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Review / Pregledni znanstveni članak

The GNSS-Levelling Local Geoid Determination Using the Conditional Adjustment with Unknown Model

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ABSTRACT. GNSS-Levelling geoid determination is defined as the mathematical relation between orthometric and ellipsoidal height. The local polynomial surface interpolation method is one of the most widely applied methods for GNSS-Levelling geoid determination. The basic solution of polynomial surface interpolation is based on the geoid undulation, the difference of orthometric and ellipsoidal height, including some undetectable measurement error. In this study, firstly, the1st, 2nd and 3rd degree polynomial surface were taken as the geoid surface to determine the appropriate polynomial degree for selected study area, Samsun province of Turkey, using ordinary least squares adjustment models. Then, the Conditional Adjustment with Unknown Model adjustment solutions are made for these real work data. At the end of application, the results were then compared to these models and some suggestions were made for future applications.

Keywords: geoid undulation, Conditional Adjustment with Unknown Model, optimization, ordinary least square adjustment.

1. Introduction

The geoid, accepted as the earth shape, is a closed surface formed by the earth gravity fields (Jekeli et al. 2013). The geoid determination is the one of important research area of geodesy, geophysics, oceanography etc. (Albayrak et al. 2020). The reference surface of height is taken as geoid and the orthometric heights (*H*) can be obtained the vertical distance from the geoid. Global Navi-

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gation Satellite Systems (GNSS) surveys provides a lot of advantages (Maciuk 2018). One of them is the ellipsoidal height (h), the vertical distance from the reference ellipsoid (Meyer et al. 2006). The relationship between the ellipsoidal and orthometric heights is called geoid undulation. The geoid undulation can be determined as globally, nationally, regionally, or locally. The local geoid provides the high accuracy according to the other geoid for study areas. The local geoid determination applications are increased and facilitated (Denker and Wenzel 1987, Kenduiywo et al. 2013, Sanso and Sideris 2013, Doganalp 2016, Albayrak et al. 2020, Oduyebo et al. 2022).

The nationally geoid determination study in Turkey has been began in the 1970s (Ayan 1978, Gürkan 1978). In the 1991, Turkey Geoid-1991 (TG-91) was calculated with gravity, global earth potential model using least square collocation method. TG-91 and GNSS-Levelling data of Turkish National Fundamental GNSS Network had been combined and Turkey Geoid-1999 (TG-99) has been obtained. TG-99 was transformed TG-99A with new measurement after 1999 Kocaeli earthquake. The TG-99A has got a 15 cm external accuracy. Then, Turkey Geoid 2003 (TG-03) was calculated with some gravity measurements, Earth Geopotential Model 96, sea gravity anomaly and digital elevation model using least square collocation method. TG-03 was to TG-07 transformed with GRACE global earth potential model using global fast Fourier transform method. The external accuracy of TG-07 has been detected as 8.8 cm. At the end The Turkey geoid-2009 (TG-09) was established. In the consequence of the external accuracy of TG-09 was 8.4 cm, it was decided that the modernization of Turkish elevation system and reinforcement of gravity infrastructure (Yildiz 2012, Simav et al. 2015). This project was completed in 2020 and Turkey Geoid-2020 (TG-20) is actual geoid for Turkey. The accuracy of TG-20 was 1.3 cm – 6.3 cm with gravity measurement (Simav et al. 2021). The local geoid determination studies are frequently determined in addition to national geoid determination in Turkey (Akyilmaz et al. 2003, Yilmaz et al. 2006, Erol et al. 2008, Tusat 2011, Soycan 2013, Doganalp and Selvi 2015, Kirici and Sisman 2015). In these studies, generally the regional suitability of different geoid determination models is analysed. The surface fitting with polynomial Interpolation using geoid undulation is one of the local geoid determination methods.

