Economic Research-Ekonomska Istraživanja

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/rero20

# On Jain's digital piracy model: horizontal vs vertical product differentiation

Michael Kúnin & Krešimir Žigić

**To cite this article:** Michael Kúnin & Krešimir Žigić (2023) On Jain's digital piracy model: horizontal vs vertical product differentiation, Economic Research-Ekonomska Istraživanja, 36:1, 1299-1316, DOI: 10.1080/1331677X.2022.2086149

**To link to this article:** https://doi.org/10.1080/1331677X.2022.2086149

© 2022 The Author(s). Published by Informa UK Limited, trading as Taylor & Francis Group.

Published online: 20 Jun 2022.

Submit your article to this journal

Article views: 448

View related articles

View Crossmark data

Citing articles: 1 View citing articles

Full Terms & Conditions of access and use can be found at
https://www.tandfonline.com/action/journalInformation?journalCode=rero20

ECONOMIC RESEARCH-EKONOMSKA ISTRAŽIVANJA
2023, VOL. 36, NO. 1, 1299–1316
https://doi.org/10.1080/1331677X.2022.2086149

Routledge
Taylor & Francis Group

OPEN ACCESS  Check for updates

# On Jain's digital piracy model: horizontal vs vertical product differentiation

Michael Kúnin and Krešimir Žigić

CERGE-EI, a joint workplace of the Center for Economic Research and Graduate Education, and the Economics Institute of Academy of Sciences of the Czech Republic, Charles University, Prague, Czech Republic

**ABSTRACT**
We study how private intellectual property rights protection affects equilibrium prices in a duopoly competition between firms that offer a product variety of distinct qualities (vertical product differentiation) in a setup that is closely related to that put forward by Jain where firms offer the same qualities in equilibrium (horizontal product differentiation). Consumers may make a choice to buy a legal version, use an illegal copy (if they want to and can), or not use a product at all. Using an illegal version violates intellectual property rights protection and is thus punishable when discovered. Thus, both private and public (copyright) intellectual property rights protection are available on scene.

**ARTICLE HISTORY**
Received 26 November 2021
Accepted 24 May 2022

**KEYWORDS**
Vertical and horizontal product differentiation; Software Piracy; Bertrand competition; Private and public intellectual property rights protection

**JEL CLASSIFICATION**
D43; L11; L21; O25; O34

## 1. Introduction

In an innovative and insightful model, Jain (2008), addresses the issue of digital piracy by end-users and shows that under some plausible conditions firms (providers of digital content) may be better off by not using private intellectual property rights (IPR) protection against piracy. Moreover, he shows that this might be the case even in the absence of any network externalities given that the presence of the latter has often been used as an argument for a lax approach to digital piracy. An important result of his model is the fact that an increase in a firm's IPR protection always leads to a decline in the equilibrium prices, and, consequently, may result in a fall in a firm's profit.

Jain (2008) uses the Hotelling model of horizontal product differentiation with two firms exogenously located at the end of the Hotelling line and explores a three-stage game. The firms choose the quality of their product in the first stage, strength of private IPR protection in the second stage, and finally, compete in prices in the last stage. Moreover, Jain (2008) assumes that there are two segments of consumers: those who potentially infringe IPR and illegally copy the software ('copier' segment) and

CONTACT Krešimir Žigić kresimir.zigic@cerge-ei.cz

© 2022 The Author(s). Published by Informa UK Limited, trading as Taylor & Francis Group.
This is an Open Access article distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/4.0/), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

those who never undertake any illegal action of software piracy ('non-copier' segment). Finally, the consumers in the 'copier' segment are assumed to be more price sensitive. Since the focus of our analysis is on IPR protection and its impact on prices, we focus on the last stage in the game and take the outcome of the first two stages as given. We build a similar model that departs from Jain's (2008) approach in two important respects. First, we replace the horizontal product differentiation setup with a vertical one (see Žigić et al., 2015, for examples of vertical product differentiation in software markets) in which two software producing firms offer low and high quality software, respectively; second, in addition to a firm's own (or private) IPR protection, we also add public protection, namely, copyright. We then study how robust Jain's (2008) findings on the impact of private IPR protection on firms' prices are in this new setup given that it is of utter importance for a firm to learn how its decision on protection of their own product affects its resulting price and also indicates in which direction the demand and the profits will change. In particular, we show that the impact of private protection on the equilibrium price in our broader setup is more complex. Unlike in Jain (2008), where an increase in private protection always negatively affects the equilibrium price due to the 'price sensitivity effect' of the 'copier' segment, it would not be necessary the case in our setup due to the presence of the other effect ('consumer base effect') not present in the Jain's (2008) model. More specifically, we show that with the copier segment 'large enough' and not 'overly' price sensitive (to discourage firms to use private IPR protection), the equilibrium prices and the profits of firms show in general non-monotonic behaviour in private IPR protection. Thus, it could be easily the case that an increase in private protection softens price competition due to the larger consumer base. Even more strikingly, it would be possible that in our setup an increase in private protection positively affects the price of low quality software but negatively the price of high quality software in the equilibrium. Such effects are not present in Jain's setup.

## 2. Model set-up

*Developers* $A$ and $B$ compete in prices on a particular market and offer product varieties of different quality. Developer $A$ releases a product of quality normalised to $q_A = 1$, while the quality of developer $B$ is $q_B = q$ and we assume, without loss of generality, that developer $A$ offers the higher quality ($q < 1$). The product qualities are exogenous and cannot be changed by the developers, and the unit variable costs are constant and normalised to zero. One may think about developer $A$ as an already established and known software producer that already operates on other markets. This fact is, in turn, reflected in the preferences of the consumers, who strictly prefer software $A$ over software $B$ if they are offered at the same price. Similarly, developer $B$ can be thought of as a local developer offering lower quality. In other words, we assume that both developers already existed before meeting and competing on the market under consideration. Consequently, both developers are assumed to have already incurred set-up fixed costs and fixed costs associated with software development (R&D costs).[1] These fixed costs are, from our perspective, general and not related to the developer's presence on the particular market under consideration, and we therefore leave them out of the profit function. The probability of being caught using an illegal version is the same for all users, and the level of the penalty is fixed. The penalty and the probability of being caught is known and independent of used product and product prices; thus all users and both developers could calculate the expected penalty for using an illegal version, which we denote as $X$. Moreover, while we implicitly assume that the regulator choice of optimal IPR is governed by an underlying objective function such as the maximisation of social welfare, we do not explicitly study the optimal choice of expected penalty since we focus on the forms of the developers' pricing and IPR protection strategies and their economic implications. Thus, the whole regulator's framework is very simple in our model and translates into one parameter: expected penalty $X$ for illegal users, which also captures the strength of the copyright protection (see Varian's, 2005, survey on the economics of copyrights).

