

BIANCHI TYPE-I STIFF-FLUID TILTED COSMOLOGICAL MODEL WITH
BULK VISCOSITY IN GENERAL RELATIVITY

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Bianchi type-I stiff-fluid tilted cosmological model with bulk viscosity is investigated. To get the deterministic model of the universe, we have assumed that the universe is filled with a stiff fluid, a condition $A = (BC)^n$ between the metric potentials A, B, C and $\zeta\theta = \text{constant}$, where ζ is the coefficient of bulk viscosity, θ the expansion and n is a constant. The physical and geometrical aspects of the model in the presence and absence of bulk viscosity are discussed.

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1. Introduction

It has been argued for a long time that in the early stage of cosmic expansion, the dissipative process may well account for the high degree of isotropy we observe today. Dissipative effects, including both the bulk and shear viscosities, play a significant role in the early stage of evolution of the universe. To study the effect of bulk viscosity, Eckart [1] developed a relativistic theory of non-equilibrium thermodynamics. Misner [2, 3] discussed the effect of viscosity on the evolution in cosmological models. Heller and Klimek [4] obtained viscous fluid cosmological models without initial singularity. Roy and Prakash [5] investigated Bianchi type-I viscous fluid cosmological model assuming uniform coefficient of viscosity. Bali and Jain [6] obtained some expanding and shearing viscous fluid cosmological models in which coefficient of shear viscosity is proportional to the expansion in the model. Padmanabhan and Chitre [7] pointed out that the presence of bulk viscosity leads to inflationary-like solutions in general relativity. Saha [9] and Saha and Rikhvit-

sky [9, 10] in a series of papers discussed Bianchi type-I cosmological models for viscous-fluid distribution. Recently, Bali and Singh [11] investigated Bianchi type-I cosmological models in the presence of bulk viscosity with time-dependent cosmological term.

Spatially-homogeneous and anisotropic cosmological models, in which the fluid flow is not normal to the hypersurface of homogeneity, have considerable interest in the studies. These models are called tilted cosmological models. The general dynamics of tilted cosmological models has been discussed by King and Ellis [12], and Ellis and King [13]. Bradley and Sviestine [14] have shown that the heat flow is expected for tilted models. Mukherjee [15] investigated Bianchi type-I cosmological model with heat flux for a perfect-fluid distribution. The tilted universes with heat flux have been investigated by a number of authors, viz. Banerjee and Sanyal [16], Coley [17], Roy and Banerjee [18] and Bali and Sharma [19]. Recently, Bali and Kumawat [20] investigated LRS Bianchi type-V tilted cosmological model with bulk viscosity for stiff fluid distribution. To get the deterministic model of the universe, a supplementary condition $A = B^n$ between the metric potentials A and B is also assumed, where n is a constant.

In this paper, we investigate Bianchi type-I tilted cosmological model with heat flux and bulk viscosity for the stiff fluid distribution. To get the deterministic model, we also assume a supplementary condition $A = (BC)^n$ between the metric potentials A , B and C , where n is a constant and $\zeta\theta = \text{constant}$, where ζ is the coefficient of bulk viscosity, θ the expansion and n is a constant. The physical and geometrical aspects of the model in the presence and absence of bulk viscosity are also discussed.

We consider Bianchi type-I metric as

$$ds^2 = A^2 dx^2 + B^2 dy^2 + C^2 dz^2 - dt^2, \quad (1)$$

where A , B and C are functions of t alone.

The energy-momentum tensor (T_i^j) for heat conduction given by Ellis [21] and for bulk viscosity given by Landau and Lifshitz [22] is given by

$$T_i^j = (\epsilon + p)v_i v^j + p g_i^j + q_i v^j v_i q^j - \zeta \theta (g_i^j + v_i v^j), \quad (2)$$

together with

$$g_{ij} v^i v^j = -1, \quad (3)$$

$$q_i q^i > 0, \quad (4)$$

$$q_i v^i = 0, \quad (5)$$

where p is the isotropic pressure, ϵ the matter density and q_i the heat conduction vector orthogonal to v_i . The fluid-flow vector v^i has the components

$\left(\frac{\sinh \lambda}{A}, 0, 0, \cosh \lambda\right)$ satisfying (3) where λ is the tilt angle. Einstein's field equations are

$$R_i^j - \frac{R}{2}g_i^j = -8\pi T_i^j \quad (6)$$

(in the geometrized units $c = 1$, $G = 1$, and taking $\Lambda = 0$).

