Printed ISSN 1330–0016
Online ISSN 1333–9133
CD ISSN 1333–8390
CODEN FIZBE7

BIANCHI TYPE-I COSMOLOGICAL MODEL IN SELF-CREATION COSMOLOGY

S. D. KATORE[a] and A. Y. SHAIKH[b]

[a]*Department of Mathematics, S. G. B. Amravati University, Amravati 446002, India*
[b]*Department of Mathematics, Dr. B. N. College of Engineering & Technology, Yavatmal 445001, India*
E-mail addresses: katoresd@rediffmail.com shaikh_2324ay@yahoo. com

Received 20 April 2010; Revised manuscript received 17 January 2011
Accepted 2 February 2011 Online 7 March 2011

A five-dimensional Bianchi type-I space-time is considered in the presence of a perfect fluid source in Barber's (Gen. Relat. Gravit. 14, 117, 1982) second self-creation theory of gravitation. The model is presented using a relation between the metric potentials and an equation of state. Some physical and kinematical properties of the model are discussed.

PACS numbers: 98.80.Es, 98.80.Jk UDC 524.83
Keywords: higher dimensional, Bianchi type-I model, self-creation cosmology

1. Introduction

In recent years, there has been a considerable interest in alternative theories of gravitation. Brans-Dicke [1] formulated a scalar-tensor theory of gravitation which incorporates Mach's principle in a relativistic framework by assuming interaction of inertial masses of fundamental particles with some cosmic scalar field coupled with the large scale distribution matter in motion. Barber [2] proposed two modified theories known as self-creation theories. His first theory is a modification of Brans-Dicke [1] theory, while the second theory is a modification of general theory of relativity. These modified theories create the universe out of self-contained gravitational and matter fields. Brans [3] pointed out that the first self-creation theory is inconsistent, in general, as it violates equivalence principle. However second theory is a modification of general relativity to a variable G-theory and predicts local effects that are within the observational limits. In this theory, the scalar field does not gravitate directly, but simply divides the matter tensor, acting as a reciprocal gravitational constant. The scalar field is postulated to couple to the trace of the energy-momentum tensor. The consistency of Barber's second theory motivates us to study cosmological model in this theory. The massless scalar field in relativistic mechanics yields some significant results regarding both the singularities involved and Mach's Principle.

Several cosmologists have studied various aspects of Friedmann-Robertson-Walker model in Barber's second self-creation cosmology with perfect fluid satisfying the equation of state $p = (\gamma - 1)\rho$, where $1 \leq \gamma \leq 2$. Hawking and Ellis [4] showed that the flat FRW model with a massless scalar field can be steady-state model as $t \to \infty$. Pimentel [5] and Soleng [6] discussed FRW models by using a power-law relation between the expansion factor of the universe and the scalar field. Singh [7], Reddy [8] and Reddy et al. [9] studied Bianchi-type space-time solutions in Barber's second theory of gravitation while Reddy and Venkateswarlu [10] presented Bianchi type-VI cosmological model in Barber's second self-creation theory of gravitation. Shanti and Rao [11] studied Bianchi type-II and III space-times in this theory, both in vacuum as well in the presence of a stiff fluid. Carvalho [12] studied a homogeneous and isotropic model of the early universe in which the parameter gamma of the 'gamma law' equation of state varies continuously with cosmological time and presented a unified description of early universe for inflationary period and radiation-dominating era. Pradhan and Pandey [13], Prandhan and Vishwakarma [14], Panigrah and Sahu [15], Vishvakarma and Narlikar [16], Sahu and Mohanty [17] and Venkateswarlu et al. [18], Katore et al. [19] are some of the authors who studied various aspects of different cosmological models in Barber's second self-creation theory. In recent years, Katore et al. [20], Reddy and Naidu [21] studied the cosmological models with the deceleration parameter and constant deceleration parameter model of the universe in the context of different aspects of different space-times.

Studies of higher-dimensional cosmological models are important because of the underlying idea that the cosmos at its early stage of evolution might have had a higher-dimensional era. This fact has attracted many researches to the field of higher-dimensional theories (Witten [22], Appelquist et al. [23], Chodos and Detweller [24]). Solutions of the field equations in higher-dimensional space-time are believed to be of physical relevance possibly at the times before the universe has undergone compactification transitions. Rathore et al. [25] studied the higher-dimensional perfect-fluid cosmological model in Barber's second self-creation theory by assuming that the component of the shear tensor σ^i_j is proportional to the scalar expansion θ.