While in principle both *developers* could have access to technology that allows product protection against copying and illegal usage,[2] we assume that only a high-quality developer may adopt the protection and this decision is dependent only on the profitability of such a step. A reason for this simplifying assumption could be that hardware protection is not available or too costly for a low quality developer, or that the level of public IPR protection is such that it would never be optimal for developer $B$ to adopt protection.

The protection against copying is imperfect, which means that a fraction of the users still has access to the illegal version. Much like in Jain (2008), we say that a developer implements protection at level $c \in [0, 1]$, whereby the level of $c$ represents the fraction of consumers 'controlled' by the high quality developer, that is, the share of consumers who are unable to use the software illegally due to the private IPR protection. (In Jain's, 2008 notation $c = 1 - \alpha$). If $c$ tends to 1 we say that protection becomes perfect and all end users are controlled, while $c$ tending to 0 represents the full public availability of an illegal version.[3] As noted in the Introduction, we treat $c$ as a given parameter.

Regarding the developers' cost of incurring protection, Jain (2008) does not give it much attention, since it would not qualitatively change his results (more specifically, adding these costs would only reinforce his findings). While these costs are also not essential for the main argument and the focus of our analysis, we still consider them important for understanding why only the high quality developer incurs these costs. This is because, given the equilibria we focus on, there is no need for a low quality developer to undertake such costs, since the public protection is high enough to protect developer $B$ from piracy.[4] In addition, the private protection of developer $A$, for whom it is optimal to make private IPR protection also enables developer $B$ to free ride on this protection.

As already indicated in the Introduction, there are two segments of users, and in each segment consumers differ in their quality sensitivity $\theta$, which has density 1 on $[0, 1]$. The first segment are the potential copiers ('copier' segment) and the second segment are consumers who never opt for an illegal version of software ('non-copier' segment). Regarding the first segment, these are consumers who are willing to copy if they were in a position to do it and, as in Jain (2008), the size of this segment is $\beta$ (which can be bigger or smaller than one). The empirical finding shows that the users in this segment are more price sensitive and have lower willingness to pay (see Cheng et al., 1997) than the consumers in the second segment; so, following Jain (2008), to account for this fact we introduce a discount factor $0<\delta<1$ for this segment. Due to the private IPR protection only some of those users have access to both a legal and illegal version, while some users have access only to a legal version. The users with access to both versions prefer the legal version only if the utility from it is higher and their proportion is $1-c$. The utility function of a user $\theta$ in the first segment is as follows:

$$U_P(\theta) = \begin{cases} \delta\theta q_i - p_i & \text{...if he buys the legal version of the software} \\ \delta\theta q_i - X & \text{...if he uses the software illegally.} \\ 0 & \text{...if he does not use the software at all.} \end{cases} \tag{1}$$

We also assume that if the price of the legal version of a product exactly equals the expected punishment for using the illegal one, $p_i = X$, then the consumers strictly prefer the legal version—in other words, second-order stochastic dominance applies.

The utility of a 'non-copier' user $\theta$ is

$$U(\theta) = \begin{cases} \theta q_i - p_i & \text{...if he buys the software.} \\ 0 & \text{...if he does not use the software at all.} \end{cases} \tag{2}$$

In addition, note that developer $A$ does not need to implement private protection when its equilibrium price is below $X$. In other words, two cases are possible.

1. Developer $A$ does not implement protection. This situation arises when $X$ does not bind in the maximisation problems of either $A$ or $B$, so that in equilibrium we have $p_B^* \leq p_A^* \leq X$.
2. Developer $A$ implements protection. This situation occurs when pure Bertrand equilibrium is not possible because $X$ would be binding for developer $A$ since $p_B^* \leq X < p_A^*$.

We focus on the case where developer $A$ has the incentive to introduce protection, that is, $p_B^* \leq X < p_A^*$. This case seems to be relevant for middle and, perhaps, some high per capita income countries, while the situation associated with zero or very low effective strength of copyright protection is typical in developing countries (see Fig. 1 in Varian, 2005). Note that in our set-up, prices are, as is typical, strategic complements (see Bulow et al., 1985; Tirole, 1988), that is, $\frac{\partial^2 \pi_i}{\partial p_B \partial p_A} > 0$.

## 3. Demand function

Before we start with solving the duopoly model backward, we have to work out the demand functions in the potential copier segment that could emerge in the setup

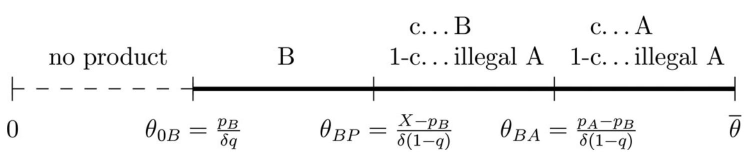


**Figure 1.** Developer $A$ introduces protection $c$ (Case 1).
Source: Authors illustration.

under consideration. In the case where $p_B^* \leq X < p_A^*$, only developer $A$ has the incentive to implement protection since the product of developer $B$ would only be used legally. As we already mentioned in our model set-up, the illegal version of product $A$ is available only to the fraction $1-c$ of the users' base in a copier segment. Product $A$ is used illegally only by users with $\frac{X}{\delta} \leq \theta$, while users with $\theta \leq \frac{X}{\delta}$ prefer not to use the product at all. The demand for product $B$ consists of users with low sensitivity $\theta$ to purchasing product $A$, who, at the same time, have no access to an illegal version of $A$, but their $\theta$ is high enough to buy product $B$. These users have $\theta \in (\frac{p_B}{\delta q}, \frac{p_A - p_B}{\delta(1-q)})$, and their fraction is $c$. Regarding the users with access to an illegal version of product $A$, there are two main sub-cases that could occur in equilibrium depending on the size of the expected penalty:

1. The first sub-case occurs when there are some users who have illegal access to product $A$ but still want to buy product $B$, or more formally, the measure of these users is strictly positive with $\theta \in \left(\frac{p_B}{\delta q}, \frac{X-p_B}{\delta(1-q)}\right)$, and so, $\frac{X-p_B}{\delta(1-q)} > \frac{p_B}{\delta q}$. These users would like to purchase product $B$ if $X$ is 'large enough' (in the sense that $X > \frac{p_B}{q}$). Looking at it from the developers' point of view, developer $B$ competes for the consumers that have illegal access to software (so called 'non-controlled' consumers) by aggressively charging a low price so that $p_B^* < qX$. The market coverage is given in Figure 1.
2. The second sub-case occurs when illegal users always prefer an illegal version of $A$ to the legal version of $B$, that is, when $\delta\theta - X > \delta\theta q - p_B$ for all $\theta$ since illegal usage is then more valuable even for the consumer with the lowest valuation. So, $X$ has to be 'low' enough, that is, $\frac{X-p_B}{\delta(1-q)} \leq \frac{p_B}{\delta q}$ (or equivalently $X \leq \frac{p_B}{q}$) given that $p_B^* \leq X$ still holds. From the perspective of the developers, developer $B$'s price is 'too high' to attract the non-controlled consumers and in this situation his profit fully depends on the protection of developer $A$. The market coverage of this case is presented in Figure 2.

Given that the products' demands on the 'non-copier' segment is straightforward (that is, $\left(1 - \frac{p_A - p_B}{1-q}\right)$ for $A$ and $\left(\frac{p_A - p_B}{1-q} - \frac{p_B}{q}\right)$ for $B$), we obtain the total demand for legal versions of both products on both segments by putting all fractions of users together (Subcase 1):

$$D_A = \beta c\left(1 - \frac{p_A - p_B}{(1-q)\delta}\right) + \left(1 - \frac{p_A - p_B}{1-q}\right) \tag{3}$$

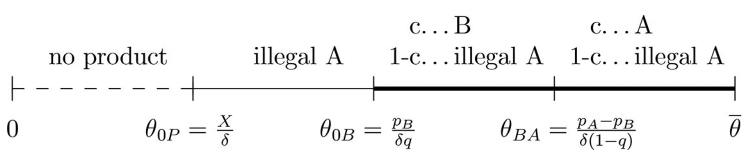


**Figure 2.** Developer $A$ introduces protection $c$ (Case 2).
Source: Authors illustration.

$$D_B = \left(1+\frac{\beta c}{\delta}\right)\left(\frac{p_A - p_B}{1-q} - \frac{p_B}{q}\right) + \frac{\beta(1-c)}{\delta}\left(\frac{X-p_B}{1-q} - \frac{p_B}{q}\right) \tag{4}$$

If only the users without access to an illegal version of $A$ buy product $B$, the demand function for developer $B$ is now (Subcase 2):

$$D_B = \left(\frac{p_A - p_B}{1-q} - \frac{p_B}{q}\right)\left(1+\frac{\beta c}{\delta}\right).$$

## 4. Solving the model—equilibrium analysis

### *4.1. Types of equilibria*

As the developers choose their prices to maximise profits, the following can be shown to hold.

**Lemma 1.** *Each developer can choose its price in a way that obtains a positive profit whatever the other developer's price is.*

**Lemma 2.** *In any equilibrium, the 'non-copier' segment of the market shares of both developers are strictly positive.*

The proofs of these Lemmas are in the Appendix.

These results mean that the equilibrium market structures are exclusively determined by what happens in the 'copier' segment. While there are several types of equilibria, including those wherein one or both developers do not enter the 'copier' segment, we like Jain (2008) concentrate on the equilibrium structures that satisfy the following two conditions.

- Both developers enter the 'copier' segment, and
- Private protection is used, which implies $p_A^* > X$.

There are three qualitatively different equilibrium outcomes that satisfy those conditions.

1. The equilibria with $p_B^* \leq Xq$, which we call *'no full dependence'* equilibria since developer $B$ competes, on the one hand, with developer $A$'s legal product in both the 'non-copier' segment and in the controlled part of the 'copier' segment, and, on the other hand, with the illegal product in the uncontrolled part of the 'copier' segment.
2. The equilibria with $Xq < p_B^* < X$, which we call *'interior full dependence'* equilibria since developer $B$ only competes in the controlled part of the 'copier' segment (as well as in the 'non-copier' segment) so that developer $B$'s consumer base in the 'copier' segment fully depends on developer $A$'s private protection. In addition, developer $B$'s profit reaches an interior maximum.
3. The equilibria with $p_B^* = X$, which we call *'corner full dependence'* equilibria which differ from the previous case in that developer $B$'s profit is maximised at

$p_B = X$, which is the maximum possible price for the consumers in the 'copier' segment to buy product $B$.

Our main focus will be on the 'no full dependence' equilibrium as the only equilibrium in which the impacts of private IPR protections on the equilibrium prices are qualitatively different than the ones in Jain (2008).

## 4.2. Deviation to not serving the 'copier' segment

Before moving to the analysis of the equilibria, we have to check the conditions that none of the developers deviate to serving only 'non-copier' segment. Jain (2008) shows that in his model the condition $\delta \geq \frac{1}{2+\beta}$ is sufficient for not deviating to but only serving the non-copier segment. It turns out that in our model the following holds for developer $A$.

**Proposition 1.** *Let $(p_A, p_B)$ be an equilibrium candidate with piracy present such that both developers serve both consumer segments while maximising their profits taking the other price as given. Then $\delta \geq \frac{1}{2+c\beta}$ is a sufficient condition for developer $A$ to serve both consumer segments rather than the 'non-copier' segment alone.*

**Proof.** First note that any equilibrium candidate with piracy present is such that developer $A$'s profit reaches an interior maximum.

Given $p_B$, developer $A$'s reaction function and profit are

$$p_A = \frac{1}{2}\left(p_B + \frac{(1+c\beta)\delta}{c\beta+\delta}(1-q)\right), \Pi_A = \frac{\left((c\beta+\delta)p_B + (1+c\beta)\delta(1-q)\right)^2}{4\delta(c\beta+\delta)(1-q)}$$

when both consumer segments are served and

$$p_A = \frac{1}{2}(p_B + (1-q)), \Pi_A^0 = \frac{(p_B + (1-q))^2}{4(1-q)}$$

when only the 'non-copier' segment is served. Then

$$\Pi_A - \Pi_A^0 = \frac{c\beta(1-q)}{4(c\beta+\delta)}((2+c\beta)\delta - 1) + \frac{1}{2}c\beta p_B + \frac{c\beta}{4\delta(1-q)}p_B^2.$$

Here the terms containing $p_B$ are non-negative whereas the first term is non-negative iff $\delta \geq \frac{1}{2+c\beta}$, which completes the proof. □

As for developer $B$, the 'no full dependence' case is analysed in the Appendix and the 'interior full dependence' case is trivial as the profit function when serving both segments, which has an interior maximum given $p_A$, is exactly $(1 + c\beta/\delta)$ times the profit function when only serving the 'non-copier' segment. However, in the 'corner full dependence' case, i.e., when $p_B = X$, it is obvious that if $X$ is sufficiently low then developer $B$ would prefer to set a price $p_B > X$ and only serve the 'non-copier' segment.