The line-element (1) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} = -8\pi \left[(\epsilon + p) \sinh^2 \lambda + p + \frac{2q_1 \sinh \lambda}{A} - K \cosh^2 \lambda \right], \quad (7)$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4C_4}{AC} = -8\pi(p - K), \quad (8)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} = -8\pi(p - K), \quad (9)$$

$$\frac{A_4B_4}{AB} + \frac{B_4C_4}{BC} + \frac{A_4C_4}{AC} = -8\pi \left[-(\epsilon + p) \cosh^2 \lambda + p - \frac{2q_1 \sinh \lambda}{A} + K \sinh^2 \lambda \right], \quad (10)$$

$$(\epsilon + p)A \sinh \lambda \cosh \lambda + q_1 \cosh \lambda + q_1 \frac{\sinh^2 \lambda}{\cosh \lambda} - KA \sinh \lambda \cosh \lambda = 0, \quad (11)$$

where

$$\zeta\theta = K, \quad (12)$$

and the subscript '4' denotes the differentiation with respect to t .

2. Solution of the field equations

Equations (7)–(11) are five equations in seven unknowns A , B , C , ϵ , p , λ and q_1 . For the complete determination of these quantities, we assume that

$$p = \epsilon \quad (\text{stiff - fluid condition}), \quad (13)$$

and

$$A = (BC)^n. \quad (14)$$

From Eqs. (7), (10), (13) and (14), we have

$$(BC)_{44} + \frac{n[(BC)_4]^2}{BC} = 8\pi K(BC). \quad (15)$$

Using $BC = \mu$ and $\frac{B}{C} = \nu$ in (15), we have

$$\frac{df^2}{d\mu} + \frac{2n}{\mu} f^2 = 16\pi K \mu, \quad (16)$$

where

$$\mu_4 = f(\mu), \quad \mu_{44} = f f' \quad \text{and} \quad f' = df/d\mu.$$

Equation (16) is a linear differential equation in f^2 . Its integrating factor is μ^{2n} . Equation (16) leads to

$$\int \frac{\mu^n d\mu}{\sqrt{8\pi K/(n+1)} \mu^{2n+2} + N} = \int dt, \quad (17)$$

where N is a constant of integration. Let $\mu^{n+1} = \xi$. From Eq. (17), we have

$$\mu = \left\{ (n+1)\sqrt{N} \frac{\sinh(at+b)}{a} \right\} = \left\{ (n+1)\sqrt{N} \frac{E_1}{a} \right\}, \quad \text{where} \quad E_1 = \sinh(at+b), \quad (18)$$

b is a constant of integration and $a^2 = 8\pi K(n+1)$. We introduced above the abbreviation E_1 for the expression $\sinh(at+b)$ because it appears repeatedly in the following equations.

Equations (8) and (9) lead to

$$\frac{\nu_4}{\nu} = \frac{L}{\mu^{n+1}}, \quad (19)$$

where L is a constant of integration. From Eqs. (19) and (18), we have

$$\nu = F \left[\operatorname{cosec}(at+b) + \operatorname{coth}(at+b) \right]^{-L/(n+1)\sqrt{N}}, \quad (20)$$

where F is a constant of integration. We assume

$$F = a^{-L/(n+1)\sqrt{N}}. \quad (21)$$

Thus the metric (1) leads to

$$ds^2 = -dt^2 + \left\{ \frac{E_1}{a} \right\}^{2n/(n+1)} dX^2 + \left\{ \frac{E_1}{a} \right\}^{1/(n+1)} \left[a \{ \operatorname{cosec}(at+b) + \operatorname{coth}(at+b) \} \right]^{-L/(n+1)\sqrt{N}} dY^2$$

$$+ \left\{ \frac{E_1}{a} \right\}^{1/(n+1)} [a \{ \operatorname{cosec}(at + b) + \operatorname{coth}(at + b) \}]^{L/(n+1)\sqrt{N}} dZ^2, \quad (22)$$

where

$$\left\{ \sqrt{N} (n + 1) \right\}^{n/(n+1)} x = X,$$

$$\left\{ \sqrt{N} (n + 1) \right\}^{1/2(n+1)} y = Y,$$

$$\left\{ \sqrt{N} (n + 1) \right\}^{1/2(n+1)} z = Z.$$

In the absence of viscosity, i.e. when $K \rightarrow 0$, the metric (1) leads to

$$ds^2 = -\ell^2 d\tau^2 + (\ell\tau)^{2n/(n+1)} dX^2 + (\ell\tau)^{1/(n+1)} \left\{ \left(\frac{2}{\ell\tau} \right)^{-L/(n+1)\sqrt{N}} dY^2 + \left(\frac{2}{\ell\tau} \right)^{L/(n+1)\sqrt{N}} dZ^2 \right\}, \quad (23)$$

where we have used the transformation

$$\sinh(at + b) = \ell \sin a\tau, \quad (24)$$

where ℓ is a constant.

