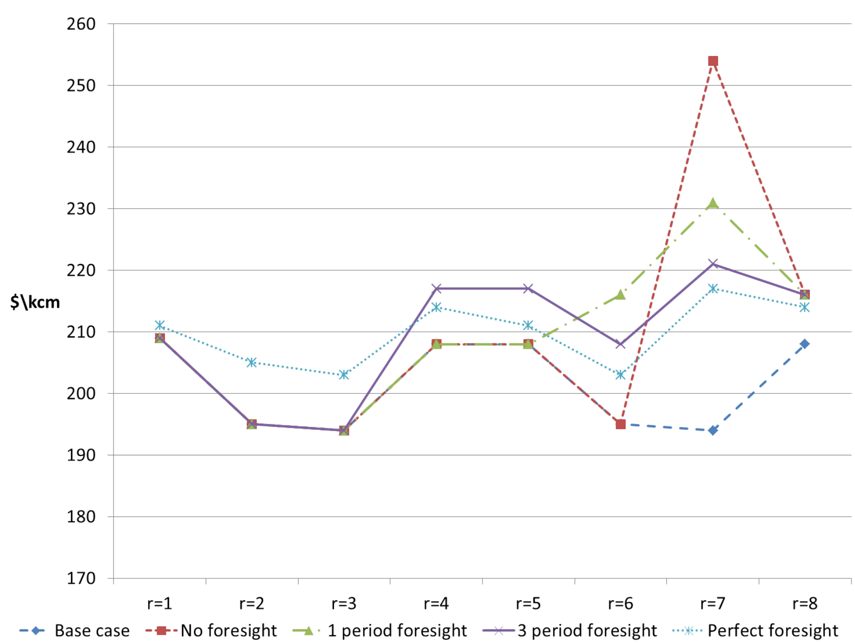


Figure 5: Prices in market $m = 1$.

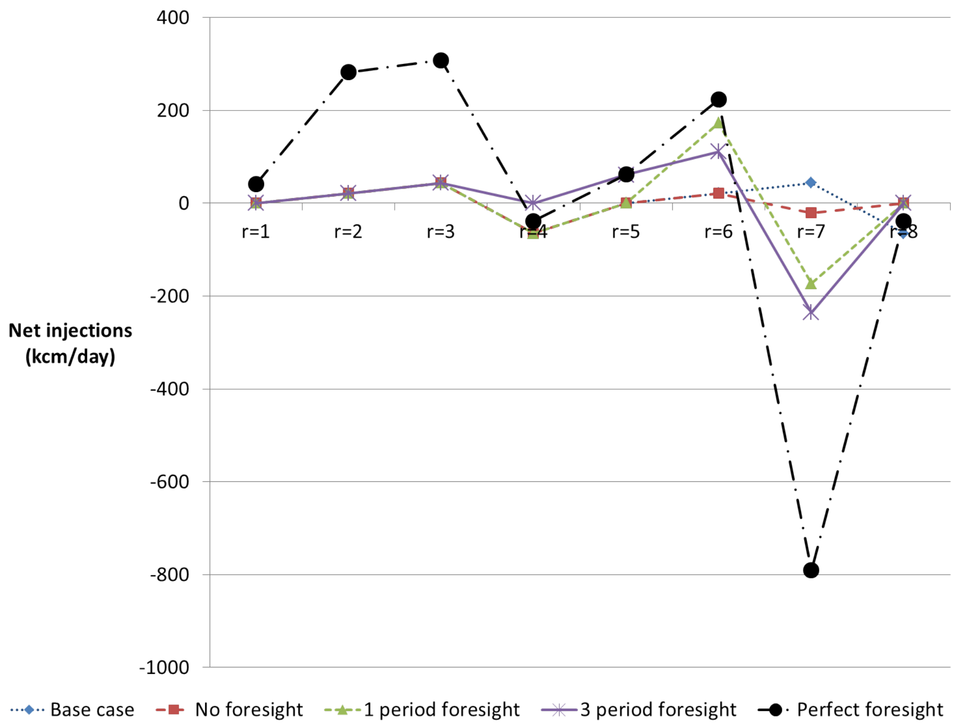


Figure 6: Injections less extractions (cumulative across all markets).

Figure 7: Prices in market $m = 1$ for different learning algorithms.

6.2. Tables

Table 1: Sets

$a \in A$	Arcs (gas pipelines, LNG, other distribution).
$p \in P$	Producers.
$a \in A(p)$	Arcs that producer p is connected to.
$r \in R$	Rolls. For each roll an MCP is solved.
$t \in T(r) = \{r, .., r + H - 1\}$	Time set for roll r (e.g., quarters) where H is the time horizon, i.e, the number of timesteps in a roll.
$s(r) \in S(r)$	Scenario for stochastic elements, depending possibly on roll r.
$m \in M$	Gas node/market m.
$a^{in}(m)$	Arcs inward to market/node m.
$a^{out}(m)$	Arcs outward from node m.
$e \in \{r, .., t\}$	Dummy time index for storage constraint that represent timesteps from r to t.

