MINISTRY OF EDUCATION OF THE REPUBLIC OF BELARUS
Belarusian National Technical University

Department “Engineering Graphics for Machine Building”

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DESCRIPTIVE GEOMETRY
A Student’s book

Recommended by the Educational and Methodological Association of Higher Education on Transport and Transport Activities for students of the 0715 field “Vehicles, transport infrastructure and technology”

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ББК 22.151.3я7
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6-05-0715-04 Hydro-pneumatic systems of mobile and technological machinery and equipment;
6-05-0715-05 Power plants;
6-05-0715-03 Cars, tractors, mobile and technological complexes;
6-05-0715-07 Operation of ground transport and technological machines and complexes;
6-05-0715-10 Technologies of transport processes;
6-05-0715-12 Operation of road and transport infrastructure

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This student's book is developed for the first-year students of engineering majors in order to improve their skills at independent work and to provide their study process by additional material. This book is a combination of a textbook and a workbook, that makes her appropriate for classwork. It can also be intended for distance learning case as it contains theoretical material on the descriptive geometry as the first and the fundamental branch for the academic course "Engineering graphics". In addition, the algorithms given here enable students to solve similar problems. The specific distinction from the other books is in providing students with professional linguacultural knowledge on the subject.

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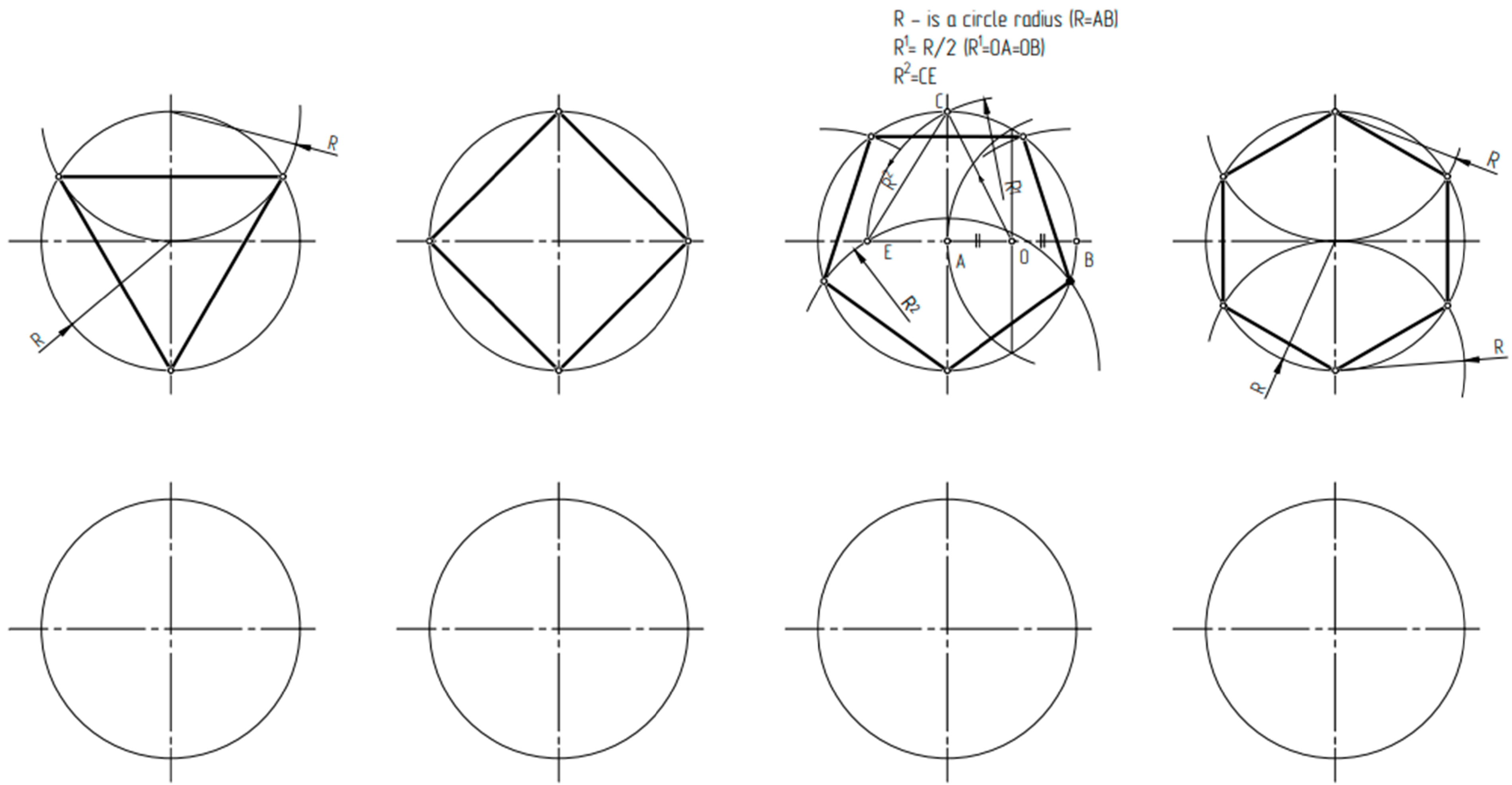
CONTENTS

LEGEND

1. Projection planes are marked with:
   – the frontal – with **V**;
   – the horizontal – with **H**;
   – the profile – with **W**.
2. Points are marked with Latin capital letters: **A, B, C, D,** etc. or numbers: **1, 2, 3,** etc.
3. Straight and curved lines are marked with lowercase Latin letters: **a, b, c, d,** etc.
4. Planes and surfaces are marked with lowercase Greek letters: **α, β,** etc.
5. Angles are marked with the following lowercase Greek letters: **φ, γ,** etc.
6. New projection planes (different from indicated above) are marked with the additional indexes **1, 2, 3, 4,** etc. in order of their appearing. Examples:
   – the first new frontal plane of projection is marked with $\mathbf{V_1}$;
   – the first new horizontal plane of projection – with $\mathbf{H_1}$;
   – the first new profile plane of projection – with $\mathbf{W_1}$.
7. Projections of points and lines are marked with the same letters as their originals in space with the relevant number of dashes and if it is needed with additional index. Examples:
   – in the frontal projection **V** with two dashes – projection of the point **A** is **A''**, projection of the line **a** is **a''**;
   – in the horizontal projection **H** with one dash – projection of the point **A** is **A'**, projection of the line **a** is **a'**;
   – in the profile projection **W** with three dashes – projection of the point **A** is **A'''**, projection of the line **a** is **a'''**.
8. Projections of planes and angles are marked with the relevant subscript. Examples:
   – for the frontal projection **V** with index $_\mathbf{V}$ – projection of the plane α is α_v, projection of the angle **φ** is $\boldsymbol{\varphi}_\mathbf{v}$;
   – for the horizontal projection **H** with index $_\mathbf{H}$ – projection of the plane α is α_H, projection of the angle **φ** is $\boldsymbol{\varphi}_\mathbf{H}$;
   – for the profile projection **W** with index $_\mathbf{W}$ – projection of the plane α is α_W, projection of the angle **φ** is $\boldsymbol{\varphi}_\mathbf{W}$.

Ex. 1. Fill the graph and white fields with the given letter patterns. Fonts N° 10.

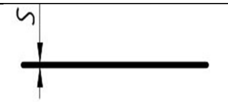
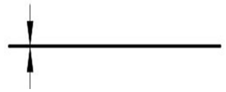
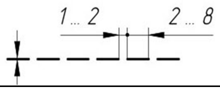
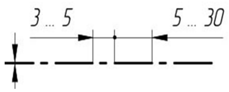
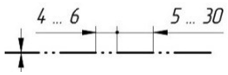
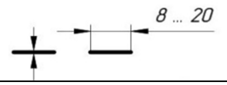
For more information see GOST 2.304.-81 [3], ISO 3098:2015 [6; 7; 8].



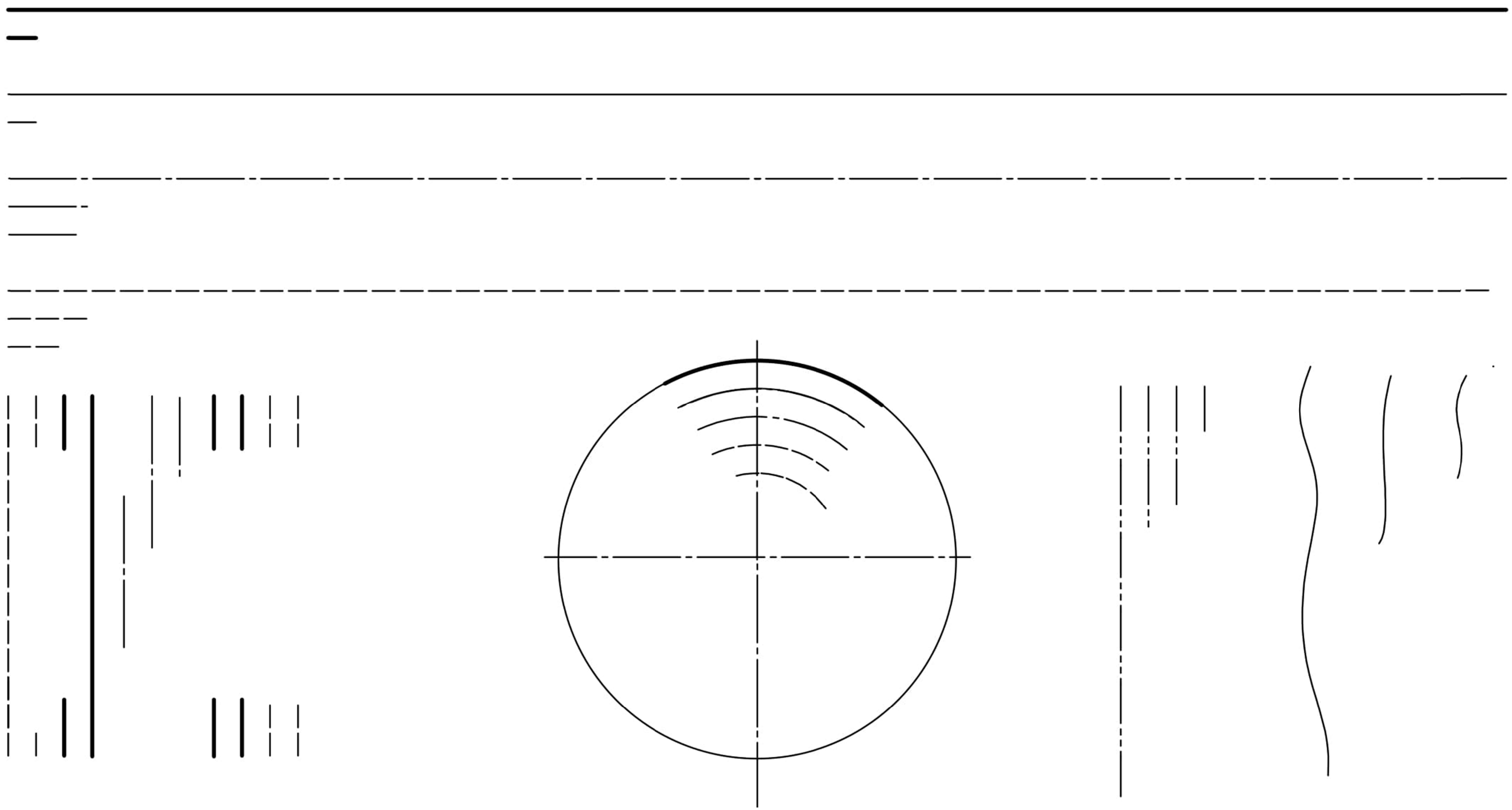
Ex. 2. Inscribe regular polygons into circles.

A regular polygon is a polygon where all sides are equal.

Lines' types

Type of line (equal to)	Designation	Recommended line thickness [mm]	Applications
1. Continuous thick line (wide line, mane line)		Š≈1 mm	1. Visible edges and outlines (visible lines) 2. Lines of the internal frame 3. Some lines of the title block
2. Continuous thin (narrow) line		S/2..S/3 0,5 mm	1. Dimension lines 2. Extension lines 3. Connection lines 4. Leader and Reference lines 5. Imaginary lines of intersection 6. Axis of coordinate system 7. Short centre lines 8. Hatching lines (Shading) 9. Diagonals for indication of flat surface 10. Projection lines 11. Grid lines
3. Continuous freehand/freeform line (curve), (wavy)		S/2..S/3 0,5 mm	Termination of partial or interrupted views, sections, or cuts provided the line is not an axis of symmetry. Preferably manually represented
4. Dashed line		S/2..S/3 0,5 mm	Hidden outlines and edges (Line of an invisible counter)
5. Long dash-dotted thin (narrow) line (long and short dashed line) (chain line) (long-dashed short-dashed line)		S/2..S/3 0,5 mm	1. Centre lines. 2. Lines of symmetry 3. Pitch circle for gears 4. Pitch circle for holes
6. Long dashed-double-dotted thin (narrow) line (with double dashes)		S/2..S/3 0,5 mm	1. Bending lines (Folding edges in developments) 2. Outlines of adjacent parts, contacting features 3. Parts are situated in front of cutting planes
7. Continuous line with zig zags		S/2..S/3 0,5 mm	1. Limits of partial or interrupted views 2. Suitable for CAD drawings provided the line is not an axis of symmetry
8. Open line (cutting plane line)		1S..1,5S 1,5 mm	Noting of cutting planes

For more information see standards [2; 5].



Ex. 3. Continue lines relevantly to the given patterns.

1. ORTHOGRAPHIC PROJECTION. POINT PROJECTION

Drawings, that are used in Descriptive Geometry, are called **projections**.

Projecting implies three components:

– **an actual object** for representing (a point is the simplest object of projection);

– **projecting rays (projectors)** that are directed from the viewer to the plane of projection and passing through the object;

– one or several **projection planes (image/picture planes)**.

A point projection is a point of the projecting ray and the projection plane intersecting.

There are two types of projections, with subclassifications according to the projecting rays' direction:

– **a central (conic) projection;**

– **a parallel projection:**

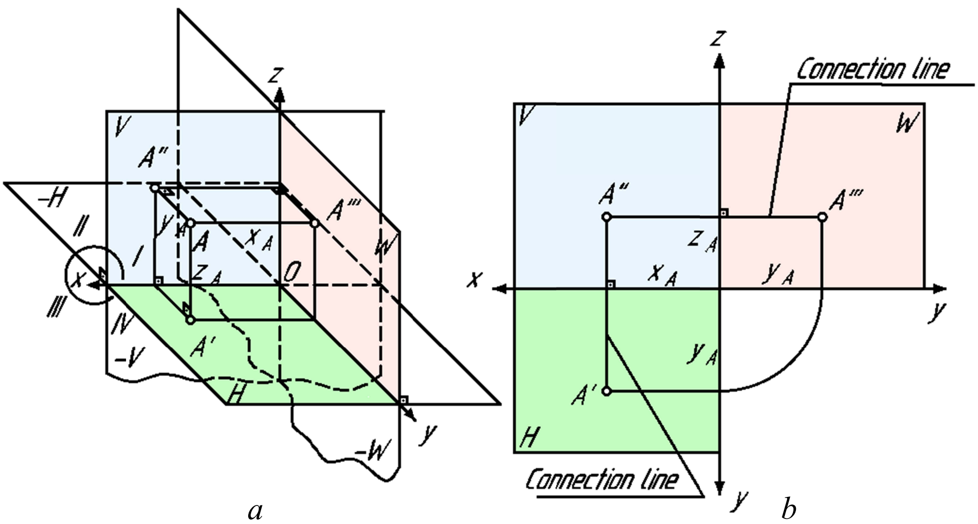
a) **an orthographic;**

b) **an oblique.**

An orthographic (orthogonal) projection is a parallel projection where all projection lines intersect the projection plane at right angles. Orthogonal projections are drawn as the multi view drawings, that provides flat representations of the subject principal.

Two projections are enough for comprehending and solving the descriptive geometry problems. Although, some problem can be solved easier, if two principal planes of projection would be supplemented by the third plane, perpendicular to them. It also corresponds to Cartesian coordinate system where the position of any point is defined by 3 coordinates: OX, OY and OZ in space. The axis lines of this system organize 3 mutually perpendicular planes – horizontal (H), frontal (V), profile (W).

The object is projected orthogonally in three mutually perpendicular planes of projection that are as mentioned combined into the one drawing. It is feasible by the next actions. The axis OY is divided (it should be cut along the OY-coordinate) and the profile (W) plane of projection to turns around about the axis OZ to the right and the horizontal (H) plane of projection to turns down around the axis OX to combine with the frontal (V) plane of projection (drawing plane) (fig. 1.1). In such a case, coordinates of a point are: OX, OY, OZ – numeric values of line segments along coordinate axes.



a *b*

Fig. 1.1. Projection planes (V, H, W):
a – a 3-dimantional model; *b* – a 2-dimantional drawing

For more information please see [1, p. 4–9].

Point projection. Practice

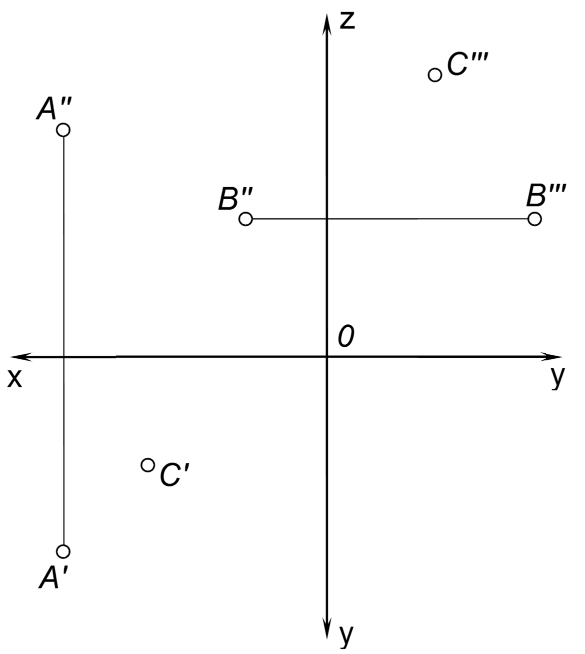


Fig. 1.2. Add the missing points' projections: A''' – ?; B' – ?; C'' – ?

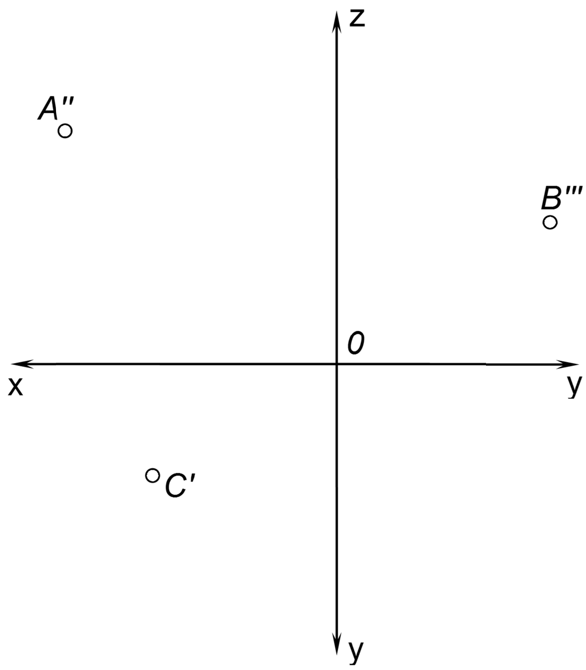


Fig. 1.3. Add the missing points' projection:
A''' – ? and A' – ? if the point A is far from both frontal and horizontal projection planes equally;
B' – ? and B'' – ? if the point B is far from both profile and frontal projection planes equally;
C'' – ? and C''' – ? if the point C is far from both frontal and horizontal projection planes equally

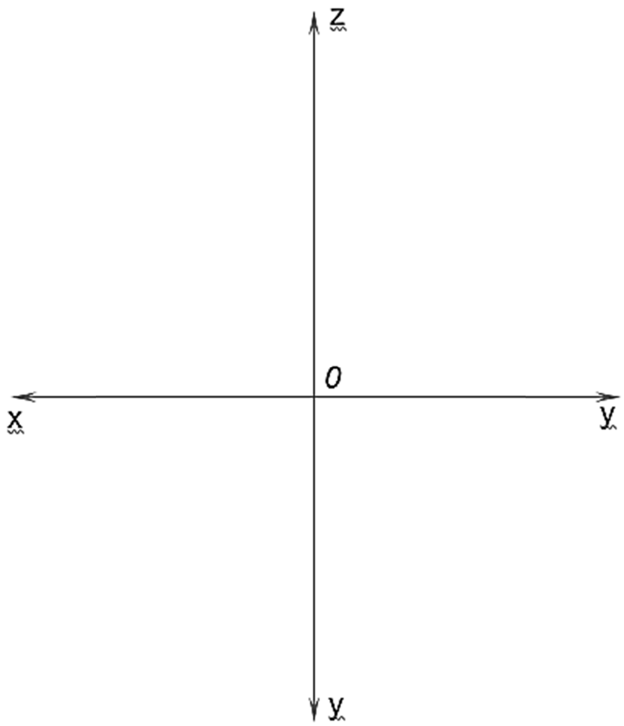


Fig. 1.4. Draw frontal, horizontal and profile points' projections in relevance with the given coordinates:
K (35, 35, 45);
M (25, 0, 20);
N (20, 35, 0)

2. STRAIGHT LINE LOCATION IN RELEVANCE WITH PROJECTION PLANES

A general line is a line that is not parallel and not perpendicular to any projection plane. Such line is not projected as true view. All three projections are distorted. It is also called **an oblique line.**

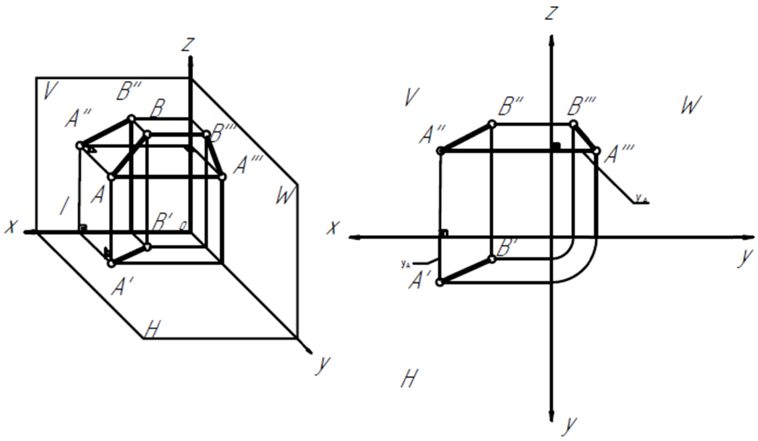


Fig. 2.1. A general line

A level line is a line that is parallel to one projection planes and perpendicular to both the other. Such line is projected as true length on the plane it is parallel to. Both the other projections are distorted and projected in parallel lines to axes. Its particular name has the same name with a projection plane it is parallel to. For instance: if a line parallel to the frontal projecting plane, it is called a frontal level line. The word "level" can be omitted.

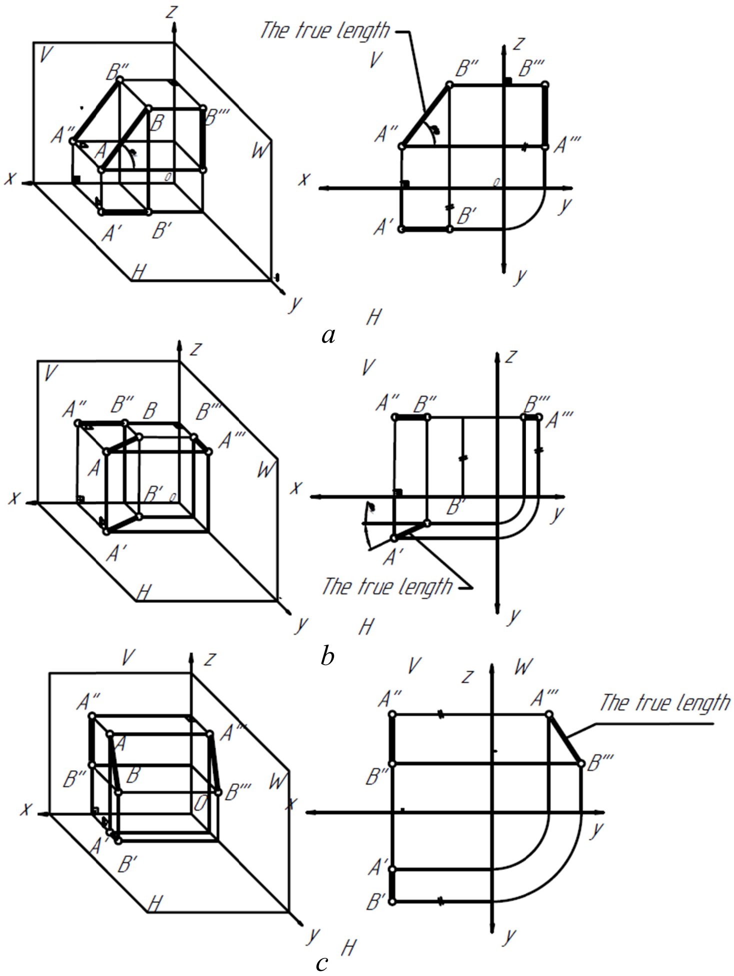
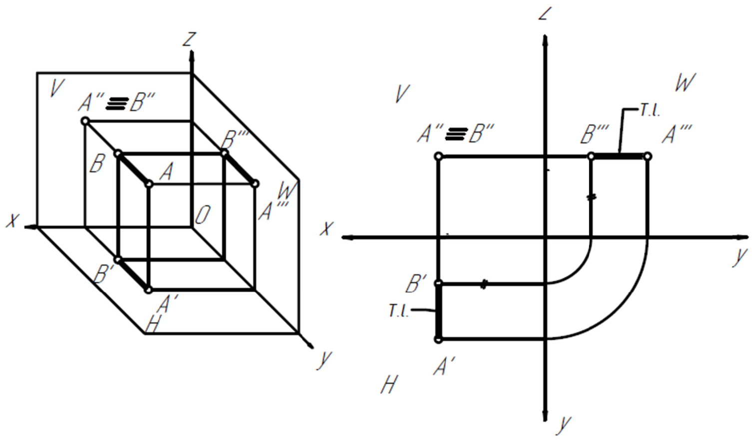
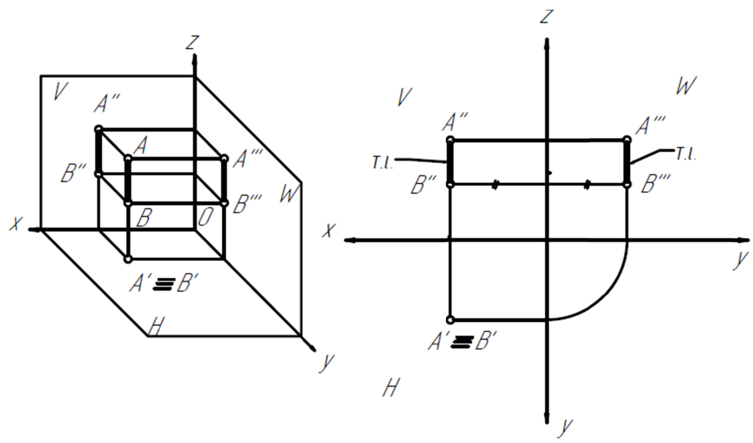


Fig. 2.2. Level lines:
a – a frontal level line; *b* – a horizontal level line; *c* – a profile level line

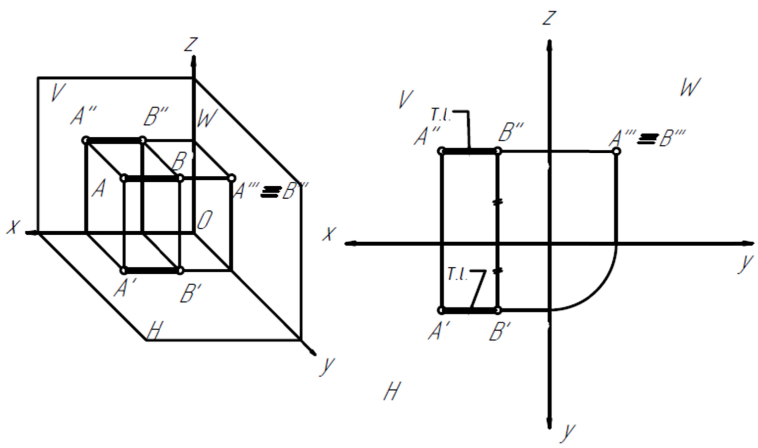
A projecting line is a line, that is parallel to two planes of projection, and perpendicular to the third. Such line is projected as true length in two projection planes it is parallel to and as a point in the third one. Such line has the same name with a projecting plane it is perpendicular to. For instance: If a line perpendicular to the profile projecting plane, it is called a profile-projecting line or profiled-projecting line. Such lines also can be named those two projections they are parallel to. For instance: if a line perpendicular to the profile projection plane it is parallel to both the other – frontal and horizontal projection planes, therefore, such line is called **a frontal horizontal line**.



a



b

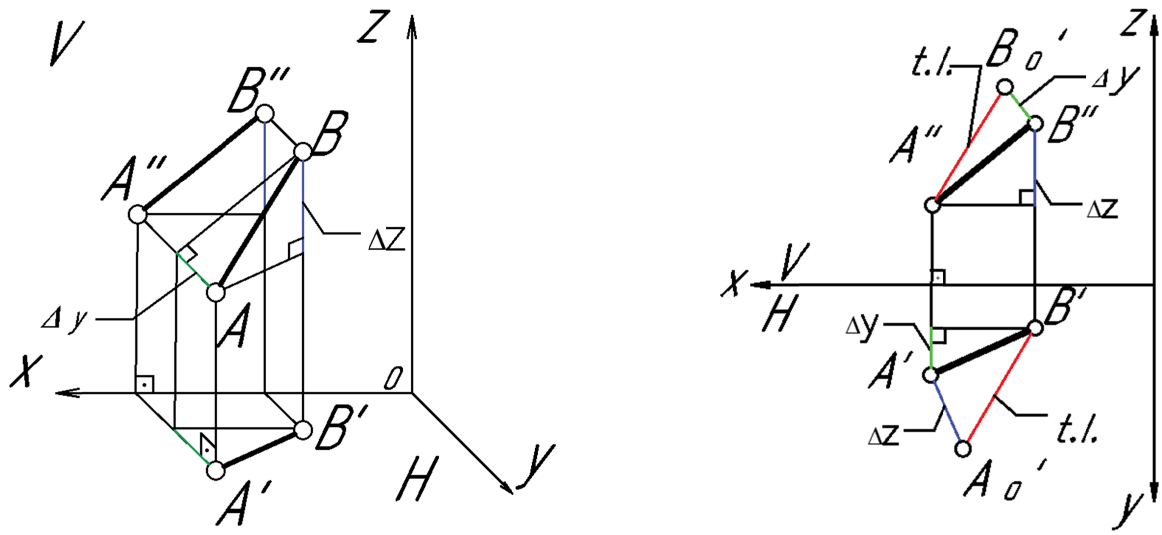


c

Fig. 2.3. Projecting lines:
a – A frontal-projecting line; *b* – a horizontal-projecting line; *c* – a profile-projecting line

The right triangle Method: A hypotenuse of a right triangle is a true length of an oblique line where one leg is a projection of the segment, and the other leg is the distance between endpoints of the other projection of the segment. In other words, the hypotenuse of the right triangle is equal to the true length of the segment AB, if one of two sides is a projection of this segment, and the other is the difference between the ends of the other projection which is supposed to be measured (see fig. 2.10).