In this paper, we obtain a five-dimensional Bianchi type-I cosmological model in Barber's second self-creation theory in the presence of a perfect fluid.

2. *Field equations and the model*

We consider a homogenous Bianchi type-I metric in the form

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + A^2\mathrm{d}x^2 + B^2(\mathrm{d}y^2 + \mathrm{d}z^2) + C^2\mathrm{d}w^2 \,, \tag{1}$$

where A, B and C are functions of t only.

The field equation in Barber's (1982) second self-creation theory is

$$R_{ij} - \frac{1}{2} g_{ij} R = -8\pi\phi^{-1} T_{ij} \tag{2}$$

and

$$\phi^{k}_{;k} = \frac{8\pi}{3}\lambda T \,, \tag{3}$$

where T is the trace of the energy-momentum tensor, λ is a coupling constant to be determined from the experiment ($|\lambda| \leq 0.1$) and semi-colon denotes covariant differentiation. In the limit $\lambda \to 0$, this theory approaches the standard general relativity theory in every respect and $G = \phi^{-1}$.

The energy-momentum tensor for a perfect-fluid distribution is given by

$$T_{ij} = (\rho + p) u_i u_j + p g_{ij} \,, \tag{4}$$

where p is the isotropic pressure, ρ the energy density and u_i represents the four-velocity of the fluid. Corresponding to the metric given by Eq. (1), the four-velocity vector u_i satisfies the equation

$$g_{ij} u^i u^j = -1 \,. \tag{5}$$

In a co-moving coordinate system, the field Eqs. (2) and (3) for the metric (1), with the help of Eqs. (4) and (5), take the form

$$2\frac{B_{55}}{B} + \frac{C_{55}}{C} + \frac{B_5^2}{B^2} + 2\frac{B_5}{B}\frac{C_5}{C} = -8\pi\phi^{-1} p \,, \tag{6}$$

$$\frac{A_{55}}{A} + \frac{B_{55}}{B} + \frac{C_{55}}{C} + \frac{A_5}{A}\frac{B_5}{B} + \frac{A_5}{A}\frac{C_5}{C} + \frac{B_5}{B}\frac{C_5}{C} = -8\pi\phi^{-1} p \,, \tag{7}$$

$$\frac{A_{55}}{A} + 2\frac{B_{55}}{B} + \frac{B_5^2}{B^2} + 2\frac{A_5}{A}\frac{B_5}{B} = -8\pi\phi^{-1} p \,, \tag{8}$$

$$2\frac{A_5}{A}\frac{B_5}{B} + \frac{B_5^2}{B^2} + 2\frac{B_5}{B}\frac{C_5}{C} + \frac{A_5}{A}\frac{C_5}{C} = 8\pi\phi^{-1} \rho \,, \tag{9}$$

$$\phi_{55} + \left(\frac{A_5}{A} + \frac{B_5}{B} + \frac{C_5}{C}\right)\phi_5 = \frac{8\pi}{3}\lambda(\rho - 4p) \,, \tag{10}$$

where the suffix '5' at a function denotes ordinary differentiation with respect to time t.

Equations (6) – (10) are five independent equations in six unknowns A, B, C, ϕ, ρ and p. For a complete determinacy of the system, one extra condition is needed.

For this purpose, we assume $A = (BC)$. Then

$$2\frac{B_{55}}{B} + \frac{C_{55}}{C} + \frac{B_5^2}{B^2} + 2\frac{B_5}{B}\frac{C_5}{C} = -8\pi\phi^{-1}p\,, \tag{11}$$

$$2\frac{B_{55}}{B} + 2\frac{C_{55}}{C} + \frac{B_5^2}{B^2} + \frac{C_5^2}{C^2} + 5\frac{B_5}{B}\frac{C_5}{C} = -8\pi\phi^{-1}p\,, \tag{12}$$

$$3\frac{B_{55}}{B} + \frac{C_{55}}{C} + 3\frac{B_5^2}{B^2} + 4\frac{C_5}{C}\frac{B_5}{B} = -8\pi\phi^{-1}p\,, \tag{13}$$

$$3\frac{B_5^2}{B^2} + \frac{C_5^2}{C^2} + 5\frac{B_5}{B}\frac{C_5}{C} = 8\pi\phi^{-1}\rho\,, \tag{14}$$

$$\phi_{55} + \left(3\frac{B_5}{B} + 2\frac{C_5}{C}\right)\phi_5 = \frac{8\pi}{3}\lambda(\rho - 4p)\,. \tag{15}$$

