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CHARMLESS AND STRANGLESS NONLEPTONIC B DECAYS

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Decay rates of the B-meson are studied through charmless and strangless transitions into π, ρ , ω and γ systems. The important features of these modes are their clean signatures. The CLEO II collaboration has recently reported the value $BR(B^0 \to \pi^+\pi^-) = (1.3\ ^{+0.8}_{-0.6} \pm 0.2) \times 10^{-5}$. This value is tests our approach to nonleptonic B-decays within the standard model (SM). Since $B \to \gamma\gamma$ is far beyond our experimental reach, we believe that the correct determination of the order of magnitude $\sim 10^{-10}$ for $BR(B \to \gamma\gamma)$ provides the most reliable value needed at this moment. The most recent experimental report by the CLEO collaboration on $BR(B \to K^*\gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$ represents a confirmation of our SM prediction for the $B \to K^*\gamma$ decay. This experimental result, despite of its great importance for the SM and the physics beyond the SM, enables us to predict the value $BR\,(B \to \rho\gamma) \cong (3.5 \pm 3.3) \times 10^{-6}$.

1. Introduction

Hadronic rare B decays have recently been the subject of both theoretical and experimental interest. The estimates for these processes involve the matrix elements of four quark operators, and these did not receive much attention in the past. In this paper we propose to study a number of B decays that follow from $b \to u, d$ and have clean signatures. The calculation proceedes in two steps. The effective short-distance interaction consists of two contributions. One contribution comes from the QCD-corrected tree-level w-exchange diagram [1], with the $b \to u$ vertex. The other contribution comes from the gluon exchanged with $b \to d$ transitions [2]. The second step is to use the factorization approximation to derive the hadronic matrix elements by saturating H_w^{eff} with the vacuum state in all possible ways. The resulting matrix elements involve quark bilinears between one meson and the vacuum and between two meson states. These matrix elements are estimated using relativistic quark model wave functions. Such a technique has been extensively used for B and D nonleptonic decays by Bauer et al [3] and the results are consistent with experiment. These methods for $b \to s$ nonleptonic modes were employed [4] in our previous study.

The short- and long-distance contribution to $B \to \rho\gamma$ and the determination of the order of magnitude for $B \to \gamma\gamma$ are described in the last two sections of this paper.

2. Effective Hamiltonian

The effective weak interaction Hamiltonian (local four-quark operator) describing the $b \to u$ transitions for charmless and strangeless B-meson decays at the tree level is

$$H_{eff}^{w} = \sqrt{2} G_F \left(c_- O_- + c_+ O_+ \right) + h.c., \tag{1}$$

$$O_\pm = \overline{u}_L^i \gamma_\mu b_L^i \overline{d}_L^j \gamma^\mu u_L^j \pm \overline{d}_L^i \gamma_\mu b_L^i \overline{u}_L^j \gamma^\mu u_L^j . \tag{2}$$

Here $c_\pm$ are the QCD coefficients and (i, j) are the colour indices. The parameters used to evaluate the $c_\pm$ and the $c_\pm$ coefficients are

$$m_b = 4.9\mathrm{GeV}, \qquad \Lambda_{QCD} = 200\,\mathrm{MeV}, \qquad \alpha_s(m_b) = 0.256, \tag{3}$$

$$c_- = 1.34, \qquad c_+ = 0.86. \tag{4}$$

Charmless and strangless B decays can also arise through a one-loop process involving gluon exchange. The relevant Hamiltonian is [2]

$$\begin{aligned} H_{eff}^{P} &= \kappa \Big(\overline{d}_L^i \gamma_\mu b_L^i \overline{q}_L^j \gamma^\mu q_L^j - 3 \overline{d}_L^i \gamma_\mu b_L^j \overline{q}_L^j \gamma^\mu q_L^i + \\ &+ \overline{d}_L^i \gamma_\mu b_L^i \overline{q}_R^j \gamma^\mu q_R^j - 3 \overline{d}_L^i \gamma_\mu b_L^j \overline{q}_R^j \gamma^\mu q_R^i \Big) , \end{aligned} \tag{5}$$

$$\kappa = \frac{\alpha_s G_F G_1}{6\pi\sqrt{2}}, \qquad \alpha_s = \frac{g_s^2}{4\pi}. \tag{6}$$

Here q runs over all quark species, although only u and d are relevant to our discussion. We have used $\alpha_s(m_b) = 0.256$ as in (3), and $G_1 = -6.52 V_{bc} V_{uc}^*$ corresponding to $m_t = 150$ GeV.

3. Factorization approximation

From experience we know that nonleptonic decays are extremely difficult to handle. For example, the $\Delta I = 1/2$ rule in $K \to \pi\pi$ decays has not yet been understood in a satisfactory way. Enormous theoretical machinery has been applied to $K \to \pi\pi$ decays producing only up to 50% agreement with experiment. For energetic decays of heavy mesons (D, B), the situation is somewhat simpler. For these decays, the direct generation of a final meson by a quark current is (probably) a good approximation.