Adjustment calculus according to the selected aim function must be realized to increase the accuracy and obtain a unique solution with observation number larger than unknown number. Generally, an ordinary least squares (OLS) solution is applied according to its objective function. The design matrix of the adjustment is considered that is set up with error-free elements. In the case where the elements of the design matrix consist of observations, this case does not occur. If the elements of the design matrix consist of observations, both the unknown of the problem and the observation errors can be calculated together for the solution. In some geodetic applications such as 2D or 3D coordinate transformation, line or surface fitting of local geoid determination, the design matrix is set from the observations. The OLS solution may be insufficient about the effect of observation errors in the design matrix on the results. In this case, a special solution should be made with a different approach. This solution is named the error in variables (EIV) model (Golub and van Loan 1980, Carroll and Ruppert 1996, Huffel 2004, Schaffrin and Snow 2019a). The EIV model can be solved according to the Total Least Square (TLS) solution. In recent years, TLS has frequently used some application of geodesy. Furthermore, the Conditional Adjustment with Unknown (CAU) Model, one of the older the EIV model (Vanicek and Krakiwsky 1986a, Schaffrin and Snow 2019b).

In this study, the authors were aimed local geoid determination in the Samsun city of Turkey using surface fitting with polynomial Interpolation. For this aim, firstly, the 1st, 2nd and 3rd degree local polynomial interpolation (LPI) geoid surface were calculated to decide which surface is suitable for study area using OLS solution. Then, the CAU model was made for found suitable geoid surface with obtained compatible observation group. The results of OLS and CAU models were compared and the advantages or disadvantages of the CAU model were discussed.

2. Methods and Material

2.1. Methods

The fundamental vertical datum used in geodesy since it is a reference surface for physical height. There geoid determination is based on geodetic and gravimetric techniques. These techniques can be classified according to the use of data and mathematical models (Vaníček and Christou 1994, Bolat 2013). The accuracy of geoid determination can be increased to use physical and geometrical height together. The Physical height, named orthometric height (H^*), is a distance along the plumb (vertical) direction from the geoid surface; the Geometrical heights, named the ellipsoidal height (h), is the distance along the ellipsoidal normal direction from the ellipsoid surface, thus H^* has a physical mean, h has a geometric mean. Therefore, h does not have a physical mean in practical engineering applications (Featherstone et al. 1998). While the H^* is measured using geometrical levelling methods, the h is measured using GNSS methods. However, the relationship between H^* and h must be determined for h to be used in engineering studies. It also means relating the reference ellipsoid to the geoid (Amalvict and Boavida 1993). This is called as geometric approach (Erol and Celik 2004).

It is quite difficult to mathematically determine the globally geoid model for the earth's surface. Therefore, the local geoid is determined using simple surfaces regionally. The local geoid surface must be determined to facilitate the transition between the physical orthometric height H^* and the geometrical ellipsoidal height h. The geoid and the reference ellipsoid, which are the reference surfaces of the height measurements, do not match. The difference in these surfaces is named geoid undulation (N), (Fig. 1) (Grafarend 1994, Erol and Celik 2004, Borowski and Banaś 2019).

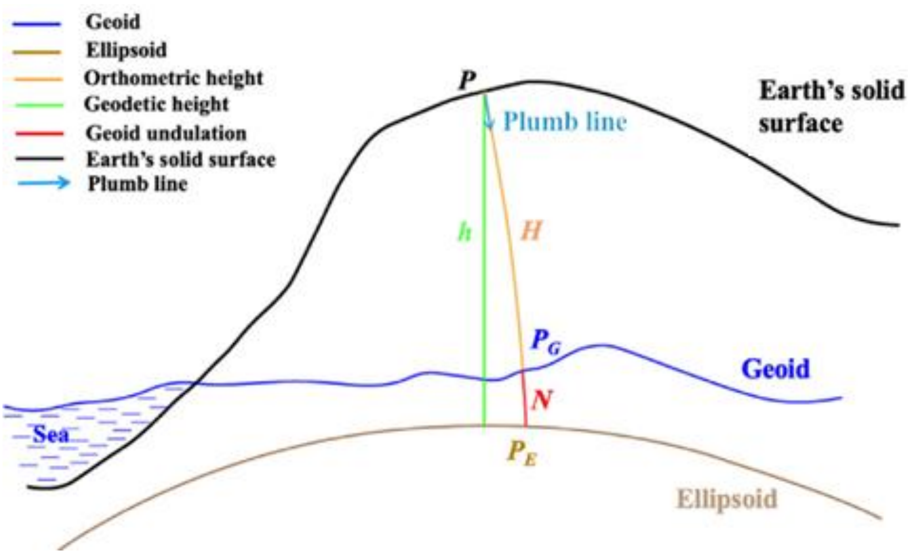

Fig. 1. *Geometric relationship between geoid undulation, orthometric and ellipsoidal heights (Xie et al. 2021).*

