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# A NEW CLASS OF PLANE-SYMMETRIC INHOMOGENEOUS COSMOLOGICAL MODELS OF PERFECT FLUID DISTRIBUTION WITH ELECTROMAGNETIC FIELD

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A new class of plane-symmetric inhomogeneous cosmological models of perfect fluid distribution with electromagnetic field is obtained. The source of the magnetic field is due to an electric current produced along the z-axis. $F_{12}$ is the non-vanishing component of electromagnetic field tensor. The free gravitational field is assumed to be of Petrov type-II non-degenerate. We have studied three cases : (i) cosine hyperbolic (ii) linear and (ii) cosine form. Some geometric and physical properties of the models in the presence and absence of magnetic field are also discussed.

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## 1. Introduction

The standard Friedman-Robertson-Walker (FRW) cosmological model prescribes a homogeneous and an isotropic distribution of matter in the description of the present state of the universe. At the present state of evolution, the universe is spherically symmetric and the matter distribution in the universe is on the whole isotropic and homogeneous. But in the early stages of evolution, it could have not

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had such a smoothed picture. Close to the big bang singularity, neither the assumption of spherical symmetry nor that of isotropy can be strictly valid. So we consider plane-symmetry, which is less restrictive than spherical symmetry and can provide an avenue to study inhomogeneities. Inhomogeneous cosmological models play an important role in understanding some essential features of the universe such as the formation of galaxies during the early stages of evolution and process of homogenization. The early attempts at the construction of such models have been done by Tolman [1] and Bondi [2] who considered spherically symmetric models. Inhomogeneous plane-symmetric models were considered by Taub [3, 4] and later by Tomimura [5], Szekeres [6], Collins and Szafron [7], and Szafron and Collins [8]. Recently, Senovilla [9] obtained a new class of exact solutions of Einstein's equations without the big bang singularity, representing a cylindrically symmetric, inhomogeneous cosmological model filled with perfect fluid which is smooth and regular everywhere, satisfying energy and causality conditions. Later, Ruiz and Senovilla [10] have examined a fairly large class of singularity-free models through a comprehensive study of general cylindrically-symmetric metric with separable function of $r$ and $t$ as metric coefficients. Dadhich et al. [11] have established a link between the FRW model and the singularity-free family by deducing the latter through a natural and simple in-homogenization and anisotropization of the former. Recently, Patel et al. [12] presented a general class of inhomogeneous cosmological models filled with non-thermalized perfect fluid by assuming that the background spacetime admits two space-like commuting Killing vectors and has separable metric coefficients. Singh, Mehta and Gupta [13] obtained inhomogeneous cosmological models of perfect fluid distribution with electromagnetic field. Recently, Pradhan et al. [14] have investigated plane-symmetric inhomogeneous cosmological models in various contexts.

The occurrence of magnetic field on the galactic scale is well-established fact today, and their importance for a variety of astrophysical phenomena is generally acknowledged as pointed out by Zeldovich et al. [15]. Also, Harrison [16] has suggested that magnetic field could have a cosmological origin. As a natural consequences, we should include magnetic fields in the energy-momentum tensor of the early universe. The choice of anisotropic cosmological models in the Einstein system of field equations leads to the cosmological models more general than Robertson-Walker model [17]. The presence of primordial magnetic field in the early stages of the evolution of the universe has been discussed by several authors [18]−[27]. Strong magnetic field can be created due to adiabatic compression in clusters of galaxies. Large-scale magnetic field gives rise to anisotropies in the universe. The anisotropic pressure created by the magnetic fields dominates the evolution of the shear anisotropy and it decays slower than the case when the pressure was isotropic [28, 29]. Such fields can be generated at the end of an inflationary epoch [30]−[34]. Anisotropic magnetic field models have significant contribution in the evolution of galaxies and stellar objects. Bali and Ali [35] obtained a magnetized cylindrically-symmetric universe with an electrically neutral perfect fluid as the source of matter. Pradhan et al. [36] have investigated magnetized viscous fluid cosmological models in various contexts.

Bali and Tyagi [37] have investigated a plane-symmetric inhomogeneous cosmological model of perfect fluid distribution with electromagnetic field. In this paper, we have revisited their solution and obtained a new class of plane-symmetric inhomogeneous cosmological models of perfect fluid distribution with electromagnetic field. We assume the free gravitational field to be of the Petrov type-II non-degenerate. The paper is organized as follows. The metric and the field equations are presented in Section 2. In Section 3, we deal with the three types of solutions of the field equations in presence of magnetic field, whereas Section 4 includes the solutions in the absence of magnetic field. Finally, the results are discussed in Section 5.

## 2. *The metric and field equations*

We consider the metric in the form

$$\mathrm{d}s^2 = A^2(\mathrm{d}x^2 - \mathrm{d}t^2) + B^2\mathrm{d}y^2 + C^2\mathrm{d}z^2, \tag{1}$$

where the metric potentials $A$, $B$ and $C$ are functions of $x$ and $t$. The energy momentum tensor is taken as

$$T_i^j = (\rho + p)v_i v^j + p g_i^j + E_i^j, \tag{2}$$

where $E_i^j$ is the electromagnetic field given by Lichnerowicz [38] as

$$E_i^j = \bar{\mu}\left[h_l h^l (v_i v^j + \frac{1}{2}g_i^j) - h_i h^j\right]. \tag{3}$$

Here $\rho$ and $p$ are the energy density and isotropic pressure, respectively, and $v^i$ is the flow vector satisfying the relation

$$g_{ij}v^i v^j = -1. \tag{4}$$

$\bar{\mu}$ is the magnetic permeability and $h_i$ the magnetic flux vector defined by

$$h_i = \frac{1}{\bar{\mu}}\ {}^*F_{ji}v^j, \tag{5}$$

where ${}^*F_{ij}$ is the dual electromagnetic field tensor defined by Synge [39]

$${}^*F_{ij} = \frac{\sqrt{-g}}{2}\epsilon_{ijkl}F^{kl}. \tag{6}$$

$F_{ij}$ is the electromagnetic field tensor and $\epsilon_{ijkl}$ is the Levi-Civita tensor density. The coordinates are considered to be comoving so that $v^1 = 0 = v^2 = v^3$ and $v^4 = 1/A$. We consider that the current is flowing along the z-axis so that $h_3 \neq 0$,

$h_1 = 0 = h_2 = h_4$. The only non-vanishing component of $F_{ij}$ is $F_{12}$. The Maxwell's equations

$$F_{ij;k} + F_{jk;i} + F_{ki;j} = 0 \tag{7}$$

and

$$\left[\frac{1}{\bar{\mu}} F^{ij}\right]_{;j} = J^i \tag{8}$$

require that $F_{12}$ be function of $x$ alone. We assume that the magnetic permeability is a function of $x$ and $t$ both. Here the semicolon represents a covariant differentiation.