### 4.3. The 'no full dependence' equilibrium

The piracy 'no full dependence' equilibrium, that is at the centre of our attention, occurs within the Subcase 1 presented above. Thus, we start with determining the range of the expected penalty values $X$ such that this sub-case is the Nash equilibrium in prices. Namely, Subcase 1 is an equilibrium if (i) each developer's profit, given the other developer's price choice, has a local maximum in the relevant price range. Also, it is an equilibrium if (ii) neither developer is better off deviating to a price outside the range (e.g., developer $A$ cannot be better off deviating to $p_A = X$). Finally, it is an equilibrium if (iii) developer $A$ does not deviate to not entering the 'copier' segment at all (see Proposition 1). Note that there is no deviation by developer $B$ to only serving the 'non-copier' segment if $c$ is not 'too low' as shown in the Appendix. More specifically, $c \geq 1/9$ is sufficient and the 'no full dependence' equilibrium typically does not occur at such low values of $c$—see the numerical example below.

Intuitively, for developer $A$ to charge a high price $p_A > X$, the value of $X$ should be small enough so that developer $A$ prefers introducing protection than simply lowering the price to $X$. For developer $B$ to charge a low price $p_B < Xq$, $X$ should be large enough so that developer $B$ prefers charging a low price to both charging an intermediate price $Xq \leq p_B \leq X$ or charging a high price $p_B > X$ and introducing protection. In addition, the 'copier' segment should be attractive enough in the sense of $\beta$ and $\delta$ being high enough.

If this equilibrium occurs, then the equilibrium prices are

$$p_A^* = \frac{\beta(c\beta+\delta)X(1-c)q + 2(1+c\beta)\delta(\beta+\delta)(1-q)}{(c\beta+\delta)\big(4(\beta+\delta)-(c\beta+\delta)q\big)},$$

$$p_B^* = q\frac{2\beta X(1-c) + \delta(1+c\beta)(1-q)}{4(\beta+\delta)-(c\beta+\delta)q}.$$

**Proposition 2.** *There is a non-empty range of the model parameters for the 'no full dependence' equilibrium to exist.*

**Proof.** See Appendix E. □

### 4.4. Equilibrium structures with 'full dependence'

As for the two 'full dependence' structures, the outcome is basically the same as in Jain (2008). Specifically, of the two effects we consider below, the consumer base effect and the price sensitivity effect, the latter always dominates under such equilibrium structures, so that an increase in private protection by developer $A$ results in a decrease in developer $A$'s price and, in the interior 'full dependence' case, in developer $B$'s price (recall that in the corner 'full dependence' case, $p_B = X$). See the Appendix for the formulae.

## 5. Vertical versus horizontal product differentiation

To make our comparison with Jain's (2008) model as insightful as possible, we, as claimed above, focus on the most interesting 'no full dependence' equilibrium.

Roughly speaking, this equilibrium occurs when the copier segment is large enough, price sensitivity not so high and the copyright protection is such that it pays off for developer $B$ to compete for the users who have access to the illegal version (i.e., $X$ is in the 'midrange' of permissible values; see the numerical example below). Following Jain (2008), we put the comparative statics analysis with respect to $c$ in the form of a Proposition:

**Proposition 3.** *In 'no full dependence' equilibria, the equilibrium prices $p_A^*(c)$, $p_B^*(c)$ show in general non-monotonic behaviour in private IPR protection c. Namely, they both increase in c when both $\delta$ and $\beta$ are high enough, i.e., when the 'copier' segment consumers are not very sensitive to prices and the 'copier' segment is relatively large enough. Then they both decrease in c when both $\delta$ and $\beta$ are low enough. As for the intermediate values, $p_A^*$ decreases in c whereas $p_B^*$ increases in c.*

The Proof can be obtained on request in the form of a Mathematica file.

The reason behind the above result is that there are two opposing effects at work at the copier segment of the market. First, strengthening of the private IPR protection by the developer $A$ enables him to broaden the base of his end users and thus to increase the price of his product. An increase in $A$'s protection, in turn, has also direct positive impact on firm $B's$ market share since $A$'s protection applies also on consumers with lower valuation who opt for the product $B$. Moreover, the competitive segment, $1-c$, on which developer $B$ competes for (potential) illegal users of product $A$, shrinks as $c$ increases, enabling developer $B$ to also increase his price. We name the above effect as the *consumer base effect*. In other words, in the absence of price sensitivity (i.e., $\delta = 1$) this would be the only effect. However, since $\delta<1$, there is also a second effect, which works in the opposite direction. Imposing private protection on the fraction of (potentially) copying consumers would tend to lower prices in equilibrium since any increase in the fraction of price sensitive consumers would, *ceteris paribus*, require a lower price in the absence of price discrimination. Clearly, if this *price sensitivity effect* is very strong then it dominates and equilibrium prices would be adversely affected by imposing IPR protection. In Jain (2008), however, the impact of private IPR protection on equilibrium price is always negative for any value of discounting factor lower than a unit (and zero for $\delta = 1$) and so the second effect (price sensitivity) dominates across all permissible values of $\delta$ and $\beta$.

There is also an intermediate range of the values of $\delta$ and $\beta$ when $p_A^*$ decreases in $c$ whereas $p_B^*$ increases in $c$. In this range, the price sensitivity effect dominates for the high-quality developer $A$ (whose prices are higher) whereas the consumer base effect dominates for the low-quality developer $B$ (whose prices are lower but this developer depends on the share of the 'controlled' consumers in the 'copier' segment).