3. Geometrical and physical properties

In the following, we use the replacement

$$E_2 = \frac{L^2}{N} - 4n - 1,$$

because it appears repeatedly in the following equations.

Using the replacements E_1 and E_2 , the pressure p , density ϵ , λ , θ , σ_{ij} , q_1 and q_4 are given by

$$8\pi p = 8\pi\epsilon = \frac{a^2}{4(n+1)^2} \left\{ (2n+3-4n^2) - \frac{E_2}{E_1^2} \right\}, \quad (25)$$

$$\cosh \lambda = \left\{ \frac{n(2n-1)E_1^2 + E_2}{E_2 - (2n-1)E_1^2} \right\}^{1/2}, \quad (26)$$

$$\theta = \frac{a \coth(at + b) \{E_2^2 - (2n - 1)^2 n E_1^4 + 2n(2n - 1)E_2 E_1^2\}}{\{E_2 - (2n - 1)E_1^2\}^{3/2} \{n(2n - 1)E_1^2 + E_2\}^{1/2}}, \quad (27)$$

$$\sigma_{11} = \frac{N_1}{3(n + 1) \{E_2 - (2n - 1)E_1^2\}^{5/2}}, \quad (28)$$

where the numerator in Eq. (28) is given by

$$\begin{aligned} N_1 &= (2n - 1)a \coth(at + b) \left\{ \sqrt{N} (n + 1)E_1/a \right\}^{2n/(n+1)} \left\{ n(2n - 1)E_1^2 + E_2 \right\}^{1/2} \\ &\quad \times \left\{ (4n^2 + n + 3)E_2 E_1^2 + E_2^2 - n(2n - 1)^2 E_1^4 \right\}, \\ \sigma_{22} &= \frac{N_2}{3 \left\{ E_2 - (2n - 1)E_1^2 \right\}^{3/2} \left\{ n(2n - 1)E_1^2 + E_2 \right\}^{1/2}}, \quad (29) \end{aligned}$$

where the numerator in Eq. (29) is given by

$$\begin{aligned} N_2 &= \left\{ \sqrt{N} (n+1)E_1/a \right\}^{1/(n+1)} a \coth(at+b) \left\{ \operatorname{acosech}(at+b) + a \coth(at + b) \right\}^{-L/(n+1)\sqrt{N}} \\ &\quad \times \left[E_2^2 \left\{ \frac{1 - 2n}{2(n + 1)} + \frac{3}{2} \frac{L}{2(n + 1)\sqrt{N}} \frac{1}{\sqrt{E_1^2 + 1}} \right\} \right. \\ &\quad \left. + (2n - 1)E_2 E_1^2 \left\{ -\frac{4n^2 + n + 3}{2(n + 1)} + \frac{3(n - 1)}{2(n + 1)\sqrt{N}} \frac{L}{\sqrt{E_1^2 + 1}} \right\} \right. \\ &\quad \left. + n(2n - 1)^2 E_1^4 \left\{ \frac{2n - 1}{2(n + 1)} - \frac{3}{2} \frac{L}{(n + 1)\sqrt{N}} \frac{1}{\sqrt{E_1^2 + 1}} \right\} \right], \\ \sigma_{33} &= \frac{N_3}{3 \left\{ E_2 - (2n - 1)E_1^2 \right\}^{3/2} \left\{ n(2n - 1)E_1^2 + E_2 \right\}^{1/2}}, \quad (30) \end{aligned}$$

where the numerator in Eq. (30) is given by

$$N_3 = \left\{ \sqrt{N} (n+1)E_1/a \right\}^{1/(n+1)} a \coth(at+b) \left\{ \operatorname{acosech}(at+b) + a \coth(at + b) \right\}^{L/(n+1)\sqrt{N}}$$