The right angle projection theorem – a right angle is projected in a plane of projection as the true size, if one side of the right angle is parallel to the plane of projection, and the other side is not perpendicular to this plane of projection.



a *b*

Fig. 2.4. True legth determintation by the Right triangle method:
a – a 3-dimentional model; *b* – a 2-dimentional drawing

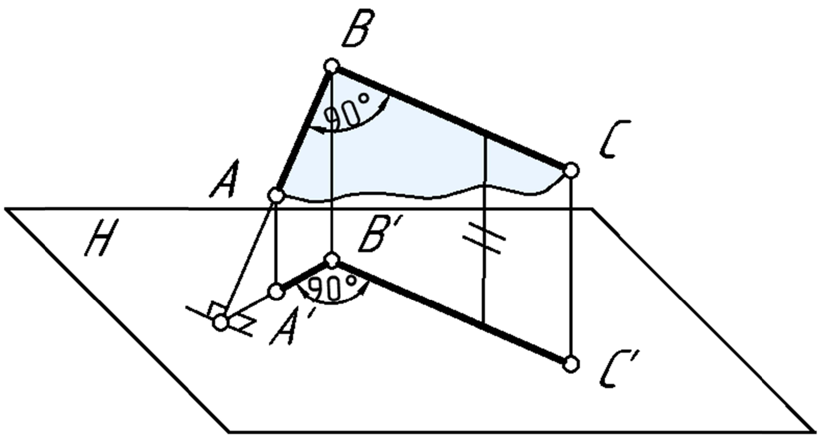


Fig. 2.5. Projection of a right angle

For more information please see [1, p. 9–13].

Projecting lines projections. True length of a line. Inclination angles of lines. Practice

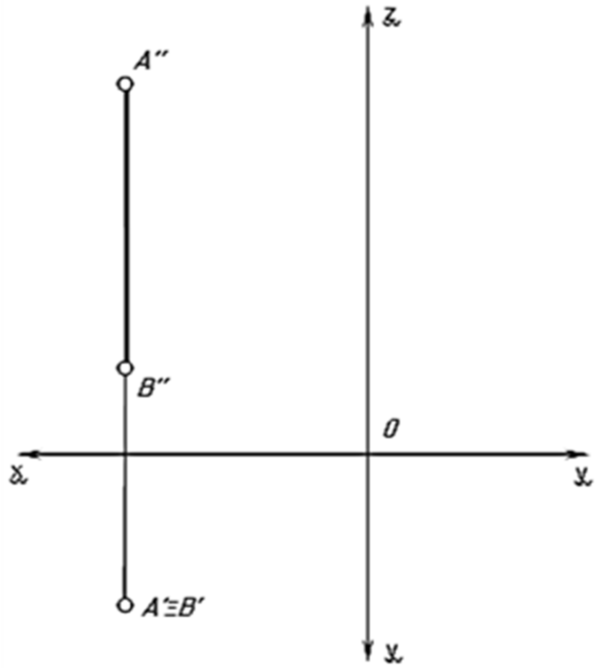


Fig. 2.6. Draw a profile projection of the horizontal-projecting segment AB. Mark with “t. l.” those projections that are projected without distortion and represent the true length of AB. Add a point on the AB in 35 mm from the horizontal projection plane

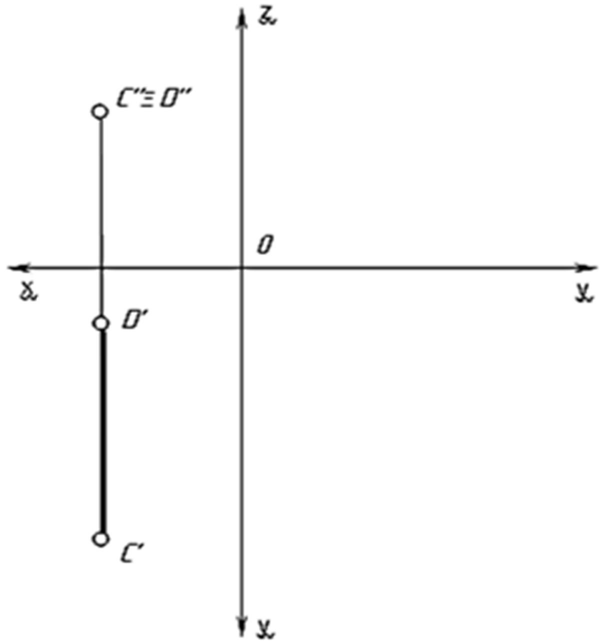


Fig. 2.7. Draw a profile projection of the frontal-projecting segment CD. Mark those projections that are projected without distortion and represent the true length of CD by “t. l.”. Add a point on the CD in 25 mm from the frontal projection plane

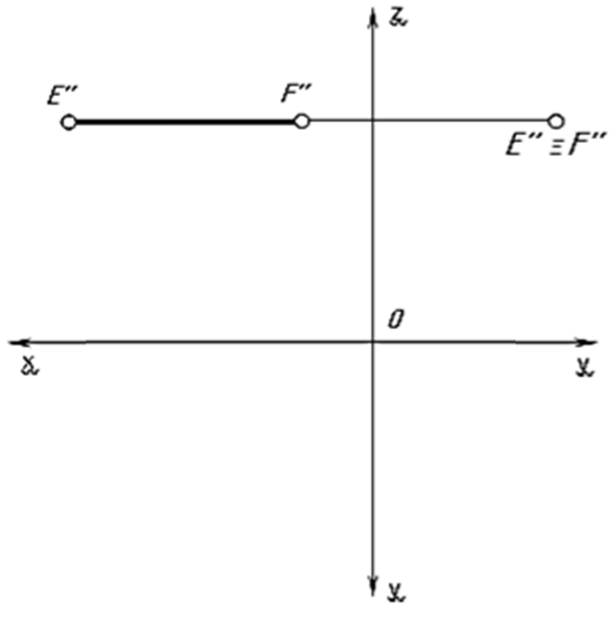


Fig. 2.8. Draw a horizontal projection of a profile-projecting segment EF. Mark with “t. l.” those projections that are projected without distortion and represent the true length of a segment. Add a point on the EF in 30 mm from the profile projection plane

Level lines projections.
Lines true length. Inclination angles of lines. Practice

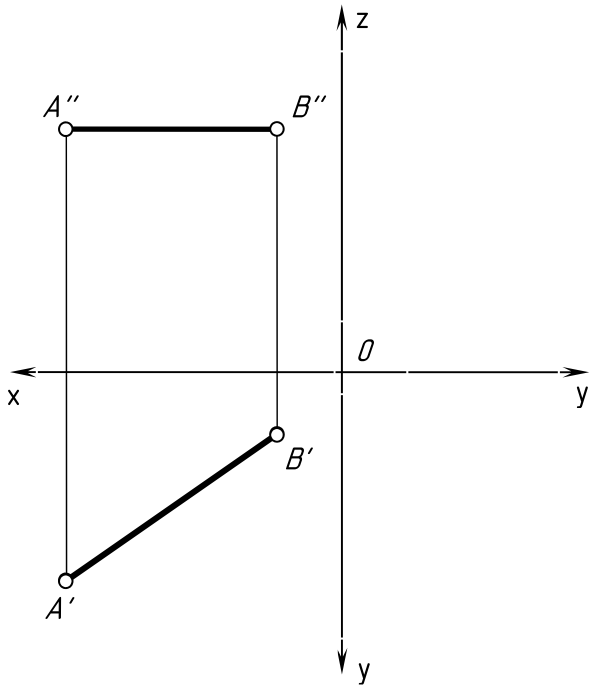


Fig. 2.9. Draw a profile projection of a horizontal level segment AB. Mark with “t. l.” that projection that is projected without distortion and represent the true length of a segment. Mark segment’s inclination angles φ_V и φ_W to projection planes V and W relevantly. Add a point on the segment in 20 mm from the frontal projection plane

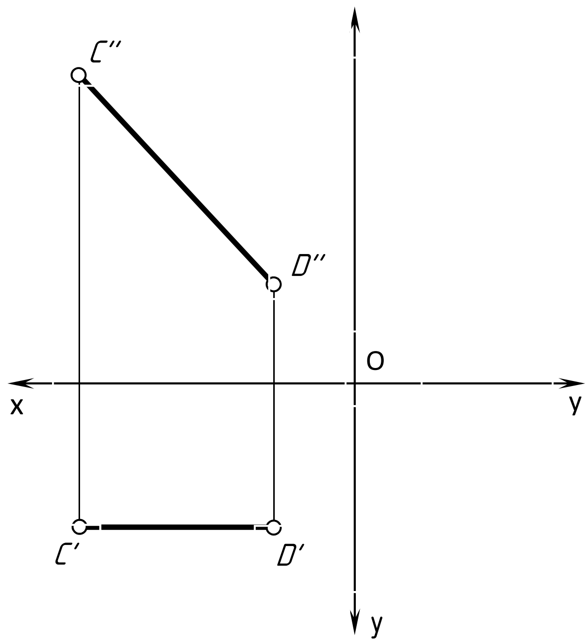


Fig. 2.10. Draw a profile projection of a frontal level segment CD. Mark with “t. l.” that projection that is projected without distortion and represent the true length of a segment. Mark segment’s inclination angles φ_H и φ_W to projection planes H and W relevantly. Add a point on the segment in 30 mm from the profile projection plane

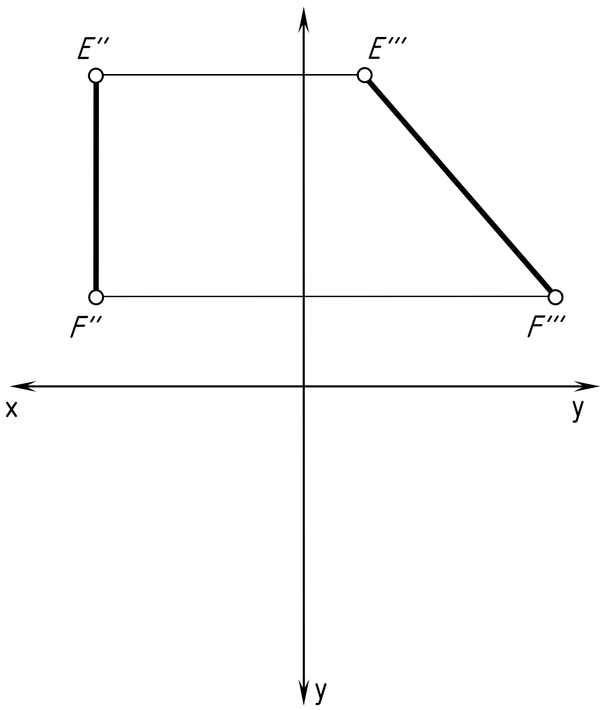


Fig. 2.11. Draw a horizontal projection of a profile level segment EF. Mark with “t. l.” that projection that is projected without distortion and represent the true length of a segment. Mark segment’s inclination angles φ_V and φ_H to projection planes V and H relevantly. Add a point on the segment in 20 mm from point E

General lines. The True length of a general line determination. The right-angle projection theorem. Practice

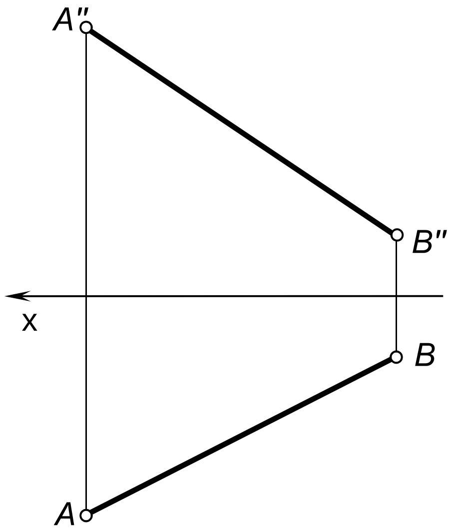


Fig. 2.12. It is necessary to determine graphically the true length of a general segment AB and its inclination angles to projection planes H and V

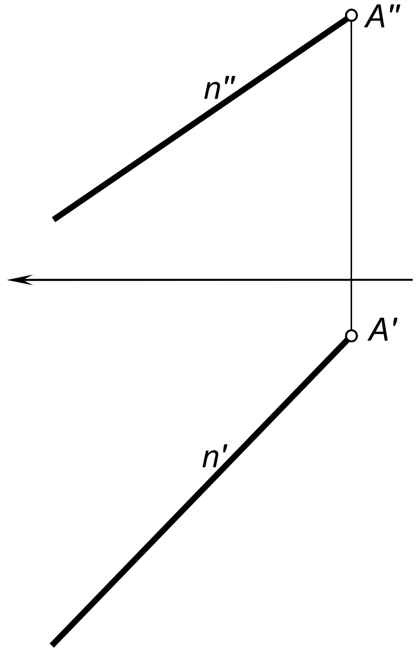


Fig. 2.13. Add point B on a general line (n", n') in 50 mm from the point A

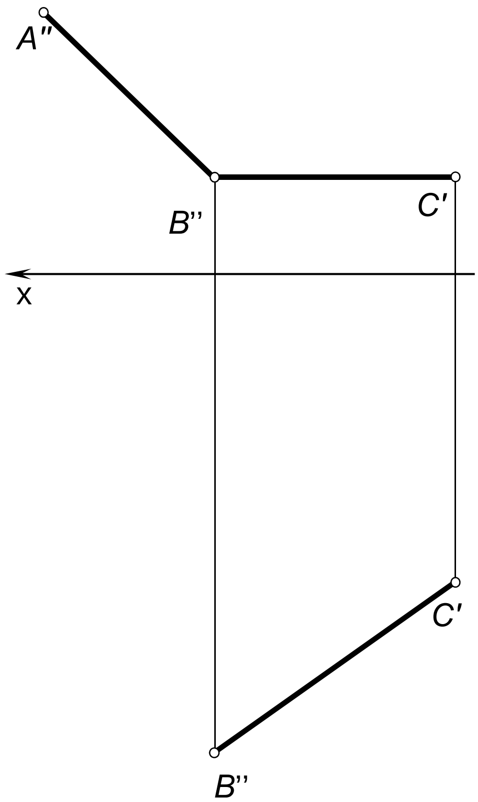


Fig. 2.14. Complete rectangle ABCD's projections

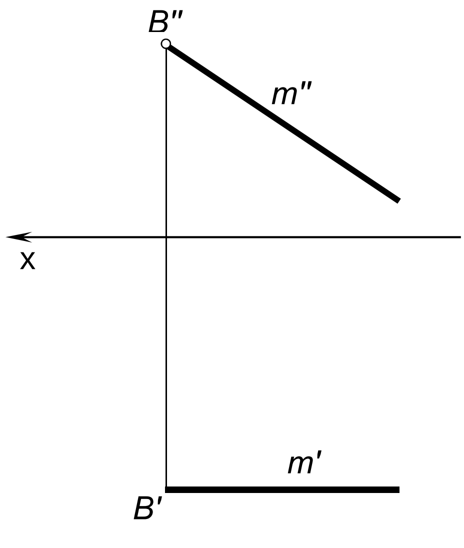


Fig. 2.15. Complete right triangle ABC's projections where its side BC lies on a frontal level line m(m", m'); angle A is on axis X; angle C is in horizontal projection plane H

3. PLANES. PLANES LOCATION

Planes are always drawn with limited size. But, in principle, the plane has indefinite extent.

A projection of a plane in a drawing can be set (limited) in the following ways:

– by a projection of three points (points do not lie on the same straight line);

– by a projection of a straight line and projection of a point (both of them do not lie on the same straight line);

– by a projection of two parallel lines;

– by a projection of two intersecting lines;

– by a projection of any shape (triangle, rectangle);

– by plane traces.

Here are some postulates of the descriptive geometry concerning to point and line belonging:

– if two points lie in a plane, then the line joining them lies in that plane;

– if a line passes through two points of a plane or if it passes through a point belonging to a plane and also parallel to a plane line, then this line belongs to the plane.

The following problems of descriptive geometry can be solved by these postulates:

– drawing a line on a plane;

– finding a point on a plane;

– adding a missing projection of a point of the plane;

– checking of a point belonging to a plane (fig. 3.1, *a*).

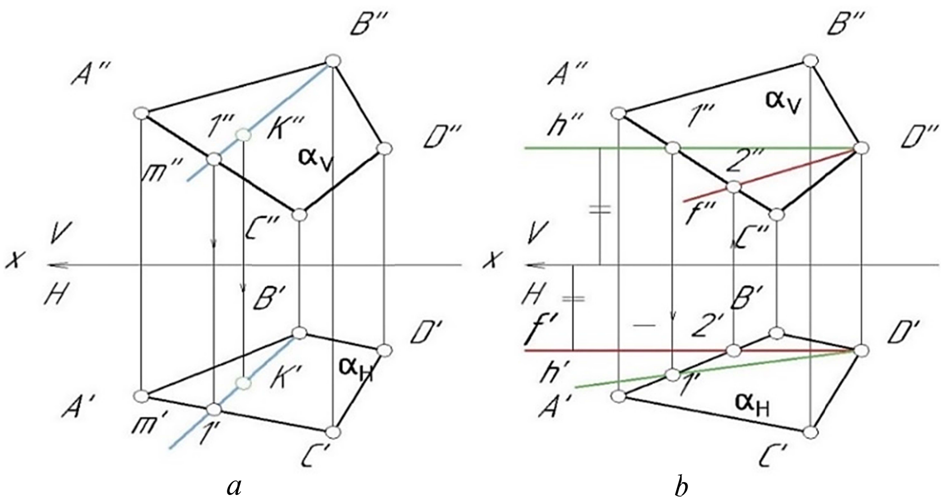


Fig. 3.1. Point, line, and plane relation
a – point and line belonging to a plane;
b – main (principle) lines of a plane

Main (principle) lines of a plane.

There are plenty of lines belonging to the plane.

Those of them which have a special or particular position should be distinguished.

The main line of a plane is a straight line of level which belongs to the plane. A frontal plane line is a frontal level line belonging to the given plane. A horizontal plane line is a horizontal level line belonging to the given plane (fig. 3.1, *b*).

A plane can have the following positions relative to the planes of projection:

– an inclined position to all planes of projection;

– a perpendicular position to the plane of projection;

– a parallel position to the plane of projection.

A general plane is a plane, that is not parallel and not perpendicular to any planes of projection, it is also called as **an oblique plane**.

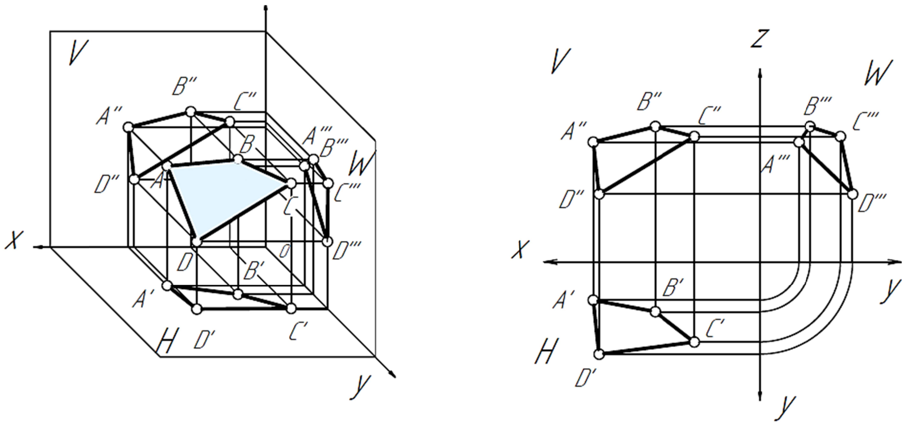


Fig. 3.2. A general plane

A level plane is a plane that is parallel to one plane of projection and perpendicular to both the other. Such planes are projected as true shape in projection plane they are parallel to and as lines paralleled to axes in others. They are named as a projection plane thay are parallel to. For instence: If a plane is parallel to the frontal projection plane it is named as a frontal level plane (see fig. 3.3). The word "level" can be omitted.

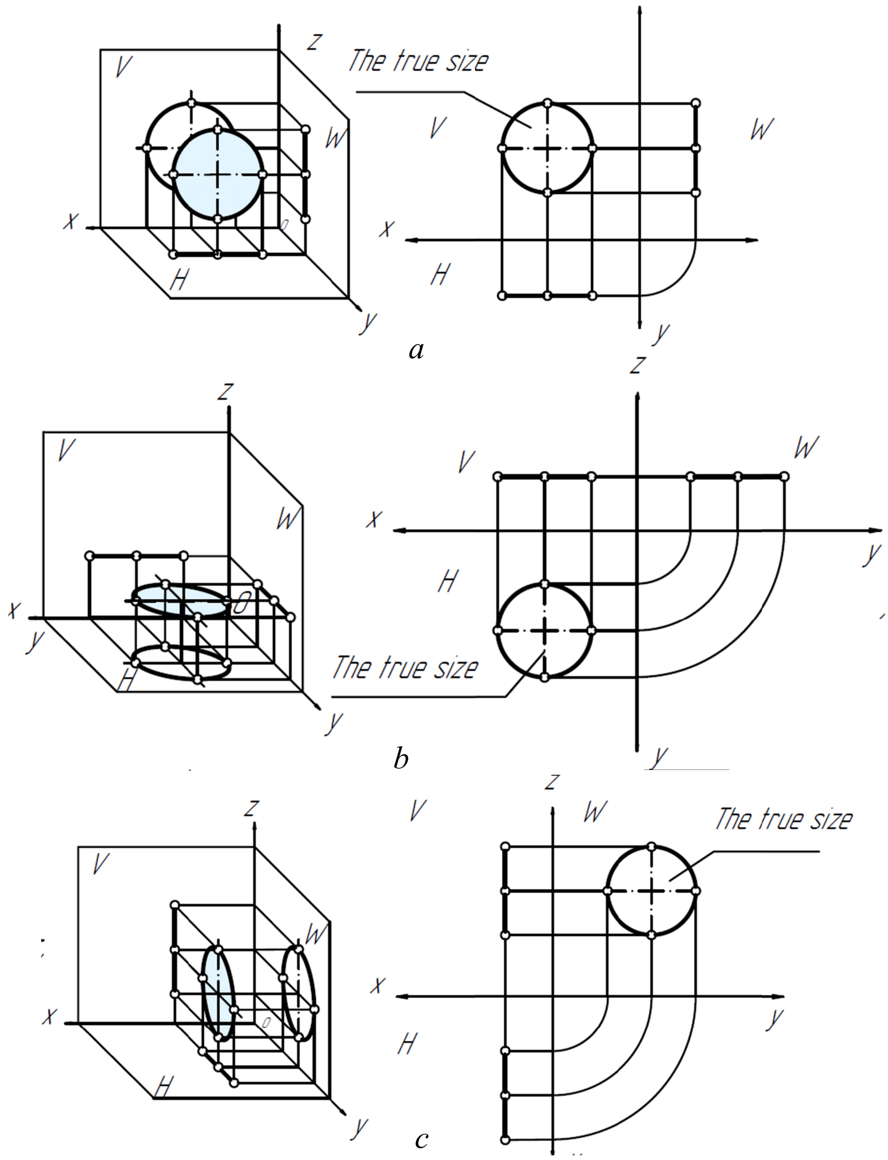
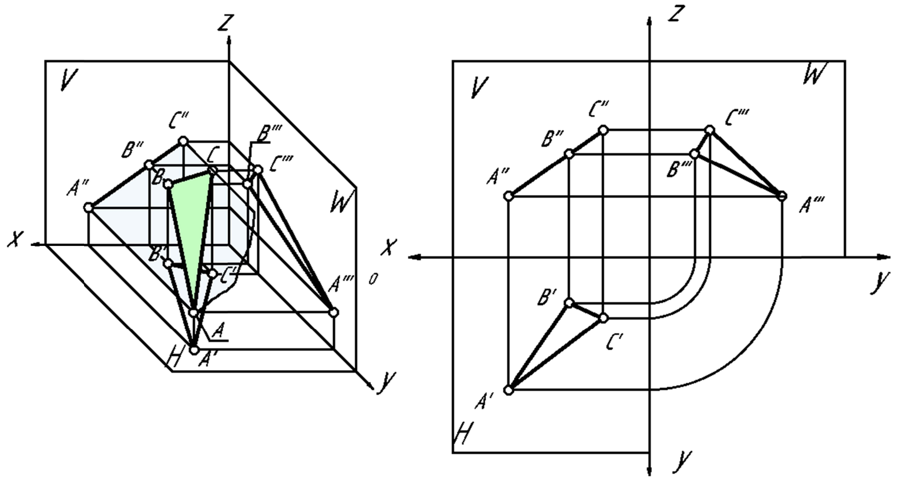
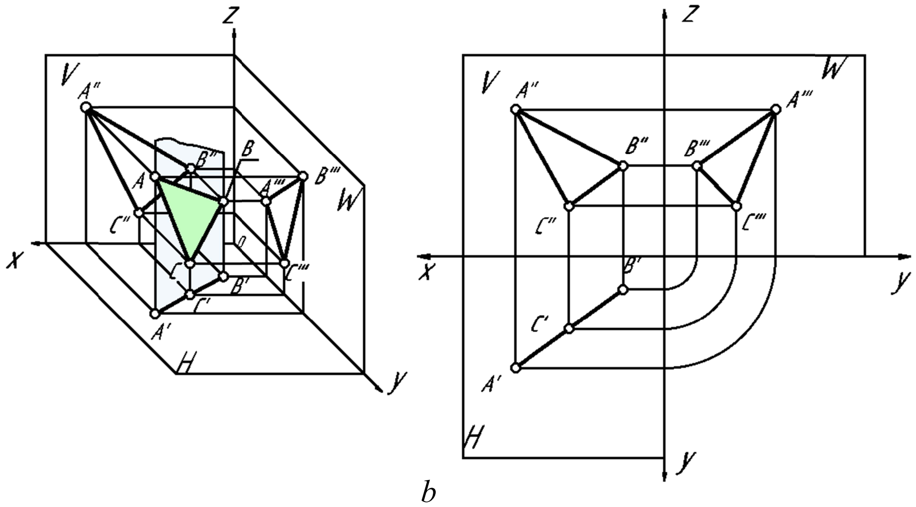


Fig. 3.3. Level planes:
a – frontal level plane; *b* – a horizontal level plane; *c* – profile level plane

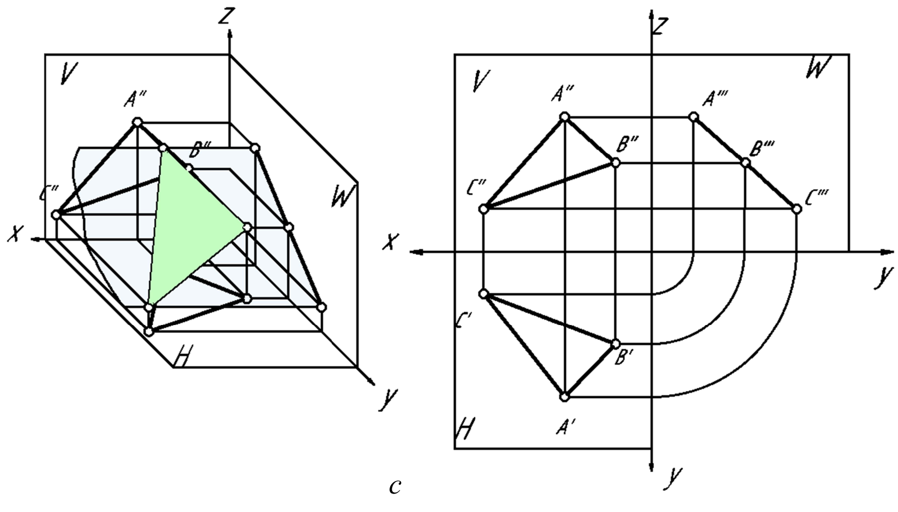
A projecting plane is a plane that is perpendicular to one plane of projection. Such planes are projected in one projection as an inclined line and as distorted shapes on the other. They are named as a projection plane they are perpendicular to. For instence: if a plane is perpendicular to the profile projection plane it is named as profile projecting plane.



a



b



c

Fig. 3.4. Projecting planes:
a – a frontal-projecting plane; *b* – a horizontal-projecting plane; *c* – profile-projecting plane

For more information please see [1, p. 17–21].