In light of the above intuition for Proposition 3 it is also insightful to qualify our findings in relation to the size of the 'copier' segment (i.e., a change in $\beta$). If the 'copier' segment gets very large ($\beta$ tends to infinity), then only it matters, and so the significance of IPR protection (consumer base effect) is of critical importance and has an undoubtedly positive effect on equilibrium prices as long as $\delta \geq \frac{1}{2+c\beta}$. If, on the other hand, the price sensitivity effect is extremely strong ($\delta$ tends to zero) it might more than offset the first, positive, consumer base effect or, when $\delta < \frac{1}{2+c\beta}$, it may lead developer $A$ to abandon the 'copier' segment setting $c = 0$ and focus only on the more profitable 'non-copier' segment (in which there is no piracy). In other words, there are critical values of $\delta$ at which there is a switch of the sign of $dp^*/dc$ from negative to positive as $\delta$ moves from zero to one, that is, $\frac{\partial^2 p_A}{\partial c \partial \delta} > 0$. (It is straightforward to show that $dp_A/dc$ is positive for $\delta = 1$ and negative for $\delta = 0$, irrespective of the size of $\beta$).

**Lemma 3.** *The higher the size of the 'copier' market segment, the more important the consumer base effect is for developer A, given the size of the price sensitivity* $\delta \geq \frac{1}{2+c\beta}$. *Thus* $\frac{\partial^2 p_A}{\partial c \partial \beta} > 0$ *when* $\beta$ *is large enough.*

The Proof can be obtained under request in the form of a Mathematica file.

Regarding developer $B$, the effect of the 'copier' segment size on $\frac{dp_B}{dc}$ is generally ambiguous due to the competition from the illegal product which may or may not be offset by the consumer base effect.

Last but not least, unlike in Jain's (2008) symmetric model where the equilibrium price is the same for both developers (so the change in equilibrium prices due to a rise in IPR protection for both firms is the same and has always the same, negative, sign); in our setup, however, it is quite possible, as we claimed above, that an increase of private IPR protection would have an opposite impact on the equilibrium prices. In particular, it is quite conceivable that $\frac{dp_B}{dc} > 0$ while $\frac{dp_A}{dc} < 0$ when the price sensitivity is strong enough to more than offset the consumer base effect for firm $A$ but still not strong enough for the same effect for firm $B$.[5] The reason for this is that developer $B$ has higher benefits from this protection at the margin than developer $A$. Moreover, a marginal increase in $c$ is not associated with any marginal costs for developer $B$.

To conclude, there is in general a non-monotonic relationship between private IPR protection and equilibrium prices in our extended model.

### 5.1. Numerical example

In order to illustrate the key findings in Proposition 2 and other important comparative statistics results, we use the following parameter values: $q = 0.2$, $c = 0.9$, $\delta = 0.8$, $\beta = 10$. Under these values, developer $A$'s unconstrained duopoly price (which is also the value of $X$ above which an unconstrained duopoly occurs), is $\bar{X} = 0.343080$ (all numerical values are approximate). We show that the piracy 'no full dependence' equilibrium for these concrete values occurs in the range $0.167267 < X < 0.240178$ within the $0 \leq X \leq \bar{X}$ interval. At these parameter values $p_B$ increases in $c$ (so does $\pi_B^*$) in the entire 'no full dependence' range, whereas $p_A$ displays a non-monotonic behaviour (see Figure 3). It increases in $c$ when $X$ is low enough but falls in $c$ for larger values given the 'no full dependency' interval (which changes with the value of $c$).

In this figure, the top thin horizontal line stands for the pure duopoly price of developer $A$ in our numerical example, so for $X$ above this value the outcome is one of pure duopoly. The second uppermost and the lowermost curves are the maximum and the minimum levels of $X$, as functions of $c$, such that neither developer wants to deviate to another market structure (recall that $\delta > \frac{1}{2+c\beta}$ in this case so there is no

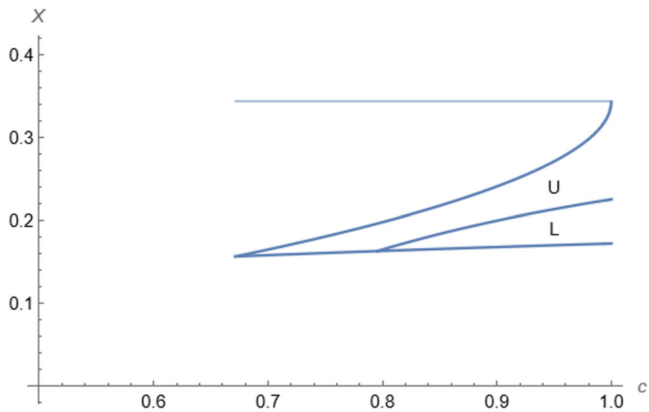


**Figure 3.** Ranges of $c$ and $X$ where there is a 'no full dependence' equilibrium: in the area $U$ the equilibrium price $p_A$ decreases in $c$, whereas in the area $L$ this price increases in $c$.
Source: Authors illustration.

deviation to not serving the 'copier' segment either). The areas labelled $U$ and $L$ correspond to the parameter ranges where $dp_A/dc<0$ and $dp_A/dc>0$ respectively.

As for the other interesting comparative statics results, we, much like in Jain (2008), find that $dp_i^*/d\beta<0$ while $dp_i^*/d\delta>0$ where $i=A,B$ under the 'no full dependence' equilibrium for all applicable $X$ and $c$ in the above example. Here an increase in $\beta$ means that mass of price sensitive consumers increases so the equilibrium prices fall (given the fact that $\delta<1$). Conversely, an increase in $\delta$ means that the price sensitivity effect weakens, so there is an increase of the equilibrium prices.

## 6. Conclusion

The main purpose of this paper is to study the impact of the firm's private IPR protection on the equilibrium pricing in a setup where there are two segments of consumers; the 'non-copier' segment, which never opts for piracy, and the 'copier' segment, which considers digital piracy as a potential option. An end-user of the 'copier' segment would use piracy i) if he is capable of circumventing the installation key (or other hardware protection) and ii) if this would be beneficial for the end user. Jain (2008) used the above setup in the symmetric horizontal differentiation duopoly model and shows that an increase in private IPR protection is always associated with a decrease in the equilibrium price, due to the existence of the (more) price sensitive 'copier' segment. Thus, the key assumption for his result is the very price sensitivity in the 'copier' segment, and, consequently, when this price sensitivity is 'very large,' it pays off not even to introduce any protection and serve only the 'non-copier' segment. Žigić et al. (2020), on the other hand, show that in the related duopoly model of end user piracy where there is only a 'copier' segment, the impact of private IPR protection on the equilibrium prices is always positive due the fact that firms increase the market base by increasing private protection and can therefore charge higher prices (see Žigić et al., 2020). Unlike Jain (2008), they use a model of the vertical product differentiation.

