SEARCH FOR OBLATE AND TRIANGULAR CONFIGURATIONS IN CARBON ISOTOPES

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Paper devoted to honour the memory of Professor Nikola Cindro

A systematic search through experimental data for the nuclei $^{12-14}$C has been made to identify states which have triangular shape. These states are oblate and must have strong α-clustering. They are expected to form rotational bands with high moments of inertia. The properties of these states are discussed on the basis of α-clusters and neutron orbitals.

1. Introduction

In the first approximation, nuclei are usually thought of as spherical or slightly deformed collections of neutrons and protons with rather uniform density. However, quite different geometrical forms have long ago been shown to exist, especially in light nuclei. It has been recognised, since a few decades, that clusters and molecular structures are essential in describing many states in the isotopes of lithium, beryllium and boron [1–3].

Recently, investigations have focused on neutron rich carbons [4–9] where strong clustering and a variety of shapes is expected to be observed. Isotopes of carbon have been intensively studied in the framework of different cluster models. The ^{12}C nucleus is certainly one of the most discussed nuclei in the last 30 years. The 3α model for ^{12}C is a typical and basic example of the three-cluster problem and has been investigated by many authors (see, e.g., Refs. [3–25]). The spectroscopy of ^{12}C above 10 MeV is rather complicated because of a number of broad overlapping states. Although experimental data are numerous [26], new efforts are being made

(see e.g. Ref. [27]) in order to get a complete picture of all excited states of this nucleus (a case of "complete spectroscopy").

The structure of the heavier carbon isotopes has not been investigated so much in terms of cluster models. Use is usually made of a weak coupling model in which neutrons are added to ^{12}C core or, with limited success, deformed shell model [28, 29]. Recently, very-large-scale shell-model calculations [30, 31] have also been performed. Having in mind the case of molecular structures in beryllium isotopes [2, 32], additional valence neutrons should be of great importance in carbon isotopes in forming molecular structures. We expect pronounced cluster states (of both triangular and linear configurations of the three α-particles) to appear in the vicinity of the particle cluster thresholds [2]. Such structures have been recently discussed for ^{13}C by the present authors [33, 34] and for ^{14}C and ^{16}C by Itagaki et al. [4–7]. In these studies, the occurrence of linear chain states is particularly intriguing, because these states can be considered as the first case of hyperdeformation (axis ratios 3:1).

We can discuss the occurrence of particular shapes and deformations related to clusters by inspection of the deformed shell diagram shown in Fig. 1 (adapted from Ref. [35]). This is similar to the work of Freer et al. [36] and Freer and Merchant [37]. Stable oblate and prolate shapes are expected for particular (magic) numbers (N) of nucleons, which are referred to as deformed shells. The level diagram indicates that peculiar shapes with stable configurations (large gaps) are expected for specific values for the deformation parameter $\beta_2>0$ and the ratios of the axes 2:1 (the dimers in beryllium isotopes) and 3:1 (chain states in carbon isotopes). Larger deformations, as α-chain states, also appear as stable shapes. For $\beta<0$ and $N=6$ (carbon), we have an oblate deformed shell with the axis ratio 1:2. The next oblate deformed shell appears at $N = 14$ (^{28}Si). Thus for ^{12}C (and also ^{13}C and ^{14}C) as well as for ^{28}Si and other silicon isotopes, the coexistence of oblate and prolate deformed states is predicted. The degeneracy for the occupation with nucleons (black dots in the diagram) is 2 for the lowest (n_z) and rises to 4 and 6 for the higher orbits.

Recently, the method of antisymmetrised molecular dynamics [22] by Horiuchi, Kanada-En'yo et al. has been applied to the light nuclei with great success, revealing the dramatic clustering structures in light nuclei. The method has also been used to reveal the 3α clustering in ^{12}C and other carbon isotopes without any preassumption of the cluster existence [8, 9]. Also, this theory describes successfully the dissociation of α-clusters in more compact shapes due to the LS-force (contrary to most of the cluster models). The results suggest that the opposite deformations between protons and neutrons may be found in the neutron-rich carbon isotopes. It has also been found that the proton density stays compact with the change of neutron number, in spite of the drastic change of neutron structure, indicating the persistence of α-clusters. The results also show the general tendency of the oblately deformed proton density in carbon isotopes.