N can be defined roughly for all points as:

$$N_i = h_i - H_i^* \tag{1}$$

This relation is named as GNSS-Levelling. There are a lot of methods to determine N geoid undulation for GNSS-Levelling. The most applied GNSS-Levelling method is the surface fitting with local polynomial interpolation (LPI) method. In the LPI model, the undulation of N geoids are determined according to the horizontal coordinates of the point using the common coordinates of the point (Borowski and Banas 2018). The horizontal coordinates and orthometric and ellipsoidal heights of the common points are known. The LPI equation can be given according to the common-point coordinates:

$$N_{(x,y)} = \sum_{i=0}^{n} \sum_{j=0}^{n} a_{ij} x^i y^j \tag{2}$$

Here; a_{ij}, x, y, and n are the polynomial coefficient, the coordinates of the points, and the number of common points. The polynomial degree is selected as 1st, the equation of 1st degree LPI surface model is followed:

$$N_{(x,y)} = a_{00} + a_{10}x + a_{01}y \tag{3}$$

It is preferred that the common point number is larger than the polynomial coefficient number to increase sensitivity and accuracy. In this case, the adjustment calculation must be made according to the aim function to obtain the best polynomial coefficient. In the adjustment procedure of the LPI model, the common point coordinates are generally considered to be error-free. In this case, the solution is made for the geoid determination with the LS solution. But this consideration is not true in real-world applications like geoid determination, coordinate transformation, map linearization, etc. (Borowski and Banasik 2020). The solution must be made according to the EIV approaches. The CAU is one of the oldest EIV models (Vanicek and Krakiwsky 1986b, Ozturk and Serbetci 1992, Zalud et al. 2015).

2.2. OLS optimization solution of geoid determination

OLS is the most popular adjustment model due to its ease of application and calculation. The mathematical models and the objective function of the 1st degree LPI according to OLS can be given the following equations:

$$V = Ax - \ell;\ Q_{\ell\ell} = P_{\ell\ell}^{-1} \tag{4}$$

$$V^T P V = \min \tag{5}$$

where (4) is the mathematical model, (5) is the objective function, V, A, x, ℓ, $Q_{\ell\ell}$ and $P_{\ell\ell}$ are the residuals vector, design matrix, unknowns (coefficient parameters of LPI) vector, remains vector, the cofactor and weight matrix common points.

$$A = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ .. & .. & .. \\ 1 & x_n & y_n \end{bmatrix} x = \begin{bmatrix} a_{00} \\ a_{10} \\ a_{01} \end{bmatrix} V = \begin{bmatrix} V_{N1} \\ V_{N2} \\ .. \\ V_{Nn} \end{bmatrix} \ell = \begin{bmatrix} N_1 \\ N_2 \\ .. \\ N_n \end{bmatrix} \tag{6}$$

The LS solution is given in (4–5):

$$x = (A^T P A)^{-1} A^T P \ell \tag{7}$$

The root mean square error (RMSE) is obtained from the following for the LS model:

$$m_0 = \pm\sqrt{\frac{V^T P V}{f}} \tag{8}$$

Here, $f = n - u$ is the redundancy number of the OLS model, n and u are the number of measurements and unknown parameters (polynomial coefficient) (Vanicek and Krakiwsky 1986a, Ghilani and Wolf 2006).