According to the current-field identities, the currents are proportional to interpolating stable or quasi-stable hadron fields. The approximation now consists only in taking the asymptotic part of the full hadron field, i.e. its "in" or "out" field. Then the weak amplitude factorizes and is fully determined by the matrix elements of another current between the two remaining hadron states. For that reason, we call this approximation the factorization approximation. Note that in replacing the interacting fields by the asymptotic fields, we have neglected any initial or final--state interaction of the corresponding particles. For B decays, this can be justified by the simple energy argument that one very heavy object decays into two light but very energetic objects whose interactions might be safely neglected. Also, diagrams in which a quark pair is created from the vacuum will have small amplitudes because these quarks have to combine with fast quarks to form the final meson. Note also that the $1/N_c$ -expansion argument provides a theoretical justification for the factorization approximation, since it follows the leading order in the $1/N_c$ expansion. Here N_c is the number of colours.

Each of the B-decay mode might receive three different contributions. As an example we give one amplitude obtained from H_{eff}^w :

$$\begin{aligned} A\big(B^+(p) \quad &\to \quad \pi^+(k)\pi^0(q)\big) = L(\pi^0)\langle\pi^+ \left|\bar{b}\gamma_\mu(1-\gamma_5)d\right| B^+\rangle\langle\pi^0 \left|\overline{u}\gamma^\mu\gamma_5 u\right| 0\rangle + \\ &+ \quad L(\pi^+)\langle\pi^+ \left|\overline{u}\gamma_\mu\gamma_5 d\right| 0\rangle\langle\pi^0 \left|\bar{b}\gamma^\mu(1-\gamma_5)u\right| B^+\rangle + \\ &+ \quad L(B^+)\langle\pi^+\pi^0 \left|\overline{u}\gamma_\mu(1-\gamma_5)d\right| 0\rangle\langle 0 \left|\bar{b}\gamma^\mu\gamma_5 u\right| B^+\rangle. \end{aligned} \tag{7}$$

The coefficients $L(\pi^0), L(\pi^+)$ and $L(B^+)$ contain the coupling constants, colour factors, flavour symmetry factors, i.e. flavour counting factors and factors resulting from the Fierz transformation of the operators in Eqs. (1) and (4). The coefficients $L(\pi^0)$ and $L(\pi^+)$ correspond to the quark decay diagram, whereas the $L(B^+)$ corresponds to the so-called annihilation diagrams. These factors are different for each decay mode, as indicated by the dependence on the final-state meson. To obtain the amplitudes for other decay modes, one has to replace the final-state particles with the particles relevant to that particular mode.

The QCD coefficients appear in two different combinations in the amplitudes of various decay modes [5]:

$$C_1 = \frac{1}{2}\left[c_+\left(1+\frac{1}{N_c}\right)+c_-\left(1-\frac{1}{N_c}\right)\right], \tag{8}$$

$$C_2 = \frac{1}{2}\left[c_+\left(1+\frac{1}{N_c}\right)-c_-\left(1-\frac{1}{N_c}\right)\right]. \tag{9}$$

The factor $1/N_c$ arises from the colour mismatch in forming colour singlets after Fierz transformation.

We proceed with the definitions of the coupling constants and the Lorentz decomposition of the typical hadronic matrix elements:

$$\langle o\left|\bar{b}\gamma_\mu\gamma_5 u\right| B^+(p)\rangle = ip_\mu f_B, \qquad f_B = 1.5 f_\pi, \qquad f_\pi = 130\,\mathrm{MeV}, \tag{10}$$

$$\langle \rho^0(k)\left|\overline{u}\gamma_\mu u\right| 0\rangle = g_{\rho^0}\epsilon_\mu(k), \quad \langle \omega(k)\left|\overline{u}\gamma_\mu u\right| 0\rangle = g_\omega\epsilon_\mu(k), \quad g_{\rho^0} = \frac{1}{\sqrt{2}}g_{\rho^+}, \tag{11}$$

$$\langle \pi^+(k)\left|\bar{b}\gamma_\mu d\right| B^+(p)\rangle = (p+k)_\mu f^{(+)}(q^2) + q_\mu f^{(-)}(q^2), \qquad q = p-k \tag{12}$$

$$\begin{aligned}\langle \rho^+(k)\left|\bar{b}\gamma_\mu(1-\gamma_5)d\right| B^+(p)\rangle &= \varepsilon_{\mu\nu\lambda\sigma}\epsilon^\nu(k)(p+k)^\lambda(p-k)^\sigma V(q^2)\\ &+i\varepsilon_\mu(k)\left[(m_B^2-m_\rho^2)A_1(q^2)-(\varepsilon\cdot q)(p+k)_\mu A_2(q^2)\right]\\ +i(\varepsilon\cdot q)(m_B+m_\rho)(q_\mu/q^2)&\left[2m_\rho A_0(q^2)-(m_B-m_\rho)\left(A_1(q^2)-A_2(q^2)\right)\right],\end{aligned} \tag{13}$$

$$\langle \pi^+(k)\rho^0(q)\left|\overline{u}\gamma_\mu d\right| 0\rangle = \langle \rho^0(q)\left|\overline{u}\gamma_\mu d\right| \pi^-(-k)\rangle. \tag{14}$$