In this article we discuss states with such oblate, triangular configurations in carbon isotopes. We review the existing experimental data for $^{12-14}$C and give some predictions for the existence of such states.
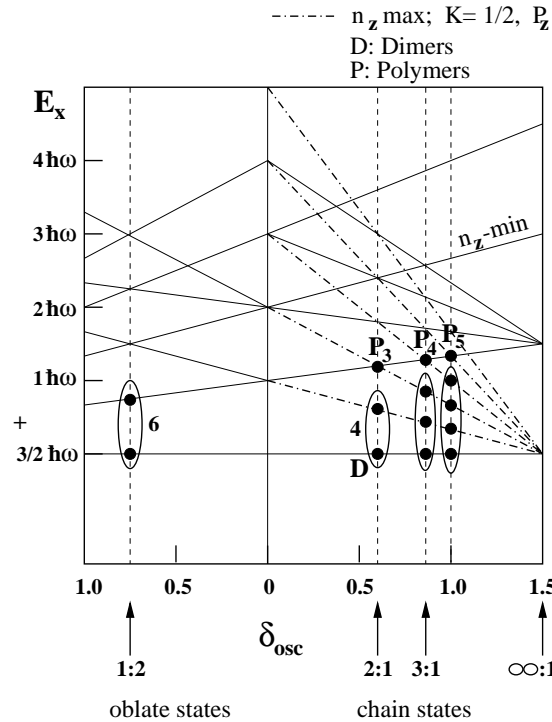
Fig. 1. Diagram of deformed shells for axially symmetric shapes; some "magic" numbers of nucleons for prolate configurations related to chain states are emphasized. The orbits are split due to the values of the relevant quantum numbers, K - the projection of angular momentum, and n_z - the number of nodes on the z-axis. The energy is counted from the lowest $(3\hbar/2)$ in units of harmonic oscillator energies. Places where oblate $(\delta<0)$, and prolate $(\delta>0)$ quadrupole deformations occur are indicated. The parameter δ_{osc} in the figure is expressed in terms of the oscillator frequencies ω_i: $\delta_{osc} = 3(\omega_p - \omega_3)/(2\omega_p + \omega_3)$ (adapted from Ref. [35]).

2. Clustering in ^{12}C

The 3α model for ^{12}C is an example of the quantum mechanical three-body problem with Coulomb interaction. The three-α system has three interaction bonds and two kinetic energies, in contrast to the two-α system which has only one bond and one kinetic energy. Thus the three α-clusters are more tightly bound than the two α clusters and the mean distance between two α's in the three-α system is smaller than in the two-α system [18].

Various models agree that the sequence of states with 0_1^+, 2_1^+ and 4_1^+ in ^{12}C is rather of shell-model-like nature while the following states 0_2^+, 3_1^-, 2_2^+ and 1_1^-

show well-developed cluster nature. A consistent understanding of both classes of states was obtained on the basis of the microscopic 3α cluster model by Horiuchi [15] with the use of the orthogonality condition model, also by Uegaki et al. [16] with the use of the generator coordinate method and by Fukushima and Kamimura [18] with the use of the resonating group method. What is important in these treatments is that the relative motion between the three α-clusters was solved with the Pauli principle taken into account correctly. The shell-model-like states and the molecule-like states of ^{12}C are obtained as two kinds of solutions of the same equation of motion; namely, as the compactly-coupled solutions and as the loosely-coupled solutions, respectively. The existence of the former type of solution is on account of the fact that the strong repulsive effect in the internal region of the α-α interaction melts away in the close presence of the third α-particle due to the total antisymmetrisation. In these studies, it has also been found that the size of the α-clusters is almost the same in all states mentioned above and is nearly equal to the size observed for the free α-particle.

^{12}C has also been extensively investigated within a framework of the shell model. With four holes in the p-shell, the ground state of ^{12}C comes out to be oblate with quadrupole deformation of $\delta_{\mathrm{osc}} \approx 0.6$ [29], which is in agreement with cluster-model results. On the other hand, recent calculations with large shell mixing were not able to reproduce some excited states. For example, the 0_2^+, 0_3^+ and 2_2^+ states are not obtained even with the largest shell model calculations [30], probably because these states have complicated configuration mixing with highly excited $n\hbar\omega$ components in their wave function. We expect that molecular-like states based on states in ^{12}C with well developed 3α-structures and with additional valence neutrons may also appear at higher excitation in ^{13}C and ^{14}C.