Principle (main) lines of a plane – frontal and horizontal. Practice

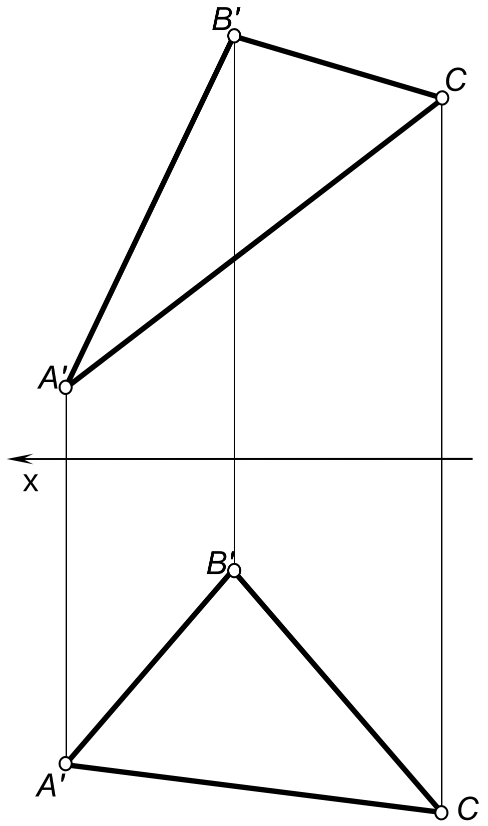


Fig. 3.5. Draw on a general plane β (ΔABC) arbitrary principle lines – frontal f (f'', f') and horizontal h(h'', h')

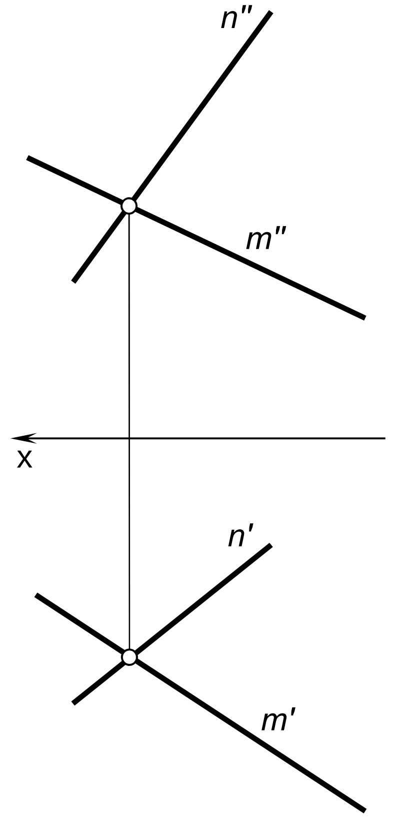


Fig. 3.6. Draw principle lines: frontal f(f'', f') and horizontal h(h'', h') for a general position plane ω (m∩n) through point of lines m and n intersection

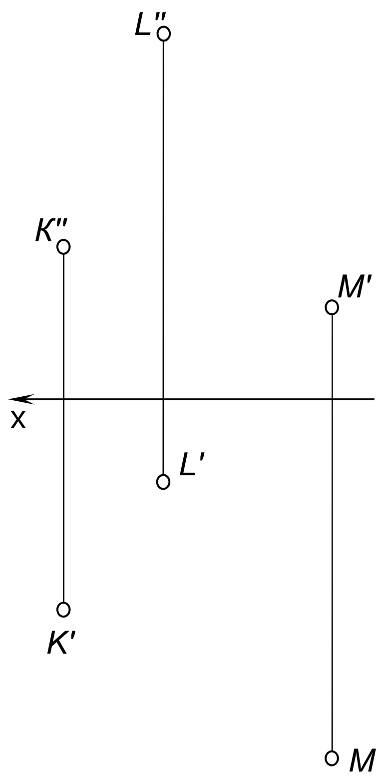


Fig. 3.7. Draw on a general plane φ (K, L, M) arbitrary principle lines – frontal f(f'', f') and horizontal h (h'', h')

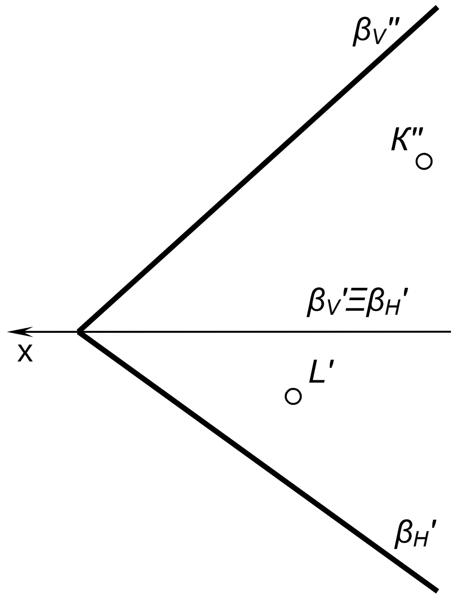


Fig. 3.8. Add a horizontal projection for point K(K') and a frontal projection for point L (L'') under condition they belong to a general plane β (β_V'', β_H') set by traces

Level planes.
True size of the level plane. Practice

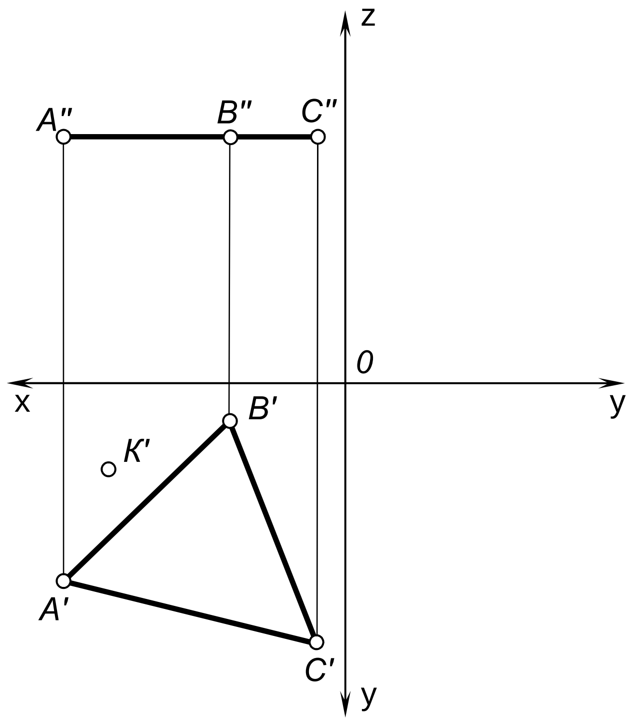


Fig. 3.9. Draw a profile projection of a horizontal level plane β(ΔABC) set by triangle ABC. Mark by "t. s." projection that equals to the true size of ABC. Add missing projections for point K (K" – ?, K', K‴ – ?), belonging to the plane

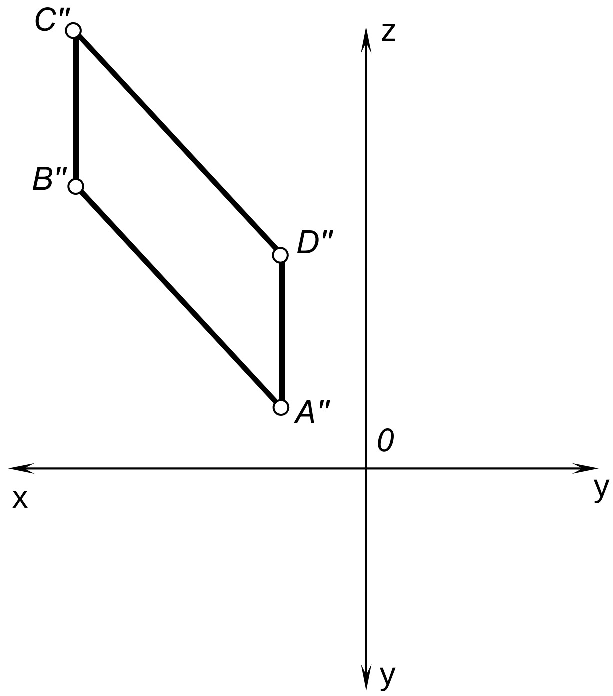


Fig. 3.10. Draw a horizontal and a profile projections of a parallelogram ABCD as a part of frontal level plane that is in 25 mm from frontal projection plane V. Mark by "t. s." projection that equals to the true size of ABCD. Draw a profile level line of the plane in 20 mm from profile projection plane

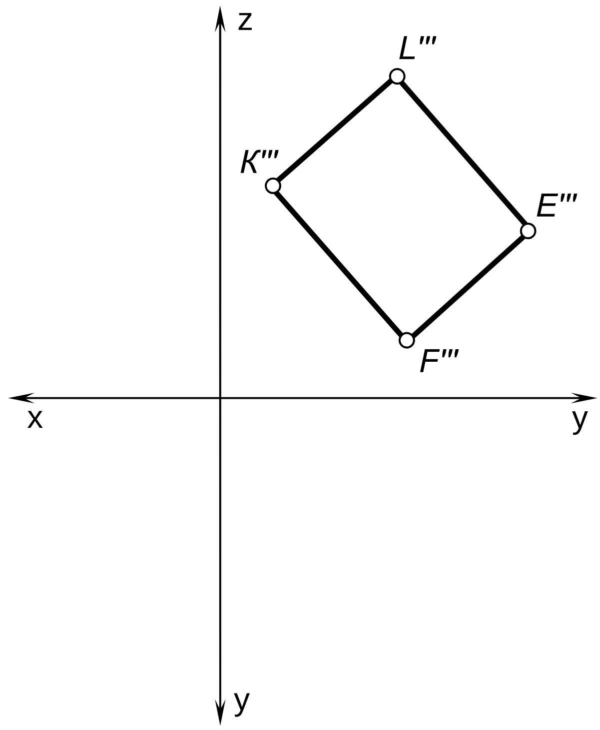


Fig. 3.11. Draw a frontal and a horizontal projections of a rectangle KLEF as a part of profile level plane located in 20 mm from profile projection. Mark by "t. s." projection that equals to the true size of KLEF. Draw frontal level line for KLEF in 30 mm from the frontal projection plane

Projecting planes.
Projecting plane's inclination angles. Practice

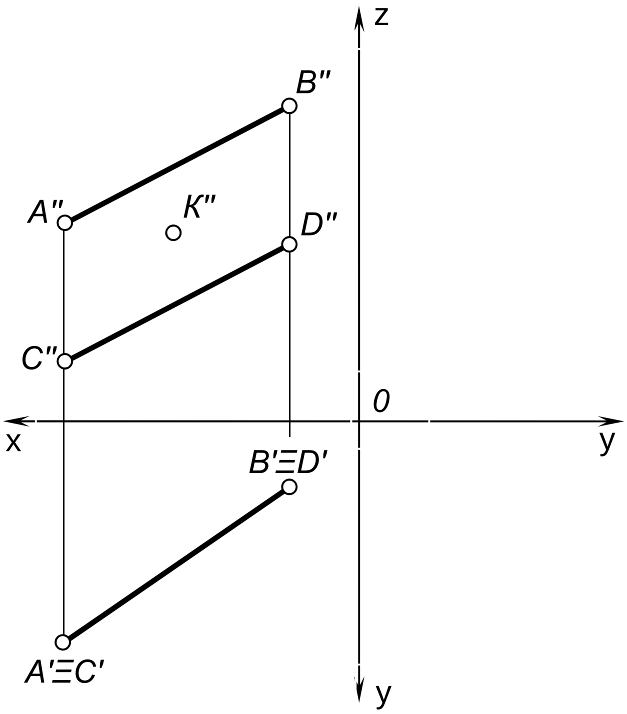


Fig. 3.12. Draw a profile projection of a horizontal-projecting plane set by parallel lines A and B β (A||B). Mark its inclination angles φ_V and φ_W to projection planes V and W relevantly. Add missing projections for point K(K'', K' – ?, K''' – ?) belonging to the plane

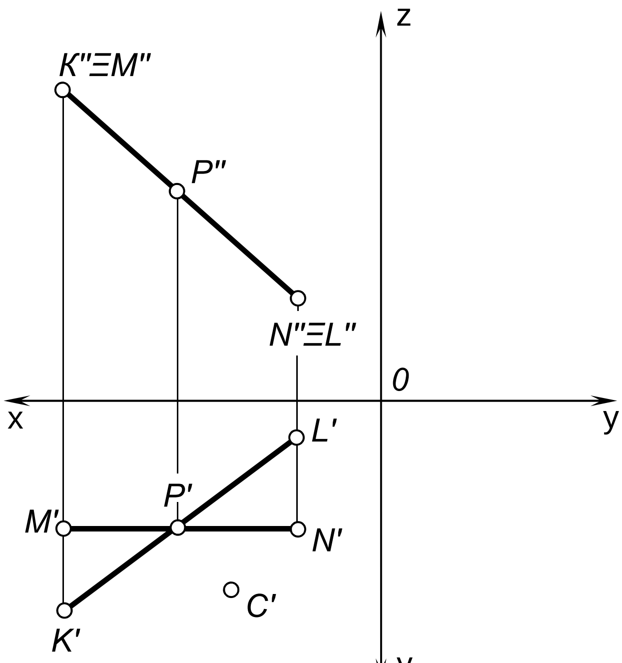


Fig. 3.13. Draw a profile projection of a frontal-projecting plane ω(KL∩MN) set by intersecting lines KL and MN, Mark its inclination angles φ_H and φ_W to projection planes H and W relevantly. Add missing projections for point C(C'' – ?, C', C''' – ?) belonging to the plane

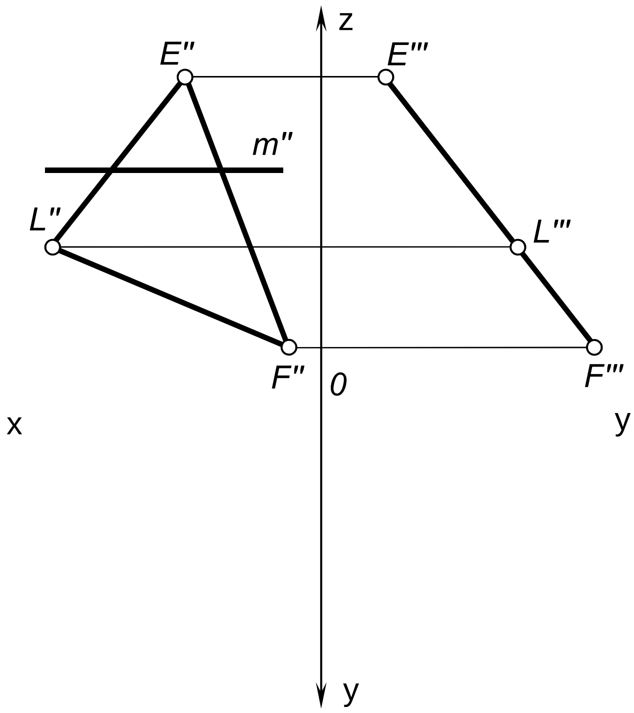


Fig. 3.14. Draw a horizontal projection of profile-projecting plane ε(ΔLEF) set by triangle LEF, Mark its inclination angles φ_V and φ_H to projection planes V and H relevantly. Add missing projections for profile-projecting plane m(m'', m' – ?, m''' – ?), belonging to the plane

Line and plane mutual location.
An intersection point of a line with a plane determination. Practice

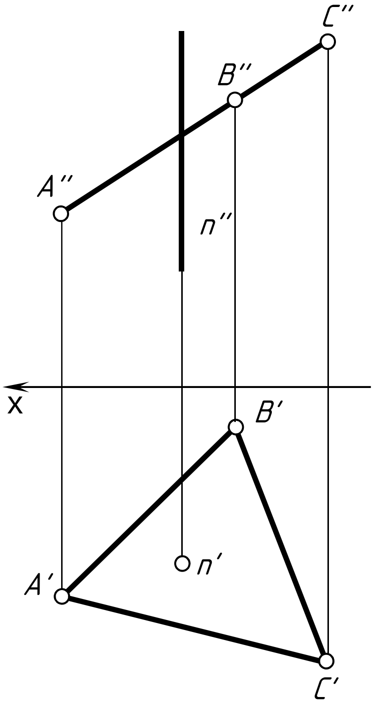


Fig. 3.15. Determine and draw both projections of intersection point K(K'' – ?, K' – ?) where K is an intersection point of the horizontal-projecting line n(n'', n') and frontal-projecting plane β(ΔABC)

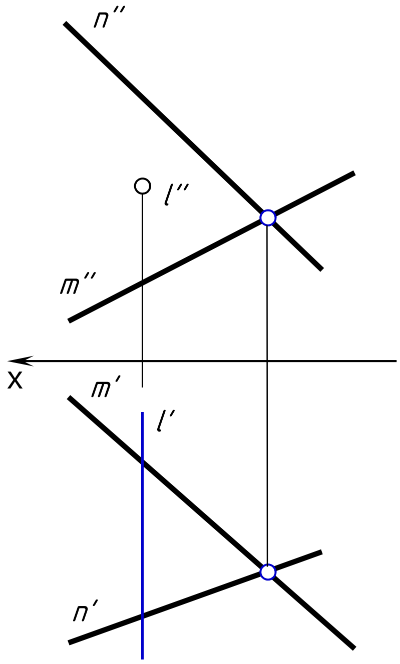


Fig. 3.16. Mark frontal and draw horizontal projections for point C(C'' – ?, C' – ?) of a frontal-projecting line l(l'', l') and general position plane β(m∩n) intersection. Design a drawing according to line visibility in relevance with a plane

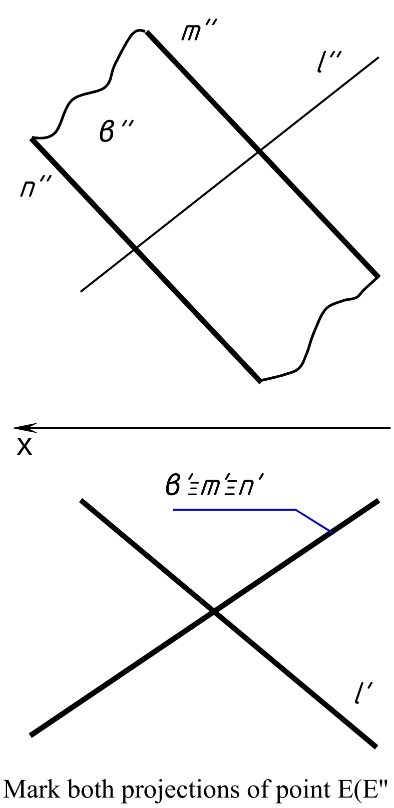


Fig. 3.17. Mark both projections of point E(E'' – ?, E' – ?) where E is an intersection point of a general position line l(l'', l') with horizontal-projecting plane β(m||n). Design a drawing according to line visibility in relevance with a plane

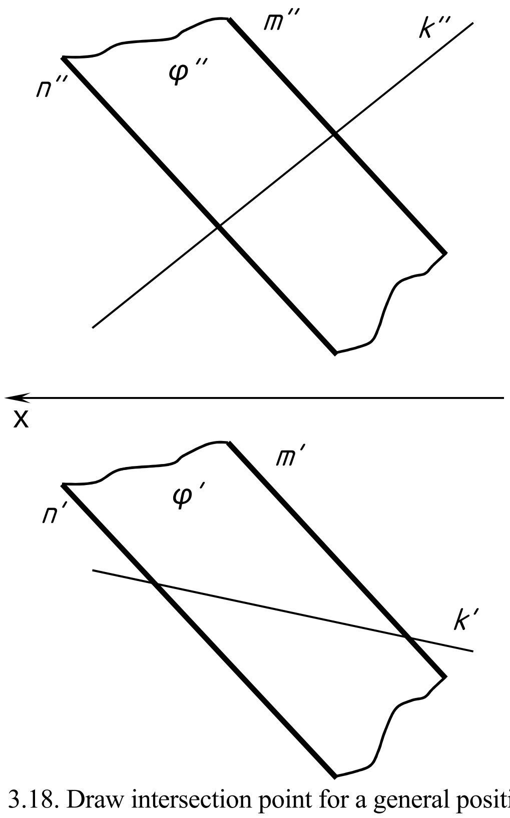


Fig. 3.18. Draw intersection point for a general position line l(l'', l') with general position plane φ(m||n). Design a drawing according to line visibility in relevance with a plane

Mutual location of two planes.
Intersection lines determination. Drawing parallel planes. Practice

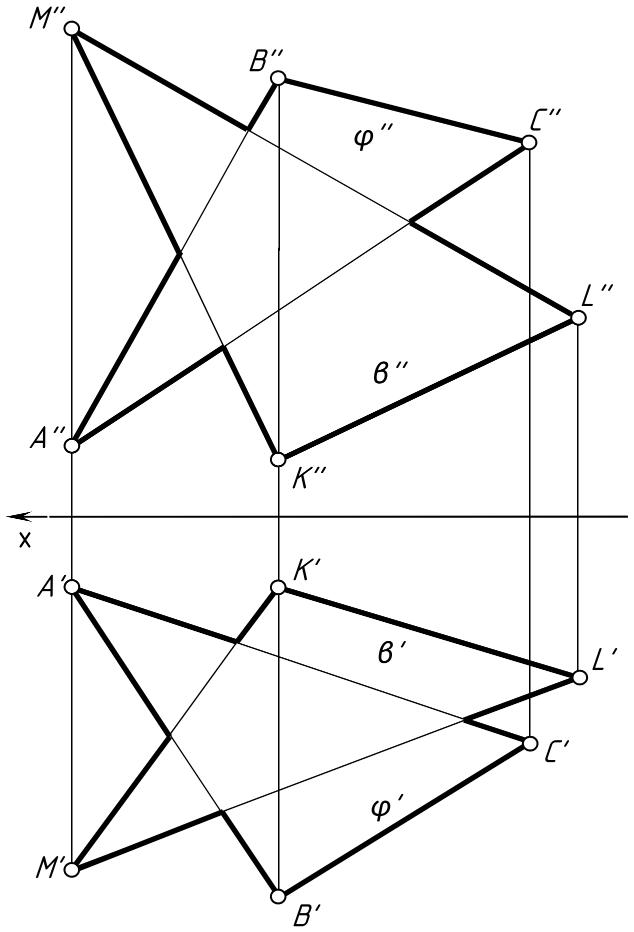


Fig. 3.19. It is necessary to determine an intersection line FE for two triangles φ(ΔABC) and β(ΔKLM), where triangles are general position planes. It is also required to determine visibility for triangles' sides and complete a drawing under these conditions

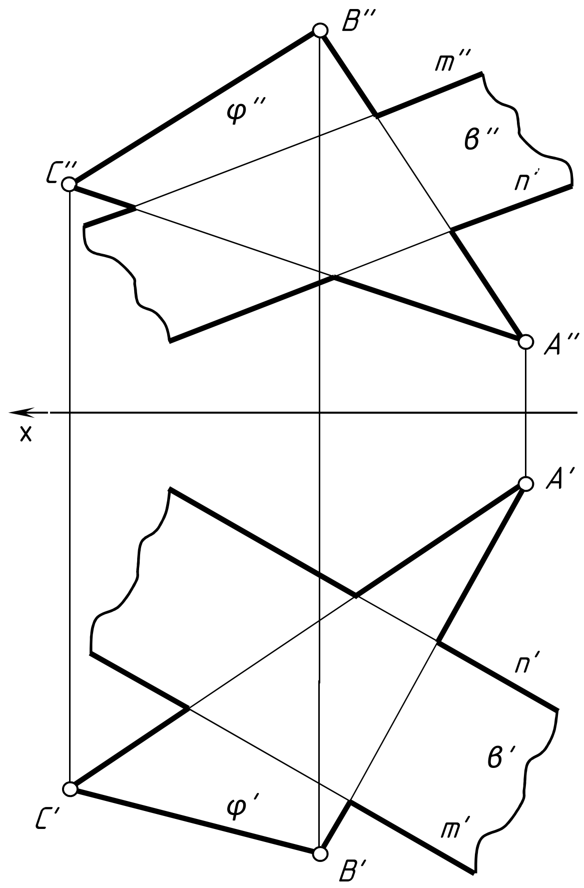


Fig. 3.20. It is necessary to draw intersection line NM for two general position planes φ(ΔABC) and β(m||n) set as triangle and two parallel lines respectively. Complete a drawing according to visibility

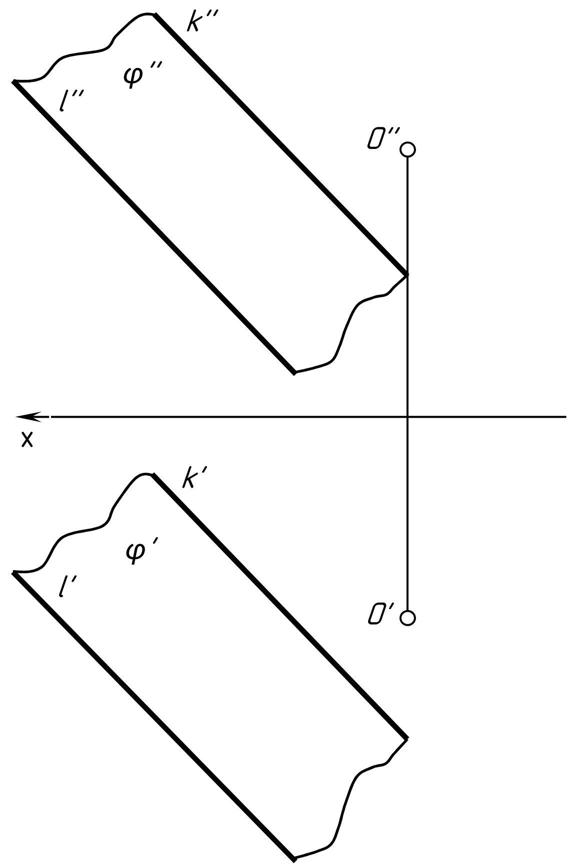


Fig. 3.21. Add a plane β(m ∩ n) parallel to a plane φ(l||k) if the plane φ is set by two intersecting lines and β is required to be set by two parallel lines

4. SURFACES

A surface is the locus of the different positions of a line or curve in space. **A surface** is the envelope surface that surrounds **a solid (body)**.

A generatrix is a line that forms a surface by motion on some direction. The movement may retain or change a shape. The movement of the generatrix can be subjected to a law, or it can have an arbitrary character. In the first case, the surface will be **legitimate or regular**, and in the second one it will be **random (irregular)**. The law of the generatrix movement is usually determined by another line, which is called a **guideline** or **a guidex** that a generatrix slides on. In some cases, one of the guidelines can be converted to a point (vertex conical surface) when one end of generatrix is fixed, or in the infinity (cylindrical surface).

4.1. Polyhedral surfaces and solids

A polyhedral surface is a surface, that is formed by the movement of a rectilinear generatrix on a broken line, for example, pyramidal (with top) and prismatic (without top) surface.

A polyhedron is a solid shape that is bounded by a polyhedral surface**.** Polyhedron outline is drawn as projections of its faces and edges in a drawing.

A prismatic surface is a surface, that is formed by the movement of a rectilinear generatrix by a broken line.

A prism is a polyhedron that has two parallel bases that are the same, and faces that are tetragons (quadrangles). A prism is called **right** (straight) if its edges (lines of intersection of the adjacent faces) are perpendicular to the base, and **oblique** (inclined) if it is not **A pyramidal surface** is a surface, that is formed by the movement of a rectilinear generatrix by a broken line with one fixed generatrix end – vertex.

A pyramid is a polyhedron having one base (**polyhedron**), **vertex (top),** faces (triangles) and **edges (ribs)** (lines of intersection of the adjacent faces) that are intersected at one point (the top of a pyramid) in general**.** A pyramid is called **truncated** if its top is cut and the pyramid has two bases. If the top base is parallel to the bottom one, a pyramid is called **frustum**.