The Einstein's field equations (in gravitational units $c = 1$, $G = 1$)

$$R_i^j - \frac{1}{2} R g_i^j + \Lambda g_i^j = -8\pi T_i^j, \tag{9}$$

for the line element (1) has been set up as

$$8\pi A^2 \left(p + \frac{F_{12}^2}{2\bar{\mu}A^2B^2}\right) = -\frac{B_{44}}{B} - \frac{C_{44}}{C} + \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right)$$

$$+\frac{A_1}{A}\left(\frac{B_1}{B} + \frac{C_1}{C}\right) + \frac{B_1C_1}{BC} - \frac{B_4C_4}{BC} - \Lambda A^2, \tag{10}$$

$$8\pi A^2 \left(p + \frac{F_{12}^2}{2\bar{\mu}A^2B^2}\right) = -\left(\frac{A_4}{A}\right)_4 + \left(\frac{A_1}{A}\right)_1 - \frac{C_{44}}{C} + \frac{C_{11}}{C} - \Lambda A^2, \tag{11}$$

$$8\pi A^2 \left(p - \frac{F_{12}^2}{2\bar{\mu}A^2B^2}\right) = -\left(\frac{A_4}{A}\right)_4 + \left(\frac{A_1}{A}\right)_1 - \frac{B_{44}}{B} + \frac{B_{11}}{B} - \Lambda A^2, \tag{12}$$

$$8\pi A^2 \left(\rho + \frac{F_{12}^2}{2\bar{\mu}A^2B^2}\right) = -\frac{B_{11}}{B} - \frac{C_{11}}{C} + \frac{A_1}{A}\left(\frac{B_1}{B} + \frac{C_1}{C}\right)$$

$$+\frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) - \frac{B_1C_1}{BC} + \frac{B_4C_4}{BC} + \Lambda A^2, \tag{13}$$

$$0 = \frac{B_{14}}{B} + \frac{C_{14}}{C} - \frac{A_1}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) - \frac{A_4}{A}\left(\frac{B_1}{B} + \frac{C_1}{C}\right), \tag{14}$$

where the sub-indices 1 and 4 in A, B, C and elsewhere indicate ordinary differentiation with respect to $x$ and $t$, respectively.

## 3. *Solutions of the field equations*

Equations (10) - (12) lead to

$$\left(\frac{A_4}{A}\right)_4 - \frac{B_{44}}{B} + \frac{A_4}{A}\left(\frac{B_4}{B}+\frac{C_4}{C}\right) - \frac{B_4C_4}{BC} =$$

$$\left(\frac{A_1}{A}\right)_1 + \frac{C_{11}}{C} - \frac{A_1}{A}\left(\frac{B_1}{B}+\frac{C_1}{C}\right) - \frac{B_1C_1}{BC} = a \text{ (a constant)} \tag{15}$$

and

$$\frac{8\pi F_{12}^2}{\bar{\mu}B^2} = \frac{B_{44}}{B} - \frac{B_{11}}{B} + \frac{C_{11}}{C} - \frac{C_{44}}{C}. \tag{16}$$

Eqs. (10) - (14) represent a system of five equations in six unknowns $A$, $B$, $C$, $\rho$, $p$ and $F_{12}$. For the complete determination of these unknowns, one more condition is needed. As in the case of general-relativistic cosmologies, the introduction of inhomogeneities into the cosmological equations produces a considerable increase in mathematical difficulty: non-linear partial differential equations must now be solved. In practice, this means that we must proceed either by means of approximations which render the non-linearities tractable, or we must introduce particular symmetries into the metric of the space-time in order to reduce the number of degrees of freedom which the inhomogeneities can exploit. In the present case, we assume that the metric is Petrov type-II non-degenerate. This requires that

$$\left(\frac{B_{11}+B_{44}+2B_{14}}{B}\right) - \left(\frac{C_{11}+C_{44}+2C_{14}}{C}\right) =$$

$$\frac{2(A_1+A_4)(B_1+B_4)}{AB} - \frac{2(A_1+A_4)(C_1+C_4)}{AC}. \tag{17}$$

Let us consider that

$$A = f(x)\nu(t),$$

$$B = g(x)\mu(t),$$

$$C = h(x)\mu(t). \tag{18}$$

Using (18) in (14) and (17), we get

$$\frac{\dfrac{g_1}{g}+\dfrac{h_1}{h}}{\dfrac{f_1}{f}} = \frac{\dfrac{2\mu_4}{\mu}}{\dfrac{\mu_4}{\mu}-\dfrac{\nu_4}{\nu}} = b \text{ (a constant)} \tag{19}$$

and

$$\frac{\frac{g_{11}}{g}+\frac{h_{11}}{h}}{\frac{g_1}{g}-\frac{h_1}{h}}-\frac{2f_1}{f}=2\left(\frac{\mu_4}{\mu}-\frac{\nu_4}{\nu}\right)=L \text{ (a constant).} \tag{20}$$

Equation (19) leads to

$$f=n(gh)^{1/b},\ b\neq 0 \tag{21}$$

and

$$\mu=m\nu^{b/(b-2)}, \tag{22}$$

where $m$ and $n$ are constants of integration.
From Eqs. (15), (18) and (19), we have

$$\frac{1}{b}\frac{g_{11}}{g}+\left(\frac{1+b}{b}\right)\frac{h_{11}}{h}-\frac{2}{b}\left(\frac{g_1^2}{g^2}+\frac{h_1^2}{h^2}\right)-\frac{(2+b)}{b}\frac{g_1h_1}{gh}=a \tag{23}$$

and

$$\frac{2}{b}\left(\frac{\mu_{44}}{\mu}+\frac{\mu_4^2}{\mu^2}\right)=-a. \tag{24}$$