All cited models agree that the wave function of the ^{12}C 0_2^+ state has a dominant three-alpha clustering nature. This state is just 288 keV above the threshold for the decay into three α-particles at 7.366 MeV. Electron inelastic scattering data indicate that this state has an abnormally large charge radius [26]. Recently, it was unambiguously shown that this state is definitely an authentic three-alpha resonance in ^{12}C [20, 21]. The first interpretations of the 0_2^+ state are due to Morinaga [10] and Brink [11] who proposed that this state can be explained as three α-particles forming a linear chain. Such interpretation faces several problems [12]: the absence of the rotational band, relatively large partial width for γ-decay to the 2_1^+ state at 4.44 MeV and fast β-decay feeding from ^{12}B and ^{12}N. Moreover, the bending mode has been shown to be an important part of the wave function [12], because the only obvious restoring force is the Coulomb repulsion which is too weak.

The 0_3^+ at 10.3 MeV state is definitely a better candidate for a linear chain configuration. Its spin and parity are still not definitely established, although the new preliminary results support 0^+ assignment [27]. There is also a rarely seen 2^+ state at 11.16 MeV [38, 39] which can be an excited state of a rotational band based on the 0_3^+ state (with the moment of inertia $\hbar^2/2\theta \approx 150$ keV that fits into systematics in this mass region [33]). The problem of such assignment is that the width of the 0^+ state at 10.3 MeV is much larger than the corresponding

width of the 2^+ state at 11.16 MeV (3 MeV compared with 0.43 MeV). The large width (exclusively α-decay) of the 0_3^+ state may be the consequence of the bending instability. The bending modes of the pure α-particle chains may be restricted once the valence neutron bonds are added (if they correspond to a covalent-binding configuration [2, 4–6]. Thus, we can expect rather stable chain-states at higher excitations in heavier carbon isotopes [4, 33, 34].

To conclude we can state that the 0_2^+ state is probably a weak-coupling state with a ^{8}Be$+\alpha$ configuration corresponding to an oblate (triangular) shape (Fig. 2). While the spatial structure of the ground state is predominantly an equal-sided triangle ($d \approx 3$ fm), the 0_2^+ state has a much larger probability to look like a flat triangle [13], with sides equal to 3.9, 4.2 and 6.1 fm [20]. Rather strong $0_2^+ \to 0_1^+$ monopole and the $0_2^+ \to 2_1^+$ E2 transitions are barely explained, unless an artificial mixture of 3-α equilateral triangle and linear chain geometries are assumed both for the 0_1^+ and 0_2^+ states [14]. The 0_2^+ state has also been investigated from the viewpoint of an α-condensation [23, 24], consistent with the statements given above. We emphasize that since this state has spin zero, no fixed geometrical shape can be defined.
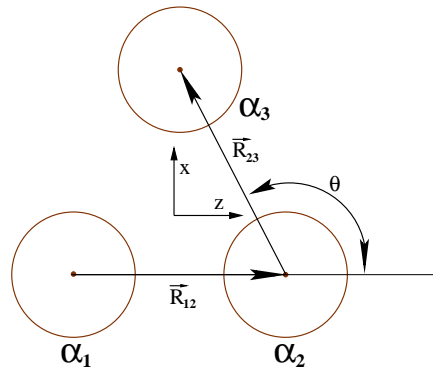

Fig. 2. Triangular configuration of three α-clusters. The origin of the coordinate system is the centre of the mass of the equilateral triangle; the y-axis is perpendicular to the x- and z-axis.

For the states for which we adopt the triangular geometry (with the zx-plane as shown on Fig. 2), the symmetry properties required for the average field are [40]:

1) invariance for the rotations by $2\pi/3$ and $4\pi/3$ around y-axis;

2) invariance for the rotation by π around x-axis;

3) invariance against the reflection with respect to zx-plane.

When all three operations are combined, one gets twelve operations, including the identity operation. These symmetry operations form a group which is denoted as D_{3h} and is a kind of a point group. The possible spins and parities for the system with D_{3h} symmetry are:

$$
\begin{array}{ll}
K^\pi = 0^+, & J^\pi = 0^+, 2^+, 4^+, \ldots \\
K^\pi = 3^-, & J^\pi = 3^-, 4^-, 5^-, \ldots \\
K^\pi = 6^+, & J^\pi = 6^+, 7^+, 8^+, \ldots
\end{array}
$$

etc. So, besides the 0^+ state, we expect an oblate rotational band based on the

3^-_1 (at 9.65 MeV). The $K=0$ rotational excitation can not remain closely packed because the binding interaction is rather weak and the system expands to a size where the α-substructures are separated by a distance similar to that in ^{8}Be.