Prisms and pyramids are called **regular** if its base is a regular polygon, that can be inscribed into the circle and has equal sides.

If points belong to edges, meridians (outlines) in other words to any specific elements of a surface, their projections can be found on their belonging by connection line.

There is a particular case when a surface is a projecting surface that enable to find missing projections of points lying on it without additional constructions, as this kind of surfaces has **a collective property**, when a projecting object (line, plane, surface) collects everything belonging to a surface on a linier outline. Right prisms and cylinders are such kind of surfaces (see fig. 4.1).

In order to find **missing points of projections** that belong to some surface (except projecting), it is necessary to build an auxiliary line on a given surface, passing through a given point of the projection. Firstly, it is necessary to construct the projection of this auxiliary line, and then to draw the required projection of the point on it. For this purpose, we can use various lines: **lines of generatrix, parallels, meridians**, etc. It is feasible by the postulates described above: **if a point belongs to a line, the projection of this point belongs to the projection of this line.** There are two methods, that a described below.

The Generatrix Method.

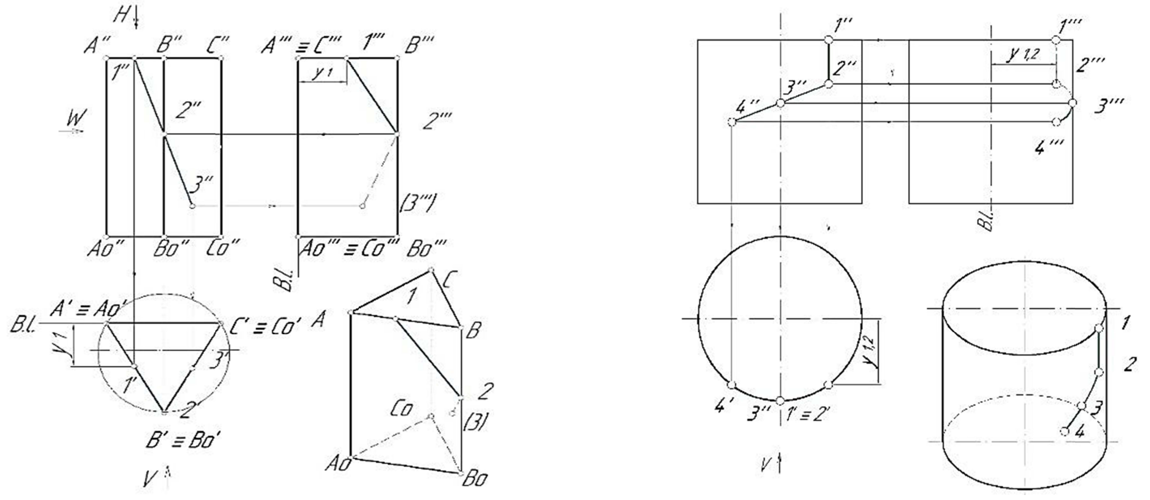
This method is also called **the method of forming.** The line m'' is one of the uncountable numbers of generatrixes is drawn from the pyramid top up to its base through the point K'' (fig. 4.2). **A line** is the shortest distance between two points. In order to draw any line, we should draw its two end points. Both these points of the line m belong to some specific elements of the surface, and their projections can be found by connection lines in accordance with their belonging and without any auxiliary actions. One of them is on the top (vertex) and the other is on the base of pyramid or cone. Their horizontal projections are the result of connection lines intersection with the relevant object's elements – the vertex and the base. Given point K can also be specified by a connection line intersection with the constructed auxiliary line.

The method of auxiliary cutting planes.

This method is also known as **the level planes method**. When an auxiliary cutting plane intersects a surface parallel to the base, it forms a shape similar to the surface's base. (a polygon or a circle that depends on the type of surface).

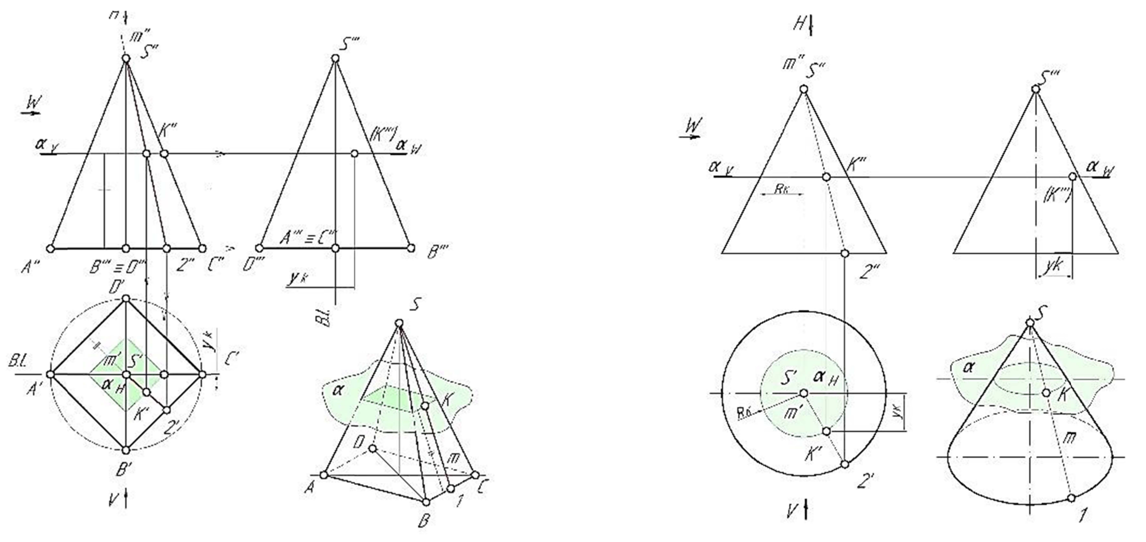
If an imaging cutting plane of the level passes through the projection of the required point P'' projections of this point belong to projections of the section. It enables to draw firstly the section and then specify the point on its side/sides.

If it is a polygon (pyramid case), a similar shape can be found by an additional point on any side edge. (Except edges that are located vertically). As the shape is similar to the base, each side of the auxiliary shape is parallel to the base sides. The shape can be drawn by drawing parallel lines from the found additional point. If a section is a circle (cone, sphere, torus, etc.) a radius should be measured (distance between the centre line (axis line) and the generatrix line (surface boundary line).



a *b*

Fig. 4.1. Specifying points on projecting surfaces:
a – prismatic surfaces; *b* – cylidrical surfaces

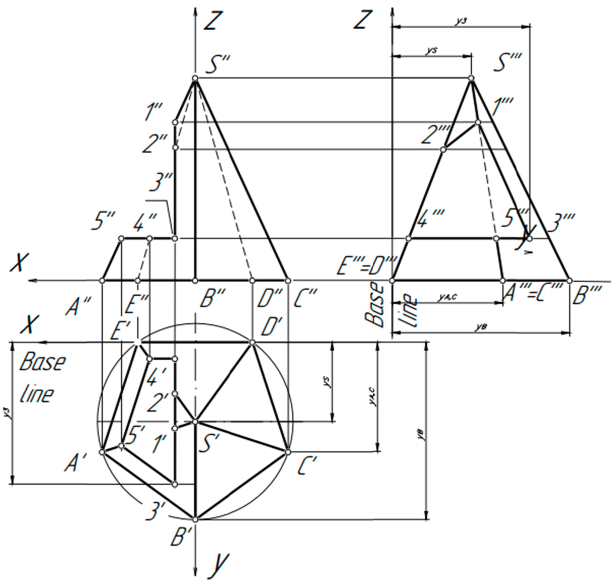


a *b*

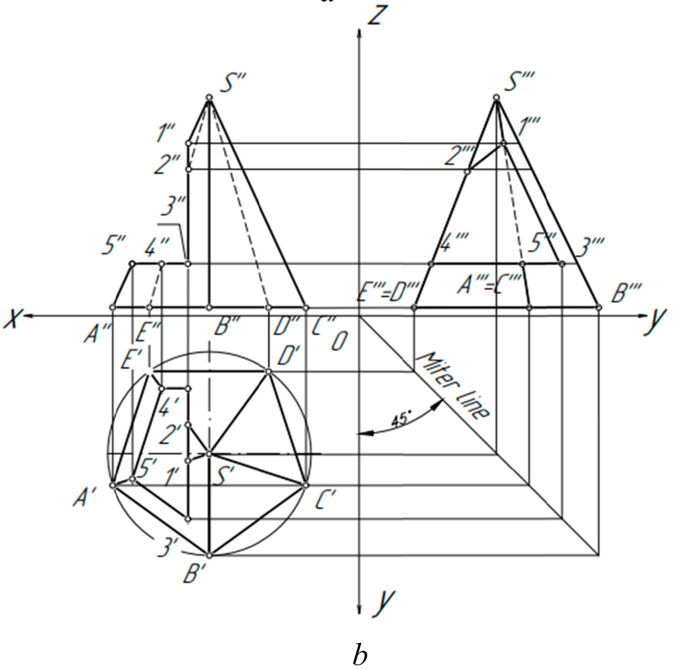
Fig. 4.2. Specifying points on non projecting surfaces eather by the Generatrix Method and The Method of auxiliary cutting planes:
a – piramidal surfaces; *b* – conical surfaces

Constructing the third projection – the profile.

A point in space has three coordinates – x, y, z as it has been mentioned before, therefore, it is possible to set axes x, y and z for completing projections. This method is called **a baseline method** (fig. 4.3, *a*) and implies setting axes separately for each projection plane, not as a Cartesian system. In this case, each projection has a pair of coordinates: the frontal projection is a combination of z and x axes, the horizontal has x and y coordinates, and the profile projection is obtained by z and y axes. Such separate comprehension allows locating projections flexibly relatively to each other. A baseline method means using the y-coordinate that is true for two projections by measuring it from one axis and taking it from another for drawing a profile projection. This method is mentioned as a kind of **measuring method** with using a divider or a rule. There is also **a miter line method** (fig. 4.3, *b*) where there is no measuring, though using an inclined in 45 degrees line that should be drawing from the front view down. This method is not considered in detail, as because of its disadvantages it is not supposed to be used.



a



b

Fig. 4.3. Methods for constructing the profile projection:
a – a base-line method; *b* – a miter method

For more information see [1, p. 34–43].

Coming back to a baseline method, if a horizontal projection of a surface has an axis of symmetry (circle, square and other), a baseline should be taken through this axis. It eases drawing by having two equal coordinates (positive and negative y) that can be measured ones and used twice. If the horizontal projection of a surface has no axis of symmetry (triangle, pentagon and other), baseline should be taken through the topmost of base in this case, all measurements will be taken to one direction (positive y).

Note! There is no obligatory in taking a base line, it can be taken anywhere. Therefore, all recommendations that have been given above are supposed to ease work as much as it is possible.

Prisms. Practice

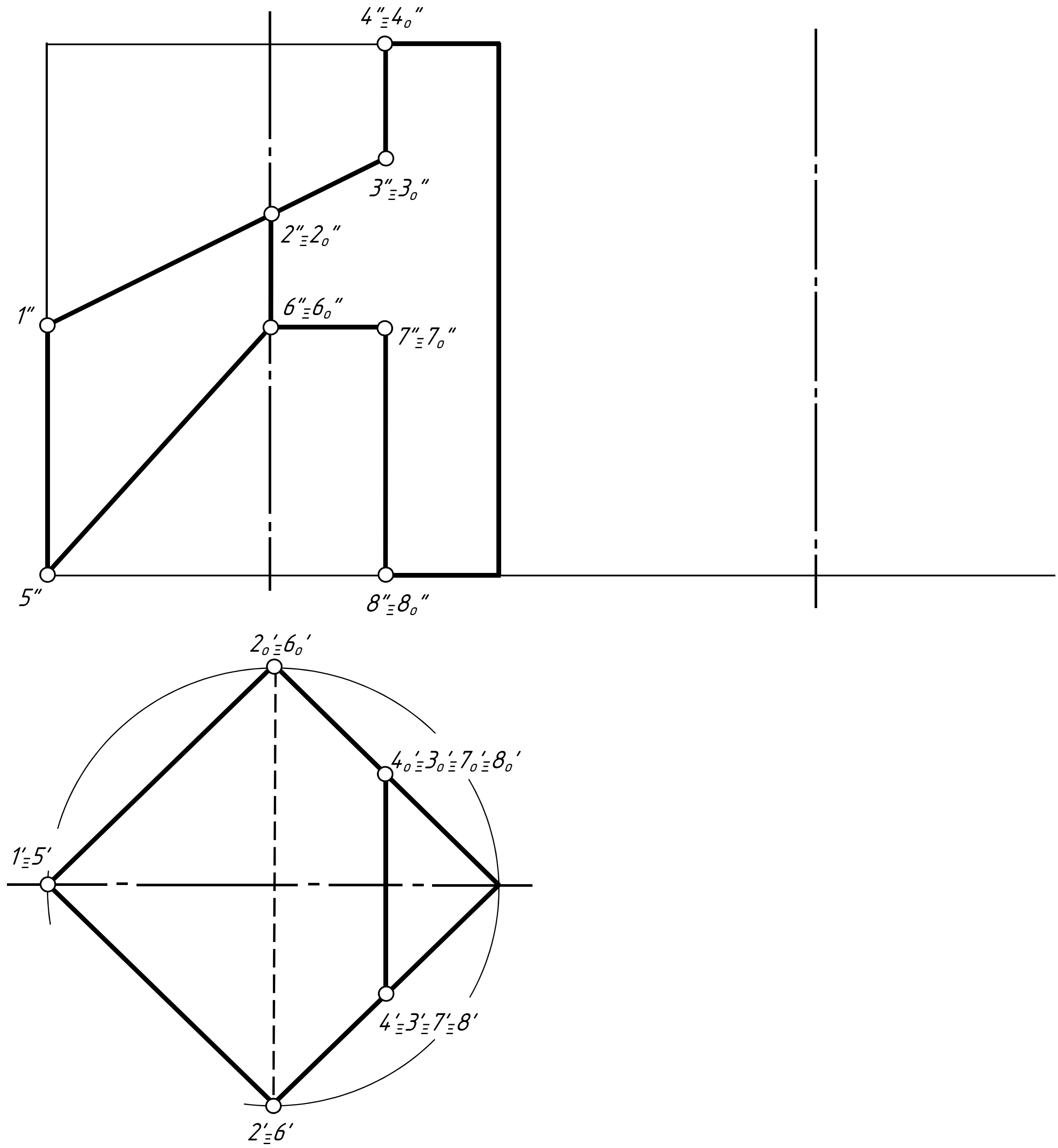


Fig. 4.4. Draw a profile projection of a right square prism with sections: by horizontal level cutting plane, two frontal-projecting cutting planes, and two profile level cutting planes

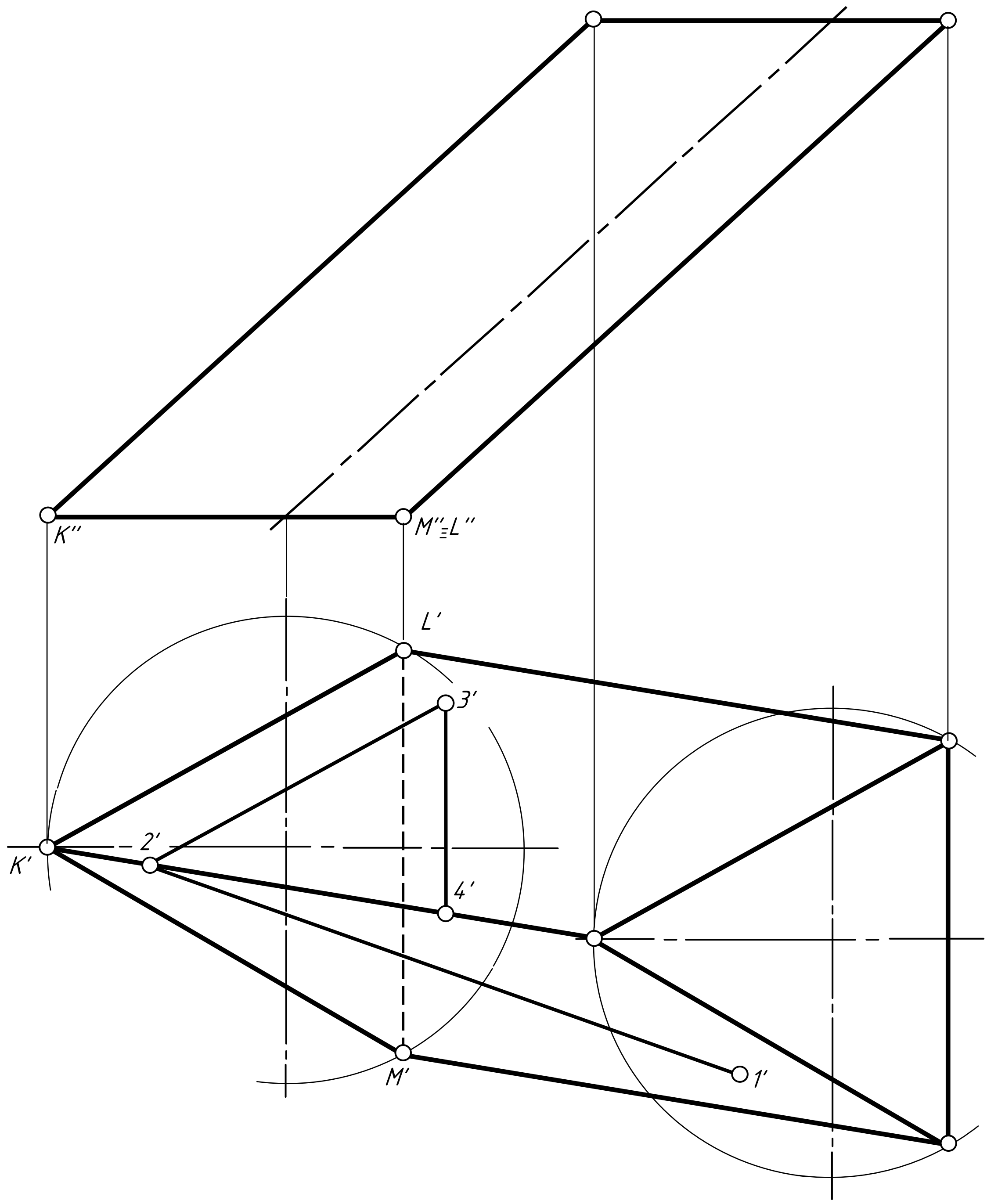


Fig. 4.5. Add frontal projection of a broken line 1234 (1'2'3'4')
on an oblique triangular prism

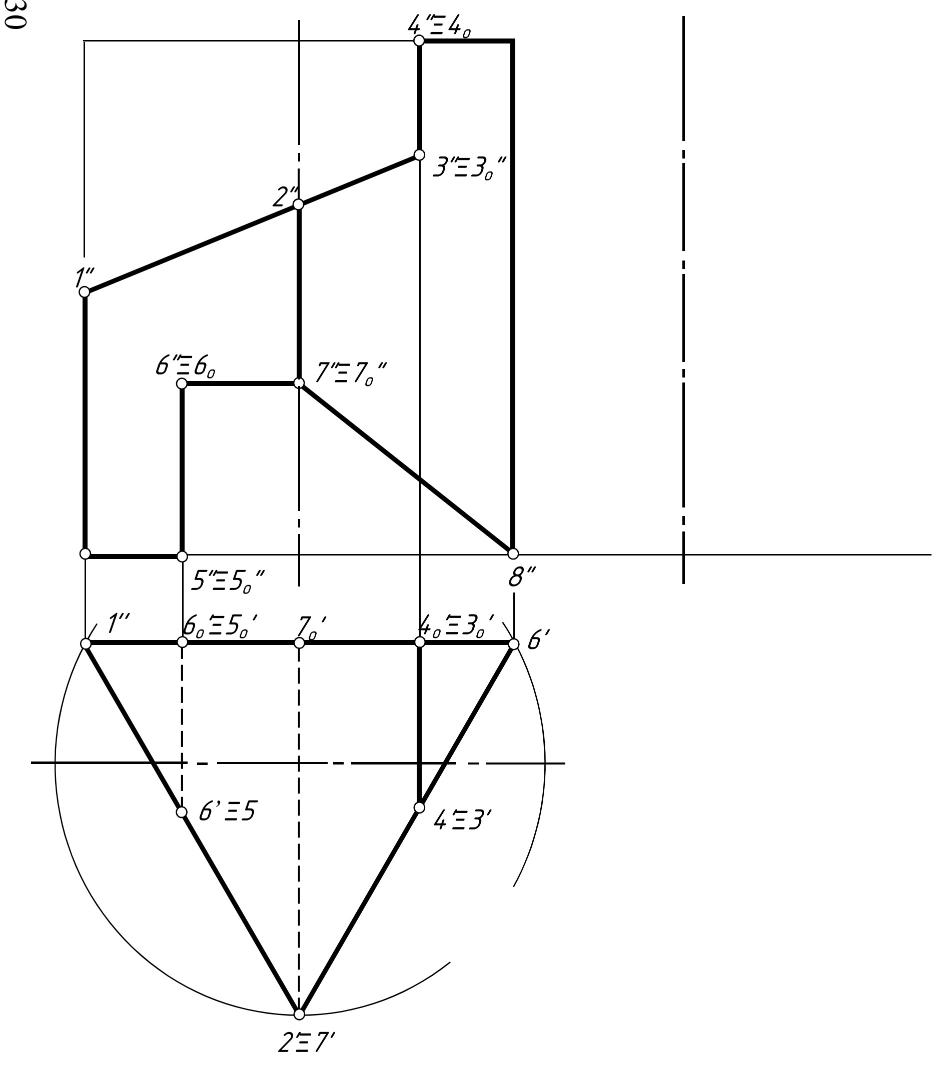


Fig. 4.6. Draw a profile projection of a right triangular prism with sections: by two frontal-projecting cutting planes, a horizontal level cutting plane, and two profile level cutting planes

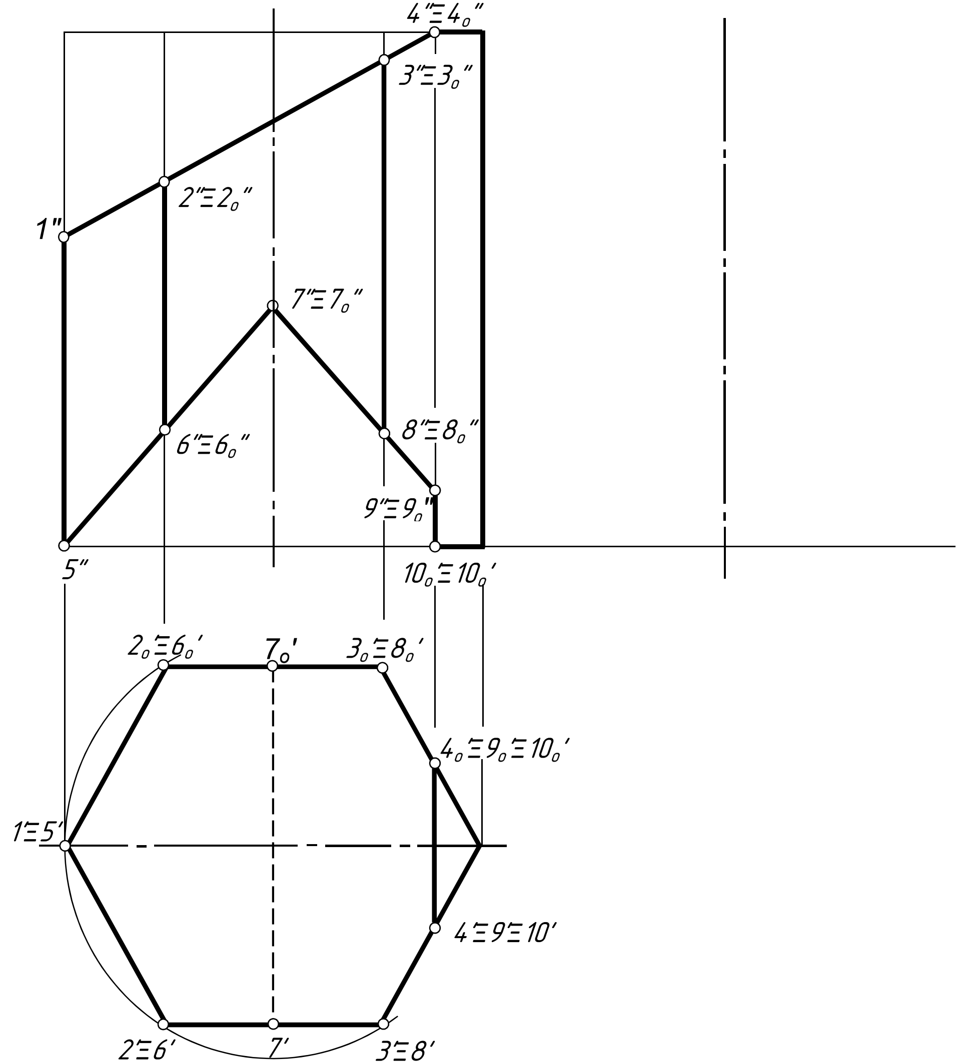


Fig. 4.7. Draw a profile projection of a right hexagonal prism with three frontal-projecting sections

Pyramids. Practice

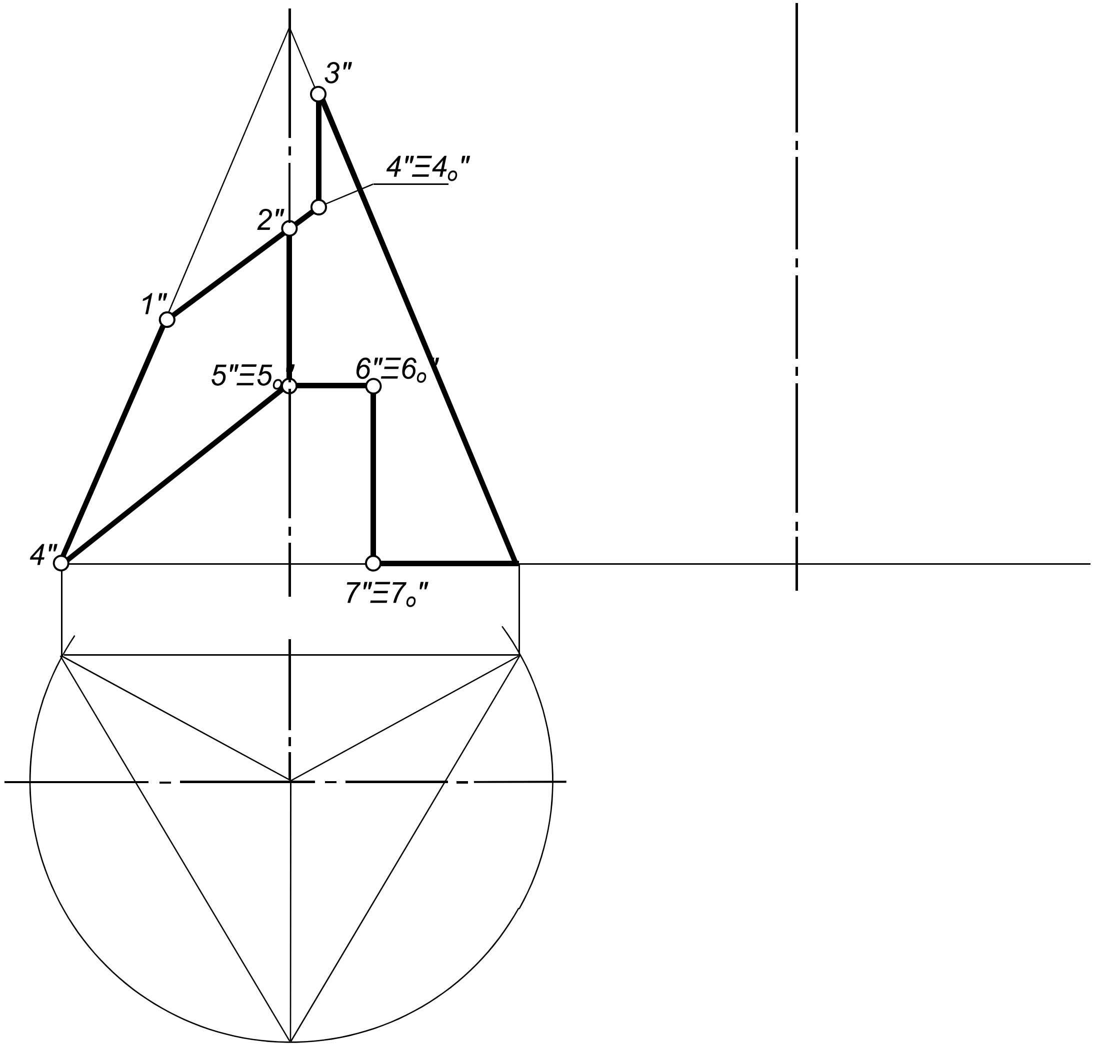


Fig. 4.8. Complete a horizontal and add a profile projection of a right triangular pyramid with sections: by horizontal cutting planes, two frontal-projecting cutting planes, and two profile level cutting planes