Let us assume

$$g=e^{U+V},\quad h=e^{U-V}. \tag{25}$$

Eqs. (20) and (25) lead to

$$V_1=M\exp\left(Lx+\frac{2(2-b)}{b}U\right), \tag{26}$$

where $M$ is an integration constant. From Eqs. (23), (25) and (26), we have

$$\left(\frac{2+b}{b}\right)U_{11}-\frac{4}{b}U_1^2-2bM\exp\left(Lx+\frac{2(2-b)}{b}U\right)-$$
$$ML\exp\left(Lx+\frac{2(2-b)}{b}U\right)+2M^2\exp\left(2Lx+\frac{4(2-b)}{b}U\right)=a. \tag{27}$$

Equation (27) leads to

$$U=\frac{Lbx}{2(b-2)},\quad b\neq 2 \tag{28}$$

Equations (26) and (28) lead to

$$V=Mx+\log N, \tag{29}$$

where $N$ is the constant of integration.
Equation (24) leads to

$$\mu = \begin{bmatrix} \beta \cosh^{1/2}(\sqrt{\mid \alpha \mid} t + t_0) & \text{when } ab < 0 \\ (c_1 t + t_0)^{1/2} & \text{when } ab = 0 \\ \beta \cos^{1/2}(\sqrt{\alpha} t + t_0) & \text{when } ab > 0 \end{bmatrix} \tag{30}$$

where $\alpha = ab$, $\beta$ is constant and $c_1$, $t_0$ are constants of integration.

### 3.1. Case(i): $ab < 0$

In this case we obtain

$$f = n \exp\left(\frac{Lx}{(b-2)}\right), \tag{31}$$

$$\mu = \beta \cosh^{1/2}(\sqrt{\mid \alpha \mid} t + t_0), \tag{32}$$

$$\nu = r \cosh^{(b-2)/2b}(\sqrt{\mid \alpha \mid} t + t_0), \tag{33}$$

$$g = N \exp\left(\frac{Lbx}{2(b-2)} + Mx\right), \tag{34}$$

$$h = \frac{1}{N} \exp\left(\frac{Lbx}{2(b-2)} - Mx\right), \tag{35}$$

where $r = (\beta/m)^{(b-2)/b}$.
Therefore, we have

$$A = E \exp\left(\frac{Lx}{(b-2)}\right) \cosh^{(b-2)/2b} (\sqrt{\mid \alpha \mid} t + t_0), \tag{36}$$

$$B = G \exp\left(\frac{Lbx}{2(b-2)} + Mx\right) \cosh^{1/2} (\sqrt{\mid \alpha \mid} t + t_0), \tag{37}$$

$$C = H \exp\left(\frac{Lbx}{2(b-2)} - Mx\right) \cosh^{1/2} (\sqrt{\mid \alpha \mid} t + t_0), \tag{38}$$

where $E = nr$, $G = N\beta$, $H = \beta/N$.

After using suitable transformation of coordinates, the metric (1) reduces to the form

$$\mathrm{d}s^2 = E^2 \exp\left(\frac{2LX}{(b-2)}\right) \cosh^{(b-2)/b} (\sqrt{\mid \alpha \mid} T)(\mathrm{d}X^2 - \mathrm{d}T^2) +$$

$$\exp\left(\frac{LbX}{(b-2)}+2MX\right)\cosh(\sqrt{\mid\alpha\mid}T)\mathrm{d}Y^2+$$

$$\exp\left(\frac{LbX}{(b-2)}-2MX\right)\cosh(\sqrt{\mid\alpha\mid}T)\mathrm{d}Z^2. \tag{39}$$

The expressions for pressure $p$ and density $\rho$ for the model (39) are given by

$$8\pi p=\frac{1}{E^2}\exp\left(\frac{2LX}{(2-b)}\right)\cosh^{(2-b)/b}\,(\sqrt{\mid\alpha\mid}T)\times$$

$$\left[\mid\alpha\mid\left\{\frac{(b+1)}{4b}\tanh^2(\sqrt{\mid\alpha\mid}T)-1\right\}+\frac{b(b+4)L^2}{4(b-2)^2}-M^2+\frac{MLb}{(b-2)}\right]-\Lambda, \tag{40}$$

$$8\pi\rho=\frac{1}{E^2}\exp\left(\frac{2LX}{(2-b)}\right)\cosh^{(2-b)/b}\,(\sqrt{\mid\alpha\mid}T)\times$$

$$\left[\frac{\mid\alpha\mid(3b-4)}{4b}\tanh^2(\sqrt{\mid\alpha\mid}T)+\frac{b(4-3b)L^2}{4(b-2)^2}-M^2+\frac{MLb}{(b-2)}\right]+\Lambda. \tag{41}$$

The non-vanishing component $F_{12}$ of electromagnetic field tensor is given by

$$F_{12}=\sqrt{\frac{\bar{\mu}}{8\pi}\frac{2MLb}{(2-b)}}G\exp\left\{\left(\frac{Lb}{b-2}+2M\right)\frac{X}{2}\right\}\cosh(\sqrt{\mid\alpha\mid}T), \tag{42}$$

where $\bar{\mu}$ remains undetermined as a function of $X$ and $T$.

The scalar of expansion $\theta$ calculated for the flow vector $v^i$ is given by

$$\theta=\frac{(3b-2)\sqrt{\mid\alpha\mid}}{2bE}\exp\left(\frac{LX}{(2-b)}\right)\cosh^{(2-b)/(2b)}\,(\sqrt{\mid\alpha\mid}T)\tanh(\sqrt{\mid\alpha\mid}T). \tag{43}$$

Now the shear scalar $\sigma^2$, acceleration vector $\dot{v}_i$ and proper volume $V^3$ are given by

$$\sigma^2=\frac{\mid\alpha\mid}{3b^2E^2}\exp\left(\frac{2LX}{(2-b)}\right)\cosh^{(2-b)/b}\,(\sqrt{\mid\alpha\mid}T)\tanh^2(\sqrt{\mid\alpha\mid}T), \tag{44}$$

$$\dot{v}_i=\left(\frac{L}{(b-2)},0,0,0\right), \tag{45}$$

$$V^3=\sqrt{-g}=E^2\exp\left(\frac{(b+2)LX}{(b-2)}\right)\cosh^{2(b-1)/b}(\sqrt{\mid\alpha\mid}T). \tag{46}$$

From Eqs. (43) and (44), we have

$$\frac{\sigma^2}{\theta^2} = \frac{4}{3(3b-2)^2} = const. \tag{47}$$

The rotation $\omega$ is identically zero.