Any hadronic matrix element needed to evaluate the branching ratios of the decays can easily be obtained from the above definitions. We assume that the momentum dependence of the form-factors $f^+(q^2)$, $V(q^2)$, etc. from Eqs. (11-13), is well described by single poles with masses of excited b-quark meson states $(1^-, 0^-, ...)$ close to m_B, i.e. we can use only the nearest pole dominance. The form factors at zero momentum transfer were calculated using the relativistic oscillator wave functions [3]. Now we give a few examples of the structure of the form-factors:

$$f^{(+)}_{\pi^+B}\left(m_\pi^2\right) = \frac{f^{(+)}_{\pi^+B}(0)}{\left(1-m_\pi^2/m_{0^+}^2\right)}, \tag{15}$$

$$V_{\rho^+B}\left(m_\rho^2\right) = \frac{V_{\rho^+B}(0)}{(m_B+m_\rho)\left(1-m_\rho^2/m_{1^-}^2\right)}, \tag{16}$$

$$A_0^{\rho^+B}\left(m_\rho^2\right) = \frac{A_0^{\rho^+B}(0)}{(m_B+m_\rho)\left(1-m_\rho^2/m_{0^-}^2\right)}, \tag{17}$$

$$A_1^{\rho^+B}\left(m_\rho^2\right) = \frac{A_1^{\rho^+B}(0)}{(m_B-m_\rho)\left(1-m_\rho^2/m_{1^+}^2\right)}, \tag{18}$$

$$A_2^{\rho^+B}\left(m_\rho^2\right) = \frac{A_2^{\rho^+B}(0)}{(m_B+m_\rho)\left(1-m_\rho^2/m_{1^+}^2\right)}. \tag{19}$$

Here we have used $m_{0^-}(b) \approx m_{0^+}(b) \approx m_{1^-}(b) \approx m_{1^+}(b) \approx m_B$. The couplings g_ρ, g_ω are determined from the $\rho, \omega \to e^+e^-$ experimental rates:

$$g_{\rho^0}^2 = 3m_\rho^3\left[\Gamma(\rho \to e^+e^-)/4\pi\alpha^2\right] = 0.0141\,\mathrm{GeV}^4, \tag{20}$$

$$g_\omega^2 = 27m_\omega^3\left[\Gamma(\omega \to e^+e-)/4\pi\alpha^2\right] = 0.0128\,\mathrm{GeV}^4. \tag{21}$$

For the ratio of the Kobayashi-Maskawa (K-M) matrix elements we use the central value $|V_{ub}/V_{bc}| = 0.1\zeta$, where $0 < \zeta < 1$. Some examples of the branching ratios that arise from the tree-level Hamiltonian are

$$BR\left(B^0 \to \pi^+\pi^-\right) = 2\left(\frac{\pi f_\pi}{m_B}\right)^2 \left|\frac{V_{ub}}{V_{bc}}\right|^2 |C_1|^2 \left|f_{B\pi}^{(+)}\left(m_\pi^2\right)\right|^2 \lambda_{\pi\pi}^{1/2}, \tag{22}$$

$$BR\left(B^0 \to \rho^0\omega\right) = \left(\frac{\pi g_{\rho^0}}{m_B}\right)^2 \left|\frac{V_{ub}}{V_{bc}}\right|^2 |C_2|^2 \left(1+\frac{g_\omega}{g_{\rho^0}}\right)^2 |F|\,\lambda_{\rho\omega}^{3/2}, \tag{23}$$

$$BR\left(B^+ \to \rho^+\omega\right) = \left(\frac{\pi g_{\rho^+}}{m_B}\right)^2 \left|\frac{V_{ub}}{V_{bc}}\right|^2 \left|C_1 - \frac{g_\omega}{g_{\rho^0}}C_2\right|^2 |F|\,\lambda_{\rho\omega}^{3/2}, \tag{24}$$

where

$$\lambda_{ab} = \left(1-\frac{m_a^2}{m_B^2}-\frac{m_b^2}{m_B^2}\right)^2 - \frac{4m_a^2m_b^2}{m_B^4}, \tag{25}$$

$$\begin{aligned} |F| &= 2V^2\left(m_\rho^2\right) + \left(\frac{3}{\lambda_{\rho\rho}}+\frac{m_B^4}{4m_\rho^4}\right)\left(1-\frac{m_\rho^2}{m_B^2}\right)^2 A_1^2\left(m_\rho^2\right) \\ &+ \frac{m_B^4}{4m_\rho^4}A_2^2\left(m_\rho^2\right) - \frac{m_B^4}{2m_\rho^4}\left(1-\frac{2m_\rho^2}{m_B^2}\right)\left(1-\frac{m_\rho^2}{m_B^2}\right)A_1\left(m_\rho^2\right)A_2\left(m_\rho^2\right). \end{aligned} \tag{26}$$