We solve the above set of field equations (11) – (15) with the transformations

$$B = \mathrm{e}^{\alpha}, \quad C = \mathrm{e}^{\beta}, \quad \mathrm{d}t = ABC\mathrm{d}T$$

The set of field equations (11) – (15) reduces to

$$2\alpha'' + \beta'' - {\alpha'}^2 - {\beta'}^2 - 4\alpha'\beta' = -8\pi\phi^{-1}p\mathrm{e}^{4\alpha+4\beta}\,, \tag{16}$$

$$2\alpha'' + 2\beta'' - {\alpha'}^2 - {\beta'}^2 - 3\alpha'\beta' = -8\pi\phi^{-1}p\mathrm{e}^{4\alpha+4\beta}\,, \tag{17}$$

$$3\alpha'' + \beta'' - {\beta'}^2 - 4\alpha'\beta' = -8\pi\phi^{-1}p\mathrm{e}^{4\alpha+4\beta}\,, \tag{18}$$

$$3{\alpha'}^2 + {\beta'}^2 + 5\alpha'\beta' = 8\pi\phi^{-1}\rho\mathrm{e}^{4\alpha+4\beta}\,, \tag{19}$$

$$\phi'' + \alpha'\phi' = \frac{8\pi}{3}\lambda(\rho - 4p)\mathrm{e}^{4\alpha+4\beta}\,. \tag{20}$$

Due to the high non-linearity of the field equations, we can introduce more conditions: either an adhoc assumption corresponding to some physical situation or an arbitrary mathematical supposition. However, both procedures have some drawbacks: physical situation may lead to differential equations which will be difficult to integrate and mathematical supposition may lead to a non-physical situation. We assume

$$\rho = 4p\,, \tag{21}$$

which is analogous to the equation of state $\rho = 3p$, which represents disordered radiation in four-dimensional space. Using Eq. (21), the field equations admit the solution given by

$$A = d(aT+b)^{1+c/a}, \quad B = (aT+b), \quad C = d(aT+b)^{c/a}, \tag{22}$$

$$8\pi\phi^{-1}(\rho) = 8\pi\phi^{-1}(4p) = \frac{h}{(aT+b)^2}, \quad \text{where} \quad h = \frac{2a^3+a^4+4a^2c+ac+c^2}{a^2}, \tag{23}$$

and

$$\phi = f\log(aT+b)^{e/a}, \tag{24}$$

where a, b, c, d, e and f are constants of integration.

Thus the five dimensional Bianchi type-I cosmological model corresponding to the above solution can be written

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + d^2(aT+b)^{2(1+c/a)}\mathrm{d}x^2 + (aT+b)^2(\mathrm{d}y^2+\mathrm{d}z^2) + d^2(aT+b)^{2c/a}\mathrm{d}w^2. \tag{25}$$

For a positive value of a, the model is free from singularities. For a negative value of a, the model has a singularity at $T = -b/a$.

3. *Physical properties of the model*

Equation (25) represents a five-dimensional Bianchi type-I cosmological model in the framework of second self-creation theory of gravitation, proposed by Barber [2], in the presence of a perfect fluid source. It is interesting to note that the model is free from singularities.

For the model (25), the physical and kinematical variables which are important in cosmology are

Spatial volume

$$V^3 = \sqrt{-g} = d^2(T)^\eta, \quad \text{where} \quad \eta = 3 + 2\frac{c}{a}. \tag{26}$$

Expansion scalar

$$\theta = U^i_{;i} = \frac{k}{T}, \quad \text{where} \quad k = 1 + 2a + 2\frac{c}{a}. \tag{27}$$

Shear scalar

$$\sigma^2 = \frac{4k^2}{9(T)^2}. \tag{28}$$

Hubble parameter

$$H = \frac{R_4}{R} = \frac{1}{3d^2(T)^{3+2c/a}}\,. \qquad (29)$$

The spatial volume increases with T and it becomes infinite for large values of T. Thus inflation is possible for the model (25). This also implies that the model is expanding.