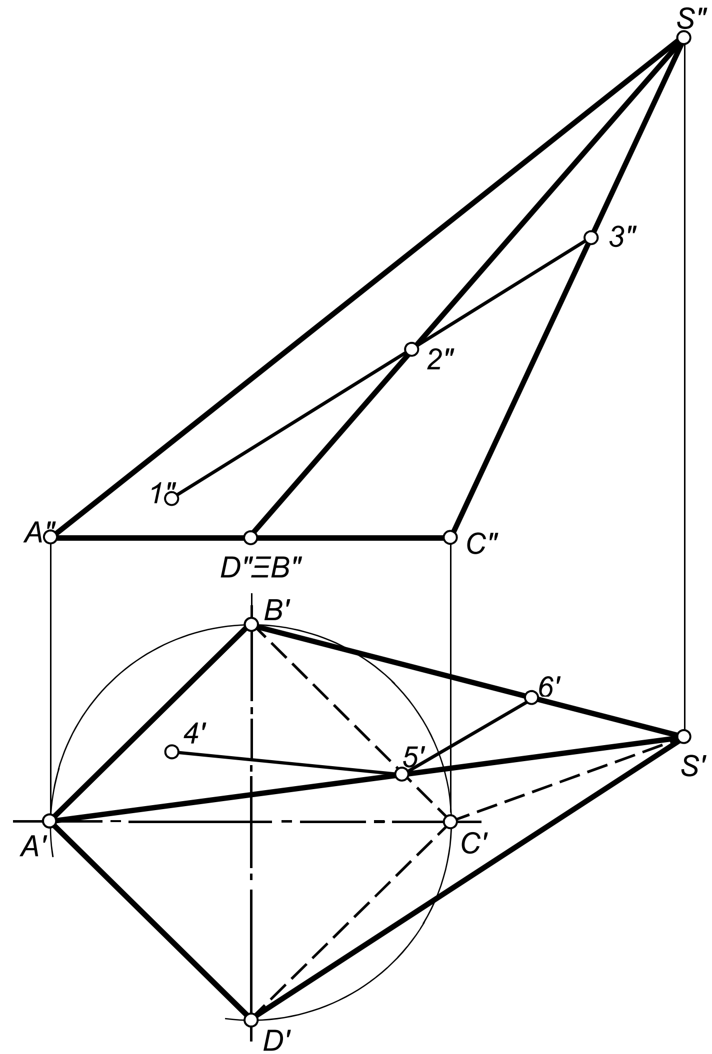


Fig. 4.9. Draw a horizontal projection of a broken line 123(1"2"3") and a frontal projection of another broken line 456(4'5'6') on an oblique square pyramid

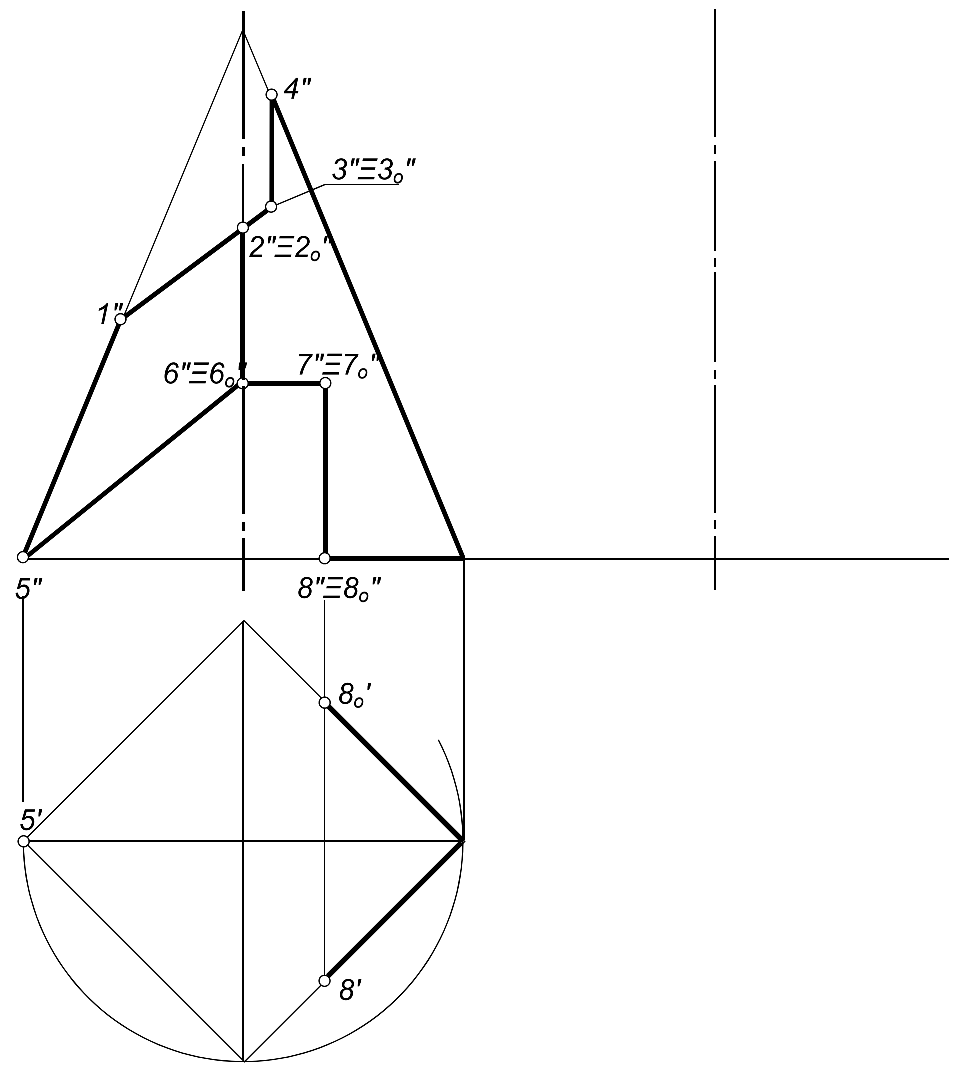


Fig. 4.10. Complete a horizontal projection and add a profile projection of a right square pyramid with sections: by horizontal level cutting plane, two frontal-projecting cutting planes, and two profile level cutting planes

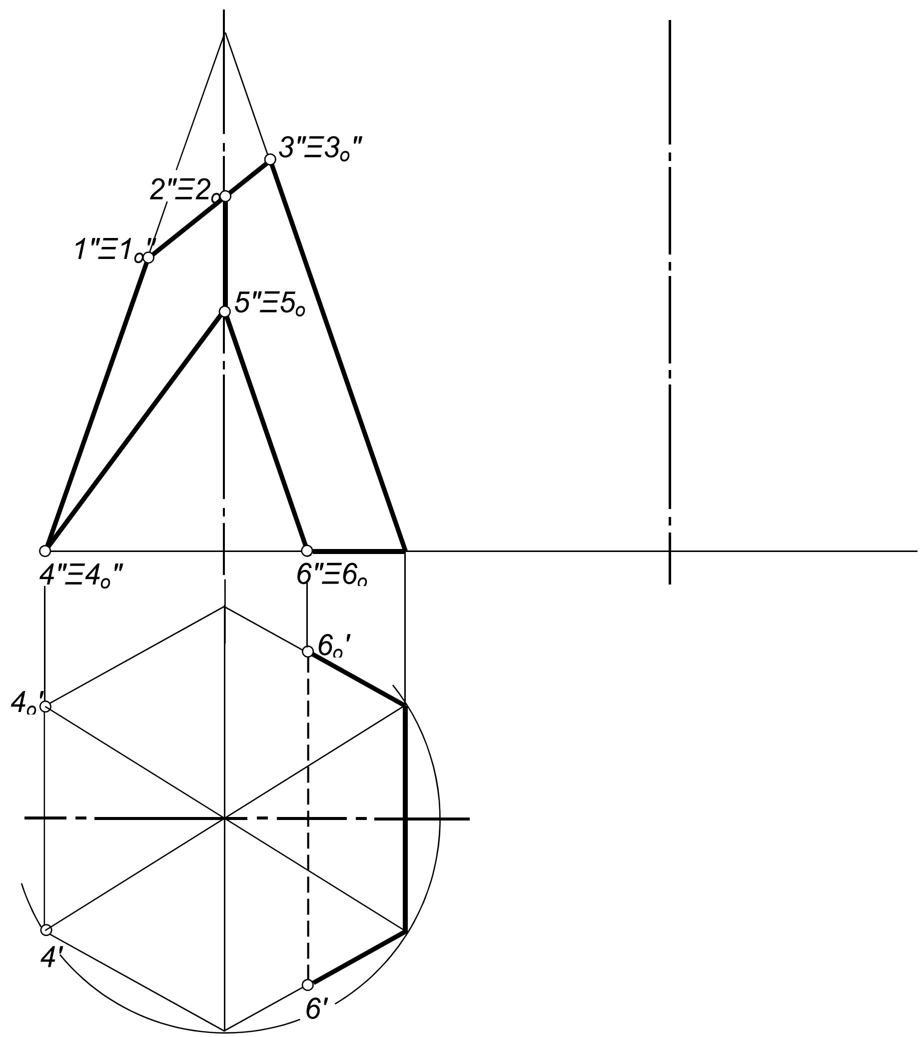


Fig. 4.11. Complete a horizontal projection and add a profile projection of a right hexagon pyramid with three frontal-projecting sections

4.2. Revolution surfaces

4.2.1. Ruled surfaces. Cylinder and cone (circular)

A revolution surface is a surface that is formed by a movement of a line (generatrix) around another fixed straight line. This fixed straight line is called an axis line of rotation. A surface is called **ruled** if the generatrix is a straight line, and it is called **curved** when the generatrix is not a straight line. All points of generatrix rotate around the axis on circles with relevant radiuses that are called **parallels.**

The parallel with the smallest diameter is called **the neck or throaght**, and with the largest diameter as **the equator**. A meridian is an outline which is formed by intersection of a surface with the plane of level – frontal or profile.

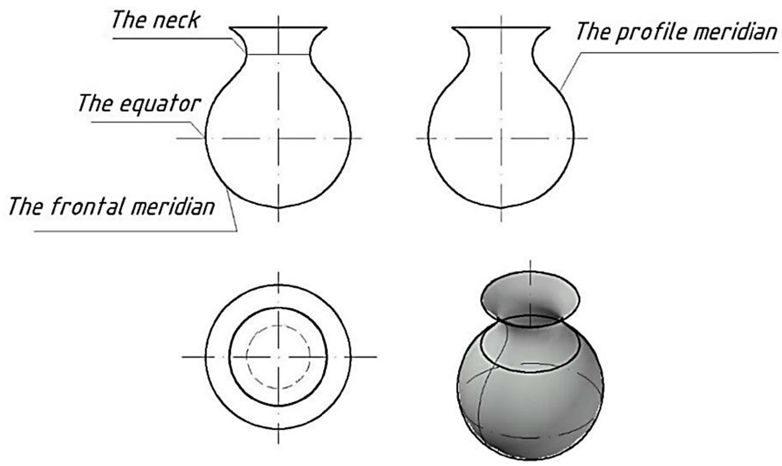


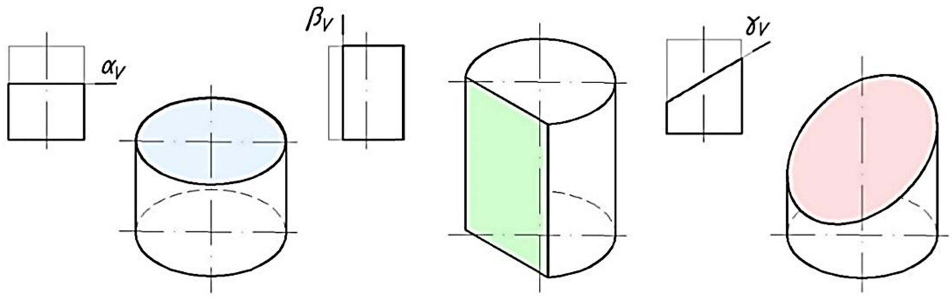
Fig. 4.12 A revolution surface

A cylindrical surface of revolution is a ruled surface formed by parallel movement of a straight line (generatrix) on a circular or elliptical generatrix. A cylinder is a body bordered by cylindrical surface of revolution (side surface) with two parallel bases which are perpendicular to the axis of rotation.

A cylinder is circular if its bases are circles, and **elliptical** if the bases are ellipses. **A cylinder** is right if the axis of rotation is perpendicular to its bases.

There are three possible cylinder **sections**:

1. If a cutting plane is parallel to the bases it cuts a shape that is similar to the base (fig. 4.13, *a*).
2. If a cutting plane is parallel to the axis of rotation it cuts some rectangle (fig. 4.13, *b*).
3. If a cutting plane is inclined to the axis of rotation (angle is not 90^{o}) it cuts an ellipse (fig. 4.13, *c*).



a *b* *c*

Fig. 4.13. Three possible cylinder sections:
a – a circle; *b* – a rectangle; *c* – an ellipse

A conical surface is produced by rotation of a straight line (generatrix) round the circular or elliptical axis (directrix) intersecting it. In other words, one end of the generatrix is fixed and referred to as a vertex (top).

A cone is a solid bordered by a conical revolution surface (side surface) with one base which. **A cone** is right if the axis of rotation is perpendicular to the base, and a cone is **oblique** when the axis is not perpendicular to the base. **The frustum cone** is formed by a cutting plane that is parallel to the base of the cone, and it has two bases. It is a particular case of **a truncated cone** where the cutting plane (top base) is not parallel to the bottom.

There are five types of **conical sections** that are subdivided in accordance with the direction of a cutting plane.

1. Cutting plane is perpendicular to the cone axis, a circle is obtained (fig. 4.14, *a*) (in case if a cone is circular).
2. Cutting plane passes through the cone vertex and cuts a pair of generatrix lines, which form a triangle (fig. 4.14, *b*).

If the cutting plane is inclined to the rotation axis and does not pass through its vertex, then it is possible to obtain three types of lines in the section.

3. If a cutting plane is not parallel to any generatrix, an ellipse is obtained (fig. 4.14, *c*).
4. If a cutting plane is parallel to one generatrix, a parabola is obtained (fig. 4.14, *d*).
5. If a cutting plane is parallel to two generatrixes, a hyperbola is obtained (fig. 4.14, *e*).

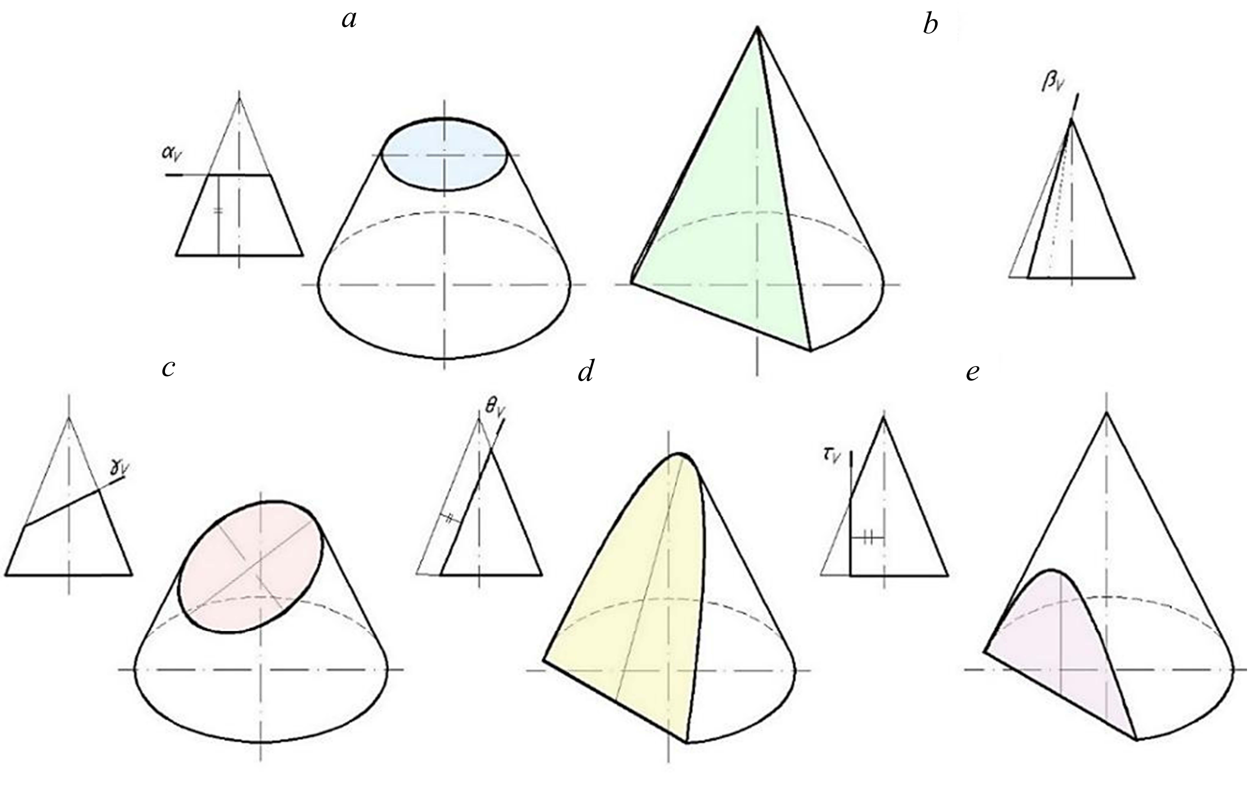


Fig. 4.14. Five possible cone’s sections:
a – by a parallel plane to its base; *b* – by a plane passed through the cone vertex;
c – by an inclined plane to all generatrixes; *d* – by a parallel plane to a generatrix;
e – by a perpendicular plane to its base

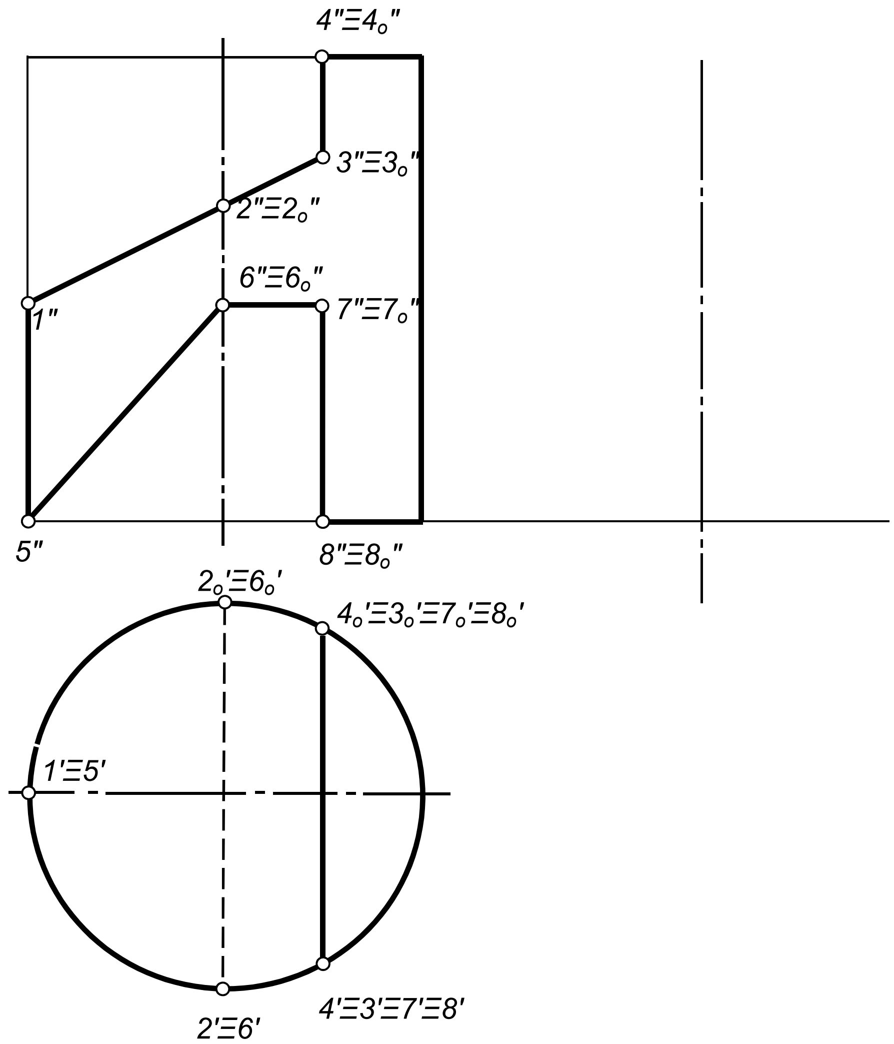


Fig. 4.15. Draw a profile projection of a right circular cylinder with sections: by a horizontal level cutting plane, two frontal-projecting cutting plane, and two profile level cutting plane

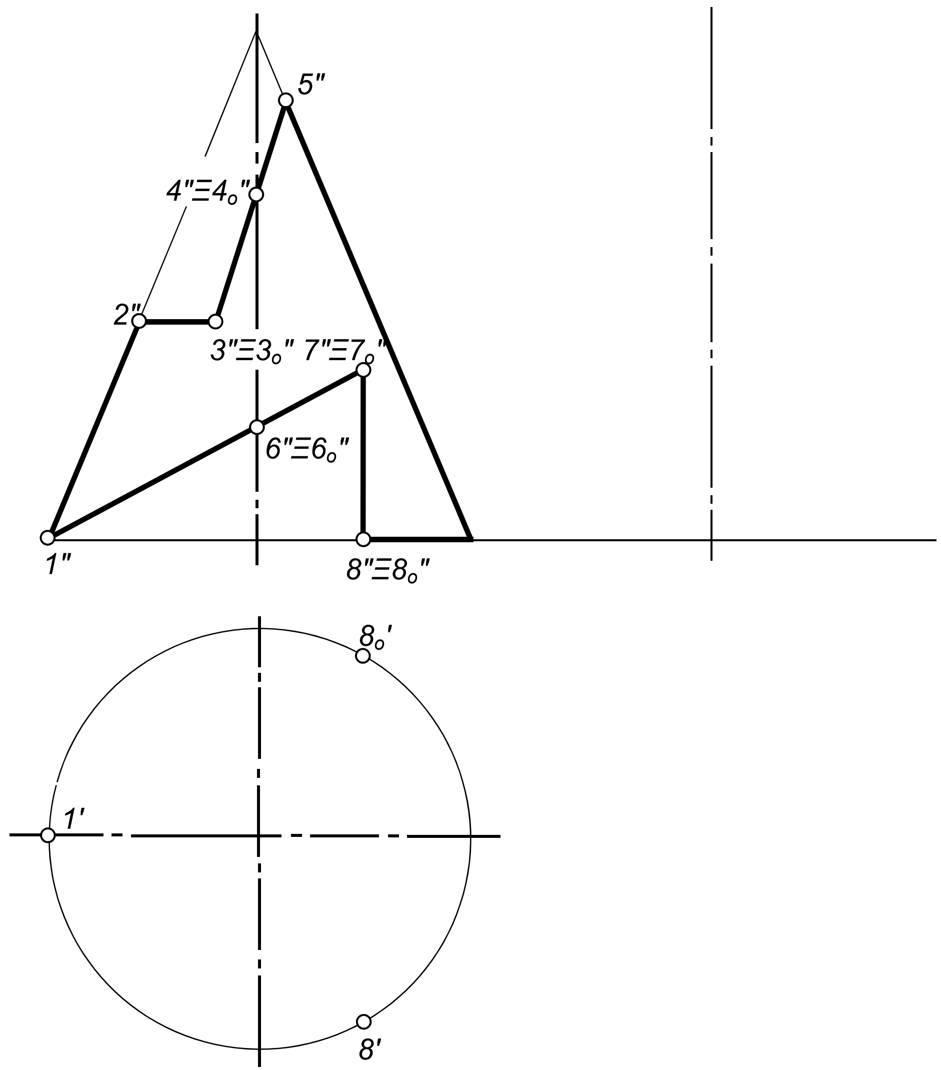


Fig. 4.16. Complete a horizontal projection and add a profile projection for a right circular cone with sections: by horizontal level cutting plane, two frontal-projecting cutting planes, and profile level cutting planes

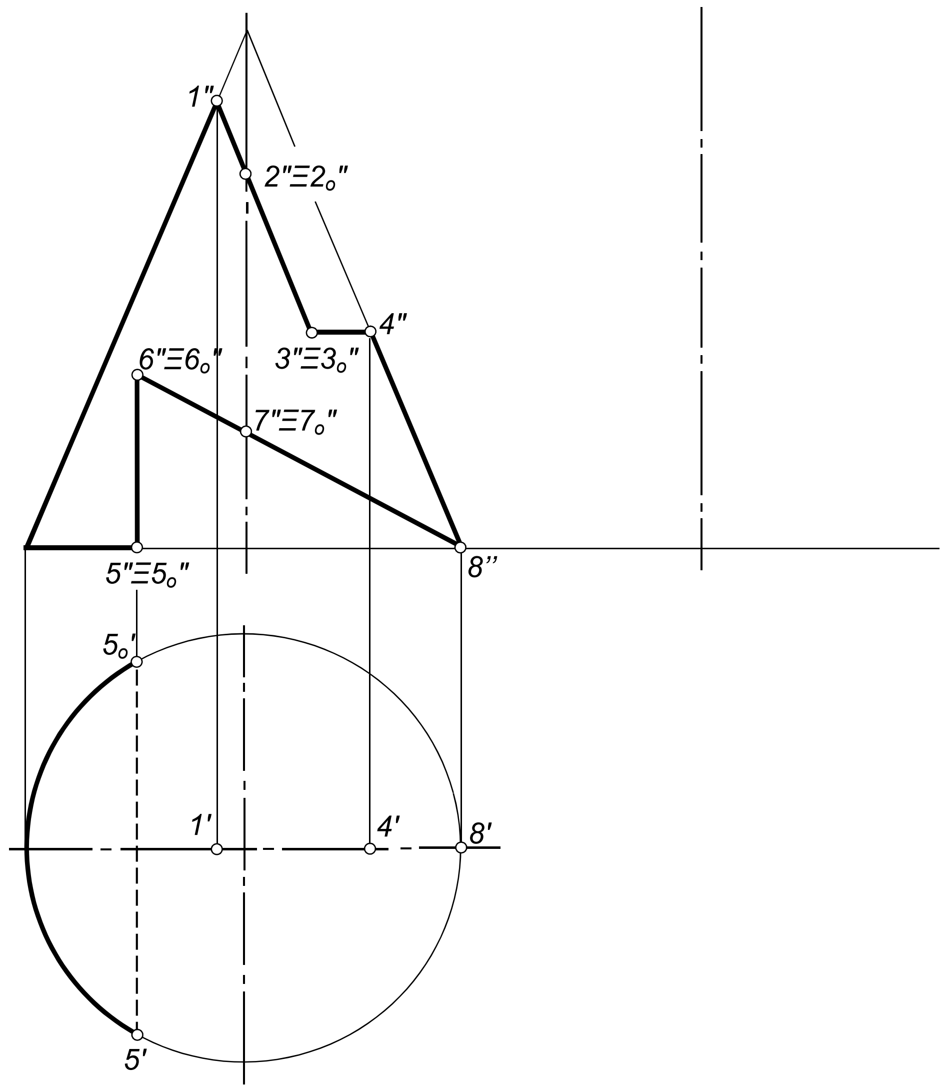


Fig. 4.17. Complete a horizontal projection and add a profile projection for a right circular cone with sections: by two frontal-projecting cutting planes, a horizontal level, and a profile cutting planes

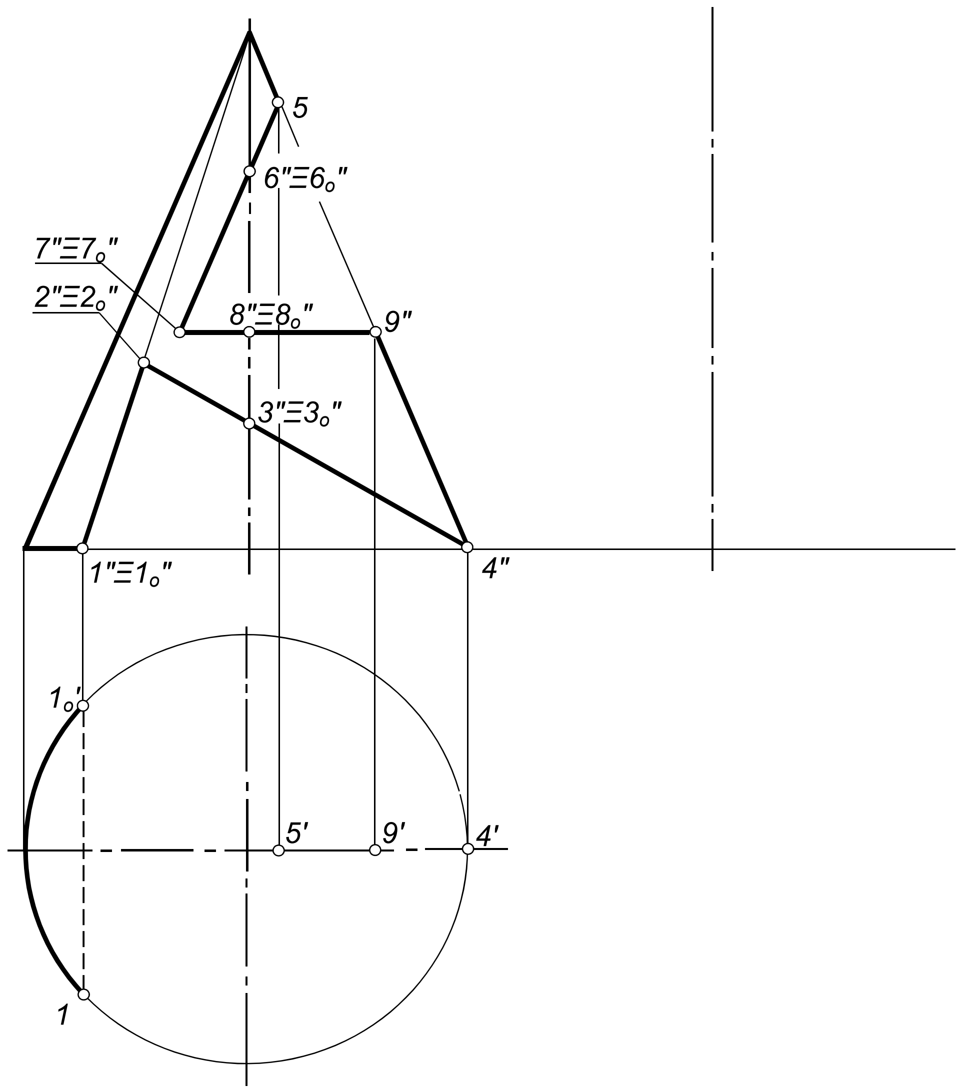


Fig. 4.18. Complete a horizontal and add a profile projection for a right circular cone with sections: by three frontal-projecting cutting planes and a horizontal level cutting plane