The results for all the modes using the tree-level Hamiltonian are given in Table 1. The values of the branching ratios presented in Table 1 have been obtained by applying the $N_c \to \infty$ limit, i.e. for $C_1 = 1.1$ and $C_2 = -0.24$. We find that the penguin contributions to these decays are generally small. We therefore estimate the contribution to a few of the more interesting modes given in Table 1. Finally note that the branching ratios BR are evaluated by normalizing the partial widths Γ to the B-meson lifetime, i.e. to the total B-meson decay width:

$$\Gamma_{tot}(B) = \frac{1}{\tau_B} \cong 3 \frac{G_F^2 m_B^5 \left|V_{cb}\right|^2}{192\pi^3}. \tag{27}$$

For details, see Refs. 8 and 9.

Table 1. The branching ratios of various charmless and strangless nonleptonic B-decay modes.

Branching Ratio Mode	Ref.3 $[10^{-5}]\zeta^2$	QCD coeff. [for the tree level graphs]	Contribution from H^w_{eff} $[10^{-5}]\zeta^2$	Contribution from H^P_{eff} $[10^{-5}]$	ARGUS/CLEO Exp. limit [6] at 90% CL $[10^{-5}]$
$B^+ \to \pi^+\pi^0$	0.6	$C_1 - C_2$	0.49	0.00	50/260
$\pi^+\rho^0$	0.6	C_2	0.18		19/17
$\pi^+\omega$		C_2	0.18		
$\pi^0\rho^+$	0.6	C_1	1.74		
$\rho^+\rho^0$	1.4	$C_1 - C_2$	1.15		
$\rho^+\omega$		eq.(17)	1.19		
$B^0 \to \pi^+\pi^-$	2.1	C_1	1.56	3×10^{-4}	$1.3^{+0.8}_{-0.6}\pm$ $\pm 0.2 \mid$ [7]
$\pi^0\pi^0$		C_2	0.03	2.5×10^{-4}	
$\pi^+\rho^-$	5.6	C_1	4.35		
$\pi^-\rho^+$		C_1	1.07		
$\pi^0\rho^0$	0.1	C_2	0.10	1.8×10^{-3}	43
$\pi^0\omega$		C_2	0.09		
$\rho^+\rho^-$	4.5	C_1	3.60	4×10^{-3}	420
$\rho^0\rho^0$	0.1	C_2	0.07		
$\rho^0\omega$		C_2	0.14		
$\omega\omega$		C_2	0.06		

4. *Short and long distance contributions to $B \to \rho\gamma$*

In April this year the CLEO collaboration announced the experimental discovery of electromagnetic-penguin B decays $B \to K^*\gamma$. The information from Wilson Lab on 13-APR-1993 was the following [10]:

Using a data sample of 2.8×10^6 B-meson decays collected by the CLEO II detector operating at the Cornell Electron Storage Ring (CESR) it was possible to observe the rare b-quark decay $b \to s\gamma$ in the modes $B^0 \to K^{*0}(892)\gamma$ and $B^- \to K^{*-}(892)\gamma$ (the charge conjugation was implied throughout). Details are given below in Table 2. This is the first unambiguous evidence for penguin-type diagrams in weak decays.

Table 2. The CLEO collaboration experimental discovery of electromagnetic--penguin B-decays [10] : $B \to K^*\gamma$.

Mode	$K^{*0}\gamma$	$K^{*-}\gamma$	
K^* decay mode	$K^-\pi^+$	$K^0_s\pi^-$	$K^-\pi^0$
Signal events	8	2	3
Sideband events	41	2	10
Sideband scale factor$^{(*)}$	1.0/37.6	1.0/40.	1.0/12.
Extrapolated background	1.1 ± 0.2	0.050 ± 0.035	0.83 ± 0.26
Binomial probability of background fluctuation	3.7×10^{-5}	0.35%	7.30%
Efficiency	(11.9 ± 1.8)%	2.0%	3.1%
Additional $B\overline{B}$ backgnd$^{(**)}$	0.30±0.15	0.01	0.10
Branching ratio $[10^{-5}]$	$4.0 \pm 1.7 \pm 0.8$	$5.7 \pm 3.1 \pm 1.1$	
Combined BR $[B \to (K^{*0} + K^{*-})\gamma]$	$(4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$		

(*)This is the number of background events in the signal region divided by the number of background events in the sideband region from off $B\overline{B}$ resonance data.