The recent analysis by Bijker and Iachello [25] of triangular configurations with D_{3h} point-group symmetry show that the low-lying spectrum of ^{12}C can be described by three alpha-particles at the vertices of an equilateral triangle, but not as a rigid structure. Large rotation-vibration interactions, Coriolis forces and vibration-vibration interactions are needed to reproduce excitation energies and form factors for inelastic electron scattering. In their work, the 3^-_1 state is placed in the ground rotational band; its calculated excitation energy is a bit too low.

To conclude, we can safely assume that the 0^+_1 state of ^{12}C has an oblate geometry, the 3^-_1 state is proposed to have a stretched triangle geometry, while the 0^+_2 state has an oblate shape of a loose arrangement of α-particles. By adding neutrons to these states, we can build oblate (triangular) states in heavier carbon states. The candidates for these states are shown in Fig. 3.
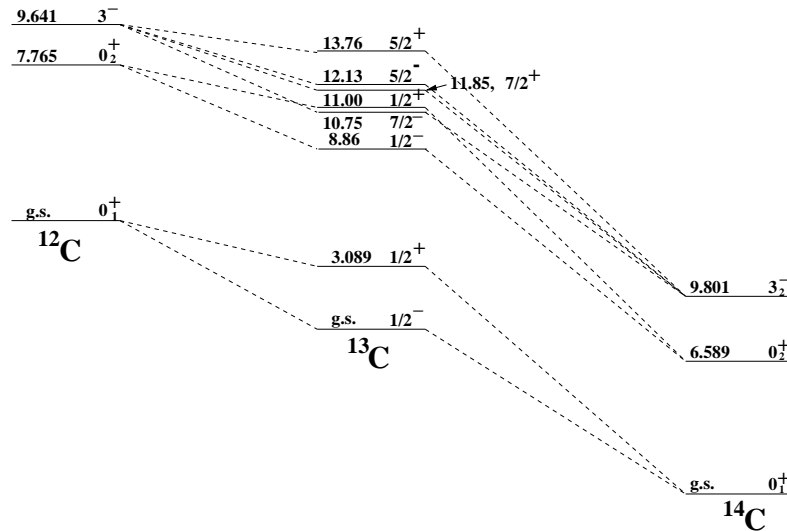

Fig. 3. Sequence of energy levels of the isotopes $^{12-14}$C which can be related to oblate and triangular configurations.

3. Molecular orbital model for oblate states in ^{13}C

Beginning with the 0^+_1 state of ^{12}C, we use the shell model and get the well known spectrum of configurations in ^{13}C with the quantum numbers ($p_{1/2}$, $s_{1/2}$, $d_{5/2}$). Starting with the excited ^{12}C 0^+_2 state, the same sequence should be repeated. Because of its large radial extension, the energy distances between these single-particle configurations must be smaller. There are in fact good candidates (Table 1)

for the doublet with $J^\pi=1/2^\pm$ based on the 0_2^+ state of ^{12}C: the 8.86 MeV ($1/2^-$) and 10.996 MeV ($1/2^+$) states of ^{13}C. Both states are below the ^{9}Be*($1/2^+$)$+\alpha$ threshold. Adding a further neutron, a corresponding triangular configuration has been recently calculated for ^{14}C by Itagaki et al. [6] with results in agreement with our suggestion for the mentioned ^{13}C states with $J^\pi=1/2^\pm$. Experimental evidence for this suggestion comes from the inelastic scattering to the 8.86 MeV state which is found to be weak and with very similar angular distributions to the excitation of the 0_2^+ state of ^{12}C [41, 42]. Those states have spherical density distribution (because of their 1/2 spins); the $1/2^-$ state will have a node and the $1/2^+$ will have a maximum in the centre. The suggestion is that these two states correspond to configurations of $p_{1/2}$ and $2s_{1/2}$ orbits (as for the two first states of ^{13}C), based now on the second 0_2^+ state, as shown in Fig. 3. The splitting is now 2.14 MeV instead of 3.089 MeV. A $d_{5/2}$ resonance built on the 0_2^+ must be expected at ≈ 11.5 MeV. We will later give arguments that these two states may also be based on a valence neutron coupled to the triangular three α-particle structure.

Table 1. Proposed states in ^{13}C with intrinsic oblate or triangular configuration.