It can be observed that for large T, the parameters vanish and diverge when $T \to 0$. Also, for large value of T, the ratio $\sigma^2/\theta^2 \neq 0$, hence the model (25) does not approach isotropy.

4. Conclusion

We have presented a five-dimensional Bianchi type-I cosmological model in Barber's second self-creation theory of gravitation in the presence of a perfect fluid source. Our model represents radiation-dominated universe in the self-creation cosmology. Our results resemble the results of R. Venkateswarlu and D. R. K. Reddy [26] in the presence of a perfect fluid distribution. The model obtained is free from initial singularity. It can be observed that for large T, the parameters θ, σ and H vanish, and diverge when $T \to 0$. Models considered in this paper are of considerable interest and may be useful in self-creation cosmology to study the large-scale dynamics of the physical universe.

Acknowledgements

The authors would like to place on record their sincere thanks to the Editor and referee for their valuable comments which have helped in improving the quality of the paper.

References

[1] C. Brans and R. H. Dicke, Phys. Rev. **124** (1961) 925.

[2] G. A. Barber, Gen. Relat. Gravit. **14** (1982) 117.

[3] C. Brans, Gen. Relat. Gravit. **19** (1987) 949.

[4] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-Time*, Cambridge University (1973) p. 94.

[5] L. O. Pimentel, Astrophys. Space Sci. **116** (1985) 935.

[6] H. H. Soleng, Astrophys. Space Sci. **139** (1987) 13.

[7] T. Singh, Astrophys. Space Sci. **102** (1984) 67.

[8] D. R. K. Reddy, Astrophys. Space Sci. **133** (1987) 389.

[9] D. R. K. Reddy, M. B. Avadhanulu and R. Venkateswarlu, Astrophys. Space Sci. **134** (1987) 201.

[10] D. R. K. Reddy and R. Venkateswarlu, Astrophys. Space Sci. **155** (1989) 135.

[11] K. Shanti and V. U. M. Rao, Astrophys. Space Sci. **179** (1991) 147.

[12] J. C. Carvalho, Int. J. Theor. Phys. **35** (1996) 2019.

[13] A. Pradhan and H. R. Pandey, arXiv:gr-qc/0207027 vl 4 Jul 2002.

[14] A. Pradhan and A. K. Vishwakarma, Int. J. Mod. Phys. D **11** (2002) 1195.

[15] U. K. Panigrahi and R. C. Sahu, Theor. Appl. Mech. **30** (2003) 163.

[16] R. G. Vishwakarma and J. V. Narlikar, Int. J. Mod. Phys. D **14** (2005) 345.

[17] R. C. Sahu and G. Mohanty, Astrophys. Space Sci. **306** (2006) 179.

[18] K. Venkateswarlu, V. U. M. Rao and K. P. Kumar, Int. J. Theor. Phys. **47** (2008) 640.

[19] S. D. Katore, R. S. Rane and K. S. Wankhade, Int. J. Theor. Phys. **49** (2009) 187; doi 10.1007/s10773-009-0191-y.

[20] S. D. Katore, R. S. Rane and V. B. Kurkure, Astrophys. Space Sci. **315** (2008) 347.

[21] D. R. K. Reddy and R. L. Naidu, Int. J. Theor. Phys. **48** (2009) 10; doi:10.1007/s100773-008-9774-2.

[22] E. Witten, Phys. Lett. B **144** (1984) 351.

[23] T. Apple, A. Chodos and P. G. O. Freund, *Modern Kaluza-Klein Theories*, Addison-Wesley, Reading (1987).

[24] A. Chodos and S. Detweller, Phys. Rev. D **21** (1980) 2167.

[25] G. S. Rathore and K. Mandawat, Adv. Studies Theor. Phys. **4**, 5 (2010) 213.

[26] D. R. K. Reddy and R. Venkateswarlu, Astrophys. Space Sci. **168** (1990) 193.

BIANCHIJEV KOZMOLOŠKI MODEL TIPA I U SAMOTVORNOJ KOZMOLOGIJI

Razmatramo Bianchijev prostor-vrijeme tipa I u pet dimenzija, uz prisustvo perfektne tekućine, u drugoj Barberovoj samotvornoj teoriji gravitacije (Gen. Relat. Gravit. 14, 117, 1982). U modelu pretpostavljamo relaciju među metričkim potencijalima i jednadžbu stanja. Raspravljamo neke fizičke i kinematičke odlike modela.