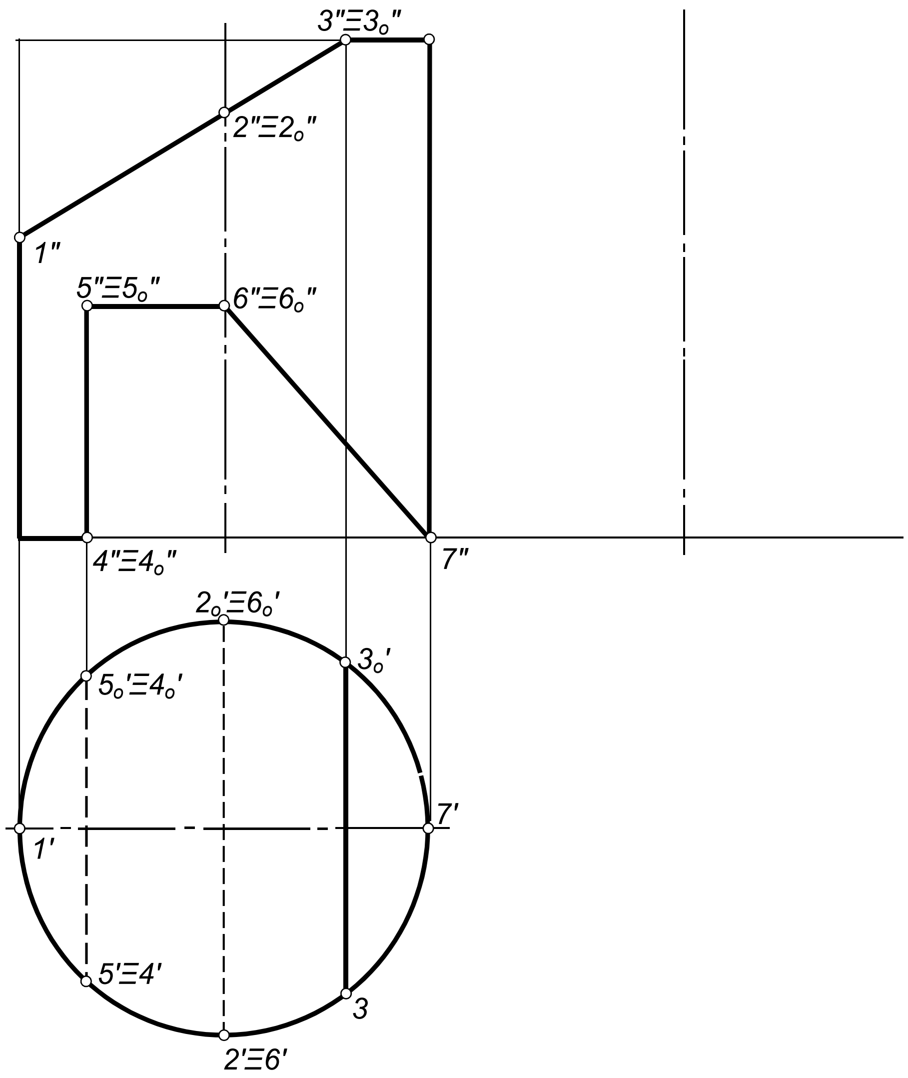


Fig. 4.19. Draw a profile projection for a right circular cylinder with sections: by a horizontal level, two frontal-projecting, and a profile level cutting planes

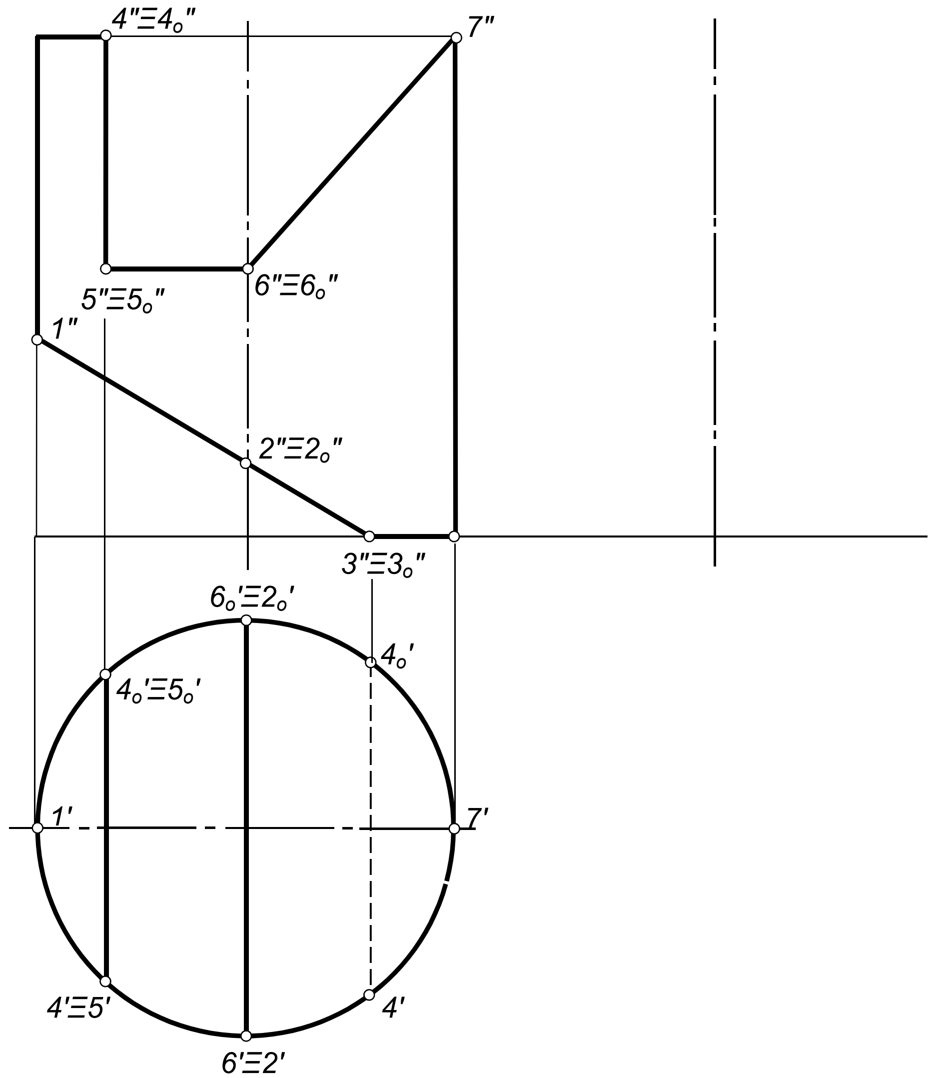


Fig. 4.20. Draw a profile projection for a right circular cylinder with sections: by a horizontal level, two frontal-projecting, and profile level cutting planes

4.2.2. Surfaces of the 2nd order. Sphere

A spherical surface is a surface formed by rotation of a circle round its diameter. This surface is a set of equally-spaced points from the centre. Sphere is the only one surface which has an infinity numbers of axis passed through the centre. This property is used in solving points on a surface and many other descriptive problems.

Spherical surfaces are drawn as a circle in all three projections with the equal diameter. In other words, all its outlines are circles. Every point on the sphere is rotated on some circles, which are called parallels.

A ball (sphere) is **a solid (body)** bordered by a spherical surface.

A frontal outline is **the main frontal meridian**. It is projected as a horizontal line on the horizontal axis line in the horizontal projection and as a vertical line is the same with the vertical axis line in the profile projection.

A horizontal outline is **the equator** of a sphere. It takes the same place with the horizontal axis in the frontal and the profile projections.

A profile outline is **the main profile meridian, and** it is combined with the vertical axis line in the horizontal projection and with the vertical axis line in the frontal projection.

The Method of auxiliary cutting planes is used for solving a point on the ball.

There is only **one type of section for a ball, and** it is a circle. But this **circle can be projected as three different images** in the drawing. It depends on the direction of a cutting plane.

1. If a cutting plane is a horizontal level plane, this section is drawn as a line in the frontal and profile projections and as a circle in the horizontal projection (fig. 4.21, *a*).
2. If a cutting plane is a profile level plane, this section is drawn as a line in the frontal and horizontal projections and as a circle in the profile projection (fig. 4.21, *a*).
3. If a cutting plane is a projecting plane, this section is drawn as a line in this plane of projection and as ellipses in two other planes (fig. 4.21, *b*).

A circle needs only specifying its centre and knowing its radius. So according to this all points on its surface can be solved by the method of auxiliary cutting planes in all planes of projections.

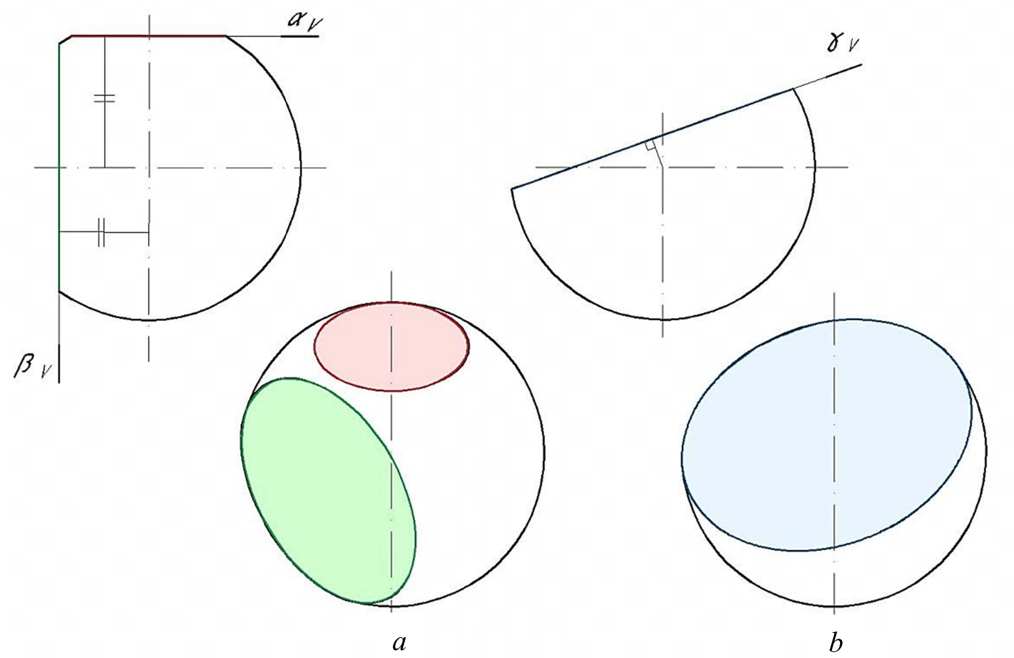


Fig. 4.21. Three possible representations of a ball section:
a – vertical and horizontal cutting planes; *b* – an inclined cuttingplane

Balls. Practice

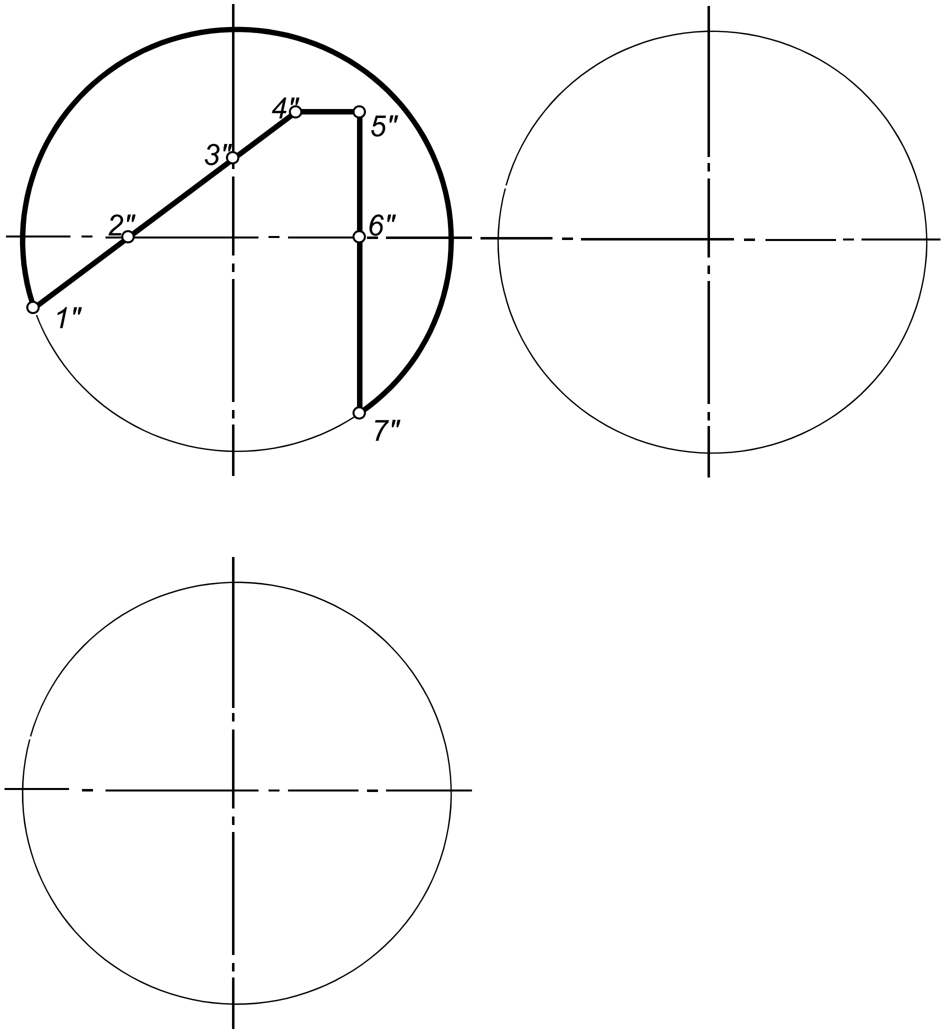


Fig. 4.22. Draw a horizontal and a profile projection for a ball with sections: by a frontal-projecting, horizontal level, and profile level cutting planes

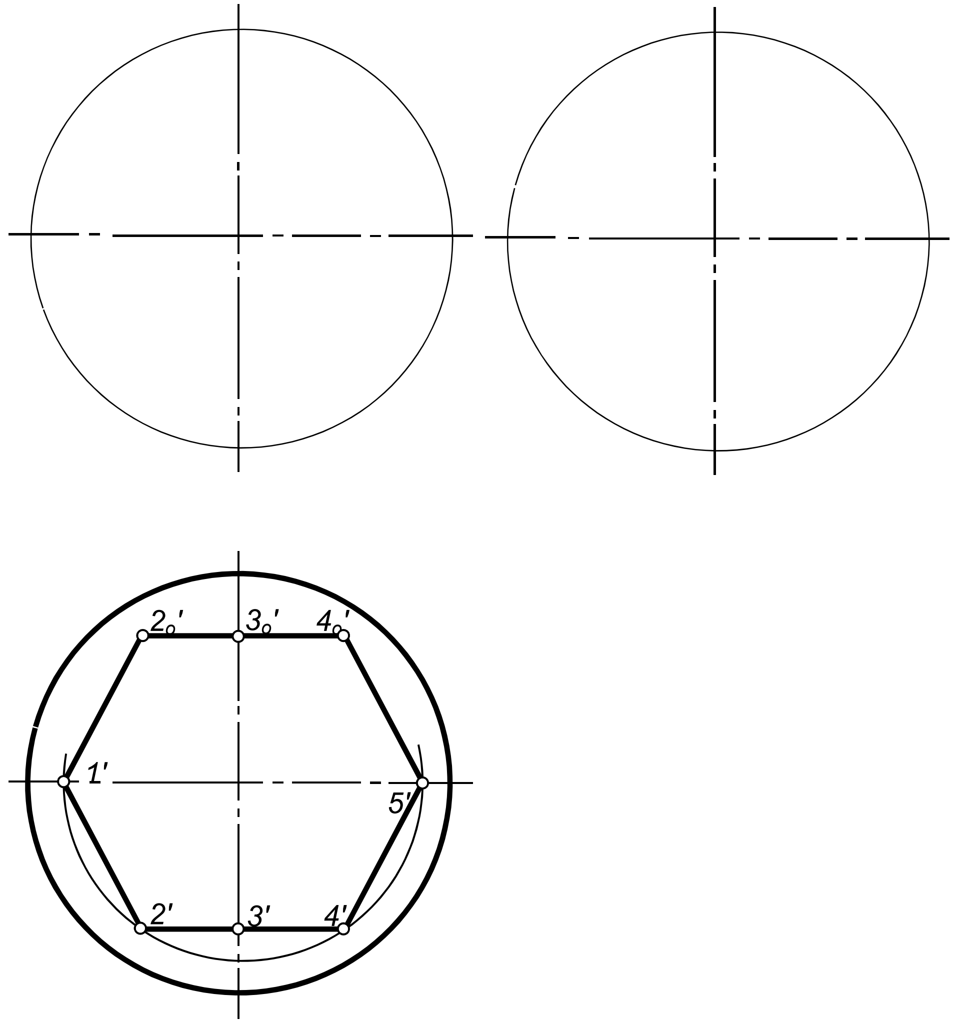


Fig. 4.23. Draw frontal and profile projections for a ball with sections: by four horizontal-projecting and two frontal level cutting planes

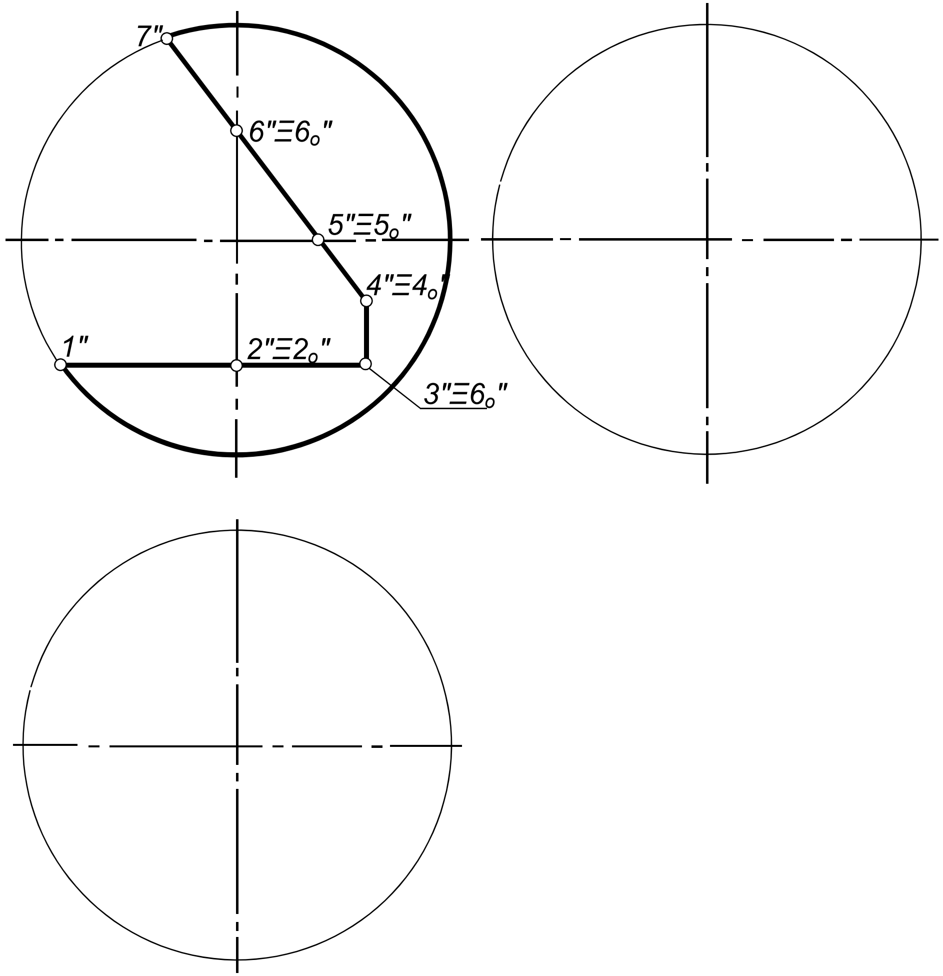


Fig. 4.24. Draw a horizontal and a profile projection for a ball with sections: by frontal-projecting, horizontal, and profile cutting planes

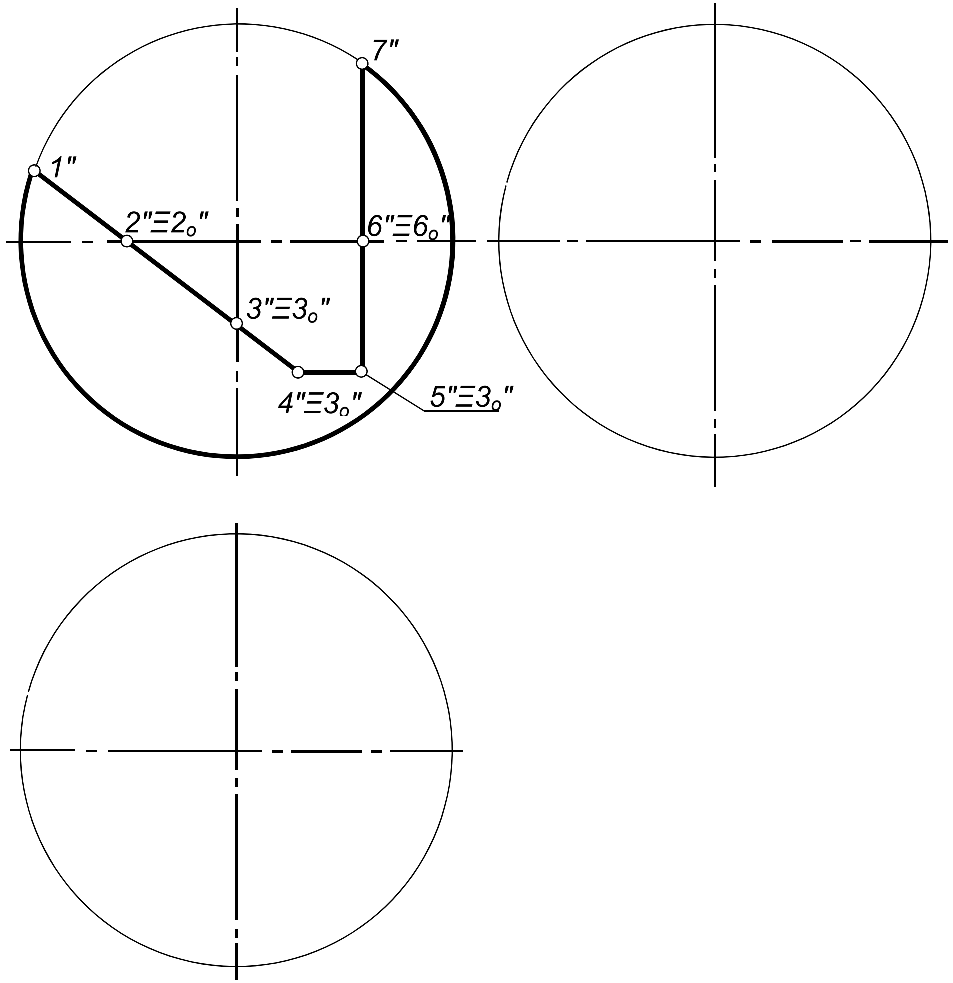


Fig. 4.25. Draw a horizontal and a profile projection for a ball with sections: by frontal-projecting, horizontal, and profile cutting planes

4.2.3. Surfaces of the 4^{th} order. Tori

A torus surface (colloquially **donut)** is formed by rotation of forming circle by the guidex round the axis that belongs to the same plane as the circle, but it does not pass through the centre of this circle. The guidex is a circle with a radius (fig. 4.22).

A torus is a solid (body) bordered by a torus surface. Torus is **open** if the outlines of the generatrix circle do not touch each other and do not intersect. Such torus is also called **a ring torus** (fig. 4.22, *a*, fig. 4.33, *a*). In other words, if R is more than r (R > r) where R is the radius of the guide circle and r is the radius of the generating circle. The internal side of the ring torus forms a **globoid** surface (fig. 4.22, *d*).

A torus is **closed** if R is equal to r (R = r), what means that outlines of the generating circle touches each other (fig. 4.22, *b*). It is also called **horn** torus. A torus is **a self-crossing** if outlines of the generating circles intersect. In this case, R is less than r (R < r).

Such torus is also called a **spindle** torus (fig. 4.22, *c*). Its inner part describes a toroid surface that can be called **a lemon** torus (fig. 4.22, *e*). If torus has the frontal-projecting axis i it is drawn as it is shown (fig. 4.22).

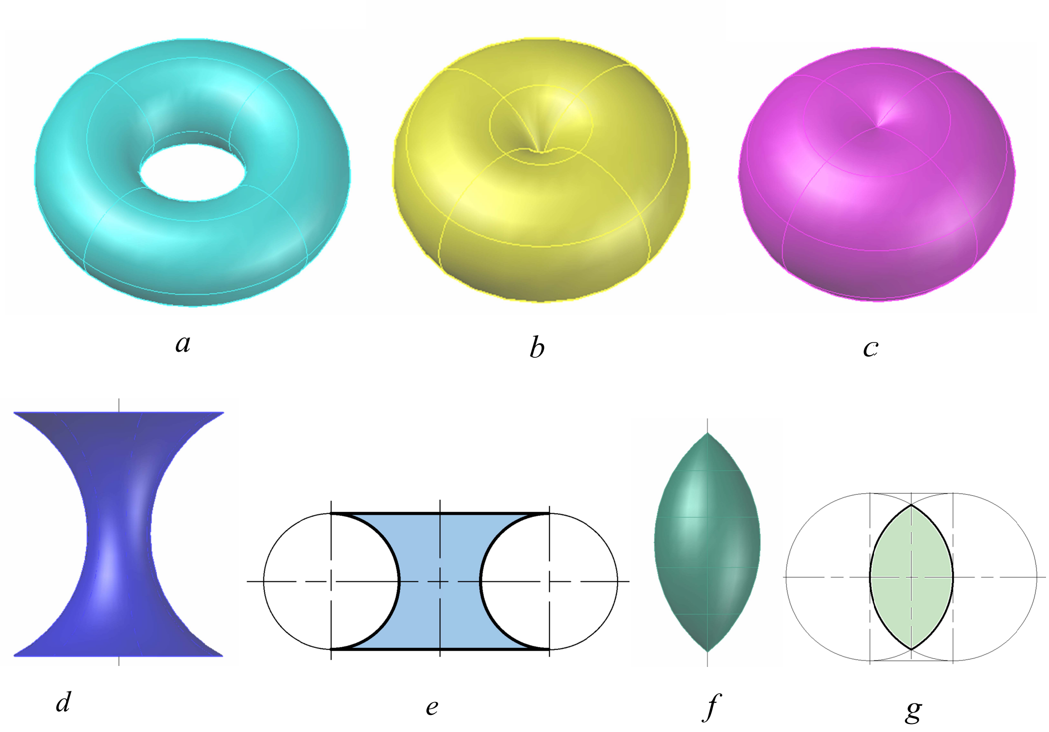
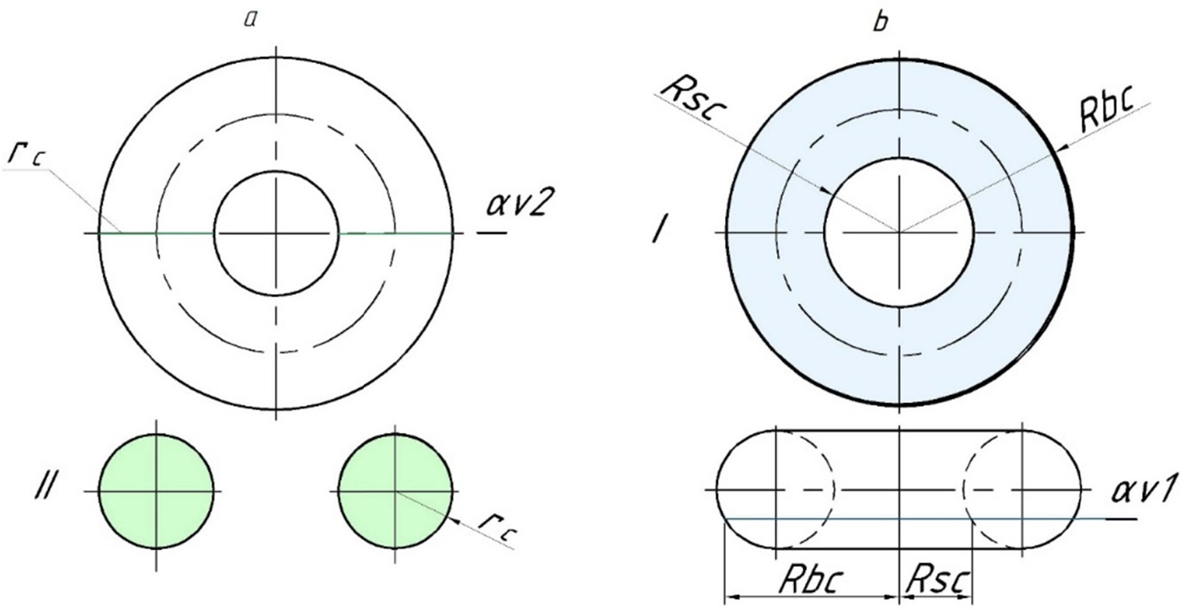


Fig. 4.26. Torus surfaces:
a – an open/ring torus; *b* – a closed/horn torus; *c* – a self-crossing/spindle torus;
d, *e* – a globoid; *f*, *g* – a lemon torus

There are **six types of section for torus**. Two of them are circles if a cutting plane passes through the axis of rotation.