H	The time horizon, i.e, the number of timesteps in a roll.
$TT = \|R\| + H - 1$	Total amount of timesteps over all rolls.
D_t	Discount factor for time t (%).
$DAYS_t$	Number of days in time period t.
τ_{at}^{REG}	Regulated price for using arc a at time t ($/kcm).
$DP_{pm}^{\max}$	Maximum daily production capacity for producer p at node m (kcm/day).
$DI_{pm}^{\max}$	Maximum daily storage injection rate for producer p at node m (kcm/day).
$DX_{pm}^{\max}$	Maximum daily storage extraction rate for producer p at market/node m (kcm/day).
$DA_{a}^{\max}$	Maximum arc capacity for arc a (mcm/day).
$TP_{pmr}^{\max}$	Total production capacity for producer p at node m over the whole time horizon (mcm).
$MINSTOR_{pm}$	Minimum amount of gas needed at storage facility at node/market m for producer p (mcm).
$INITSTOR_{pmr}$	Initial amount of gas in storage facility at node/market m for roll r for producer p (mcm).
$MAXSTOR_{pm}$	Maximum amount of gas allowed in storage facility at node/market m for producer p (mcm).
$LOSS_m$	Injection to storage loss factor for node m (%).
$LOSS_a$	Arc a loss factor (%).
$CONTRACTS_{pat}$	Contracted gas that producer p must provide through arc a at time t (mcm/day).

Z^s_{mr}	Fixed demand (demand curve intercept) at market/node m for roll r and scenario s (kcm).
B^s_{mr}	Demand curve slope at market/node m for roll r and scenario s (\$/kcm).
RU^*_{pmr}	Recourse/penalty cost associated with underestimating for producer p in market/node m for roll r (\$/kcm). There are different costs for each primal variable.
RO^*_{pmr}	Recourse/penalty cost associated with overestimating for producer p in market/node m for roll r (\$/kcm). There are different costs for each primal variable.
RU^{flows}_{ar}	Recourse/penalty cost for TSO associated with underestimating flows through arc a for roll r (\$/kcm).
RO^{flows}_{ar}	Recourse/penalty cost for TSO associated with overestimating flows through arc a for roll r (\$/kcm).
$PROB^s_{pr}$	probability producer p associates with scenario s at roll r.
$PROB^s_r$	probability TSO associates with scenario s at roll r.

Table 2: Parameters. Note: Any variables with "*previous*" superscript are parameters. Also, the superscript $*$ is a placeholder for multiple superscripts.

Table 3: Functions

$C^{production}_{pmtr}(.)$	Production cost function for producer p at node m at time t roll r (\$/day).
$C^{storage}_{pmtr}(.)$	storage cost function for producer p at node m at time t roll r (\$/day).
$C^{arc}_{atr}(.)$	operations cost function for arc a at time t roll r (\$/day).

Table 4: Primal variables: Each of these primal variables have multiple superscripts which are described below. The superscript $*$ is a placeholder for the superscripts in Table 5.

$sales^{*}_{pmtr}$	Amount producer p, at node m, sells at time t. Decision made at roll r (kcm/day).
$prod^{*}_{pmtr}$	Amount produced by producer p, at node m, at time t. Decision made at roll r (kcm/day).
inj^{*}_{pmtr}	Amount injected into storage by producer p, at node m, at time t. Decision made at roll r (kcm/day).
xtr^{*}_{pmtr}	Amount extracted from storage by producer p, at mode m, for at time t. Decision made at roll r (kcm/day).
$flows^{*,prod}_{patr}$	Producer p's flows through arc a at time t. Decision made at roll r (kcm/day).
$flows^{*,tso}_{atr}$	TSO flows through arc a at time t. Decision made at roll r (kcm/day).

Table 5: Superscripts

s	Scenario s.
$previous$	Decisions made in the most immediately previous MCP, at roll $r-1$. These represent parameters of the model.
$adj+$	Balancing decision made at roll r for time period $t = r$. Represents an increase.
$adj-$	Balancing decision made at roll r for time period $t = r$. Represents a decrease.
FS	First-stage decisions that are made at roll r for time period $t = r + 1$. These are scenario-independent.
$SS+$	Second-stage decisions that are made at roll r for time period $t = r + 1$. These represent an increase and are scenario-dependent.
$SS-$	Second-stage decisions that are made at roll r for time period $t = r + 1$. These represent an increase and are scenario-dependent.