The dominant energy conditions given by Hawking and Ellis [40],

$$(i) \quad \rho - p \geq 0\,,$$

$$(ii) \quad \rho + p \geq 0\,,$$

lead to

$$\exp\left(\frac{2LX}{2-b}\right)\left[\frac{(2b-5)\mid\alpha\mid}{4b}\tanh^2(\sqrt{\mid\alpha\mid}T) + \mid\alpha\mid - \frac{L^2b^2}{(b-2)^2}\right]$$

$$+2\Lambda E^2\cosh^{\frac{(b-2)}{b}}(\sqrt{\mid\alpha\mid}T) \geq 0 \tag{48}$$

and

$$\frac{(4b-3)\mid\alpha\mid}{4b}\tanh^2(\sqrt{\mid\alpha\mid}T) \geq \mid\alpha\mid + 2M^2 - \frac{2MLb}{(b-2)} + \frac{L^2b(b-4)}{2(b-2)^2} \tag{49}$$

The reality conditions given by Ellis [41],

$$(i) \quad \rho + p > 0\,,$$

$$(ii) \quad \rho + 3p > 0\,,$$

lead to

$$\frac{(4b-3)\mid\alpha\mid}{4b}\tanh^2(\sqrt{\mid\alpha\mid}T) > \mid\alpha\mid + 2M^2 - \frac{2MLb}{(b-2)} + \frac{L^2b(b-4)}{2(b-2)^2} \tag{50}$$

and

$$\exp\left(\frac{2LX}{2-b}\right)\left[\frac{(6b-1)\mid\alpha\mid}{4b}\tanh^2(\sqrt{\mid\alpha\mid}T) - \mid\alpha\mid + \frac{4bL^2}{(b-2)^2} - 4M^2 + \frac{4MLb}{(b-2)}\right]$$

$$> 2\Lambda E^2\cosh^{(b-2)/b}(\sqrt{\mid\alpha\mid}T). \tag{51}$$

The model (39) represents an expanding, shearing and non-rotating universe. Since $\sigma/\theta$ is constant, the model does not approach isotropy.

### 3.2. Case(ii): $ab = 0$

In this case we obtain

$$f = n \exp\left(\frac{Lx}{(b-2)}\right), \tag{52}$$

$$\mu = (c_1 t + t_0)^{1/2}, \tag{53}$$

$$\nu = \left(\frac{1}{m}\right)^{(b-2)/b} (c_1 t + t_0)^{(b-2)/(2b)}, \tag{54}$$

$$g = N \exp\left(\frac{Lbx}{2(b-2)} + Mx\right), \tag{55}$$

$$h = \frac{1}{N} \exp\left(\frac{Lbx}{2(b-2)} - Mx\right). \tag{56}$$

Therefore, we have

$$A = E_0 \exp\left(\frac{Lx}{(b-2)}\right)(c_1 t + t_0)^{(b-2)/(2b)}, \tag{57}$$

$$B = N \exp\left(\frac{Lbx}{2(b-2)} + Mx\right)(c_1 t + t_0)^{1/2}, \tag{58}$$

$$C = H_0 \exp\left(\frac{Lbx}{2(b-2)} - Mx\right)(c_1 t + t_0)^{1/b}, \tag{59}$$

where $E_0 = nm^{(2-b)/b}$ and $H_0 = 1/N$.

After using suitable transformation of coordinates, the metric (1) reduces to the form

$$\mathrm{d}s^2 = E_0^2 \exp\left(\frac{2LX}{(b-2)}\right)(c_1 T)^{(b-2)/b}\ (\mathrm{d}X^2 - \mathrm{d}T^2) +$$

$$\exp\left(\frac{LbX}{(b-2)} + 2MX\right)(c_1 T)\mathrm{d}Y^2 + \exp\left(\frac{LbX}{(b-2)} - 2MX\right)(c_1 T)\mathrm{d}Z^2, \tag{60}$$

The expressions for the pressure $p$ and density $\rho$ for model (60) are given by

$$8\pi p = \frac{1}{E_0^2} \exp\left(\frac{2LX}{(2-b)}\right)(c_1 T)^{(2-b)/b} \times$$

$$\left[\frac{1}{4T^2} + \frac{(b-2)}{2b} + \frac{b(5b-8)L^2}{4(b-2)^2} - M^2 + \frac{MLb}{(b-2)}\right] - \Lambda, \tag{61}$$

$$8\pi\rho = \frac{1}{E_0^2} \exp\left(\frac{2LX}{(2-b)}\right)(c_1T)^{(2-b)/b}\times$$

$$\left[\frac{(3b-4)}{4b}\frac{1}{T^2} + \frac{b(4-3b)L^2}{4(b-2)^2} - M^2 + \frac{MLb}{(b-2)}\right] + \Lambda. \tag{62}$$

The non-vanishing component $F_{12}$ of electromagnetic field tensor is given by

$$F_{12} = \sqrt{\frac{\bar{\mu}}{8\pi}\frac{2MLb}{(2-b)}} N \exp\left\{\left(\frac{Lb}{b-2} + 2M\right)\frac{X}{2}\right\}(c_1T)^{1/2}, \tag{63}$$

where $\bar{\mu}$ remains undetermined as a function of $X$ and $T$.

The scalar of expansion $\theta$ calculated for the flow vector $v^i$ is given by

$$\theta = \frac{c_2(3b-2)}{2bE_0} \exp\left(\frac{LX}{(2-b)}\right)T^{(2-3b)/(2b)}, \tag{64}$$

where $c_2 = c_1^{(2-b)/(2b)}$. Now the shear scalar $\sigma^2$, acceleration vector $\dot{v}_i$ and proper volume $V^3$ are given by

$$\sigma^2 = \frac{c_2^2}{3b^2E_0^2} \exp\left(\frac{2LX}{(2-b)}\right)T^{(2-3b)/(2b)}, \tag{65}$$

$$\dot{v}_i = \left(\frac{L}{(b-2)}, 0, 0, 0\right), \tag{66}$$

$$V^3 = \sqrt{-g} = E_0^2 \exp\left(\frac{(b+2)LX}{(b-2)}\right)(c_1T)^{2(b-1)/b}. \tag{67}$$

From Eqs. (64) and (65), we have

$$\frac{\sigma^2}{\theta^2} = \frac{4}{3(3b-2)^2} = const. \tag{68}$$

The rotation $\omega$ is identically zero.