$$\begin{aligned}
& \times \left[E_2^2 \left\{ \frac{1-2n}{2(n+1)} - \frac{3L}{2(n+1)\sqrt{N}} \frac{1}{\sqrt{E_1^2+1}} \right\} \right. \\
& + (2n-1)E_2E_1^2 \left\{ -\frac{4n^2+n+3}{2(n+1)} - \frac{3(n-1)}{2(n+1)\sqrt{N}} \frac{L}{\sqrt{E_1^2+1}} \right\} \\
& \left. + n(2n-1)^2E_1^4 \left\{ \frac{2n-1}{2(n+1)} + \frac{3L}{2(n+1)\sqrt{N}} \frac{1}{\sqrt{E_1^2+1}} \right\} \right], \\
\sigma_{44} = & \frac{(2n-1)^2E_1^2a \coth(at+b) \left[E_2^2 + (4n^2+n+3)E_2E_1^2 - n(2n-1)^2E_1^4 \right]}{3 \left\{ E_2 - (2n-1)E_1^2 \right\}^{5/2} \left\{ n(2n-1)E_1^2 + E_2 \right\}^{1/2}}, \quad (31)
\end{aligned}$$

$$\sigma_{14} = \frac{N_4}{3(n+1) \left\{ E_2 - (2n-1)E_1^2 \right\}^{5/2}}, \quad (32)$$

where the numerator in Eq. (32) is given by

$$\begin{aligned}
N_4 = & (2n-1)a \coth(at+b) \left\{ \sqrt{N} (n+1)E_1/a \right\}^{n/(n+1)} \\
& \times \left\{ (2n-1)(n+1)E_1^2 \right\}^{1/2} \left\{ - (4n^2+n+3)E_2E_1^2 - E_2^2 + n(2n-1)E_1^4 \right\}, \\
v^1 = & \frac{\left\{ (2n-1)(n+1)E_1^2 \right\}^{1/2}}{\left\{ E_2 - (2n-1)E_1^2 \right\}^{1/2} \left\{ (n+1)\sqrt{N} E_1/a \right\}^{n/(n+1)}}, \quad (33) \\
v^4 = & \frac{\left\{ n(2n-1)E_1^2 + E_2 \right\}^{1/2}}{\left\{ E_2 - (2n-1)E_1^2 \right\}^{1/2}}.
\end{aligned}$$

The shear tensor (σ_{ij}) satisfies the trace-free condition

$$\sigma_{ij}v^j = 0,$$

which leads to

$$\sigma_{11}v^1 + \sigma_{14}v^4 = 0. \quad (34)$$

The components of rotation tensor (ω_{ij}) are given by

$$\omega_{11} = \omega_{22} = \omega_{33} = \omega_{44} = \omega_{14} = 0. \quad (35)$$

The components of heat-conduction vector (q_i) are given by

$$q_1 = \frac{[(\sqrt{N})^n a^{(n+2)}]^{1/(n+1)} \sqrt{2n-1} E_1 \{n(2n-1)E_1^2 + E_2\}}{16\pi [(n+1)^{(n+3)/2} E_1^{(n+2)}]^{1/(n+1)} \left\{ E_2 - (2n-1)E_1^2 \right\}^{1/2}}, \quad (36)$$

$$q_4 = \frac{-a^2(2n-1)^2 \{n(2n-1)E_1^2 + E_2\}^{1/2}}{16\pi(n+1) \left\{ E_2 - (2n-1)E_1^2 \right\}^{1/2}}. \quad (37)$$

The deceleration parameter (\bar{a}) is given by

$$\bar{a} = -\frac{R_{44}/R}{R_4^2/R^2} = \frac{2 \coth^2(at+b) - 3}{\coth^2(at+b)}. \quad (38)$$

The heat conduction vector (q_i) satisfies the condition

$$q_i v^i = 0,$$

i.e.

$$q_1 v^1 + q_4 v^4 = 0. \quad (39)$$

The rotation tensor (ω_{ij}) satisfies the condition $\omega_{ij}v^j = 0$. This leads to

$$\omega_{11}v^1 + \omega_{44}v^4 = 0. \quad (40)$$

In the absence of viscosity, the above mentioned quantities lead to

$$8\pi p = 8\pi\epsilon = \frac{1}{4(n+1)^2} \frac{(4n+1 - L^2/N)}{\ell^2\tau^2}, \quad (41)$$

$$\cosh \lambda = 1, \quad (42)$$

$$\theta = \frac{1}{\ell\tau}, \quad (43)$$

$$\sigma_{11} = \frac{2n-1}{3(n+1)} \{ \sqrt{N}(n+1) \}^{2n/(n+1)} (\ell\tau)^{(n-1)/(n+1)}, \quad (44)$$

$$\sigma_{22} = \frac{(\sqrt{N})^{1/(n+1)} \{ 1 - 2n + 3L/\sqrt{N} \}}{3(n+1)^{n/(n+1)} 2^{L+\sqrt{N}(n+1)}/\sqrt{N}(n+1)} (\ell\tau)^{(n\sqrt{N}-L)/\sqrt{N}(n+1)}, \quad (45)$$