J^π Ref. [26]	E_x (MeV)	Γ (keV)	Structure
$1/2^-$	8.86	150	^{12}C(0_2^+)$\otimes 1p_{1/2}$
$7/2^-$	10.754	55	^{12}C(3^-)$\otimes \sigma_+$
$1/2^+$	10.996	37	^{12}C(0_2^+)$\otimes 2s_{1/2}$
$7/2^+$	11.848	68	^{12}C(3^-)$\otimes \sigma_-$
$5/2^-$	12.13	80	^{12}C(3^-)$\otimes \sigma_+$
($5/2^+$,$3/2^+$)	13.76	≈ 300	^{12}C(3^-)$\otimes \sigma_-$

For the cluster states, the ^{13}C wave functions can be based on the asymptotic ^{9}Be$+\alpha$ structure; in this case, the strongly-deformed nature of the ground state of ^{9}Be($3/2^-$) enters. By choosing a specific orientation of ^{9}Be relative to the third α-particle, we arrive at the ^{13}C triangular structure. In this case, a particular difficulty may arise if we choose the ground state of ^{9}Be($3/2^-$) to construct the wave function. The ^{9}Be($3/2^-$) wave function is described by a molecular neutron orbit in a π-configuration, in which the valence particle occupies the space spanned by the x- and y-coordinates, perpendicular to the symmetry axis (z-axis) of ^{9}Be (see, e.g., Ref. [4]). The third α-particle approaching along these x- or y-coordinates will thus find occupied neutron states. If the antisymmetrization between valence particles and the nucleons in the α-particle is taken into account, the Pauli-principle will inhibit this configuration or make it energetically less favourable. The situation changes if the first excited state ^{9}Be*($1/2^+$) is chosen. In this case, the valence-neutron molecular orbit is in a σ-configuration, with the wave function concentrated along and on the z-axis (the symmetry axis of ^{9}Be), the x- and y-coordinates are

thus free for the approaching third α-particle. We thus conclude that the *triangular shapes will be favoured with the first excited state* $^9\text{Be}^*(1/2^+)$.

Based on the arguments for the symmetry operations (Sect. 2), we give in Fig. 4 a pictorial illustration of the wave functions based on the $\Psi_{13} = {}^9\text{Be}^*(\sigma_{1/2^+}) \otimes \Psi_\alpha$ configuration. The spin/parity projection needs a superposition of three components, giving rise eventually to parity doublets

$$\Psi^{J^\pi}({}^{13}C) = [N_1\Psi_1 D(0°) + N_2\Psi_2 D(120°) + N_3\Psi_3 D(240°)] = \Psi_{\text{tri}}^{3^-}({}^{12}\text{C}) \otimes \Psi_{1/2^+} \tag{1}$$

Using the same approach as for the states based on $^{12}\text{C}(0_2^+) \otimes \sigma$-orbits, we predict $^{12}\text{C}(3_1^-) \otimes \sigma_{1/2}^+$ states which form doublets with $J^\pi = 7/2^\pm$ and $5/2^\pm$. Good candidates for the 7/2 doublet are the states at 10.75 MeV and at 11.85 MeV, and for the 5/2 doublet the states at 12.13 MeV and at 13.76 MeV (see Table 1). All these states were not included in the recent systematics of chain states in ^{13}C [33]. These states must generally be populated by two-step processes (inelastic + transfer). They are seen in all types of reactions (though always weakly populated), in contrast to single-particle states which are very weak in multi-nucleon transfer reactions. The latter reactions populate strongly chain states, which are not seen in one-nucleon transfer reactions [33].

4. *Oblate and triangular shapes in* ^{14}C

The ground state of ^{14}C is semi-magic with a spherical shape and the first excited states lie as high as in ^{16}O, at approximately 6 – 7 MeV excitation energy. These states are related to 1p-1h excitations or to 4p-2h and 6p-4h structures (the 0_2^+). The two-particle excitation for protons naturally produces an α-^{10}Be configuration, which can give rise to oblate ("pancakes") or prolate shapes (chain-configurations).