(**) This is the number of background events expected from $B\overline{B}$ events from the Monte Carlo simulation. Note that the background is dominated by non-$B\overline{B}$ processes.

The branching fractions were calculated by summing signal events, subtracting summed background events and dividing by summed efficiencies. In particular, the combined K^{*0} and K^{*-} number was NOT obtained by averaging the individual results, weighted by the statistical (or any other) error. The combined number was obtained by summing all signal events, subtracting all backgrounds and dividing by the sum of all efficiencies.

The short distance (SD) contribution to $B \to K^*\gamma$ was proposed and obtained in several papers [11, 12]. The branching ratio obtained is of the form

$$BR^{SD}(B \to K^*\gamma) = \frac{\alpha}{2\pi}\left(\frac{m_b}{m_B}\right)^2 \left|\frac{V_{ts}^* V_{tb}}{V_{bc}}\right|^2 \left|f_1^{BK^*}(0)\right|^2 \left|\tilde{F}_2^{bs}(m_t)\right|^2. \qquad (28)$$

Here $\tilde{F}_2^{bs}(m_t)$ is the one-loop flavour-changing $b \to s\gamma$ vertex with QCD corrections, and $f_1^{BK^*}(0)$ is the form-factor of the operator $\bar{s}\sigma_{\mu\nu}q^\nu b_R$ between the spin-one kaon resonance and the B meson, responsible for the $b \to s\gamma$ transition at $q^2 = o$, i.e. for the real photon. Details of the calculations are given in Ref. 8. The BR^{SD} is obtained by normalizing the partial width $\Gamma^{SD}(B \to K^*\gamma)$ to the B-meson lifetime (see Eq. (28)). For the top-quark mass $m_t = 150$ GeV, we have found [8]:

$$BR^{SD}(B \to K^*\gamma) = 2.6 \times 10^{-5}, \qquad (29)$$

which is in excellent agreement with the recent theoretical estimate [13] and is within the error of the measured branching ratio published recently by the CLEO collaboration [10].

The short distance contribution to $B \to \rho\gamma$ is dominated by the flavour-changing vertex $b \to d\gamma$, which proceeds in one loop through the exchange of u,c and t quarks and the W-boson. For the emission of the real photon the only contributing operator $\bar{d}\sigma_{\mu\nu}q^\nu b_R$ gives the following branching ratio:

$$BR^{SD}(B \to \rho\gamma) = \frac{\alpha}{2\pi}\left(\frac{m_b}{m_B}\right)^2 \left|\frac{V_{td}^* V_{tb}}{V_{bc}}\right|^2 \left|f_1^{B\rho}(0)\right|^2 \left|\tilde{F}_2^{bd}(m_t)\right|^2. \qquad (30)$$

Here $\tilde{F}_2^{bd}(m_t)$ is the one-loop flavour-changing $b \to d\gamma$ vertex with QCD corrections, and the $f_1^{B\rho}(0)$ is the form-factor of the operator $\bar{d}\sigma_{\mu\nu}q^\nu b_R$ between the ρ-meson and the B-meson. The BR^{SD} is obtained by normalizing the partial width $\Gamma^{SD}(B \to \rho\gamma)$ to the B-meson lifetime (see Eq. (28)).

Careful analysis of the procedure for the one-loop QCD calculations of the $\tilde{F}_2^{bs}(m_t)$ coefficient [11,12] shows that up to the mass symmetry-breaking effects, one can write

$$\tilde{F}_2^{bs}(m_t) = \tilde{F}_2^{bd}(m_t). \qquad (31)$$

The calculations of the matrix element $\langle K^{*+}(k)\left|\bar{s}\sigma_{\mu\nu}q^\nu b_R\right| B^+(p)\rangle$, in the Relativistic Constituent Quark Model (RCQM) [8] gives approximately the same value as the calculation of this matrix element [13] in an effective chiral theory for mesons using flavour and spin symmetries of the Heavy Quark Effective Theory [HQET].

