Frequency-domain stability tools for hybrid integrator-gain systems

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Frequency-Domain Stability Tools for Hybrid Integrator-Gain Systems

S.J.A.M. van den Eijnden, M.F. Heertjes, and H. Nijmeijer
Department of Mechanical Engineering, Eindhoven University of Technology, The Netherlands
s.j.a.m.v.d.eijnden@tue.nl

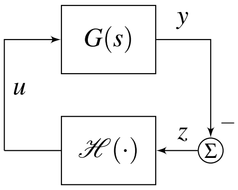
1 Introduction

Due to the inherent switching behaviour, stability analysis of hybrid integrator-gain systems (HIGS) is a challenging task. The vast majority of available tools rely on finding suitable (quadratic) Lyapunov functions, for example by solving a set of linear matrix inequalities (LMIs) [1] or using circle-criterion-like arguments [2]. Although LMI-based methods can be extremely useful, these rely on accurate parametric plant models, which are hard to obtain for complex industrial systems. Contrarily, the circle-criterion provides conditions that are graphically verifiable on the basis of measured frequency response data, and therefore is often considered favourable in practice. However, this approach may render a rather conservative estimate on closed-loop stability.

For reset control systems, the H_β-condition has been proposed in [3] as a frequency-domain tool for stability analysis. Inspired by this result, new frequency-domain stability conditions for HIGS-controlled systems are presented.

2 Main Results

Consider the feedback configuration depicted in Figure 1. Here, $G(s) \in \mathbb{C}$ is a linear time-invariant system with input $u \in \mathbb{R}$ and output $y \in \mathbb{R}$, and $\mathscr{H}(\cdot)$ denotes the hybrid integrator-gain system.



$$\mathscr{H}: \begin{cases} \dot{x}_h = \omega_h z & \text{if } q \in \mathscr{F}_1, \\ x_h = k_h z & \text{if } q \in \mathscr{F}_2, \\ u = x_h. \end{cases}$$

Figure 1: System configuration (left) and HIGS description (right) with $q = (z, u, \dot{z})$, and where $\omega_h, k_h \in \mathbb{R}$ denote the integrator frequency and gain, respectively. Here $\mathscr{F}_1, \mathscr{F}_2$ are the integrator and gain mode regions.

Definition 1 *The HIGS-controlled system in Figure 1 is said to satisfy the $H_{\beta,k}$-condition if there exist $\beta, k \geq 0$ such that the transfer function*

$$H_{\beta,k}(s) := (C(s) + \beta)S(s) - \beta, \tag{1}$$

with $C(s) = \frac{\omega_h + k k_h}{s+k}$, $S(s) = \frac{1}{1+C(s)G(s)}$, is strictly positive real (SPR).

Theorem 1 *The HIGS-controlled system in Figure 1 is exponentially stable if it satisfies the $H_{\beta,k}$-condition.*

3 Example

Consider the following mass-spring-damper system

$$G(s) = \frac{k_p}{ms^2 + bs + c}, \tag{2}$$

with $m = 1$ kg, $b = 0.2$ Ns/m, $c = 100$ N/m and $k_p = 1.5$. From the circle criterion, it is concluded that this system is stable for $k_h \in (0, 2.69)$ and $\omega_h \in (0, \infty)$. Next, stability is verified on the basis of Theorem 1 by conducting a search for finding β, k that render $H_{\beta,k}(s)$ in (1) SPR. The result for different pairs (k_h, ω_h) are shown in Figure 2. Clearly for this example, the $H_{\beta,k}$-condition provides less conservative results as compared to the circle-criterion.

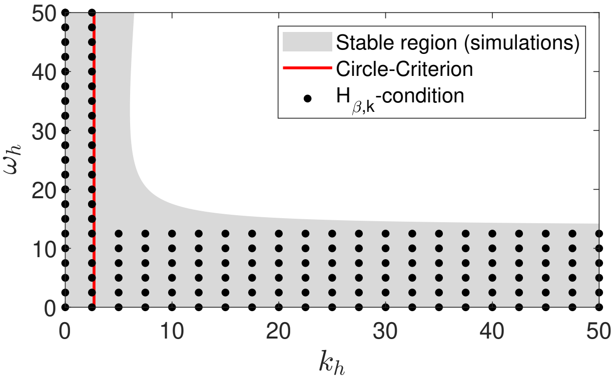


Figure 2: Stability evaluated using the circle-criterion (red), $H_{\beta,k}$-condition (black dots), and simulations (grey area).

4 Conclusion

New frequency-domain stability conditions for hybrid integrator-gain systems are presented, and the potential is demonstrated by means of example.

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