The dominant energy conditions given by Hawking and Ellis [40],

$$(i) \quad \rho - p \geq 0\,,$$

$$(ii) \quad \rho + p \geq 0\,,$$

lead to

$$\exp\left(\frac{2LX}{2-b}\right)\left[\frac{(b-2)}{2b}\frac{1}{T^2}+\frac{b(3-2b)L^2}{(b-2)^2}+\frac{(2-b)}{2b}\right]$$

$$+2\Lambda E_0^2(c_1T)^{(b-2)/b}\geq 0, \tag{69}$$

and

$$\frac{(b-1)}{b}\frac{1}{T^2}+\frac{b(b-2)}{2(b-2)^2}+\frac{(b-2)}{2b}\geq 2M^2+\frac{2MLb}{(2-b)}. \tag{70}$$

The reality conditions given by Ellis [41],

$$(i) \quad \rho+p>0\,,$$

$$(ii) \quad \rho+3p>0\,,$$

lead to

$$\frac{(b-1)}{b}\frac{1}{T^2}+\frac{b(b-2)}{2(b-2)^2}+\frac{(b-2)}{2b}> 2M^2+\frac{2MLb}{(2-b)}. \tag{71}$$

and

$$\exp\left(\frac{2LX}{2-b}\right)\left[\frac{(3b-2)}{2T^2}+\frac{3(b-2)}{2b}+\frac{b(3b-5)L^2}{(b-2)^2}-4M^2+\frac{4MLb}{(b-2)}\right]$$

$$>\ 2\Lambda E_0^2(c_1T)^{(b-2)/b}. \tag{72}$$

For $2/b>3$, the model (60) represents an expanding, shearing and non-rotating universe. For $2/b<3$, the model has singularity at $T=0$. It starts from a big bang at $T=0$ and continues to expand untill $T=\infty$. Since $\sigma/\theta = const.$, the model does not approach isotropy.

### 3.3. *Case(iii): $ab>0$*

In this case we obtain

$$f=n\exp\left(\frac{Lx}{(b-2)}\right), \tag{73}$$

$$\mu=\beta\cos^{1/2}(\sqrt{\alpha}t+t_0), \tag{74}$$

$$\nu=r\cos^{(b-2)/(2b)}(\sqrt{\alpha}t+t_0), \tag{75}$$

$$g=N\exp\left(\frac{Lbx}{2(b-2)}+Mx\right), \tag{76}$$

$$h = \frac{1}{N} \exp\left(\frac{Lbx}{2(b-2)} - Mx\right). \tag{77}$$

Therefore, we have

$$A = E \exp\left(\frac{Lx}{(b-2)}\right) \cos^{(b-2)/2b} \, (\sqrt{\alpha}t + t_0), \tag{78}$$

$$B = G \exp\left(\frac{Lbx}{2(b-2)} + Mx\right) \cos^{1/2}(\sqrt{\alpha}t + t_0), \tag{79}$$

$$C = H \exp\left(\frac{Lbx}{2(b-2)} - Mx\right) \cos^{1/2}(\sqrt{\alpha}t + t_0). \tag{80}$$

Here $E$, $G$ and $H$ are already defined in subsection 3.1.

After using suitable transformation of coordinates, the metric (1) reduces to the form

$$\mathrm{d}s^2 = E^2 \exp\left(\frac{2LX}{(b-2)}\right) \cos^{(b-2)/b}(\sqrt{\alpha}T)(\mathrm{d}X^2 - \mathrm{d}T^2) +$$

$$\exp\left(\frac{LbX}{(b-2)} + 2MX\right) \cos(\sqrt{\alpha}T)\mathrm{d}Y^2 +$$

$$\exp\left(\frac{LbX}{(b-2)} - 2MX\right) \cos(\sqrt{\alpha}T)\mathrm{d}Z^2, \tag{81}$$

The expressions for pressure $p$ and density $\rho$ for model (39) are given by

$$8\pi p = \frac{1}{E^2} \exp\left(\frac{2LX}{2-b}\right) \cos^{(2-b)/b}(\sqrt{\alpha}T) \times$$

$$\left[\alpha\left\{\frac{(3b-4)}{4b}\tan^2(\sqrt{\alpha}T) + 1\right\} + \frac{b(b+4)L^2}{4(b-2)^2} - M^2 + \frac{MLb}{(b-2)}\right] - \Lambda, \tag{82}$$

$$8\pi \rho = \frac{1}{E^2} \exp\left(\frac{2LX}{(2-b)}\right) \cos^{(2-b)/b}(\sqrt{\alpha}T) \times$$

$$\left[\frac{(3b-4)\alpha}{4b}\tan^2(\sqrt{\alpha}T) + \frac{b(4-3b)L^2}{4(b-2)^2} - M^2 + \frac{MLb}{(b-2)}\right] + \Lambda. \tag{83}$$

The non-vanishing component $F_{12}$ of electromagnetic field tensor is given by

$$F_{12} = \sqrt{\frac{\bar{\mu}}{8\pi}\frac{2MLb}{(2-b)}} G \exp\left\{\left(\frac{Lb}{b-2} + 2M\right)\frac{X}{2}\right\} \cos(\sqrt{\alpha}T), \tag{84}$$

where $\bar{\mu}$ remains undetermined as function of $X$ and $T$.

The scalar of expansion $\theta$ calculated for the flow vector $v^i$ is given by

$$\theta = \frac{(2-3b)\sqrt{\alpha}}{2bE} \exp\left(\frac{LX}{(2-b)}\right) \cos^{(2-b)/(2b)}(\sqrt{\alpha}T) \tan(\sqrt{\alpha}T). \tag{85}$$

Now the shear scalar $\sigma^2$, acceleration vector $\dot{v}_i$ and proper volume $V^3$ are given by

$$\sigma^2 = \frac{\alpha}{3b^2E^2} \exp\left(\frac{2LX}{(2-b)}\right) \cos^{(2-b)/b}(\sqrt{\alpha}T) \tan^2(\sqrt{\alpha}T), \tag{86}$$

$$\dot{v}_i = \left(\frac{L}{(b-2)}, 0, 0, 0\right), \tag{87}$$

$$V^3 = \sqrt{-g} = E^2 \exp\left(\frac{(b+2)LX}{(b-2)}\right) \cos^{2(b-1)/b}(\sqrt{\alpha}T). \tag{88}$$

From Eqs. (85) and (86), we have

$$\frac{\sigma^2}{\theta^2} = \frac{4}{3(3b-2)^2} = const. \tag{89}$$

The rotation $\omega$ is identically zero.