The 2nd and 3rd degree LPI surface model can be given following equations

$$N_{(x,y)} = a_{00} + a_{10}x + a_{01}y + a_{20}x^2 + a_{02}y^2 + a_{11}xy \tag{9}$$

$$\begin{aligned} N_{(x,y)} = a_{00} + a_{10}x + a_{01}y + a_{20}x^2 + a_{02}y^2 + a_{11}xy + a_{30}x^3 \\ + a_{21}x^2y + a_{12}xy^2 + a_{03}y^3 \end{aligned} \tag{10}$$

The solution of 1st, 2nd and 3rd degree LPI surface model according to OLS can be done using equations (4–7). Although the OLS has got a lot of advantages, it has got some disadvantages. The most important advantage is weakness of determination outlier detection. The outliers have got a different distribution from the observation groups, and they are corrupt the results of adjustment. The outlier detection is realized some approaches. The conventional outlier detection method is used frequently (Baarda 1968, Koch 1999, Berné Valero and Baselga Moreno 2005). The outlier detection can be done according to student t-test using hypothesis testing. In this process;

Step 1: to establish null and alternative hypothesis

H_o: There is no outlier in observation group;

H_S: There is at least one outlier in observation group;

Step 2: to calculate the t test value using V, residuals and Q_{VV} the cofactor matrix of residuals

$$t_i = \frac{|v_i|}{m_0\sqrt{Q_{v_i v_i}}}$$

Step 3: to find the critical values using two-way t-test table

$$q = t_{f,1-\alpha/2}$$

Step 4: to compare the test and critical values

If $t_i < q$ then there is no outliers in observation group,

If $t_i > q$ then there is at least one outlier in observation group. The observation, have biggest t_i test value, is determined as outlier. Then, the adjustment and outlier detection procedures are repeated using new observation group until null hypothesis is valid.

2.3. CAU model optimization of geoid determination

The design matrix of adjustment holds the coordinates of the points in the determination of geoids. In this case, it is not true that the common-point coordinates should be taken as error-free. Therefore, a solution should be made for the residuals of the common-point coordinates using a different adjustment model. The CAU optimization model can be used for the solution. In the solution, the conditional equations and associated unknowns are added to the model. The solution of CAU can be listed as follows (Vanicek and Krakiwsky 1986b, Ozturk and Serbetci 1992, Zalud et al. 2015):

1. determine the number of measurements, unknowns, correlates, and condition equations,
2. set up the condition and calculate the normal equations,
3. extend the objective function for conditional equations,
4. calculate k correlates and the residuals for common point coordinates,
5. obtain the adjusted measurement,
6. obtain the sensitivity values.

The mathematical model and the objective function of the CAU model can be given below (Fotiou 2018), where (10) is the mathematical model, (11) the objective function:

$$Ax + BV + W = 0;\ Q_{\ell\ell} = P_{\ell\ell}^{-1} \tag{11}$$

$$V^T PV - k^T (Ax + BV + W) = \text{min.} \tag{12}$$

where k, B, V, W and a_{ij}^0 are the correlates vector, design matrices of common point coordinates, residuals vector of common point coordinates, remains vector and the approximate value of LPI coefficient parameters (Vanicek and Krakiwsky 1986a, Schaffrin and Snow 2019b):

$$A = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ .. & .. & .. \\ 1 & x_n & y_n \end{bmatrix} \; V = \begin{bmatrix} V_{x1} \\ V_{y1} \\ V_{N1} \\ V_{x2} \\ V_{y2} \\ V_{N2} \\ .. \\ V_{xn} \\ V_{yn} \\ V_{Nn} \end{bmatrix} \; W = \begin{bmatrix} N_1 \\ N_2 \\ \ldots\ldots \\ N_n \end{bmatrix} \; x = \begin{bmatrix} a_{00} \\ a_{10} \\ a_{01} \end{bmatrix} \tag{12}$$

$$B = \begin{bmatrix} a_{10}^0 & a_{01}^0 & -1 & 0 & 0 & 0 & .. & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{10}^0 & a_{01}^0 & -1 & .. & 0 & 0 & 0 \\ .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\ 0 & 0 & 0 & 0 & 0 & 0 & .. & a_{10}^0 & a_{01}^0 & -1 \end{bmatrix}$$