$$\sigma_{33} = \frac{(\sqrt{N})^{1/(n+1)} 2^{L-\sqrt{N}(n+1)}/\sqrt{N}(n+1) \{ 1 - 2n - 3L/\sqrt{N} \}}{3(n+1)^{n/(n+1)} (\ell\tau)^{(n\sqrt{N}+L)/\sqrt{N}(n+1)}}, \quad (46)$$

$$\sigma_{44} = 0, \quad (47)$$

$$\sigma_{14} = 0, \quad (48)$$

$$v^1 = 0, \quad (49)$$

$$v^4 = 1, \quad (50)$$

$$q_1 = 0, \quad (51)$$

$$q_4 = 0, \quad (52)$$

$$\omega_{11} = \omega_{22} = \omega_{33} = \omega_{14} = 0. \quad (53)$$

4. Discussion

The model (22) starts with a big-bang at $t = -b/a$ and the expansion in the model decreases as time increases. The shear tensor (σ_{ij}) and the rotation tensor (ω_{ij}) satisfy the trace-free conditions

$$\sigma_{ij}v^j = 0,$$

i.e. $\sigma_{11}v^1 + \sigma_{14}v^4 = 0$, and

$$\omega_{ij}v^j = 0,$$

i.e. $\omega_{11}v^1 + \omega_{14}v^4 = 0$.

The heat conduction vector (q_i) satisfies the condition

$$q_i v^i = 0,$$

i.e. $q_1 v^1 + q_4 v^4 = 0$.

The reality conditions (i) $(\epsilon + p) > 0$ and (ii) $(\epsilon + 3p) > 0$ given by Ellis [23] lead to

$$\left(4n + 1 - \frac{L^2}{N}\right) > \sinh^2(at + b)(4n^2 - 2n + 3),$$

$$\sinh^2(at + b) < \frac{4n + 1 - L^2/N}{4n^2 - 2n - 3}.$$

In the presence of bulk viscosity, the model represents accelerating universe if $\text{sech}^2(at + b) < \frac{1}{3}$ and it represents decelerating universe if $\text{sech}^2(at + b) > \frac{1}{3}$.

In the absence of bulk viscosity, the model starts with a big-bang at $\tau = 0$ and the expansion in the model decreases as time increases. The shear tensor (σ_{ij}) and the rotation tensor (ω_{ij}) satisfy the trace-free conditions

$$\sigma_{ij}v^j = 0 \quad \text{and} \quad \omega_{ij}v^j = 0,$$

i.e. $\sigma_{11}v^1 + \sigma_{14}v^4 = 0$ and $\omega_{11}v^1 + \omega_{14}v^4 = 0$.

The tilt angle $\lambda = 0$. Thus in the absence of bulk viscosity, no tilt model is possible for stiff-fluid distribution. The reality conditions $\epsilon + p > 0$ and $\epsilon + 3p > 0$ lead to $4n + 1 > L^2/N$. The model (23) has a cigar-type singularity at $\tau = 0$ (MacCallum [24]). In the absence of bulk viscosity, it represents a decelerating universe.

5. Conclusion

In the models (22) and (23), in the presence and absence of bulk viscosity, shear tensor and rotation tensor satisfy the trace-free conditions i.e. $\sigma_{ij}v^j = 0$ and $\omega_{ij}v^j = 0$. The heat conduction vector q_i satisfies orthogonality condition $q_i v_i = 0$. The model (22) represents accelerating and decelerating universe under certain conditions while the model in the absence of bulk viscosity represents a decelerating universe. The model (22) has point-type singularity at $t = -b/a$, while the model (23) has cigar-type singularity at $\tau = 0$. In the absence of bulk viscosity, no tilt model is possible for the stiff-fluid distribution.

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BIANCIJEV KOZMOLOŠKI MODEL TIP A S UKOČENOM TEKUĆINOM, NAHERENOŠĆU I VOLUMNOM VISKOZNOŠĆU U OPĆOJ RELATIVNOSTI

Istražujemo Bianchijev kozmološki model tipa I s ukočenom tekućinom, naherenošću i volumnom viskoznošću. Radi postizanja određenosti, pretpostavlja se da je svemir ispunjen ukočenom tekućinom, tj. da vrijedi $A = (BC)^n$ za metričke potencijale A , B , C te $\zeta\theta = \text{konst.}$, gdje je ζ koeficijent volumne viskoznosti, θ širenje a n je stalnica. Raspravljaju se fizička i geometrijska svojstva modela s i bez viskoznosti.