The dominant energy conditions given by Hawking and Ellis [40],

$$(i) \quad \rho - p \geq 0\,,$$

$$(ii) \quad \rho + p \geq 0\,,$$

lead to

$$2\Lambda E^2 \cos^{(b-2)/b}(\sqrt{\alpha}T) \geq \exp\frac{2LX}{(2-b)} \left[\frac{b^2L^2}{(b-2)^2} + \alpha\right], \tag{90}$$

and

$$\alpha\left[\frac{(3b-4)}{2b}\tan^2(\sqrt{\alpha}T) + 1\right] + \frac{b(4-b)L^2}{2(b-2)^2} + \frac{2MLb}{(b-2)} - 2M^2 \geq 0. \tag{91}$$

The reality conditions given by Ellis [41],

$$(i) \quad \rho + p > 0\,,$$

$$(ii) \quad \rho + 3p > 0\,,$$

lead to

$$\alpha\left[\frac{(3b-4)}{2b}\tan^2(\sqrt{\alpha}T)+1\right]+\frac{b(4-b)L^2}{2(b-2)^2}+\frac{2MLb}{(b-2)}-2M^2>0. \tag{92}$$

and

$$\exp\left(\frac{2LX}{2-b}\right)\left[\frac{(3b-4)\alpha}{b}\tan^2(\sqrt{\alpha}T)+3\alpha+\frac{4bL^2}{(b-2)^2}-4M^2+\frac{4MLb}{(b-2)}\right]$$
$$>2\Lambda E^2\cos^{(b-2)/b}(\sqrt{\alpha}T). \tag{93}$$

For $2/b > 3$, the model (81) starts expanding at $T = 0$ and attains its maximum value at $T = \pi/(4\sqrt{\alpha})$. After that $\theta$ decreases to attain its minimum negative value at $T = 3\pi/(4\sqrt{\alpha})$. The model oscillates with the period $\pi/(2\sqrt{\alpha})$. The model is shearing and non-rotating. Since $\sigma/\theta = const.$, the model does not approach isotropy.

## 4. *Solutions in absence of magnetic field*

We consider the following three cases:

### 4.1. *Case(i): $ab < 0$*

When $L = 0$ and $M = 0$, we observe that the magnetic field in the model (39) vanishes and the geometry of the spacetime takes the form

$$ds^2 = E^2\cosh^{(b-2)/b}(\sqrt{|\alpha|}T)(dX^2-dT^2)+$$
$$\cosh(\sqrt{|\alpha|}T)dY^2+\cosh(\sqrt{|\alpha|}T)dZ^2, \tag{94}$$

The expressions for the pressure $p$ and density $\rho$ for model (94) are given by

$$8\pi p=\frac{1}{E^2}\cosh^{(2-b)/b}(\sqrt{|\alpha|}T)\left[|\alpha|\left\{\frac{(b+1)}{4b}\tanh^2(\sqrt{|\alpha|}T)-1\right\}\right]-\Lambda, \tag{95}$$

$$8\pi\rho=\frac{1}{E^2}\cosh^{(2-b)/b}(\sqrt{|\alpha|}T)\left[\frac{|\alpha|(3b-4)}{4b}\tanh^2(\sqrt{|\alpha|}T)\right]+\Lambda. \tag{96}$$

The scalar of expansion $\theta$ calculated for the flow vector $v^i$ is given by

$$\theta=\frac{(3b-2)\sqrt{|\alpha|}}{2bE}\cosh^{(2-b)/(2b)}(\sqrt{|\alpha|}T)\tanh(\sqrt{|\alpha|}T). \tag{97}$$

Now the shear scalar $\sigma^2$, acceleration vector $\dot{v}_i$ and proper volume $V^3$ are given by

$$\sigma^2 = \frac{\mid \alpha \mid}{3b^2 E^2} \cosh^{(2-b)/b}(\sqrt{\mid \alpha \mid} T) \tanh^2(\sqrt{\mid \alpha \mid} T), \tag{98}$$

$$\dot{v}_i = (0,0,0,0)\,, \tag{99}$$

$$V^3 = \sqrt{-g} = E^2 \cosh^{2(b-1)/b}(\sqrt{\mid \alpha \mid} T). \tag{100}$$

From Eqs. (97) and (98), we have

$$\frac{\sigma^2}{\theta^2} = \frac{4}{3(3b-2)^2} = const. \tag{101}$$

The rotation $\omega$ is identically zero.

The dominant energy conditions given by Hawking and Ellis [40],

$$(i) \quad \rho - p \geq 0\,,$$

$$(ii) \quad \rho + p \geq 0\,,$$

lead to

$$\left[\frac{(2b-5)\mid \alpha \mid}{4b} \tanh^2(\sqrt{\mid \alpha \mid} T) + \mid \alpha \mid\right] + 2\Lambda E^2 \cosh^{(b-2)/b}(\sqrt{\mid \alpha \mid} T) \geq 0 \tag{102}$$

and

$$\tanh^2(\sqrt{\mid \alpha \mid} T) \geq \frac{4b}{(4b-3)}. \tag{103}$$

The reality conditions given by Ellis [41],

$$(i) \quad \rho + p > 0\,,$$

$$(ii) \quad \rho + 3p > 0\,,$$

lead to

$$\tanh^2(\sqrt{\mid \alpha \mid} T) > \frac{4b}{(4b-3)}, \tag{104}$$

and

$$\left[\frac{(6b-1)\mid \alpha \mid}{4b} \tanh^2(\sqrt{\mid \alpha \mid} T) - \mid \alpha \mid\right] > 2\Lambda E^2 \cosh^{(b-2)/b}(\sqrt{\mid \alpha \mid} T). \tag{105}$$

### 4.2. *Case(ii): $ab = 0$*

In the absence of magnetic field, the geometry of the spacetime of the model (60) takes the form

$$\mathrm{d}s^2 = E_0^2 (c_1 T)^{(b-2)/b} (\mathrm{d}X^2 - \mathrm{d}T^2) + (c_1 T) \left(\mathrm{d}Y^2 + \mathrm{d}Z^2\right). \tag{106}$$