The CAU model is made according to the objective function named the Lagrange function given in (9):

$$x = -\left(A^T\left(BP^{-1}B^T\right)^{-1}A\right)^{-1} A^T\left(BP^{-1}B^T\right)^{-1}W \tag{13}$$

$$k = \left(BP^{-1}B^T\right)^{-1}Ax - \left(BP^{-1}B^T\right)^{-1}W \tag{14}$$

$$V = P^{-1}B^T k \tag{15}$$

The RMSE is obtained from the following for the CAU model;

$$m_0 = \pm\sqrt{\frac{V^T PV}{f}} \tag{16}$$

Here, the $f = n - u + q$ is the redundancy number of CAU model, q is the number of correlates and $q = \mathrm{rank}\left(\mathrm{B}P^{-1}B^T\right)$ (Schaffrin and Snow 2019b).

2.4. Materials

Data were taken from studies to demonstrate an urban information system of Samsun province, located in the middle Black Sea region of Turkey. The longitude and latitude of the study area are between 41°0' – 41°30' North and 36°0' – 36°40' East. The range of point orthometric heights varies between 1.118 m and 921.858 m. The study area was given in Fig. 2.

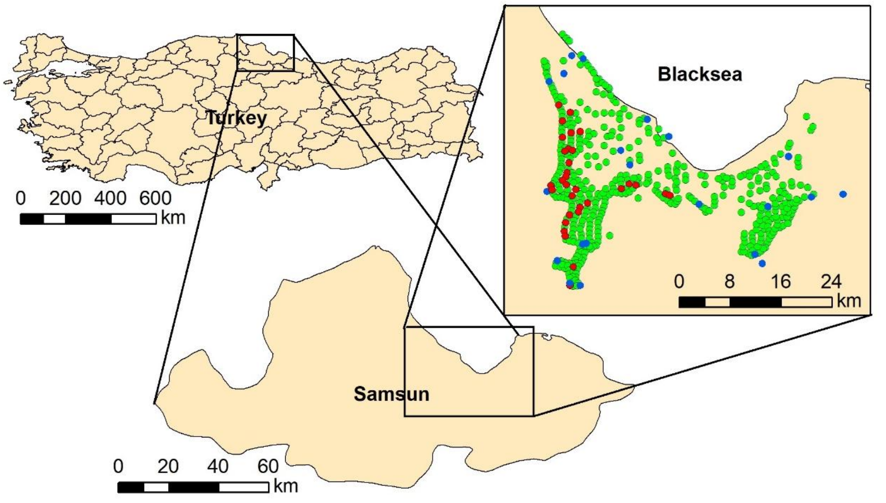

Fig. 2. *The study area.*

The 482 point was measured by the Samsun Metropolitan Municipality for the urban information system The GNSS coordinates of the points were measured in WGS84 Datum as (*phi*, *Lamda*, *h*) using two different network (Fig. i–ii). Then the horizontal Cartesian and grid point coordinates were computed in the ITRF96 Datum (Table 1). The orthometric point heights (H^*) were determined in the Turkey National Vertical Control Network 1999 in GRS80 Ellipsoid. The accuracy of ellipsoidal and orthometric height were 9.6 mm and 8.3 mm.

Table 1. *The coordinates of the common points, heights and undulation.*

PN	Y [m]	X [m]	H^* [m]	h [m]	$N = h - H^*$ [m]
360667	525295.360	4577109.000	3.071	31.556	28.485
360742	521127.440	4572212.890	122.108	151.358	29.250
…	…	…	…	…	…
F373H077	544254.3866	4563349.7289	14.1150	42.4610	28.3460
F37G001	556111.5575	4565271.2133	15.1960	43.2130	28.0170

The 53 points in a suitable distribution were selected as common point, the other 429 points were taken as control point from Table 1 for this study. The all coordinates (observations) were taken as same accuracy in the adjustment process. Thus, the $Q_{\ell\ell}$ and $P_{\ell\ell}$ matrix of observations were equal to unit matrix.