Detailed analysis of both calculations shows that up to the quark mass difference $m_s - m_d$, (which is negligible compared to the b-quark mass), we have

$$\langle K^{*+}(k) \left| \bar{s}\sigma_{\mu\nu}q^\nu b_R \right| B^+(p)\rangle \cong \langle \rho^+(k) \left| \bar{d}\sigma_{\mu\nu}q^\nu b_R \right| B^+(p)\rangle. \tag{32}$$

Equation (32) in the RCQM then gives

$$f_1^{BK^*}(0) \cong f_1^{B\rho}(0), \qquad f_2^{BK^*}(0) \cong f_2^{B\rho}(0), \qquad f_2^{BK^*}(q^2) \cong \frac{1}{2} f_1^{BK^*}(q^2). \tag{33}$$

From Eqs. (28), (30), (31) and (33) it is easy to find that

$$BR^{SD}(B \to \rho\gamma) \cong \left| \frac{V_{td}}{V_{ts}} \right|^2 BR^{SD}(B \to K^*\gamma). \tag{34}$$

The ratio of the K-M matrix elements [9] $|V_{td}/V_{ts}| = 0.10$ to 0.33 and the last line in Table 2 gives the following range of values for $BR^{SD}(B \to \rho\gamma)$:

$$BR^{SD}(B \to \rho\gamma) \cong (4.5 \text{ to } 50) \times 10^{-7}. \tag{35}$$

It is appropriate here to comment on the long-distance (LD) contribution to $B \to \rho\gamma$. The long-distance contribution to $B \to K^*\gamma$ is discussed in detail in Ref.14. Despite of the conclusion in Ref.14 that the rate for $B \to K^*\psi$ cannot be used to give a unique value for $B \to K^*\gamma$, it is necessary to evaluate $BR^{LD}(B \to \rho\gamma)$ by applying exactly the same procedure as in Ref.14 as a check of consistency. We use the vector-meson dominance which leads to the vector-meson - photon conversion mechanism. This mechanism is known as a long-distance effect.

Owing to the gauge-condition and current-conservation requirements, for the real outgoing photon in the $B \to \rho\gamma$ mode we have that [8] $A_0 = 0$ and $A_2 = A_1 \equiv A$. This gives

$$\begin{aligned} BR^{LD}\big(B^0 \quad \to \quad \rho^0\gamma\big) = & \left(\frac{\pi g_{\rho^0}^2}{m_\rho^2 m_B^2} \right)^2 \left| \frac{V_{ub}}{V_{bc}} \right|^2 |C_2|^2 \times \\ \times \quad & \left(1 + \frac{g_\omega^2 m_\rho^2}{g_{\rho^0}^2 m_\omega^2} \right)^2 \left(1 - \frac{m_\rho^2}{m_B^2} \right)^3 \big(V^2(0) + A^2(0)\big), \end{aligned} \tag{36}$$

$$\begin{aligned} BR^{LD}\big(B^+ \quad \to \quad \rho^+\gamma\big) = & \left(\frac{\pi g_{\rho^+}^2}{m_\rho^2 m_B^2} \right)^2 \left| \frac{V_{ub}}{V_{bc}} \right|^2 \left(1 - \frac{m_\rho^2}{m_B^2} \right)^3 \times \\ \times \quad & \left| C_1 \left(1 + \frac{g_\omega m_\rho^2}{g_{\rho^0} m_\omega^2} \right) - C_2 \left(1 + \frac{g_\omega^2 m_\rho^2}{g_{\rho^0}^2 m_\omega^2} \right) \right|^2 \big(V^2(0) + A^2(0)\big) \end{aligned} \tag{37}$$

In the $N_c \to \infty$ limit, i.e. for $C_1 = 1.1$ and $C_2 = -0.24$ and for $|V_{ub}/V_{bc}| = 0.1$, we have

$$BR^{LD}\left(B^0 \to \rho^0\gamma\right) = BR^{LD}\left(B^0 \to \omega\gamma\right) = 0.02 \times 10^{-7}, \tag{38}$$

$$BR^{LD}\left(B^+ \to \rho^+\gamma\right) = 1.37 \times 10^{-7}. \tag{39}$$

Assuming that the recently quoted result [15] $|V_{ub}/V_{bc}| \cong 0.08$ is of reasonable accuracy, the above $BR^{LD}(B \to \rho\gamma)$ should become smaller by at least a factor of two. This smaller branching ratio, together with Eq. (35), gives

$$BR^{SD}(B \to \rho\gamma) > 6 \times BR^{LD}(B \to \rho\gamma). \tag{40}$$

This is similar to our previous result [14] obtained with the top quark mass $m_t = = 150$ GeV for $B \to K^*\gamma$:

$$BR^{SD}(B \to K^*\gamma) > 5 \times BR^{LD}(B \to K^*\gamma), \tag{41}$$

In the best case, the LD contributions to the total branching ratios represent $< 20\%$ corrections for both $B \to K^*\gamma$ and $B \to \rho\gamma$.

The similarity of the results in Eqs. (40) and (41) shows that the procedure used in this paper to obtain $BR^{LD}(B \to \rho\gamma)$ is consistent with the procedure in Ref.14. This means that all conclusions drawn in Ref.14 are directly applicable to this work and that the rates for $B \to \rho\rho, \rho\omega, \omega\omega$ cannot be used to give a unique value for $B \to \rho\gamma$, i.e. Eq. (35) fully represents our prediction for the decay rate:

$$2 \times 10^{-7} < BR(B \to \rho\gamma) < 7 \times 10^{-6}. \tag{42}$$

Here we have also take into account the errors in the measurements [10] of $B \to K^*\gamma$ from Table 2.