Our main finding is summarised in Proposition 3 which states that the impact of private protection on the equilibrium pricing crucially depends on the intensity of the price sensitivity and the size of the 'copier' segment. The bigger the 'copier' segment is, and the less price sensitive consumers are in this segment, the more the market base effect would dominate and so stronger private IPR protection would result in higher prices. Alternatively, for strong price sensitivity and a 'not so large' 'copier' segment, Jain's (2008) negative effect of protection on prices would prevail. Thus, the model we put forward in this paper nests in a sense both Jain's (2008) and Žigić et al.'s (2020) findings on the impact of private IPR on firms' pricing.

Since our above results crucially hinge on the equilibria in which both firms are active in both segments, as an insightful aside to our analysis we provide rigorous conditions for the firms not to deviate to only serving the 'non-copier' segment and summarise these findings in the form of Proposition 1 and related Appendices.

The important reason that our results are somewhat different than those of Jain (2008), is that, besides private IPR protection, we also include public IPR protection (copyright) in our analysis, which enhances the magnitude of the first effect—increasing the market base. Recall that, unlike in Jain (2008), in our model private protection of level $c$ by firm $A$ also applies to the subsegment of potential copiers with low valuation who would then opt to buy product $B$. In Jain (2008), however, private IPR protection of one firm does not directly protect the other firm from the end users' piracy. Thus, the effect of an increase in $c$ is much larger in our asymmetric model of vertical product differentiation than in Jain's (2008) model of symmetric horizontal product differentiation, where the firms fully cover the market in equilibrium and share it equally.[6] More specifically, an increase in $c$ in our setup not only directly increases both firms' share but also shrinks the competitive subsegment, $1-c$, of developer $B$, where the size of public protection $X$ enables firm $B$ to compete for the (potential) low-end illegal users who are capable of acquiring the high quality software but may prefer the legal, unprotected version of the low quality software if the price is low enough[7] (that is, $p_B<Xq$).

Finally, having both private and public IPR protection in the model, it would be possible to study another very important issue, e.g., the optimal level of private IPR protection and the interaction of the two forms of protections within and across the different equilibria discussed above. More specifically, it would be important for policy makers to know when these two forms of protections are complements and when they act as substitutes to each other. Žigić et al. (2020) focus on this important subject. Looking at their results from the perspective of this paper, we can say that, by a continuity argument, their findings would also hold (at least) in this enlarged model for the situation where there is a large 'copier' segment and not 'too many' price sensitive consumers.

## Notes

1. Alternatively, one can think of these costs as neccessary expenditures to inform the consumers about the existence and quality of their product (like marketing and advertisement). In the language of Duchêne and Waelbroeck (2006) approach, developers rely on information push technologies to diffuse the above pieces of information.

2. Neither legal nor licence restrictions are assumed for the developer in the case of implementing protection against copying.
3. The availability of an illegal version and the ability to break it differs significantly among users and is more dependent on technical skill than on sensitivity to price and quality. The uniform distribution is an analytical simplification that does not harm the nature of the paper.
4. Alternately, we can assume that developer $B$ does not have the technological capability to protect his software from piracy.
5. For $\frac{dp_B}{dc}<0$ to hold, the discount factor has to be substantially lower than the critical $\delta$ for $\frac{dp_A}{dc}<0$ since developer $B$ benefits even more from $A's$ protection. For example, it can be shown that $\frac{dp_A}{dc}<0$ for the entire 'no full dependence' equilibrium range for $\delta<\frac{5-\beta}{6}$, but the corresponding condition for $\frac{dp_B}{dc}<0$ is $\delta<\frac{1-2\beta}{3}$.
6. Note that in our asymmetric equilibrium $c_A = c^*$ and $c_B = 0$ whereas in Jain's (2008) symmetric equilibrium $c_A = c_B = c^*$.
7. If, however, we, like Jain (2008), exclude public IPR protection, and have, like him only private IPR protection together with the segment of never copying consumers with higher willingness to pay than the potential copiers, then Jain's result carries over qualitatively in our vertical differentiation setup.

## Acknowledgements

The authors are grateful to Andrea Downing for her superb English editing assistance.

## Disclosure statement

No potential conflict of interest was reported by the authors.

## References

Bulow, J., Geanakoplos, J., & Klemperer, P. (1985). Multimarket oligopoly: Strategic substitutes and strategic complements. *Journal of Political Economy*, *93*(3), 488–511. https://doi.org/10.1086/261312

Cheng, H., Sims, R., & Teegen, H. (1997). To purchase or to pirate software: An empirical study. *Management Information Systems*, *13*(4), 49–60. https://doi.org/10.1080/07421222.1997.11518142

Duchêne, A., & Waelbroeck, P. (2006). The legal and technological battle in the music industry: Information-push versus information-pull technologies. *International Journal of Law and Economics*, *26*, 565–580.

Fernández-Márquez, C. M., Vázquez, F. J., & Watt, R. (2020). Social influence on software piracy. *Management and Decision Economics*, *2020*, 1–14.

Jain, S. (2008). Digital piracy: A competitive analysis. *Marketing Science*, *27*(4), 610–626. https://doi.org/10.1287/mksc.1070.0313

Tirole, J. (1988). *The theory of industrial organization*. The MIT Press.

Varian, H. (2005). Copying and copyright. *Journal of Economic Perspectives*, *19*(2), 121–138. https://doi.org/10.1257/0895330054048768

Žigić, K., Střelický, J., & Kúnin, M. (2015). Modeling software piracy protection: Monopoly versus duopoly. *CERGE-EI Working Papers*, *551*, 1–56.

Žigić, K., Střelický, J., & Kúnin, M. (2020). Private and public IPR protection in a vertically differentiated software duopoly. *CERGE-EI Working Papers*, *671*, 1–50.

# Appendices

## Appendix A: General notes for all appendices

Most of the calculations in this paper were performed using *Mathematica* and other similar software. The *Mathematica* file is available upon request.

In almost all model situations here, profit functions are concave (quadratic, or, in singular cases, linear) in the respective choice variables, so that an interior solution is always a (local) maximum. In the remaining situations, profit functions are explicitly assumed to be concave in the main text. Thus, second-order conditions always hold in equilibrium, so they are omitted everywhere below.