A method of cutting planes is used for solution of the projection points on a ring tori. When a guidex circle is parallel to the frontal projection, auxiliary cutting planes are planes of frontal level which are projected as circles in the frontal projection and as lines in the horizontal projection (fig. 4.23).

For more information see [1, p. 43–45].



a *b*

Fig. 4.27. Possible cases of torus sections:
a – two circles which are the same with generatrix circle; *b* – a plane bordered by two circles (with a big and small radii)

1. If a cutting plane is perpendicular to the axis line, the section is a plane which is bordered by two circles with a big radius (**Rbc**) and a small radius (**Rsc**) (fig. 4.23, *a*).
2. If a cutting plane combines (passes through the axis line) with the axis line, the section is two circles which have the same radius with the generatrix circle (fig. 4.23, *b*). Distance **t** is zero in this case. This distance **t** is taken for explanation. It is a distance which describe the location of the cutting plane.
3. When $\mathbf{R} \leq \mathbf{t} < \mathbf{R_2}$, where **R** is the radius of the guide circle and $\mathbf{R_2}$ is the radius of the biggest circle of a torus, the section is an ellipse with two axes of symmetry (vertical and horizontal) (fig. 4.23, *c*).
4. When $\mathbf{R_1} < \mathbf{t} < \mathbf{R}$, where $\mathbf{R_1}$ is the radius of the smallest circle of the torus, the section is a wavy curve (fig. 4.23, *d*).
5. When $\mathbf{t} = \mathbf{R_1}$, the section is a double petal curve. This section is called a lemniscate of Bernoulli (fig. 4.23, *e*).
6. When $\mathbf{t} < \mathbf{R_1}$, the section is two ellipses with one line of symmetry (fig. 4.23, *f*).

All last four sections are called **curves of Perseus** (fig. 4.23).

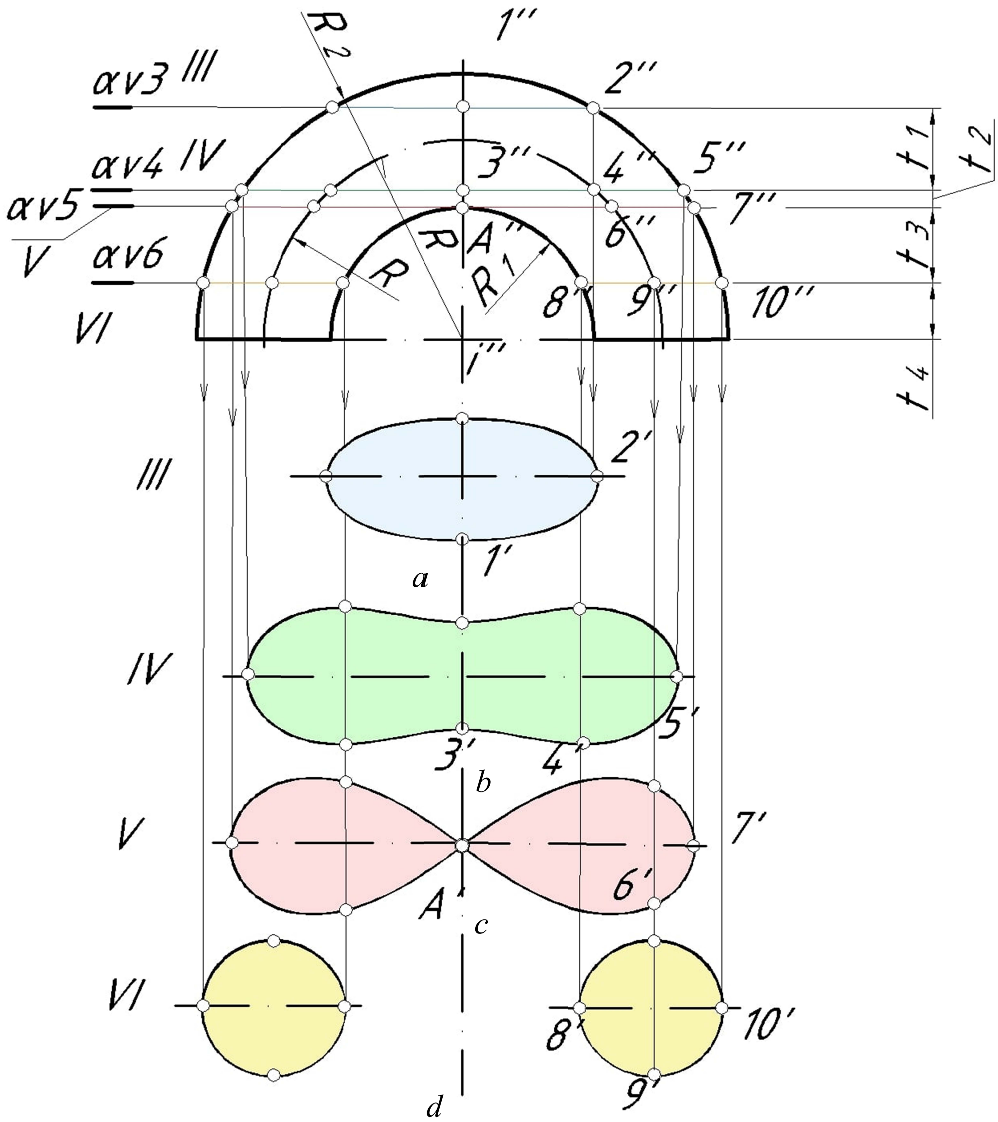


Fig. 4.28. Possible cases of torus sections:
a – an ellipse with two symmetry lines; *b* – a closed curve line; *c* – a double petal curve;
d – two ellipses with one (horizontal) symmetry line

For more information see [1, p. 45–48].

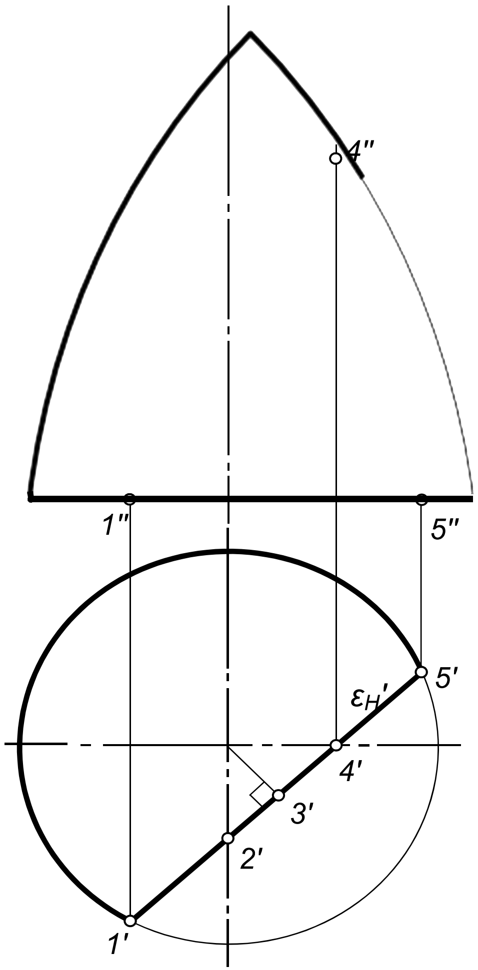


Fig. 4.29. Complete a frontal projection of a spindle torus with a horizontal-projecting section ε(ε_H)

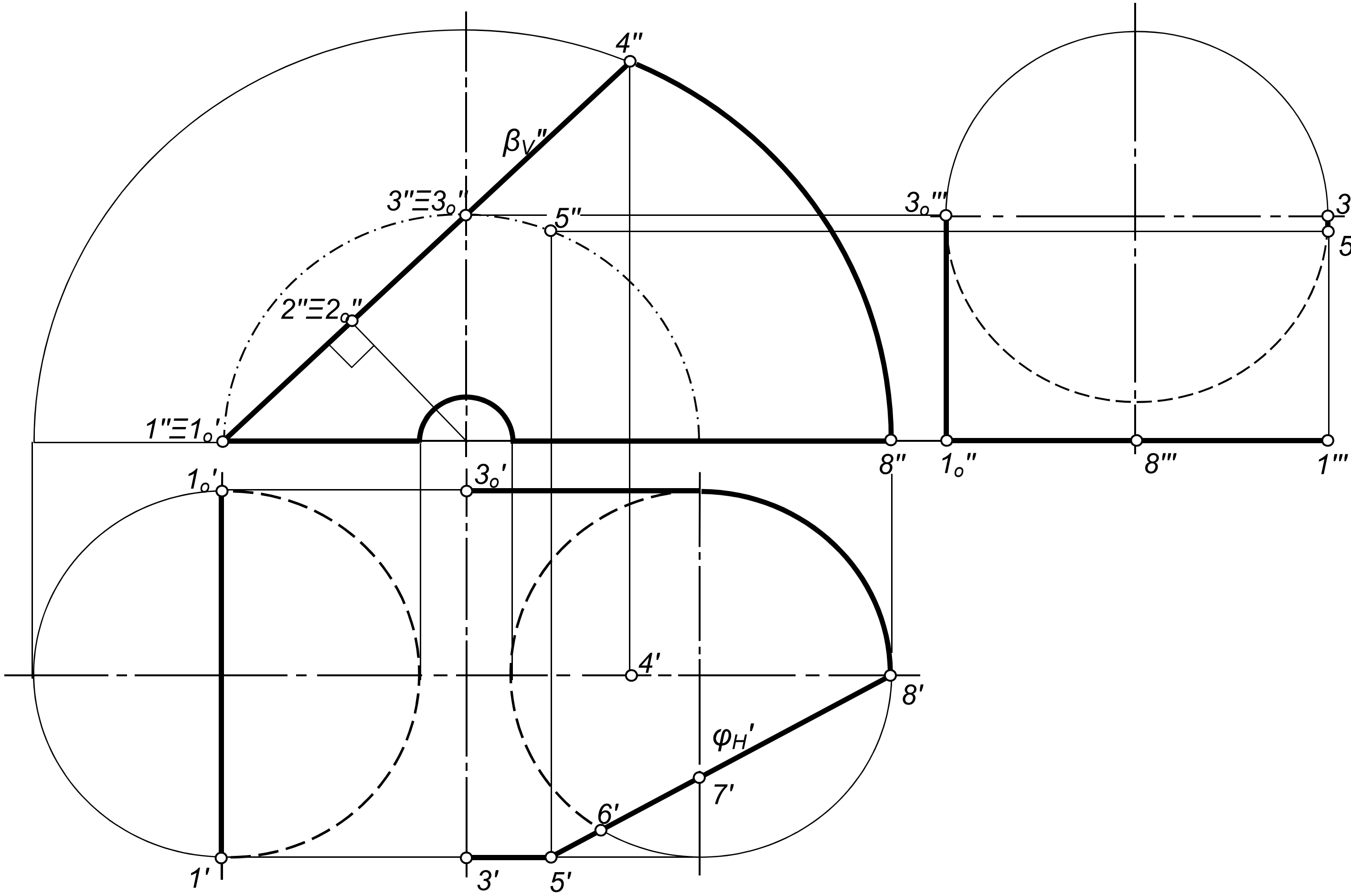


Fig. 4.30. Complete all three principal projections of a ring torus with a horizontal-projecting φ(φ_H') and frontal-projecting β(β_V'') sections

5. COMPOSITE SOLIDS

Composite solids with projecting sections. Practice

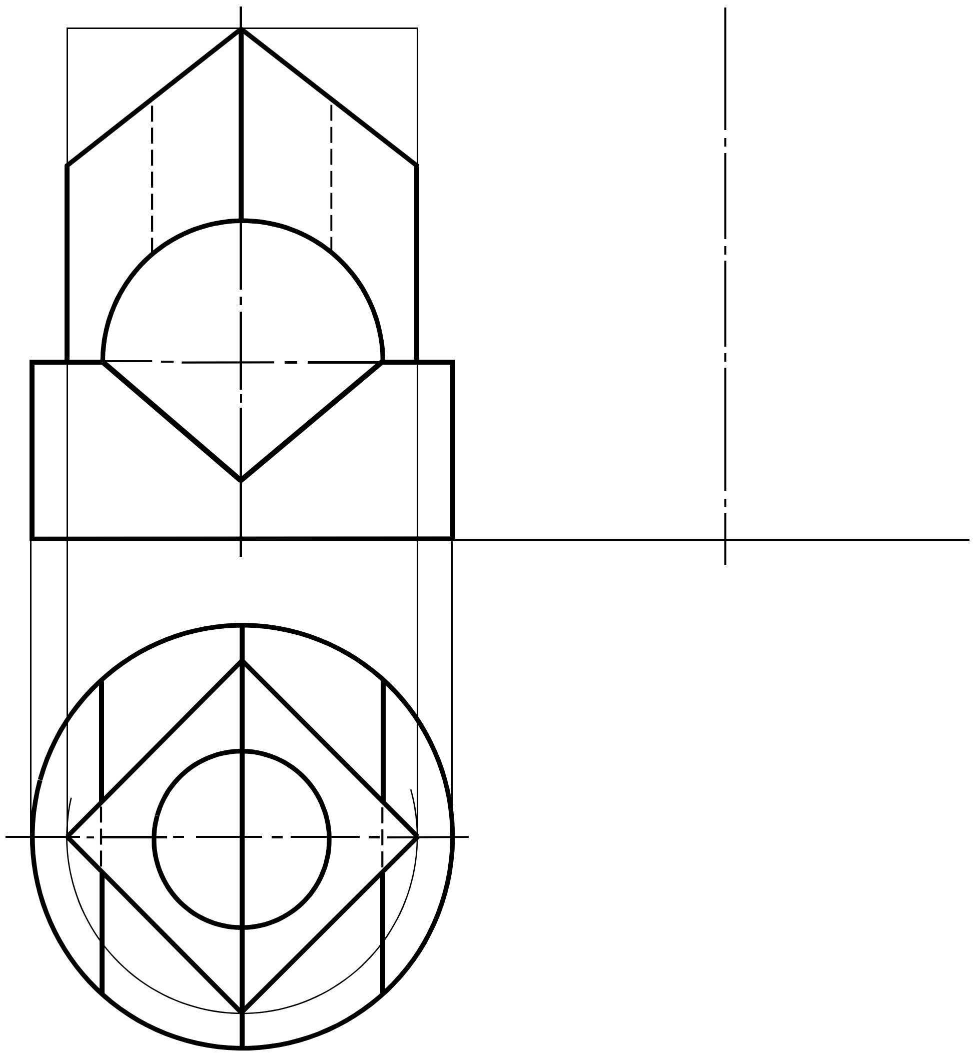


Fig. 5.1. Draw a profile projection for the composite solid

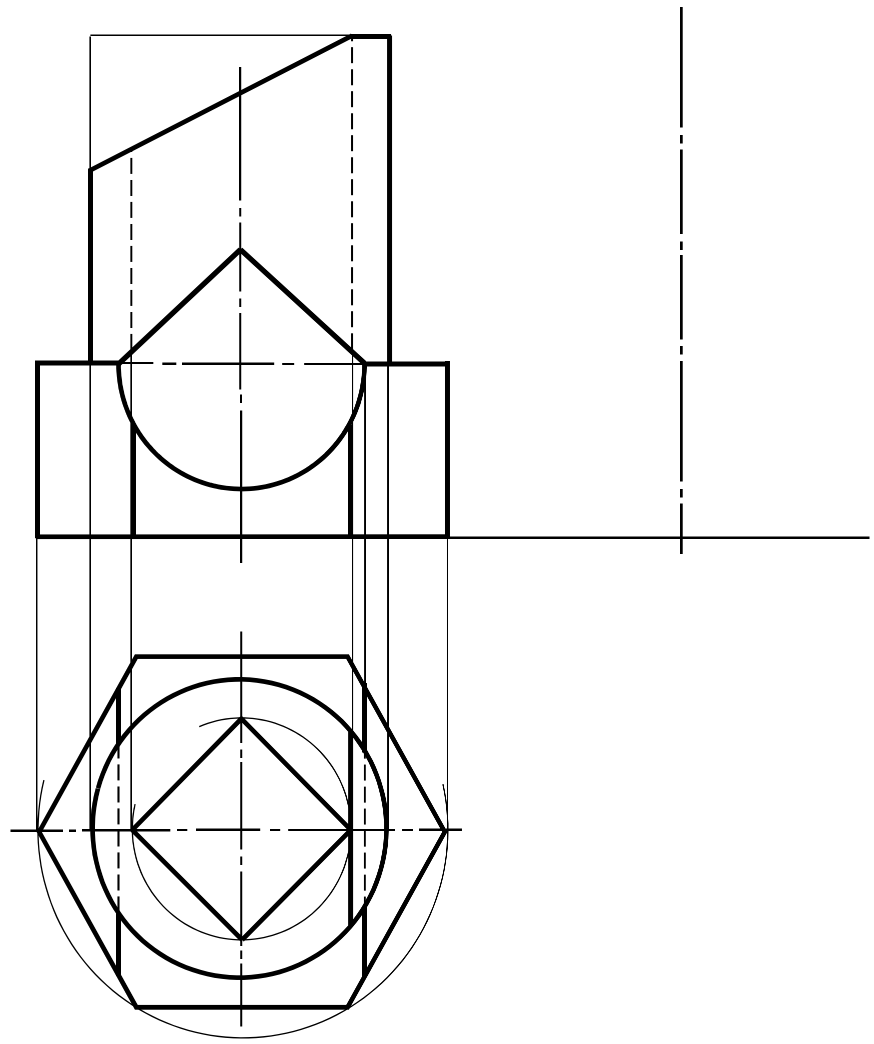


Fig. 5.2. Draw a profile projection for the composite solid

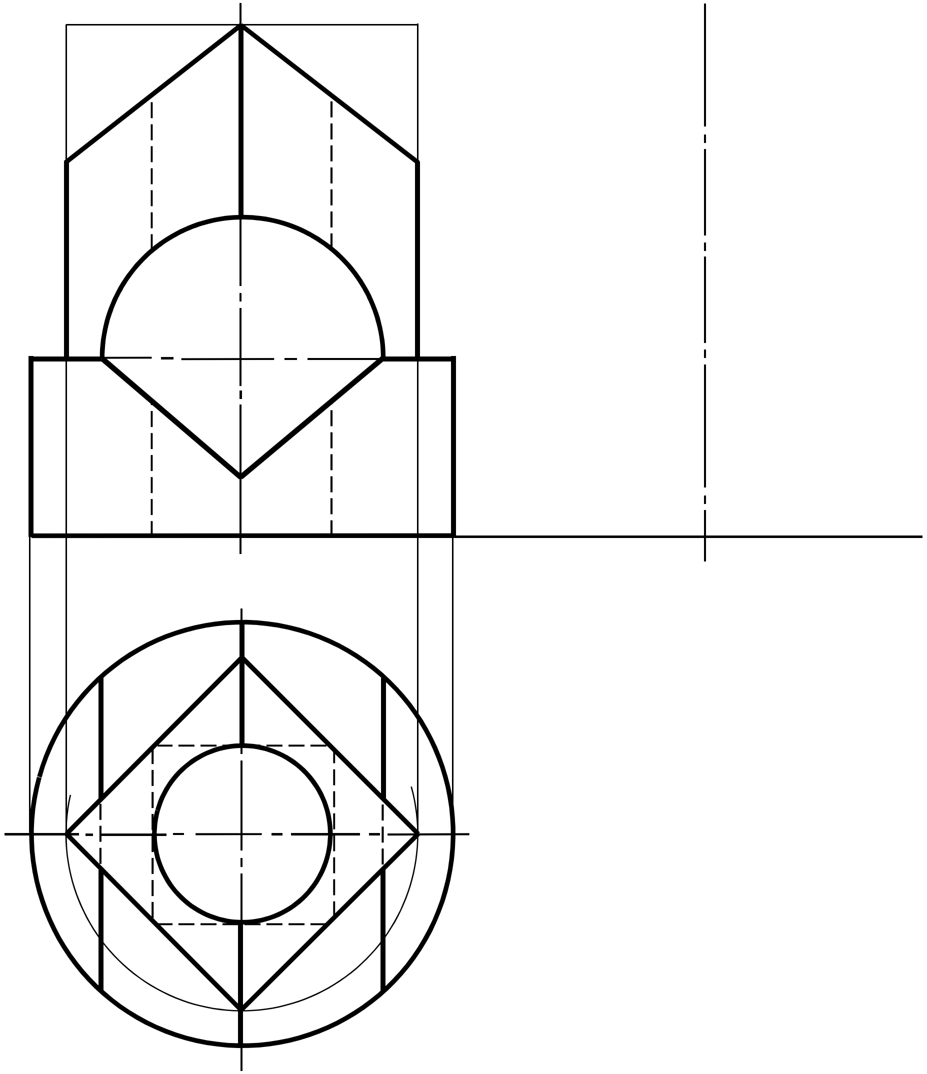


Fig. 5.3. Draw a profile projection for the composite solid

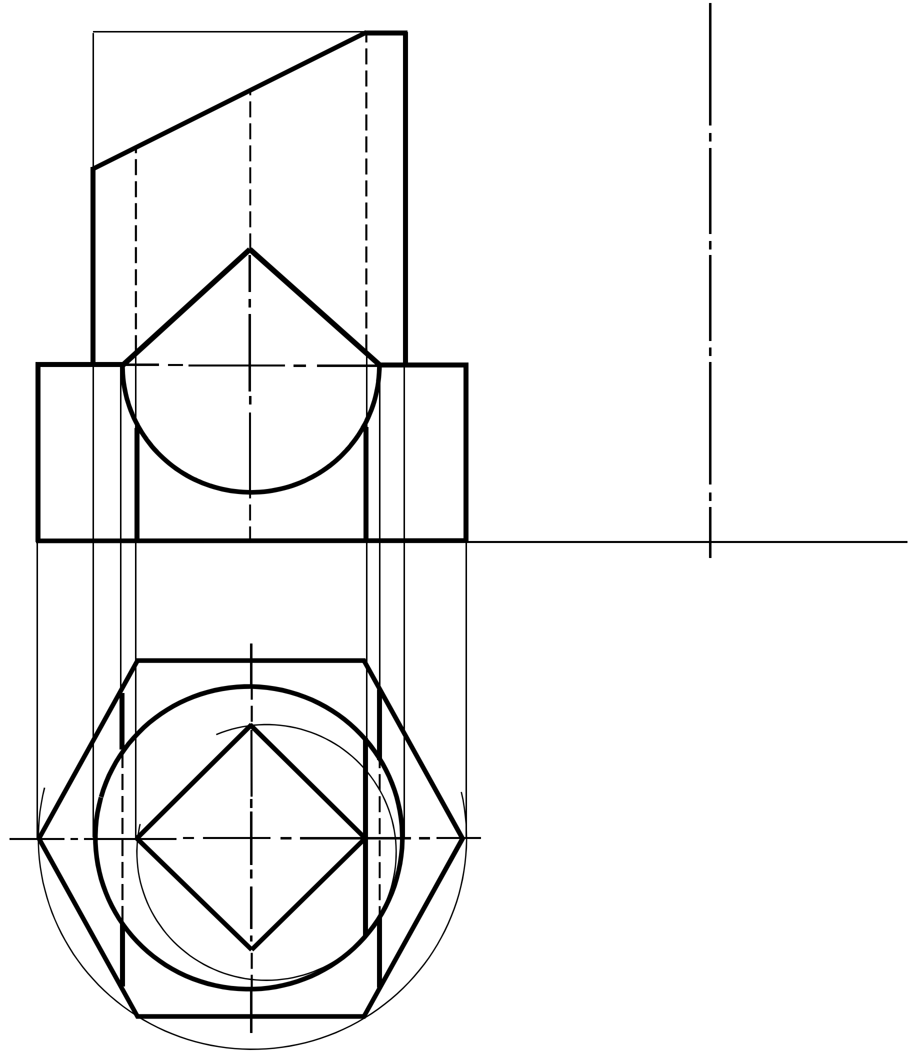


Fig. 5.4. Draw a profile projection for the composite solid

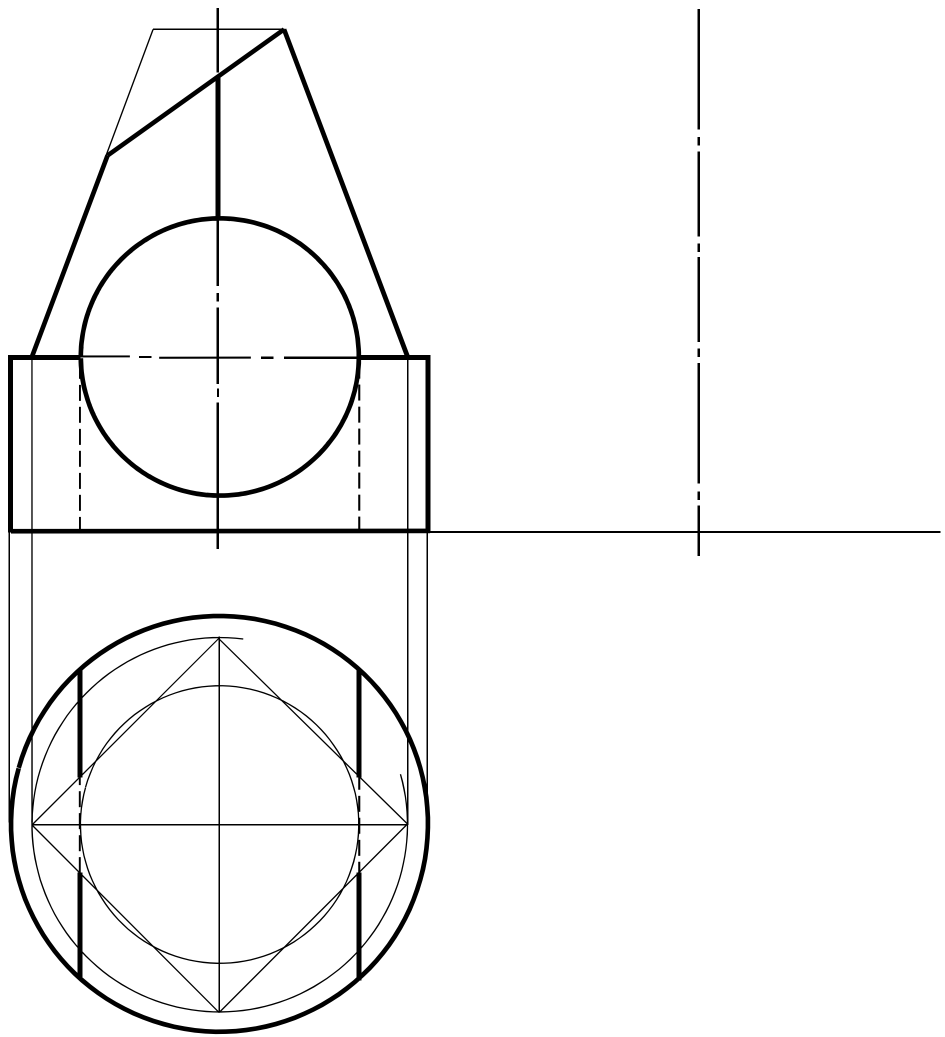


Fig. 5.5. Complete the horizontal and add a profile projection
for the composite solid