5. B^0 decay into the two-photon final state

In this case, we should only estimate the order of magnitude for $BR(B \to \gamma\gamma)$.

First, applying the vector-meson - photon conversion mechanism to the neutral B decay modes ($\rho^0\rho^0, \rho^0\omega, \omega\omega$), we may evaluate $BR^{LD}(B \to \gamma\gamma)$ as

$$BR^{LD}\left(B^0 \to \gamma\gamma\right) = \left(\frac{\pi g^3_{\rho^+}}{m_B^2 m_\rho^4}\right)^2 \left|\frac{V_{ub}}{V_{bc}}\right|^2 |C_2|^2 \left(1 + \frac{g^3_\omega m^4_\rho}{g^3_{\rho^0} m^4_\omega}\right)^2 \left(V^2(0) + A^2(0)\right). \tag{43}$$

For $C_2 = -0.24$ (in the $N_c \to \infty$ limit) and $|V_{ub}/V_{bc}| = 0.1$, we have

$$BR^{LD}\left(B^0 \to \gamma\gamma\right) \cong 10^{-10}. \tag{44}$$

Next, comparing the experimentally measured ratio of widths for K_s decays [9] with the ratio of our theoretical estimates from Eq. (44) and Table 1, we find that

$$[\Gamma(K_s \to \gamma\gamma)/\Gamma(K_s \to \pi^+\pi^-)]_{exp} \cong 3.5 \times 10^{-6}, \quad (45)$$

$$[\Gamma(B^0 \to \gamma\gamma)/\Gamma(B^0 \to \pi^+\pi^-)]_{th} \cong 6.4 \times 10^{-6}. \quad (46)$$

Since the two rates are in reasonable agreement, it is justified to use the conversion mechanism to estimate the order of magnitude for the $B^0 \to \gamma\gamma$ rate to be

$$BR\left(B^0 \to \gamma\gamma\right) \sim 10^{-10}. \quad (47)$$

6. *Discussion and conclusion*

One can see from Table 1 that our values are considerably lower than the present limits obtained at 90% CL. On the other hand, our results are in reasonable agreement with the predictions of Ref. 3 summarized in the second column in Table 1. Note also that the rates involving the (helicity-suppressed) combination $(c_- - 2c_+)^2$ (or $(c_- - c_+)^2$ in the $N_c \to \infty$ limit) are sensitive to the values of the QCD coefficients. Some experimental limits [6,7] are also presented in Table 1.

The CLEO II (Ref.7) collaboration has recently reported the $BR(B^0 \to \to \pi^+\pi^-) = (1.3^{+0.8}_{-0.6} \pm 0.2) \times 10^{-5}$ at 90% CL. This is a significant improvement upon the old results (CLEO 1.5) $BR(B^0 \to \pi^+\pi^-) < 9.0 \times 10^{-5}$ and is welcome because it is a test of our approach within the standard model.

Before discussing $B \to \rho\gamma$, let as briefly comment on $B \to \gamma\gamma$. Since the $B \to \gamma\gamma$ decay is far beyond our experimental reach, we believe that the correct determination of the order of magnitude $\sim 10^{-10}$ for $BR(B \to \gamma\gamma)$ provides the most suitable value needed at this moment.

The most recent CLEO report [10] on $BR(B \to K^*\gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$ represents a confirmation of our SM prediction for the $B \to K^*\gamma$ decay given six years ago [8]. This experimental result, despite of its great importance for the SM and the physics beyond the SM, enables us to predict the following value for the $B \to \rho\gamma$ decay within the SM:

$$BR\,(B \to \rho\gamma) \cong (3.5 \pm 3.3) \times 10^{-6}. \quad (48)$$

The range of the order of magnitude in Eqs. (42) and/or (48) is due to the range of values in the $|V_{td}/V_{ts}|$ ratio [9].

Assuming that some other experiment might determine the ratio $|V_{td}/V_{ts}| \cong 0.1$, then at least 10^8 $B^0\bar{B}^0$ pairs have to be produced at CESR and collected by the CLEO detector to discover the electromagnetic penguin B-decays $B \to \rho\gamma$. It is clear that 10^8 B-meson decays cannot be collected in the near future. However, it is encouraging that Cornell has an asymmetric B Factory [16] proposed as a further CESR upgrade.

On the other hand, if $|V_{td}/V_{ts}|$ turns out to be $\cong 0.3$, then only 10^7 $B^0\bar{B}^0$ pairs have to be collected by the CLEO detector to discover the $B \to \rho\gamma$ decay. This scenario is much more promising because Cornell is currently upgrading the luminosity of CESR by at least a factor of five [17].