3. Application and Results

The basic aim of the study is investigating the availability of CAU model in local geoid determination. All of the computations were made using MatLab programming Languages. Firstly, it is necessary to determine the suitable local geoid for this aim. Therefore, there was two stages in application. The first stage: the 1st, 2nd and 3rd degree local polynomial geoid surface were calculated to decide suitability for study area. In this stage the OLS solution was used, and the outlier detection was made to obtain the compatible control point data. The second stage: The CAU model solution was made for found suitable geoid surface using compatible observation group.

3.1. The first stage and results

In this study the 1st, 2nd and 3rd degree local polynomial geoids were determined using OLS for Samsun province according to Eq. (3) and Eq. 14. The OLS model solution was realized on the basis of the common point. For the application, firstly the N geoid undulations of the points were calculated using Eq. (1) and the horizontal coordinates of common points were normalized. The mathematical model given Eq. 4 was created using the common points given in Fig. 2 according to objective function given in Eq. (5). In the solution only N was considered erroneous, the horizontal coordinates were taken without errors. The outlier detection was made to obtain the compatible common point group for the 1st, 2^{nd,} and 3rd degree LPI surface according to significant level 0.05. The results of first stage were given in Table 2.

Table 2. *The results of the OLS model for 1st, 2^{nd,} and 3rd degree LPI.*

Polynom Degree	Common Point	RMSE (cm)	Outlier (Y/N)	Outlier Point	Polynom Degree	Common Point	RMSE (cm)	Outlier (Y/N)	Outlier Point
1st	53	19.51	Yes	F3710003	3rd	47	4.39	Yes	F37H051
1st	52	16.37	No	–	3rd	46	4.40	Yes	F36H234
2nd	53	15.84	Yes	F3710003	3rd	45	3.94	Yes	F36H196
2nd	52	10.67	Yes	F373H077	3rd	44	3.41	Yes	F36H150
2nd	51	8.26	Yes	F373H018	3rd	43	3.12	Yes	F36H149
2nd	50	7.77	Yes	F37G001	3rd	42	2.27	Yes	F36G001
2nd	49	7.19	Yes	F373H051	3rd	41	1.92	Yes	360067
2nd	48	6.16	Yes	F36G001	3rd	40	1.90	Yes	F36H282
2nd	47	5.47	Yes	360667	3rd	39	1.79	Yes	F36H270
2nd	46	4.70	No	–	3rd	38	1.73	Yes	F36H190
3rd	53	12.92	Yes	F371003	3rd	37	1.49	Yes	F36H132
3rd	52	5.91	Yes	F37H053	3rd	36	1.42	Yes	F361004
3rd	51	5.74	Yes	F37H018	3rd	35	1.19	Yes	360742
3rd	50	5.42	Yes	F37H077	3rd	34	1.04	Yes	360801
3rd	49	5.01	Yes	F36H176	3rd	33	1.00	Yes	361021
3rd	48	4.51	Yes	F37G001	3rd	32	0.99	No	–

The control, common and outliers' points of study area can be given in Fig. 3.

