

An overview of spatial spectral methods with complex-plane deformations for the representation of waves in homogeneous and layered media without absorbing boundary conditions

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An overview of spatial spectral methods with complex-plane deformations for the representation of waves in homogeneous and layered media without absorbing boundary conditions

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Abstract: The prevention of reflections from the edge of the computational domain is a challenge in computational electromagnetics. Although ways exist to absorb/negate such reflections, we recently proposed an entirely different strategy. Based on a representation in the spectral domain, we analytically represent waves on the entirety of space, but with accuracy focused only on a certain region. Therefore, we can employ formulations without worrying about boundary conditions. We show several examples of this technique, including simulations in layered media.

In the past, several methods have been developed for the computation of electromagnetic fields that employ a Fourier transform of the spatial dimension(s). An important reason for employing the spectral domain is that the Green function for electromagnetic scattering can be expressed easily in it. Especially for layered media this is true, since an analytical expression for the Green function is only available in the spectral domain, and a spatial-domain Green function can only be computed through tedious Sommerfeld integrals [1]. Additionally, spatial derivatives are more easily represented in the spectral domain.

Examples of computational methods include the Fourier Modal Method [2], which uses (truncated) analytic expressions within layers and then couples these layers with each other by employing the layer-boundary conditions. Another example is the spectral domain volume integral equation in [3]. Here, analytical expressions for reflection and transmission coefficients are employed to incorporate the boundary conditions. However, both these methods have in common that they are usually employed for periodically repeating structures.

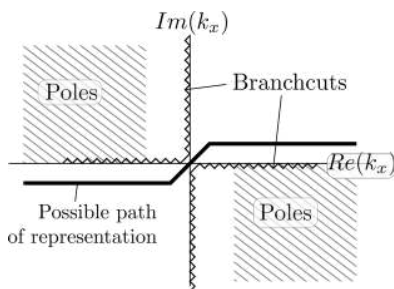


Figure 1: Typical locations of poles and branch cuts for layered media

One reason that spectral-domain methods are usually employed for periodically repeating scatterers is that employing a simple Fourier-series discretization then leads to simple and fast algorithms. However, the generalization to aperiodic scatterers is challenging. Absorbing boundaries can help to mitigate waves that travel over the unit-cell-boundary [4], but these can lead to numerical instabilities. Fully aperiodic discretization methods that are compatible with Fourier transforms exist, e.g. Gabor frames [5] and Hermite-interpolations [6]. Still the behavior of both Green function and fields are difficult, since radiation/guided waves can extend to infinity in the spatial domain. In the spectral domain, this is

represented by branch cuts and poles.

In Figure 1 we indicate the location of such poles and branch cuts for a typical reflection coefficient of a layered-medium stack – a function that is typically required to be represented in a computational method for layered media. As long as the full stack is made of passive materials, the poles and branch points will always be a certain range away from the origin, and will not exceed a range from the origin on the real k -axis. To efficiently discretize this singular behavior is hard, especially when using discretizations that are meant for continuous functions, such as is the case with discretizations that work well in both spatial and spectral domain.

We solve this issue by representing electromagnetic fields and Green functions on a deformed contour in the complex plane [6,7,9]. This contour then circumvents the poles and branch cuts, so that all desired quantities can be properly discretized. However, the contour does represent fields in the numerically unstable regions of the complex plane that represent gain-media. In [8, Chapter 9] we have shown that a stable representation can always be reached, when the deformation contour is chosen smartly, depending on the size of the simulation domain. Several formulations for complex-plane path deformations have been developed.

We will show the current status of our ongoing research and show some examples of cases relevant to the meta-materials community.

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