We expect the oblate and triangular shapes to appear at lower excitation energies than the linear chain states. The 0_2^+ state of ^{14}C is often described, by an orthogonalisation to the ground state 0_1^+, as due to a pairing state with the configurations based on configuration mixing: $^{12}\text{C}(0_1^+) \otimes [a(p_{1/2})^2 + b(s_{1/2})^2 + c(d_{5/2})^2]$. It can easily be based also on the $^{12}\text{C}(0_2^+)$ state, and it is expected to have oblate shape. Actually, Itagaki et al. [6] predict the "triangular" state in ^{14}C with spin 0^+ at 7.85 MeV excitation energy, very close to the observed energy of the 0_2^+-state at 6.589 MeV. This state is thus also strongly related to the 0_2^+ in ^{12}C and to the special states with $1/2^-$ and $1/2^+$ at 8 – 10 MeV excitation in ^{13}C, discussed before (see Fig. 3) and in Ref. [33]. For the "triangular" 0^+-states, no particular orientation of the α-clusters can be defined. For a state with a fixed geometry, a lowest spin of 3^- state is predicted; it is most likely the 3_2^- state at 9.801 MeV, predicted also by Itagaki et al. at 9.45 MeV [6]. The state with good spin and parity is obtained again by the spin/parity projection operation as discussed above for the 3^- state in ^{12}C.
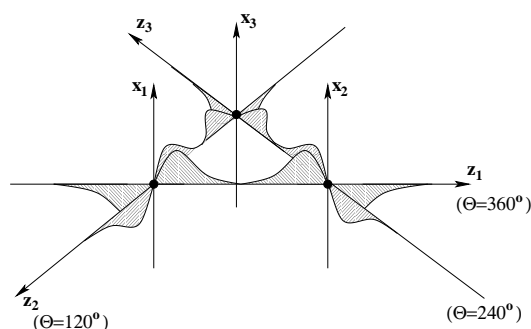
Fig. 4. Schematic illustration of the wave function for valence neutrons in the triangular configuration in ^{13}C. It is obtained via superposition of three ^{9}Be*(1/2$^+$) molecular wave functions; only one possible choice of the relative phase in Eq. 1 is shown.

As in the approach of Itagaki *et al.* [4, 6], we may use an explicit molecular orbital model for the triangular states in ^{14}C, as done for ^{13}C. The states would consist (as illustrated in Fig. 4) of a linear combination of states rotated by 120° giving a 3$^-$ state, as for ^{12}C. The basis state would be the ^{9}Be(1/2$^+$)$\otimes\Psi_\alpha$, as for ^{13}C. For ^{10}Be(σ^2)$\otimes\Psi_\alpha$, however, the energy could be lowered due to the pairing effect caused by the overlap of the neutron wave function at the centre of the oblate shape. Inspecting Fig. 2 we find that the pairing effect is not visible, like for the ^{10}Be (g.s., π^2). We can conclude, that due to the smeared density distribution of the linear combinations of the σ^2 configurations, very little pairing energy is gained.

Two bands (Table 2) can be identified in the old [26] and new [34] experimental data as very good candidates for the oblate spin/parity partners; the detailed

TABLE 2. Proposed doublet of bands in ^{14}C with intrinsic oblate or triangular configuration.

J^π proposed	E_x (MeV)	J^π Ref. [26]	Γ or τ
$K = 3^-$			
3$^-$	9.801	3$^-$	45 keV
4$^-$	11.67	4$^-$	20 keV
5$^-$	14.66	(4$^+$)	?
6$^-$	17.3		ref. [34]
$K = 3^+$			
0$^+$	6.589	0$^+$	3 ps
2$^+$	8.318	2$^+$	3.4 keV
4$^+$	12.863		20 keV
6$^+$	19.7		ref. [34]

explanation of each state is given elsewhere [34] and here we list a few arguments for the two band-heads. The **6.589 MeV state** (0_2^+) is strongly populated in the ^{9}Be(^{7}Li,d) reaction [34] in particular considering its low spin. It is not seen in one nucleon transfer reactions ^{13}C(d,p) [45] (S<0.18) and also not in ^{15}N(d,^{3}He) [46]; so it definitely has a strong $(sd)^2$ character [47]. The **9.801 MeV state** (3^-) is also strongly populated in the ^{9}Be(^{7}Li,d) reaction [34]. It corresponds to the strongest peak in (α, α') scattering [45]. On the other hand, it is barely visible in the ^{13}C(d,p) reaction [45].

5. *Conclusion*

We can conclude that the cluster model and the molecular orbital model allow us to make rather firm predictions for the parity doublet of states with oblate and triangular shapes in ^{13}C and ^{14}C.

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POTRAGA ZA SPLJOŠTENIM I TROKUTNIM KONFIGURACIJAMA UGLJIKOVIH IZOTOPA

Proveli smo sustavnu potragu kroz postojeće eksperimentalne rezultate za jezgre $^{12-14}$C da bismo raspoznali stanja trokutnog oblika. Ta su stanja spljoštena, pokazuju izrazitu nakupinsku strukturu i grupiraju se u rotacijske vrpce velikog momenta inercije. Njihova se svojstva raspravljaju na temelju α-nakupina i neutronskih orbitala.