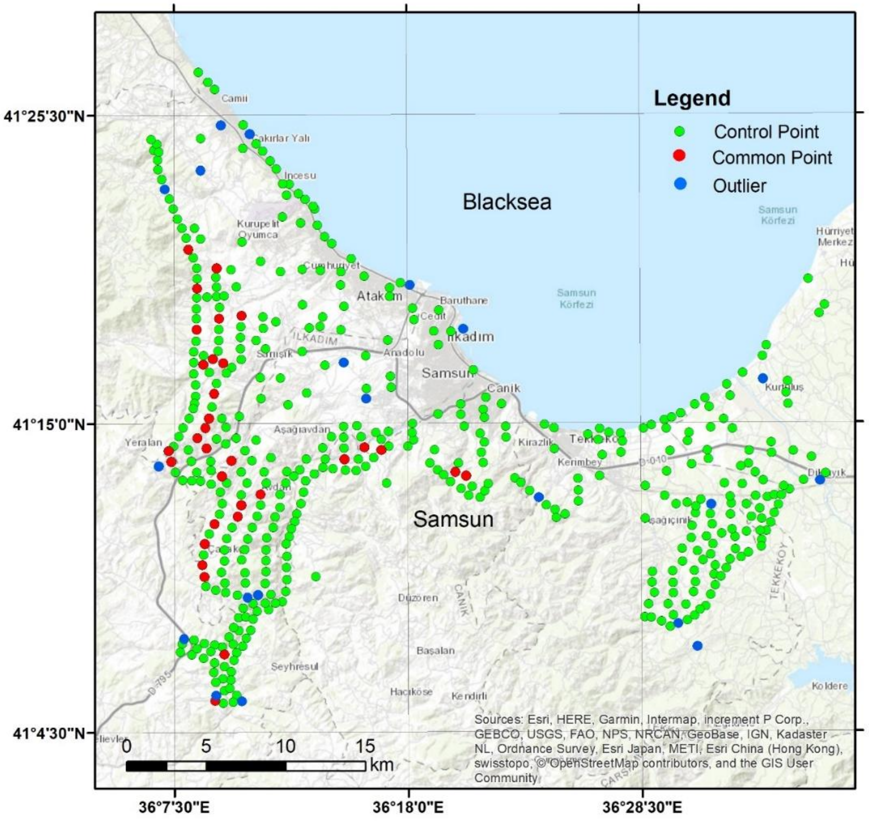

Fig. 3. *The Control, Common and Outlier points.*

1st, 2nd and 3rd degree LPI geoid surfaces and equations of OLS model can be given following equations:

$$N_{(x,y)} = 29.5853 - 0.0739x - 0.0596y \tag{17}$$

$$N_{(x,y)} = 29.6474 - 0.1270x - 0.0877y - 0.0011x^2 - 0.0021y^2 - 0.0050xy \tag{18}$$

$$N_{(x,y)} = 29.6548 - 0.1053x - 0.0623y + 0.0063x^2 + 0.0015y^2 + 0.0072xy + 0.0001x^3 + 0.0001y^3 + 0.0006x^2y + 0.0009xy^2 \tag{19}$$

When these equations were examined, it can be decided that there were not significant differences between 1st, 2nd and 3rd degree surface. Also, it was seen that the most inclusive common point group consist of 32 point was in the 3rd degree LPI geoid surfaces from Table 2. In this case, the applications of the 1st, and 2nd degree local polynomial geoid determination were made with compatible common point group of 3rd degree. The results of this applications were given in Table 3.

Table 3. *The RMSE for 1st, 2nd and 3rd degree with compatible common point group.*

Local Polynomial Degree	**Common Point Number**	**RMSE (cm)**	**Outlier Measurement**
1st	32	9.41	No
2nd	32	2.61	No
3rd	32	0.99	No

From Tale 3, it was seen that the RMSE value of 1st degree LPI geoid surfaces solution was near acceptable the accuracy of TG-09 (8.4 cm) and TG-20 (6.3 cm). Thus, it is decided that 1st degree LPI geoid surface was suitable for study area. The residuals of first stage for 1st degree LPI geoid surface using compatible common point group were given in Table 4.

Table 4. *The results of OLS model.*

PN	V_N **[cm]**
F361H005	24.29
F363H085	–7.72
…	…
F363H286	–6.77
F36H0171	–1.87

3.2. The second stage and results

For the application of the CAU model, the N geoid undulations and normalized horizontal coordinates of compatible common point group were taken as observation like OLS model. The mathematical model and objective function of the CAU model were taken as Eq. (8) and (9) respectively. In this application, the N and horizontal coordinates of the common points were considered to be erroneous. In the CAU model need to a_{ij}^0 approximate value of the parameter's coefficient. These values were taken from the OLS solution. In the CAU model solution, the residuals of N and the horizontal coordinates were calculated from Eq. (12). Also, the RMSE of CAU model was calculated using Eq. (13). The results of second stage were given in Table 5.