The expressions for pressure $p$ and density $\rho$ for model (106) are given by

$$8\pi p = \frac{1}{E_0^2} (c_1 T)^{(2-b)/b} \left[ \frac{1}{4T^2} + \frac{(b-2)}{2b} \right] - \Lambda, \tag{107}$$

$$8\pi \rho = \frac{1}{E_0^2} (c_1 T)^{(2-b)/b} \left[ \frac{(3b-4)}{4b} \frac{1}{T^2} \right] + \Lambda. \tag{108}$$

The scalar of expansion $\theta$ calculated for the flow vector $v^i$ is given by

$$\theta = \frac{c_2(3b-2)}{2bE_0} T^{(2-3b)/(2b)}. \tag{109}$$

Now the shear scalar $\sigma^2$, acceleration vector $\dot{v}_i$ and proper volume $V^3$ are given by

$$\sigma^2 = \frac{c_2^2}{3b^2 E_0^2} T^{(2-3b)/(2b)}, \tag{110}$$

$$\dot{v}_i = (0, 0, 0, 0)\,, \tag{111}$$

$$V^3 = \sqrt{-g} = E_0^2 (c_1 T)^{2(b-1)/b}. \tag{112}$$

From Eqs. (109) and (110), we have

$$\frac{\sigma^2}{\theta^2} = \frac{4}{3(3b-2)^2} = const. \tag{113}$$

The rotation $\omega$ is identically zero.

The dominant energy conditions given by Hawking and Ellis [40],

$$(i) \quad \rho - p \geq 0$$

$$(ii) \quad \rho + p \geq 0$$

lead to

$$\left[ \frac{(b-2)}{2b} \frac{1}{T^2} + \frac{b(3-2b)L^2}{(b-2)^2} + \frac{(2-b)}{2b} \right] + 2\Lambda E_0^2 (c_1 T)^{(b-2)/b} \geq 0, \tag{114}$$

and

$$\frac{(2-b)(b-1)}{b(b^2-2b+2)} \geq T^2. \tag{115}$$

The reality conditions given by Ellis [41],

$$(i) \quad \rho + p > 0\,,$$

$$(ii) \quad \rho + 3p > 0\,,$$

lead to

$$\frac{(2-b)(b-1)}{b(b^2-2b+2)} > T^2, \tag{116}$$

and

$$\left[\frac{(3b-2)}{2T^2} + \frac{3(b-2)}{2b} + \frac{b(3b-5)L^2}{(b-2)^2}\right] > 2\Lambda E_0^2 (c_1 T)^{(b-2)/b}. \tag{117}$$

## *4.3. Case(iii): ab > 0*

In the absence of the magnetic field, the geometry of the spacetime for the model (81) takes the form

$$\mathrm{d}s^2 = E^2 \cos^{(b-2)/b}(\sqrt{\alpha}\,T)(\mathrm{d}X^2 - \mathrm{d}T^2) +$$

$$\cos(\sqrt{\alpha}T)\left(\mathrm{d}Y^2 + \mathrm{d}Z^2\right). \tag{118}$$

The expressions for the pressure $p$ and density $\rho$ for model (118) are given by

$$8\pi p = \frac{1}{E^2}\cos^{(2-b)/b}(\sqrt{\alpha}T)\left[\alpha\left\{\frac{(3b-4)}{4b}\tan^2(\sqrt{\alpha}T) + 1\right\}\right] - \Lambda, \tag{119}$$

$$8\pi\rho = \frac{1}{E^2}\cos^{(2-b)/b}(\sqrt{\alpha}T)\left[\frac{(3b-4)\alpha}{4b}\tan^2(\sqrt{\alpha}T)\right] + \Lambda. \tag{120}$$

The scalar of expansion $\theta$ calculated for the flow vector $v^i$ is given by

$$\theta = \frac{(2-3b)\sqrt{\alpha}}{2bE}\cos^{(2-b)/(2b)}(\sqrt{\alpha}T)\tan(\sqrt{\alpha}T). \tag{121}$$

Now the shear scalar $\sigma^2$, acceleration vector $\dot{v}_i$ and proper volume $V^3$ are given by

$$\sigma^2 = \frac{\alpha}{3b^2E^2}\cos^{(2-b)/b}(\sqrt{\alpha}T)\tan^2(\sqrt{\alpha}T), \tag{122}$$

$$\dot{v}_i = (0,0,0,0)\,, \tag{123}$$

$$V^3 = \sqrt{-g} = E^2 \cos^{2(b-1)/b}(\sqrt{\alpha}T). \tag{124}$$

From Eqs. (121) and (122), we have

$$\frac{\sigma^2}{\theta^2} = \frac{4}{3(3b-2)^2} = const. \tag{125}$$

The rotation $\omega$ is identically zero.

The dominant energy conditions given by Hawking and Ellis [40],

$$(i) \quad \rho - p \geq 0\,,$$

$$(ii) \quad \rho + p \geq 0\,,$$

lead to

$$2\Lambda E^2 \cos^{(b-2)/b}(\sqrt{\alpha}T) \geq \alpha, \tag{126}$$

and

$$\tan^2(\sqrt{\alpha}T) \geq \frac{2b}{(4-3b)}. \tag{127}$$

The reality conditions given by Ellis [41],

$$(i) \quad \rho + p > 0\,,$$

$$(ii) \quad \rho + 3p > 0\,,$$

lead to

$$\tan^2(\sqrt{\alpha}T) > \frac{2b}{(4-3b)}, \tag{128}$$

and

$$\cos^{(2-b)/b}(\sqrt{\alpha}T)\left[3b\sec^2(\sqrt{\alpha}T) - 4\tan^2(\sqrt{\alpha}T)\right] > \frac{2b\Lambda E^2}{\alpha}. \tag{129}$$

## 5. *Conclusion*

We have obtained a new class of plane-symmetric inhomogeneous cosmological models of electromagnetic perfect fluid as the source of matter. Generally, the model represents expanding, shearing, non-rotating and Petrov type-II non-degenerate universe in which the flow vector is geodetic. In the model (39), we observe that the expansion starts at $T = 0$ and it continues till $T = \infty$. In the model (60), we also observe the similar behaviour when $2/b > 3$ but in the case when $2/b < 3$, we obtain singularity at $T = 0$. The model (81) is oscillatory. In all models $\frac{\sigma}{\theta} = const.$, and hence they do not approach isotropy.

It is worth mentioning here that in presence of magnetic field, all models are inhomogeneous, whereas in the absence of magnetic field they become homogeneous.