## Appendix B: Proof of Lemma 1

It is sufficient to show that each developer can attain a positive market share in the 'non-copier' segment.

Developer $A$ can do so by setting $p_A = p_B$.

Developer $B$ can do so by setting $p_B = p_A q/2$.

## Appendix C: Proof of Lemma 2

The proof is by contradiction: we show that if either developer is out of the 'non-copier' segment, then this developer is out of the 'copier' segment as well, so it has zero market share, which contradicts Lemma 1.

Developer $A$ is out of the 'non-copier' segment if even the consumer with the highest valuation prefers product $B$, i.e., $p_A \geq p_B + (1-q)$. This implies $p_A \geq p_B + \delta(1-q)$, whence no consumer in the 'copier' segments buys product $A$ either. Thus, developer $A$ is present in the 'non-copier' segment in any equilibrium.

Developer $B$ is out of the 'non-copier' segment if $p_B \geq p_A q$. However, in this case no consumer in the 'copier' segment buys product $B$ either. Thus, developer $B$ is present in the 'non-copier' segment in any equilibrium.

## Appendix D: Equilibrium prices and profits

The detailed calculations can be found in the *Mathematica* file available upon request.

### D.1. 'No full dependence' equilibria

If this equilibrium occurs, then the equilibrium prices and profits are

$$p_A^* = \frac{\beta(c\beta+\delta)X(1-c)q + 2(1+c\beta)\delta(\beta+\delta)(1-q)}{(c\beta+\delta)\big(4(\beta+\delta)-(c\beta+\delta)q\big)},$$

$$p_B^* = q\frac{2\beta X(1-c)+\delta(1+c\beta)(1-q)}{4(\beta+\delta)-(c\beta+\delta)q},$$

$$\Pi_A^* = \frac{\big(\beta(c\beta+\delta)X(1-c)q + 2(1+c\beta)\delta(\beta+\delta)(1-q)\big)^2}{(1-q)\delta(c\beta+\delta)\big(4(\beta+\delta)-(c\beta+\delta)q\big)^2},$$

$$\Pi_B^* = q(\beta+\delta)\frac{\big(2\beta X(1-c)+\delta(1+c\beta)(1-q)\big)^2}{(1-q)\delta\big(4(\beta+\delta)-(c\beta+\delta)q\big)^2}.$$

### D.2. 'Interior full dependence' equilibria

The prices can easily be shown to decrease in $c$.

$$\begin{aligned}
p_A^* &= 2\frac{(1-q)}{(4-q)}\frac{(1+c\beta)\delta}{(c\beta+\delta)}, \\
p_B^* &= q\frac{(1-q)}{(4-q)}\frac{(1+c\beta)\delta}{(c\beta+\delta)}, \\
\Pi_A^* &= 4\frac{(1-q)}{(4-q)^2}\frac{(1+c\beta)^2\delta}{(c\beta+\delta)}, \\
\Pi_B^* &= q\frac{(1-q)}{(4-q)^2}\frac{(1+c\beta)^2\delta}{(c\beta+\delta)}.
\end{aligned}$$

### D.3. 'Corner full dependence' equilibria

In these equilibria, $p_B^* = X$ so $p_A^* = \frac{1}{2}\left(X + (1-q)\delta\frac{1+c\beta}{c\beta+\delta}\right)$, which can easily be shown to decrease in $c$. The equilibrium profits are given by

$$\begin{aligned}
\Pi_A^* &= \frac{((c\beta+\delta)X + (1-q)(1+c\beta)\delta)^2}{4(1-q)\delta(c\beta+\delta)}, \\
\Pi_B^* &= \frac{X}{2}\left((1+c\beta) - \frac{(2-q)(c\beta+\delta)X}{(1-q)q\delta}\right).
\end{aligned}$$

## Appendix E: Proof of Proposition 2

Here we show that the equilibrium structure in question occurs at $\delta = \beta = 1$ when $c$ is high enough, and then by continuity it occurs at a range of parameters.

When $\delta = \beta = 1$, the entire market is equivalent to a homogeneous market with the share of the uncontrolled consumers equal to $c' = \frac{1+c}{2}$. In addition, the condition (iii) in the main text holds: for developer $A$, $\delta \geq \frac{1}{2+c\beta}$ definitely holds when $\delta = \beta = 1$, and for developer $B$, the outcome here guarantees that $c \geq 1/9$. Thus, it remains to investigate when (i) and (ii) hold.

For (i) to hold, we show that a necessary condition on $X$ is $X_{cl} < X < X_{cu}$, where $X_{cl} = \frac{c'(1-q)}{2(1+c')-c'q}$, and $X_{cu} = 2\frac{1-q}{4-q}$. Note that the upper bound $X_{cu}$ intuitively coincides with the equilibrium price in the case of the pure Bertrand equilibrium, whereas $X > X_{cl}$ follows from $p_B^* < Xq$, with the latter equivalent to $p_A^* < X\left(1 + \frac{1}{c'}\right)$, (note that $X_{cl} < X_{cu}$). Then, both developers' profits reach the internal local maxima in the parameter ranges corresponding to our Subcase 1, with the prices equal to

$$p_A^* = \frac{X(1-c')q + 2(1-q)}{4-c'q}, \quad p_B^* = q\frac{2X(1-c') + c'(1-q)}{4-c'q}. \tag{5}$$

For (ii) to hold, we have to verify that neither developer has an incentive to unilaterally deviate, given that the other developer sets the equilibrium price, $p_i^*$. For developer $A$, it can be profitable to deviate to $p_A = X$ (given that developer $B$ sets $p_B^*$) if the decrease in price from $p_A^*$ to $X$ is more than compensated for by an increase in the number of consumers that is no longer confined to fraction $c'$, and for $X$ large enough, such a deviation would yield a higher profit than choosing the protection. As for developer $B$, if $p_B^*$ is close enough to $Xq$, then it may pay off to jump to a higher price $p_B \in (Xq, X)$ given that developer $A$ sets $p_A^*$, as in this case the effect of such a price increase would more than offset the loss of the consumer base.

