Robust Optimization using a new Volume-Based Clustering approach

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We introduce an uncertainty set construction method based on a newly developed **volume-based clustering** approach.

Robust optimization

In robust optimization, We consider the following constraint:

$$(\tilde{a} + Pu)^\top x \leq b \quad \forall u \in \mathcal{U}$$

Uncertainty set ($\mathcal{U}$)

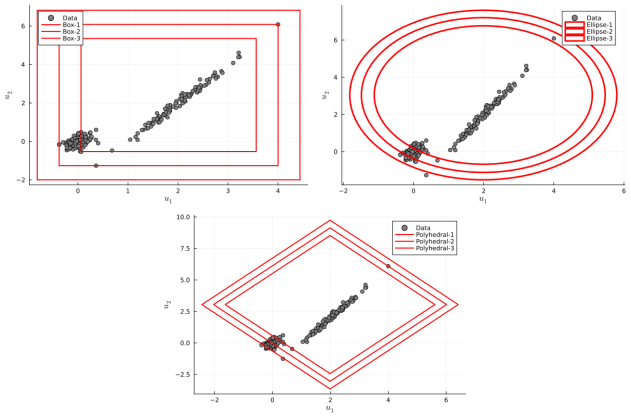

Figure 1: Fundamental uncertainty sets

Box	$\mathcal{U} = \{u : \|\|u\|\|_\infty \leq \rho\}$
Ellipsoidal	$\mathcal{U} = \{u : \|\|u\|\|_2 \leq \rho\}$
Polyhedral	$\mathcal{U} = \{u : \|\|u\|\|_1 \leq \rho\}$

The proposed uncertainty set construction method resulting in **less-conservative** solutions with minimizing volume of the uncertainty set and clustering.

Minimum volume ellipsoid Clustering (MVEC)

For an ellipsoid

$$\{x : \|\|Ax + b\|\| \leq 1\}$$

which is represented by (A, b), the volume is proportional to $\frac{1}{\det A}$.

Hence, MVEC can be represented in the following mathematical optimization:

$$\max_{A_j, b_j} \quad \sum_{j=1}^{K} \log\det(A_j)$$

$$\text{subject to} \quad \|\|A_j x^i + b_j\|\| \leq 1 \quad \forall x^i \in C_j, j = 1, \ldots, K$$

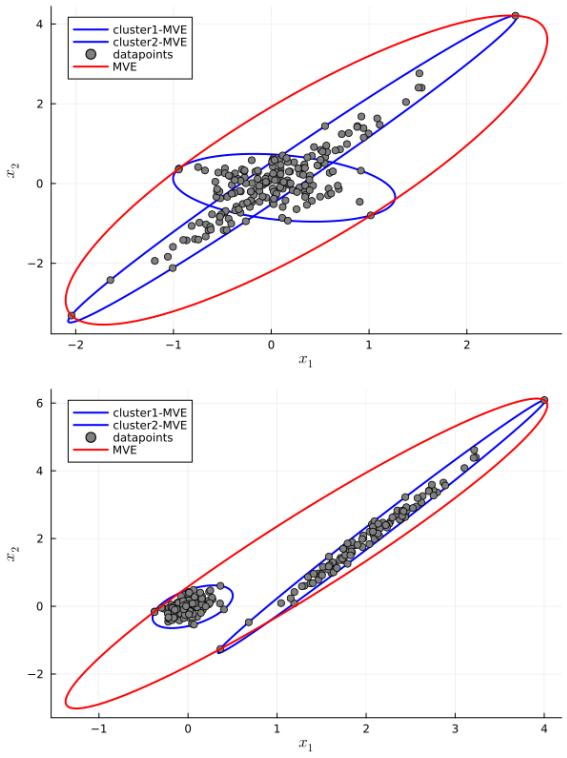

Figure 2: Uncertainty set based on MVEC

Then the proposed uncertainty set is:

$$\mathcal{U} = \bigcup_{j=1}^{k} U_j,$$

where

$$U_j = x : A_j x + b_j \leq 1$$