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References

[1] R. C. Tolman, Proc. Nat. Acad. Sci. **20** (1934) 169.

[2] H. Bondi, Mon. Not. R. Astro. Soc. **107** (1947) 410.

[3] A. H. Taub, Ann. Math. **53** (1951) 472.

[4] A. H. Taub, Phys. Rev. **103** (1956) 454.

[5] N. Tomimura, II Nuovo Cimento B **44** (1978) 372.

[6] P. Szekeres, Commun. Math. Phys. **41** (1975) 55.

[7] C. B. Collins and D. A. Szafron, J. Math. Phys. **20** (1979a) 2347; J. Math. Phys. **20** (1979b) 2362.

[8] D. A. Szafron and C. B. Collins, J. Math. Phy. **20** (1979) 2354.

[9] J. M. M. Senovilla, Phys. Rev. Lett. **64** (1990) 2219.

[10] E. Ruiz and J. M. M. Senovilla, Phys. Rev. D **45** (1990) 1995.

[11] N. Dadhich, R. Tikekar and L. K. Patel, Curr. Sci. **65** (1993) 694.

[12] L. K. Patel, R. Tikekar and N. Dadhich, Pramana-J. Phys. **49** (1993) 213.

[13] G. Singh, P. Mehta and S. Gupta, Astrophys. Space Sc. **281** (1990) 677.

[14] A. Pradhan, V. K. Yadav and N. N. Saste, Int. J. Mod. Phys. D **11** (2002) 857;
A. Pradhan and H. R. Pandey, Int. J. Mod. Phys. D **12** (2003) 941;
A. Pradhan and P. Pandey, Czech. J. Phys. **55** (2005) 749;
A. Pradhan, K. D. Thengane and J. K. Jumale, Spacetime & Substance **4(29)** (2005) 145;
A. Pradhan, P. Pandey and S. K. Singh, Int. J. Theor. Phys. **46** (2007) 1584;
A. Pradhan, P. Pandey, K. Jotania and M. K. Yadav, Int. J. Theor. Phys. **46** (2007) 2774.

[15] Ya. B. Zeldovich, A. A. Ruzmainkin and D. D. Sokoloff, *Magnetic field in Astrophysics*, Gordon and Breach, New York (1993).

[16] E. R. Harrison, Phys. Rev. Lett. **30** (1973) 188.

[17] H. P. Robertson and A. G. Walker, Proc. London Math. Soc. **42** (1936) 90.

[18] C. W. Misner, K. S. Thorne and J. A. Wheeler, *Gravitation*, W. H. Freeman, New York (1973).

[19] E. Asseo and H. Sol, Phys. Rep. **6** (1987) 148.

[20] M. A. Melvin, Ann. New York Acad. Sci. **262** (1975) 253.

[21] R. Pudritz and J. Silk, Astrophys. J. **342** (1989) 650.

[22] K. T. Kim, P. G. Tribble and P. P. Kronberg, Astrophys. J. **379** (1991) 80.

[23] R. Perley and G. Taylor, Astrophys. J. **101** (1991) 1623.

[24] P. P. Kronberg, J. J. Perry and E. L. Zukowski, Astrophys. J. **387** (1991) 528.

[25] A. M. Wolfe, K. Lanzetta and A. L. Oren, Astrophys. J. **388** (1992) 17.

[26] R. Kulsrud, R. Cen, J. P. Ostriker and D. Ryu, Astrophys. J. **380** (1997) 481.

[27] E. G. Zweibel and C. Heiles, Nature **385** (1997) 131.

[28] J. D. Barrow, Phys. Rev. D **55** (1997) 7451.

[29] Ya. B. Zeldovich, Sov. Astron. **13** (1970) 608.

[30] M. S. Turner and L. M. Widrow, Phys. Rev. D **30** (1988) 2743.

[31] J. Quashnock, A. Loeb and D. N. Spergel, Astrophys. J. **344** (1989) L49.

[32] B. Ratra, Astrophys. J. **391** (1992) L1.

[33] A. D. Dolgov and J. Silk, Phys. Rev. D **47** (1993) 3144.

[34] A. D. Dolgov, Phys. Rev. D **48** (1993) 2499.

[35] R. Bali and M. Ali, Pramana-J. Phys. **47** (1996) 25.

[36] I. Chakrabarty, A. Pradhan and N. N. Saste, Int. J. Mod. Phys. D **5** (2001) 741;
A. Pradhan and O. P. Pandey, Int. J. Mod. Phys. D **7** (2003) 1299;
A. Pradhan, S. K. Srivastav and K. R. Jotania, Czech. J. Phys. **54** (2004) 255;
A. Pradhan and S. K. Singh, Int. J. Mod. Phys. D **13** (2004) 503;
A. Pradhan, P. Pandey and K. K. Rai, Czech. J. Phys. **56** (2006) 303.

[37] R. Bali and A. Tyagi, Astrophys. Space Sci. **173** (1990) 233.

[38] A. Lichnerowicz, *Relativistic Hydrodynamics and Magnetohydrodynamics*, W. A. Benjamin. Inc. New York, Amsterdam (1967) p. 93.

[39] J. L. Synge, *Relativity: The General Theory*, North-Holland Publ., Amsterdam (1960) p. 356.

[40] S. W. Hawking and G. F. R. Ellis, *The Large-scale Structure of Space Time*, Cambridge University Press, Cambridge (1973) p. 94.

[41] G. F. R. Ellis, *General Relativity and Cosmology*, ed. R. K. Sachs Clarendon Press (1973) p. 117.

## NOVA KLASA RAVNINSKI-SIMETRIČNIH NEHOMOGENIH KOZMOLOŠKIH MODELA S PERFEKTNOM RASPODJELOM TEKUĆINE I ELEKTROMAGNETSKIM POLJEM

Izveli smo novu klasu ravninski-simetričnih nehomogenih kozmoloških modela s perfektnom raspodjelom tekućine i elektromagnetskim poljem. Izvor magnetskih polja je električna struja u smjeru $z$-osi. Različita od nule je jedino komponenta tenzora elektromagnetskog polja $F_{12}$. Pretpostavili smo nedegenerirano slobodno gravitacijsko polje tipa Petrov-II. Razmatrali smo tri slučaja: (i) hiperbolni kosinus, (ii) linearni i (iii) kosinusni oblik. Raspravljamo također neka geometrijska i fizička svojstva modela u prisustvu i bez magnetskog polja.