Developer $A$ can be shown *not* to switch to $p_A = X$ given $p_B = p_B^*$ if

$$X \le X_c^+ = \frac{2(1-q)\left(4 - c'(2-c)q - \sqrt{1-c'}(4-c'q)\right)}{16 - 8q + (3c' - 3c'^2 + c'^3)q^2},$$

which is smaller than $X_{cu}$ when $c'<1$. It turns out that $X_{cl} \lesseqgtr X_c^+$ iff $c' \gtreqless c^o = \frac{\sqrt{5}-1}{2} \approx 0.618034$, i.e., the (sub)case in question cannot occur if $c' \le \frac{\sqrt{5}-1}{2}$.

As for developer $B$, cases $c' \ge \left(\frac{q}{2-q}\right)^2$ and $c' < \left(\frac{q}{2-q}\right)^2$ are distinguished. In the former case, the condition to check is $p_A^* \le X\left(1 + \frac{1}{\sqrt{c'}}\right)$, which is equivalent to

$$X \ge X_c^- = 2\frac{\sqrt{c'}(1-q)}{\left(1+\sqrt{c'}\right)\left(4-\sqrt{c'}q\right)},$$

which is bigger than $X_{cl}$ when $c'<1$. It can be shown that $X_c^- \lesseqgtr X_c^+$ iff $c' \gtreqless \underline{c}$, where

$$\underline{c} = \frac{1}{3}\left(4 - 8\left(6\sqrt{33}-26\right)^{-1/3} + \left(6\sqrt{33}-26\right)^{1/3}\right) \approx 0.704402,$$

so the lower bound on $c'$ can be improved to $\underline{c}$ when $c' \ge \left(\frac{q}{2-q}\right)^2$. In the other case, $c' < \left(\frac{q}{2-q}\right)^2$, a direct comparison between $\pi_B^*$ and $\pi_B^X(p_A^*)$ yields a lower bound on $X$ located between $X_{cl}$ and $X_c^-$, which translates into a lower bound on $c'$ located between $\frac{\sqrt{5}-1}{2}$ and $\underline{c}$. Note that given the lower bounds on $c'$, case $c' \ge \left(\frac{q}{2-q}\right)^2$ occurs with certainty if $q$ is not too high, namely, if $q \le \approx 0.912622$.

## Appendix F: Deviation to not serving the non-copier segment by developer *B* in the 'no full dependence' case

Recall that developer $B$'s profit when both consumer segments are served (which implies $p_B \le X$) is given by

$$\Pi_B(p_B) = \begin{cases} \Pi_B^N(p_B), & p_B \le Xq, \\ \Pi_B^F(p_B), & Xq < p_B \le X, \end{cases}$$

where the cases that cannot occur in equilibrium are omitted and

$$\Pi_B^N(p_B) = \left(1 + \frac{\beta c}{\delta}\right)\left(\frac{p_A - p_B}{1-q} - \frac{p_B}{q}\right)p_B + \frac{\beta(1-c)}{\delta}\left(\frac{X - p_B}{1-q} - \frac{p_B}{q}\right)p_B,$$
$$\Pi_B^F(p_B) = \left(1 + \frac{\beta c}{\delta}\right)\left(\frac{p_A - p_B}{1-q} - \frac{p_B}{q}\right)p_B.$$

(Here '$N$' and '$F$' stand for 'no full dependence' and 'full dependence' respectively.) If developer $B$ only serves the non-copier segment, then

$$\Pi_B(p_B) = \Pi_B^0(p_B) = \left(\frac{p_A - p_B}{1-q} - \frac{p_B}{q}\right)p_B.$$

It is possible to show that the maximum when both segments are served is never attained at $p_B = Xq$, so a 'no full dependence' equilibrium candidate implies an interior local maximum of $\Pi_B^N(p_B)$,

$$p_B = p_B^N = \frac{\left(p_A(c\beta+\delta) + X(1-c)\beta\right)q}{2(\beta+\delta)},$$

which satisfies $p_B \leq Xq$ iff

$$p_A \leq p_A^N = X\left(1 + \frac{\beta+\delta}{c\beta+\delta}\right)$$

and results in the profit of

$$\Pi_B^N\left(p_B^N\right) = \frac{\left(p_A(c\beta+\delta) + X(1-c)\beta\right)^2 q}{4\delta(\beta+\delta)(1-q)}.$$

Also, this interior maximum should be global for $p_B \leq X$, i.e., there should be no profitable deviation to the 'full dependence' range. The maximum in the 'full dependence' range can be either interior or corner at $p_B = X$. In the former case, there is no deviation to not serving the 'copier' segment since the argument in the main text, $\Pi_B^F(p_B) = \left(1 + \frac{\beta c}{\delta}\right)\Pi_B^0(p_B)$, applies. In the latter case, the condition $p_A \leq p_A^N$ cannot be improved without further assumptions on the model parameters and a direct comparison is needed.

The deviation price and profit are given by

$$p_B^0 = \frac{p_A q}{2}, \;\; \Pi_B^0\left(p_B^0\right) = \frac{p_A^2 q}{4(1-q)}.$$

Then $\Pi_B^N\left(p_B^N\right) - \Pi_B^0\left(p_B^0\right)$ is a positive multiple of

$$(1-c)^2\beta X^2 + 2(1-c)(c\beta+\delta)Xp_A + \left(c^2\beta - \delta + 2c\delta\right)p_A^2$$

This expression is quadratic in $p_A$ as well as non-negative and non-decreasing in $p_A$ at $p_A = 0$, so if it is non-negative at $p_A = p_A^N$ then it is non-negative at all applicable values of $p_A$. Substituting $p_A = p_A^N$ results in a positive multiple of

$$4c^2 + \left(3c^2 + 6c - 1\right)(\delta/\beta) + 4c(\delta/\beta)^2.$$

The last expression is always positive when $3c^2 + 6c - 1 \geq 0$, i.e., when $c \geq \frac{2\sqrt{3}-3}{3} \approx 0.154701$. Otherwise, its minimum occurs at $(\delta/\beta) = \frac{1-6c-3c^2}{8c}$, and the minimum value equals

$$\frac{(1-c)^3(9c-1)}{16c}.$$

Thus, a sufficient condition for developer $B$ to never deviate from a 'no full dependence' equilibrium candidate to not serving the copier segment is $c \geq \frac{1}{9}$. Note that while it is shown in Žigić et al. (2020), that such equilibria can only occur at much higher values of $c$, this is not the case here due to the presence of the 'non-copier' segment. It is also interesting that the condition only depends on $c$, just like several 'no full dependence' equilibria–related conditions in Žigić et al. (2020).