Table 6: Dual variables

τ^s_{atr}	price for arc a at time t at scenario s. Decision made at roll r (\$/kcm).
π^s_{mtr}	market-clearing price of gas for node m, time t, scenario s and roll r (\$/kcm).
$\lambda^{*,p\#}_{\cdot}$	Lagrange multiplier associated with constraint # in producer's problem (unit depends on the constraint).
$\lambda^{*,tso\#}_{\cdot}$	Lagrange multiplier associated with constraint # in TSO's problem (unit depends on the constraint).

Table 7: Values for DP^{max}_{pm} (mcm/day).

	$m=1$	$m=2$	$m=3$
$p=1$	3.5	4.5	3.9
$p=2$	10.7	39	4.5
$p=3$	8.9	0.5	26.9

Table 8: Values for α_{pm}

	$p=1$	$p=2$	$p=3$
$m=1,2,3$	60	48	60

Table 9: Values for β_{pm}

	$m=1$	$m=2$	$m=3$
$p=1$	2.2	0.25	1.8
$p=2$	122	60	2.6
$p=3$	0.42	0.75	0.4

Table 10: Values for γ_{pm}

	$m=1$	$m=2$	$m=3$
$p=1$	6.9	6.9	6.9
$p=2$	6.8	6.6	6.8
$p=3$	6.9	6.9	6.9

Table 11: Values for DI^{max}_{pm} (mcm/day).

	$m=1$	$m=2$	$m=3$
$p=1,2,3$	150.1	423.2	91.7

Table 12: Values for DX^{max}_{pm} (mcm/day).

	$m=1$	$m=2$	$m=3$
$p=1,2,3$	300.2	846.4	183.4

Table 13: Values for $DA_a^{\max}$ (mcm/day).

$a = 1$	$a = 2$	$a = 3$	$a = 4$
13	241	46	78

Table 14: Values for C_a^{MARG} ($/kcm).

$a = 1$	$a = 2$	$a = 3$	$a = 4$
1	2	2	2

Table 15: Values for the inverse demand function slope $\bar{B}_{mtr}$ ($/kcm).

	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$	$t = 7$	$t = 8$
$m = 1$	0.024	0.028	0.029	0.022	0.024	0.028	0.029	0.022
$m = 2$	0.018	0.02	0.021	0.016	0.018	0.02	0.021	0.016
$m = 3$	0.013	0.015	0.015	0.012	0.013	0.015	0.015	0.012

Table 16: Values for VAR_t^s for the numerical example described in Section 4.1.

	$s = 1$	$s = 2$	$s = 3$
$t = 1$	1	1	1
$t = 2$	1	0.95	1.05
$t = 3$	1	0.9	1.1
$t = 4$	1	0.8	1.2

Table 17: VoRH values for increased demand examples in Section 4.2.

	VoRH	Relative VoRH	VoRH without large values	Relative VoRH without large values
No foresight	292616.95	2.69	93.40	0.90
One period ahead foresight	216355.14	1.99	83.07	0.80
Three periods ahead foresight	230268.63	2.12	65.00	0.63

Table 18: Profits, consumer surplus and VoRH for no-learning case plus % change (compared with no-learning case) for learning algorithms & perfect foresight.

Learning Algorithms	$Profit_{p=1}$	$Profit_{p=2}$	$Profit_{p=3}$	CS	VoRH
No Learning	2.32×10^7	2.49×10^6	3.34×10^7	1.05×10^{10}	1.07×10^5
Learning Alg.-1	−0.61%	−31.17%	1.14%	−0.84%	1626.83%
Learning Alg.-2	10.13%	11.52%	14.59%	12.96%	1.61%
Learning Alg.-3	7.83%	−16.81%	11.59%	8.75%	793.07%
Learning Alg.-4	16.61%	−35.86%	12.87%	12.02%	1089.56%
Perfect foresight	1.72%	0.01%	2.10%	2.26%	N/A

Table 19: Minimum, Median and Maximum CPU time (in seconds) associated with Perfect Foresight model with scenario obtained from the fast forward selection scenario-reduction algorithm.

No. of Scenarios	Model Variables	Min CPU	Median CPU	Max CPU
3	5095	0.2	0.21	0.22
10	16547	1.93	2.03	7.71
50	81987	270.41	7468.91	17632.71
100	163787	> 21600	> 21600	> 21600

Table 20: Minimum, Median and Maximum CPU time (in seconds) associated with Rolling Horizon model.

No. of Scenarios	Model Variables	Min CPU	Median CPU	Max CPU
3	2386	0.66	0.81	1.49
10	7517	2.16	2.74	7.04
50	36837	45.01	673.06	1596.71
100	73487	1596.71	12214.74	17549.27