In the case of known top-quark mass, the measurement of $BR(B \to \rho\gamma)$ is an excellent tool for fixing the weak flavour-mixing parameters more precisely, such as the $|V_{td}/V_{ts}|$ ratio.

As is well known, radiative B-meson decays are of importance as a test of the SM electroweak theory in one loop, because they verify the gauge structure of the theory. In addition, these decays might contribute to the discovery of new physics beyond the SM. Finally, note that the relative smallness of the branching ratio for $B \to \rho\gamma$ makes it more suitable for possible discovery of new physics than might be expected from the recently measured branching ratio for $B \to K^*\gamma$.

References

1) M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. **33** (1974) 108;
G. Altarelli and L. Maiani, Phys. Lett. **52B** (1974) 3512;

2) N. G. Deshpande and M. Nazerimonfared, Nucl. Phys. **B213** (1983) 390;
T. Inami and C. S. Lim, Prog. Theor. Phys. **65** (1981) 297;

3) M. Bauer, B. Stech and M. Wirbel, Z. Phys. **C29** (1985) 637; ibid. **C34** (1987) 103;

4) N. G. Deshpande and J. Trampetic, Phys. Rev. **D41** (1990) 895;

5) D. Tadic and J. Trampetic, Phys. Lett. **B114** (1982) 179;

6) P. Avery et al., CLEO 89-2 preprint CLNS 89/886;

7) P. Kim, talk presented at the 5th Int. Symposium on Heavy Flavor Physics (Montreal, Canada, 1993);

8) N.G. Deshpande, P. Lo and J. Trampetić, Z.Phys. **C40** (1988) 369;

9) Review of Particle Properties, Phys. Rev. **D45**, No.11 - Part II, 1992;

10) S. Stone, private communication;
CORNELL CLEO discovers B meson penguins, CERN Courier, Vol.33, No.5, 1993;

11) N. G. Deshpande, P. Lo and J. Trampetić, G. Eilam and P. Singer, Phys. Rev. Lett. **59** (1987) 183;
S. Bertolini, F. Borzumati and A. Masiero, Phys. Rev. Lett. **59** (1987) 180;
N. G. Deshpande, P. Lo and J. Trampetić, Z. Phys. **C40** (1988) 369;

12) B. Grinstein, R. Springer and M. B. Wise, Nucl. Phys. **B339** (1990) 269;
R. Grigjanis, P. J. O'Donnell, M. Sutherland and H. Navalet, Phys. Lett. bf B237 (1990) 355;
G. Cella, G. Curci, G. Ricciardi and A. Vicere, Phys. Lett. **B248** (1990) 181;
M. Misiak, Phys. Lett. **B269** (1991) 161;

13) R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto and G. Nardulli, preprint UGVA-DPT 1993/04-816, April 1993;

14) N. G. Deshpande, J. Trampetić and K. Panose, Phys. Lett. **B214** (1988) 467;

15) D. Cassel, Proceedings of the 1992 DPF Conference, Fermilab Batavia, Illinois, USA. For details, see also the invited talk given by R.D. Peccei at the 1993 Rencontre de Physique de la Valle d'Aosta, La Thuile, Italy, March 1993: *CP and CPT Violation: Status and Prospects*;

16) CERN Courier, Vol. **31**, No.6, p.8, 1991;

17) CERN Courier, Vol. **32**, No.10, p.17, 1992.

NELEPTONSKI B RASPADI BEZ PROMJENE ŠARMA I STRANOSTI

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Proučavani su raspadi B-mezona bez promjene šarma i stranosti u π, ρ, ω i γ konačna stanja. Važna osobina ovih modova su njihovi čisti eksperimentalni signali. CLEO II kolaboracija nedavno je objavila vrijednost $BR(B^0 \to \pi^+\pi^-) = (1.3\ ^{+0.8}_{-0.6} \pm 0.2) \times 10^{-5}$. Ta se vrijednost može upotrijebiti za test našeg pristupa objašnjenju neleptonskih B-raspada unutar standardnog modela (SM). Budući je $B \to \gamma\gamma$ daleko izvan našeg eksperimentalnog dosega, vjerujemo da korektno određeni red veličine $\sim 10^{-10}$ za $BR(B \to \gamma\gamma)$ predstavlja najrealniju vrijednost potrebnu u ovom trenutku. Najnoviji eksperimentalni izvještaj CLEO kolaboracije o mjerenju $BR(B \to K^*\gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$ predstavlja potvrdu naše teoretske (SM) predikcije $B \to K^*\gamma$ raspada. Taj eksperimentalni rezultat i pored važnosti za standardni model i fiziku izvan tog modela omogućuje nam predikciju jakosti raspada $BR(B \to \rho\gamma) \simeq (3.5 \pm 3.3) \times 10^{-6}$.