Table 5. *The results of CAU model.*

PN	V_x [cm]	V_y [cm]	V_N [cm]	$RMSE$ [cm]
F361H005	1.55	1.30	24.12	4.52
F363H085	–0.49	–0.41	–7.67	
…	…	…	…	
F363H286	–0.43	–0.36	–6.72	
F36H0171	–0.12	–0.10	–1.86	

It was seen from Table 5 that the V_x and V_y residuals were quite lower than the V_N residuals. Additionally, although the V_N residuals of models were nearly equal, the RMSE values of the CAU models were very different. The RMSE of OLS was 9.41 cm, the RMSE of CAU was 4.52 cm for compatible common point group. Also, the OLS and CAU model solutions were applied for 429 control points using determined 1st degree polynomial equation. The results of these applications were given in Table 6.

Table 6. *The results of OLS and CAU model for control point using 1st degree LPI geoid surface.*

Type of observation groups	Point Number	OLS	CAU
		RMSE (cm)	RMSE (cm)
Common	32	9.41	4.52
Control	429	51.28	16.19

The main reason for the difference was the redundancy of the models. Although the redundancy of OLS model was equal to $(n-u)$, the redundancy of CAU model was equal to $(n-u+q)$. The differences of RMSE value of OLS and CAU models was bigger than control point group because of the point number (32 and 429 respectively). In this case, it can be said that the significant difference between OLS and CAU solution of local polynomial surface determination application cannot be obtained for study area. The same results mentioned in the previous paragraph was seen in Table 6.

4. Conclusions

The 1st degree polynomial equation of local geoid was determined using OLS and CAU models using compatible common point group. The coordinates of an urban information system of Samsun province were used for application and separated as test (53 points) and control (429 points). The outlier detection was realized, and the compatible common points group was obtained as 32 points. The main difference between OLS and CAU models is to take some observations as error-free. While the horizontal point coordinates and geoid undulation data were taken as erroneous in the CAU model, only the geoid undulation data was taken as erroneous in the OLS model. While the RMSE value of common point group was 9.41 cm in CAU solution, the RMSE value of the same group was 4.52 cm. This difference was dependent on the redundancy value of models. The V_x and V_y residuals of CAU were very little according to V_N residuals. Also, the V_N residuals of these models were found nearly the same. At the end of application, it was not found a big difference between OLS and CAU model solution in local polynomial geoid determination. The main reason for this situation can be the redundancy of models and the observation accuracy difference of horizontal and vertical coordinates. In this case, the CAU model should be applied with the same accuracy observation as 2D coordinate transformation or geo-referencing. As a result of this study, it is seen that the variability of geoid determination is generally connected to the geoid undulations. But it is also seen that the approaches of taking error-free data used in the design matrix or any stage of solution should not apply the real work application. The author recommends that the CAU model should be applied in especially real work study with same accuracy observation or using the weight matrix of observations.

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Određivanje lokalnog geoida pomoću GNSS-nivelmana primjenom uvjetnog izjednačenja s nepoznatim modelom

SAŽETAK. Određivanje geoida pomoću GNSS-nivelmana definirano je kao matematički odnos između ortometrijske i elipsoidne visine. Metoda interpolacije polinomne površine jedna je od najšire primjenjivanih metoda za određivanje geoida GNSS-nivelmanom. Osnovno rješenje interpolacije polinomne površine temelji se na undulaciji geoida, razlici između ortometrijske i elipsoidne visine uključujući neke pogreške mjerenja koje se ne mogu otkriti. U ovoj studiji najprije su kao površina geoida uzete polinomne površine 1., 2. i 3. stupnja kako bi se odredio odgovarajući polinomni stupanj za odabrano područje proučavanja, Pokrajine Samsun u Turskoj, primjenom uobičajenog modela izjednačenja metodom najmanjih kvadrata. Zatim se rješenja izjednačenja izvode za dobivanje ovih stvarnih podataka. Na kraju primjene rezultati su bili uspoređeni s ovim modelima te su izrađeni prijedlozi za buduće primjene.

Ključne riječi: undulacija geoida, uvjetno izjednačenje s nepoznatim modelom, optimizacija, uobičajeno izjednačenje metodom najmanjih kvadrata.

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