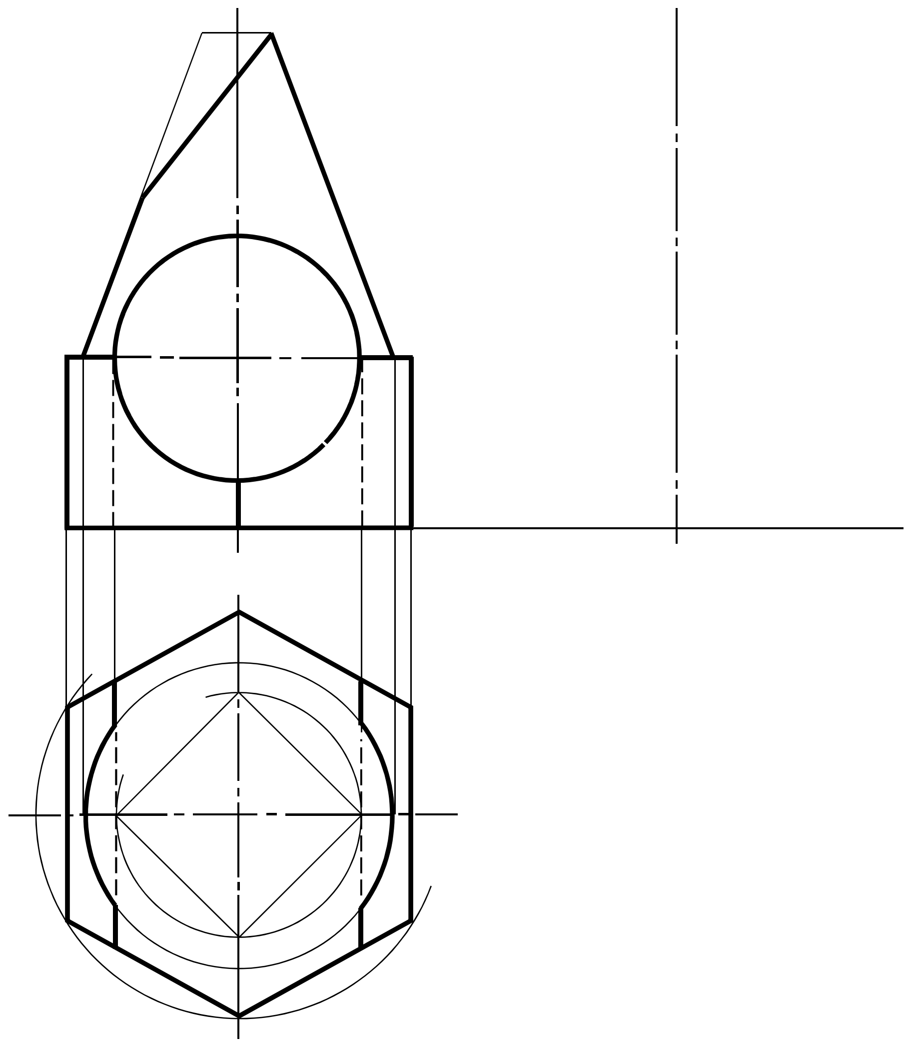


Fig. 5.6. Complete horizontal and add profile projections
for the composite solid

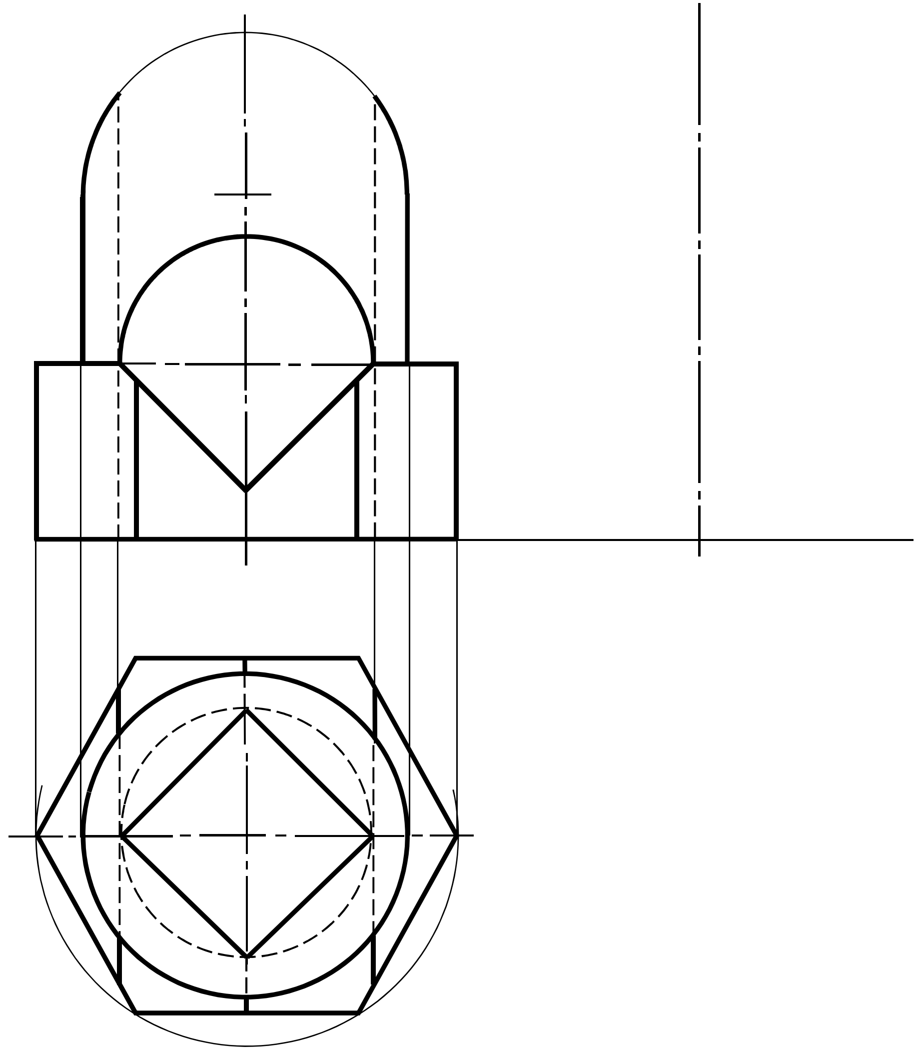


Fig. 5.7. Complete the frontal projection and add a profile projection for the composite solid

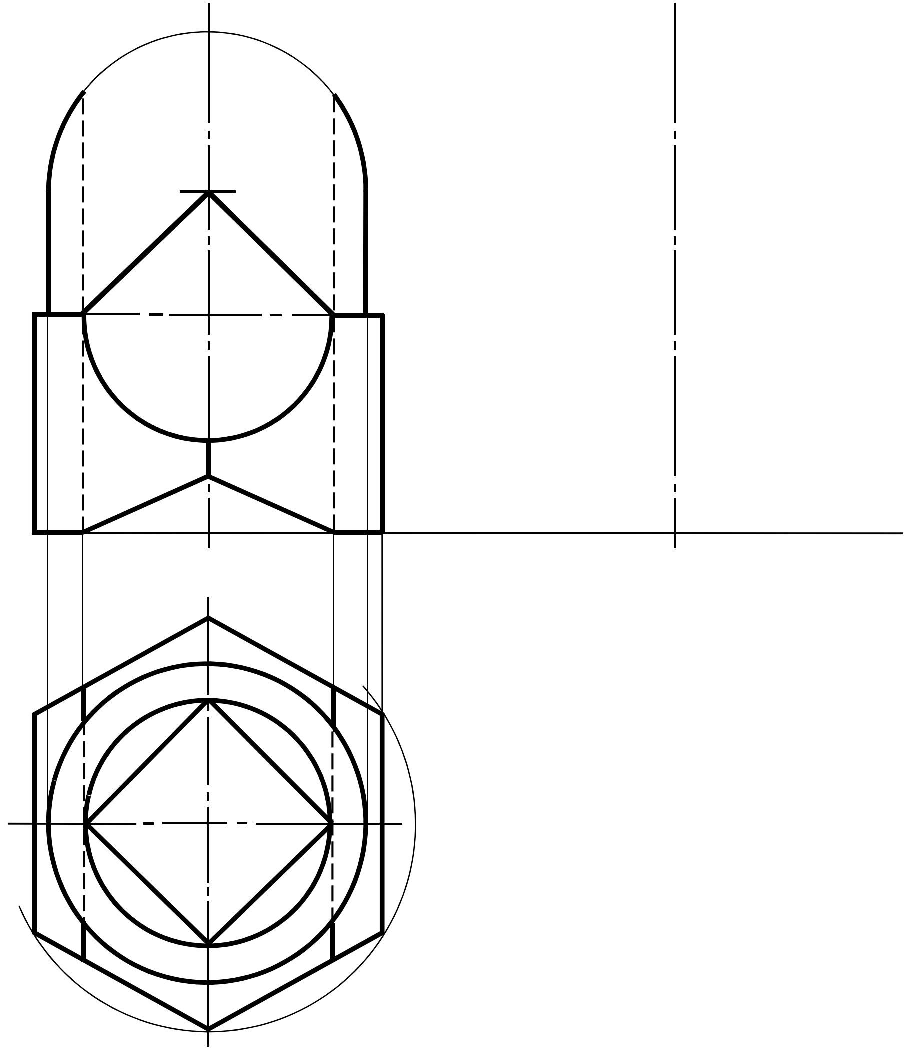


Fig. 5.8. Complete the frontal projection and add a profile projection for the composite solid

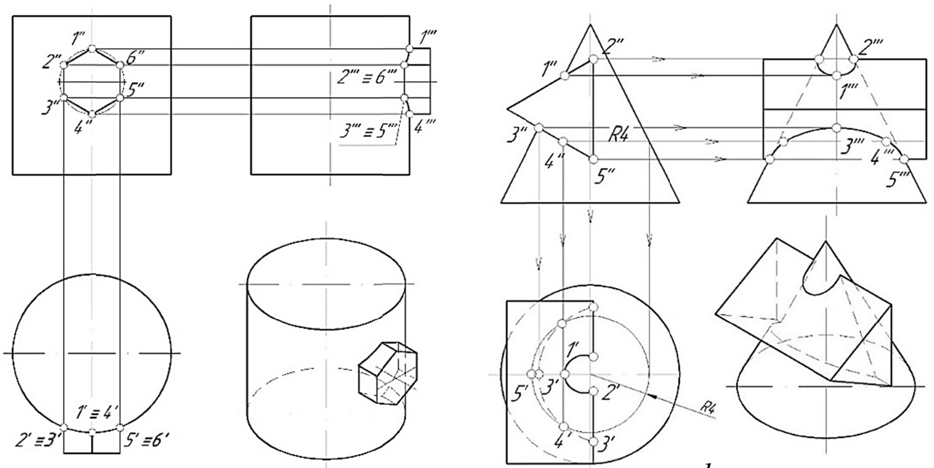
6. SURFACE INTERSECTION

6.1. Particular cases

There are particular problems and general. Particular cases can be solved by surfaces property without auxiliary actions.

Four particular cases:

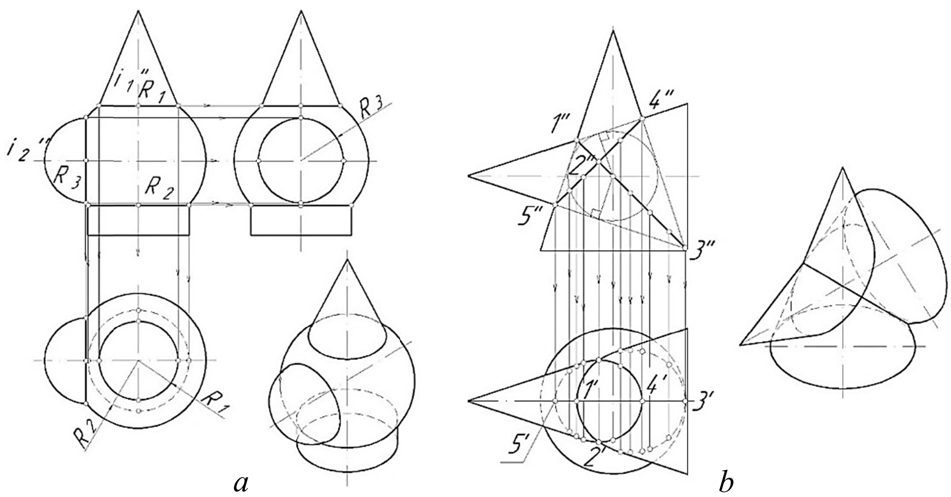
1. Intersection of two projecting surfaces, where the side surface is perpendicular to a projection plane (fig. 6.1, *a*).
2. Intersection of some surfaces when only one surface is projecting (fig. 6.1, *b*).



a *b*

Fig. 6.1. Intersection of surfaces, particular cases:
a – intersection of two projecting surfaces (a prism with a cylinder); *b* – intersection of one projecting and one non-projecting surfaces (a cone with a prism)

3. Intersection of some surfaces of revolution with the same axis of rotation for them (fig. 6.2, *b*). Such surfaces are called **coaxial** and intersect in a circular intersection line, which is perpendicular to the axis of rotation surfaces. At that, if the axis of rotation surfaces is parallel to the projection plane, the intersection line is projected onto this plane as a line-segment.
4. **Monge's Theorem.** If two surfaces of the second order are described around or inscribed into the third surface of the second order, the line of their mutual intersection decomposes into two plane curves. The planes of these curves pass through a straight line connecting the intersection points of the tangent lines.



a *b*

Fig. 6.2. Intersection of surfaces, particular cases:
a – intersection of coaxial surfaces (spheres, a cone, and a cylinder);
b – intersection of two cones which are described around one sphere

For more information, see [1, p. 49–50].

Projecting surfaces intersection. Practice

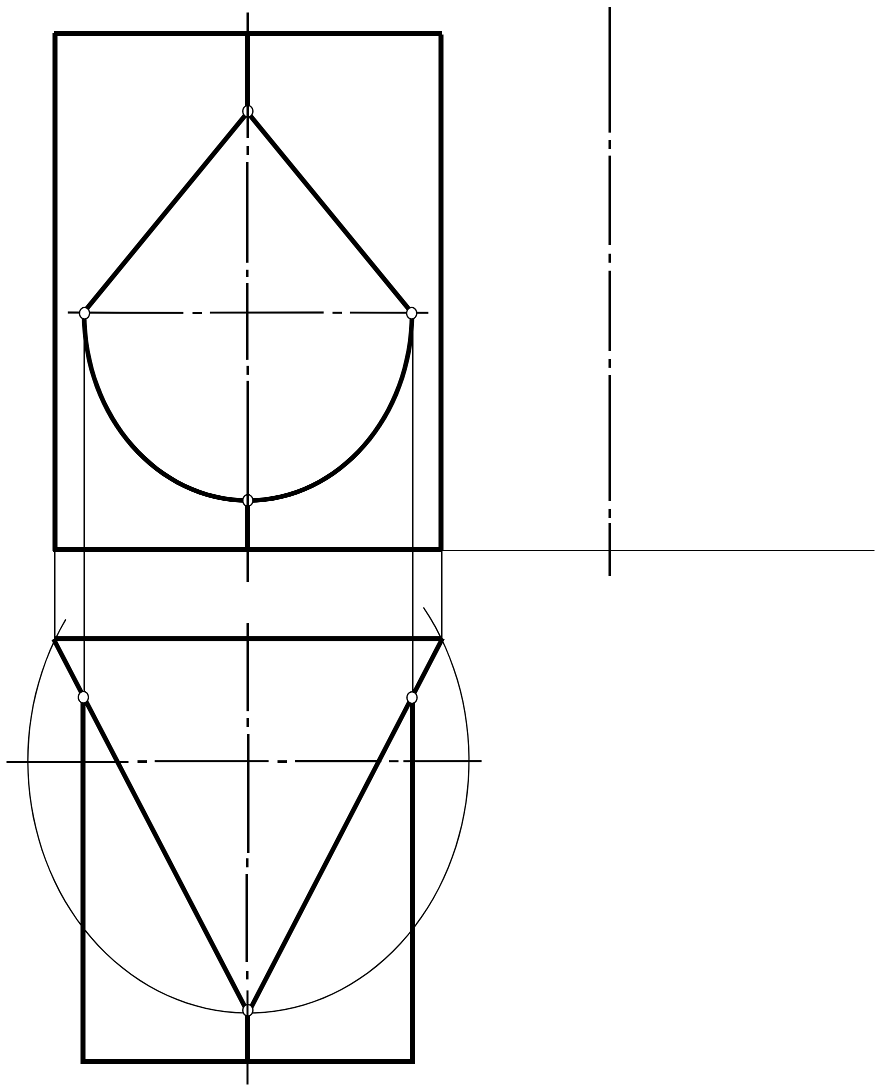


Fig. 6.3. Draw a profile projection of the composite solid

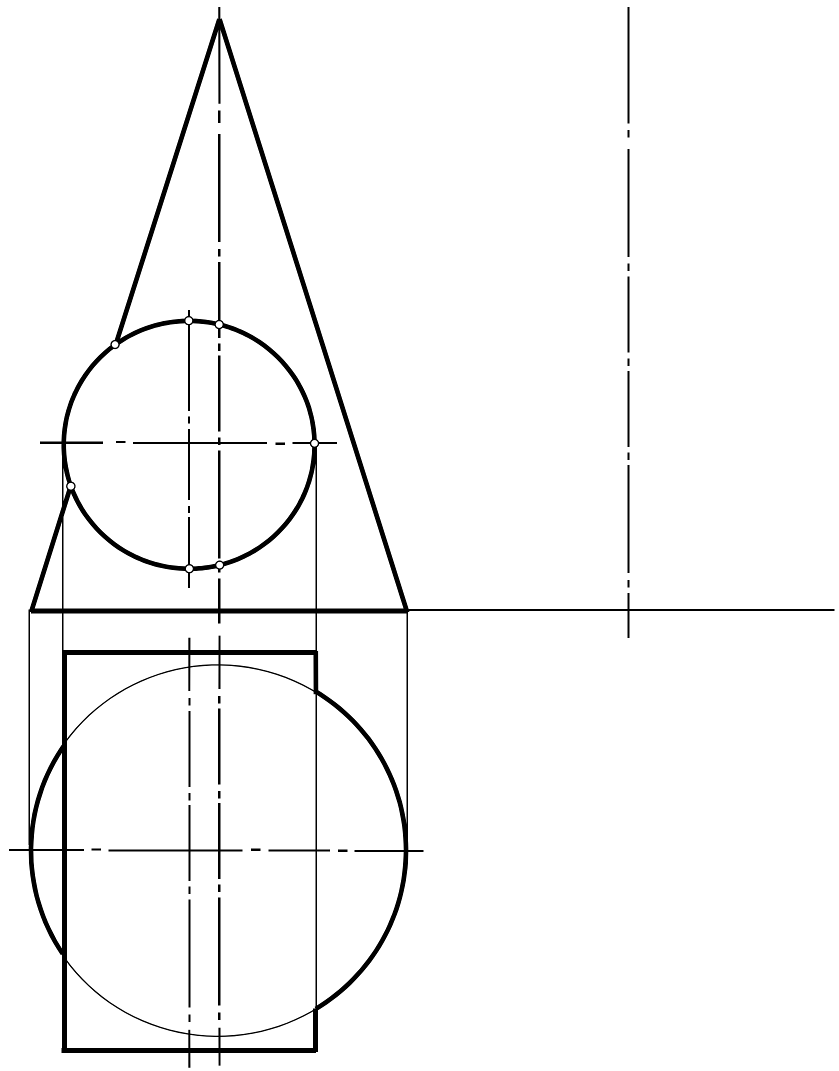


Fig. 6.4. Complete the horizontal projection and add a profile projection for the given composite solid

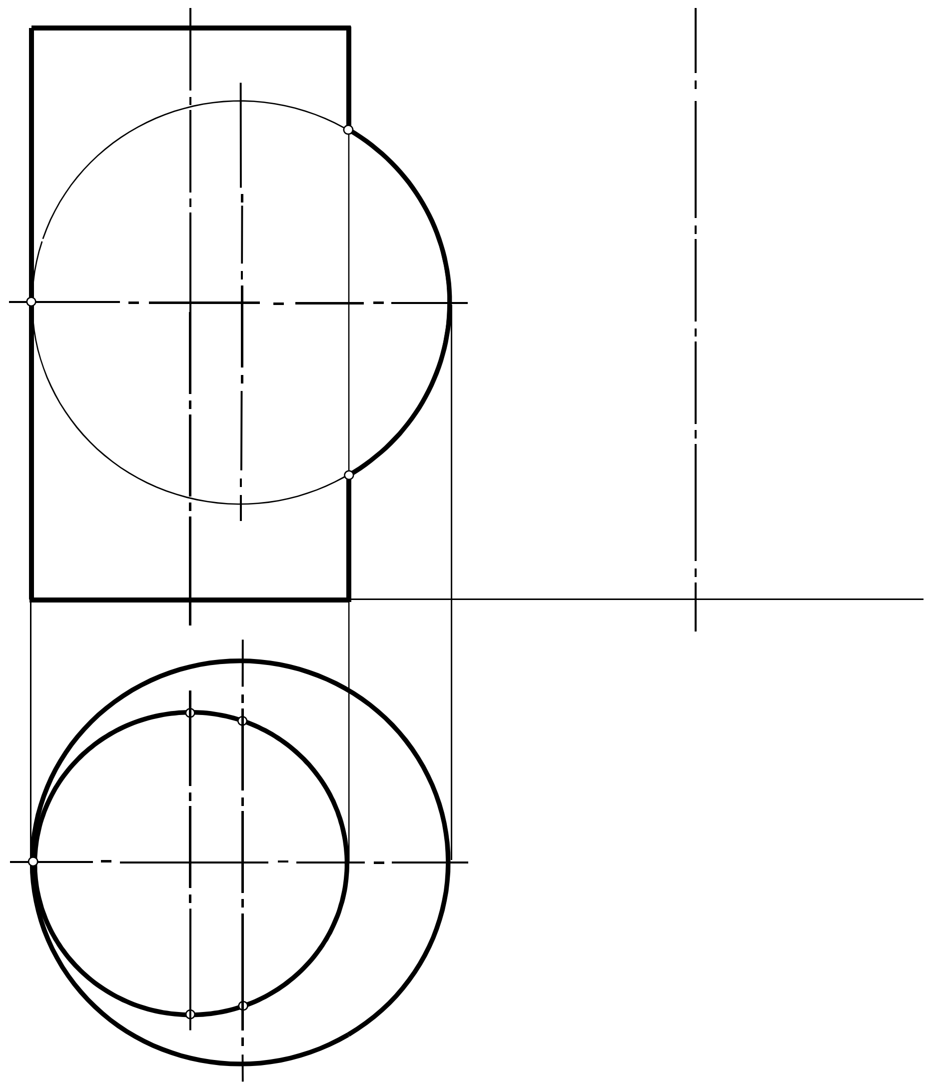


Fig. 6.5. Complete the frontal projection and add a profile projection for the composite solid

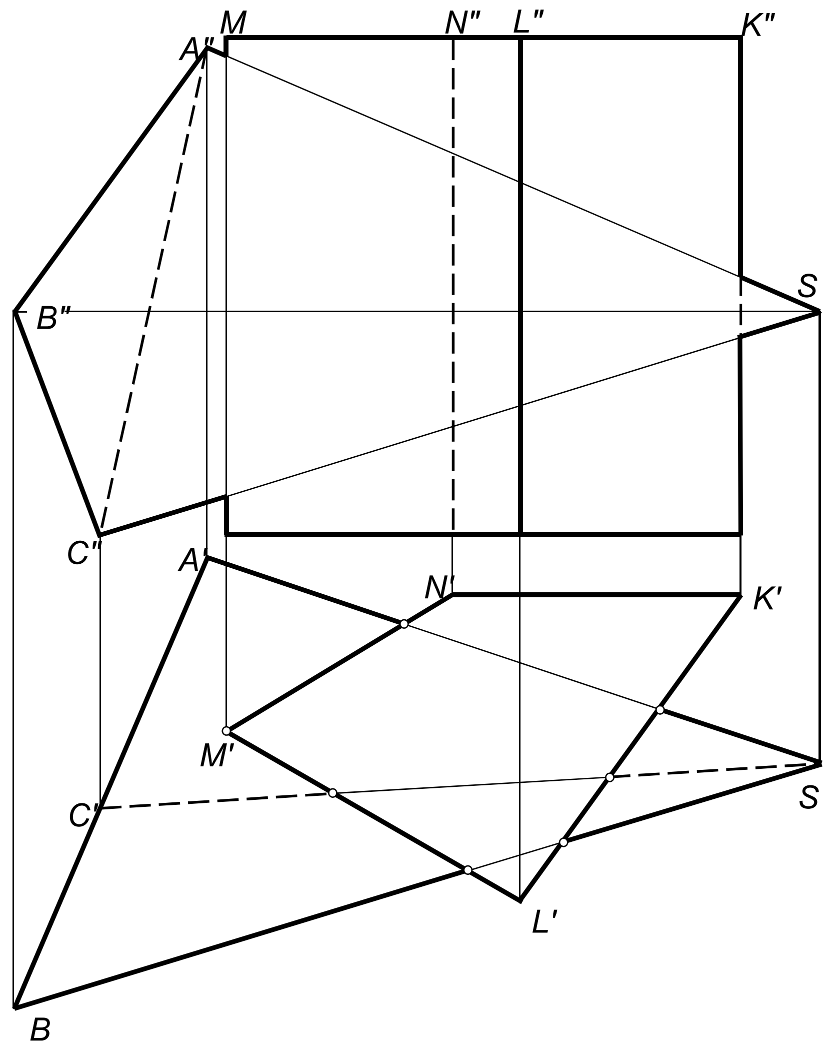


Fig. 6.6. Complete the frontal projection for two intersecting polyhedral solids

6.2. General cases

6.2.1. Method of the auxiliary cutting planes

This method is also called **the way of auxiliary planes, and** it is applicable if:

– the common symmetry plane of two intersecting surfaces is a level plane;

– sections obtained by a level plane for both surfaces are an easy-drawn lines – straight lines or circles.

By the first circumstance, intersection points of the surface's outlines belong to a required line of intersection and set the beginning and the end of it. The intersection line is supposed to be found by a set of points as an approximate line. Then more points are then more precise line. These points are common points for both surfaces. Therefore, it is necessary to find common points in one level plane. That means auxiliary planes which are level planes should be set for solving this problem. Such planes cut simple shapes from the surfaces which can be specified on the other projection, as it's been discussed above. Required points are points of intersection of the constructed sections. Their second projection can be found by their belonging: if the point was detected by a level plane, this point belongs to this plane and its projection belongs to the plane projection. Therefore, the second projection can be specified by a connection line only. The connection line should be drawn to interaction with the auxiliary level plane projection.

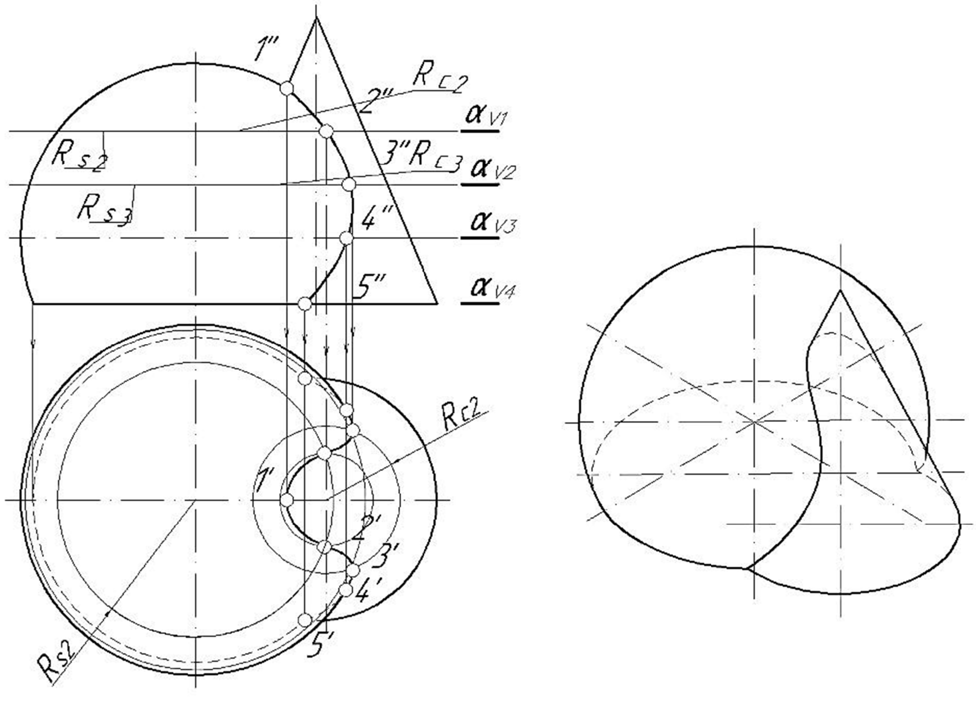


Fig. 6.7. Determining of an intersection line of a straight circular cone with a ball by the Method of auxiliary cutting level plane

For constructing the intersection line of the given surfaces it is advisable to introduce the frontal plane and a number of horizontal planes (α_{V1}, α_{V2}, α_{V3}, α_{V4}) as the auxiliary planes or vice versa. At the end, all points are connected in accordance with the given or taken order and obtain a smooth curve line in accordance with visibility.

For more information, see [1, p. 52].

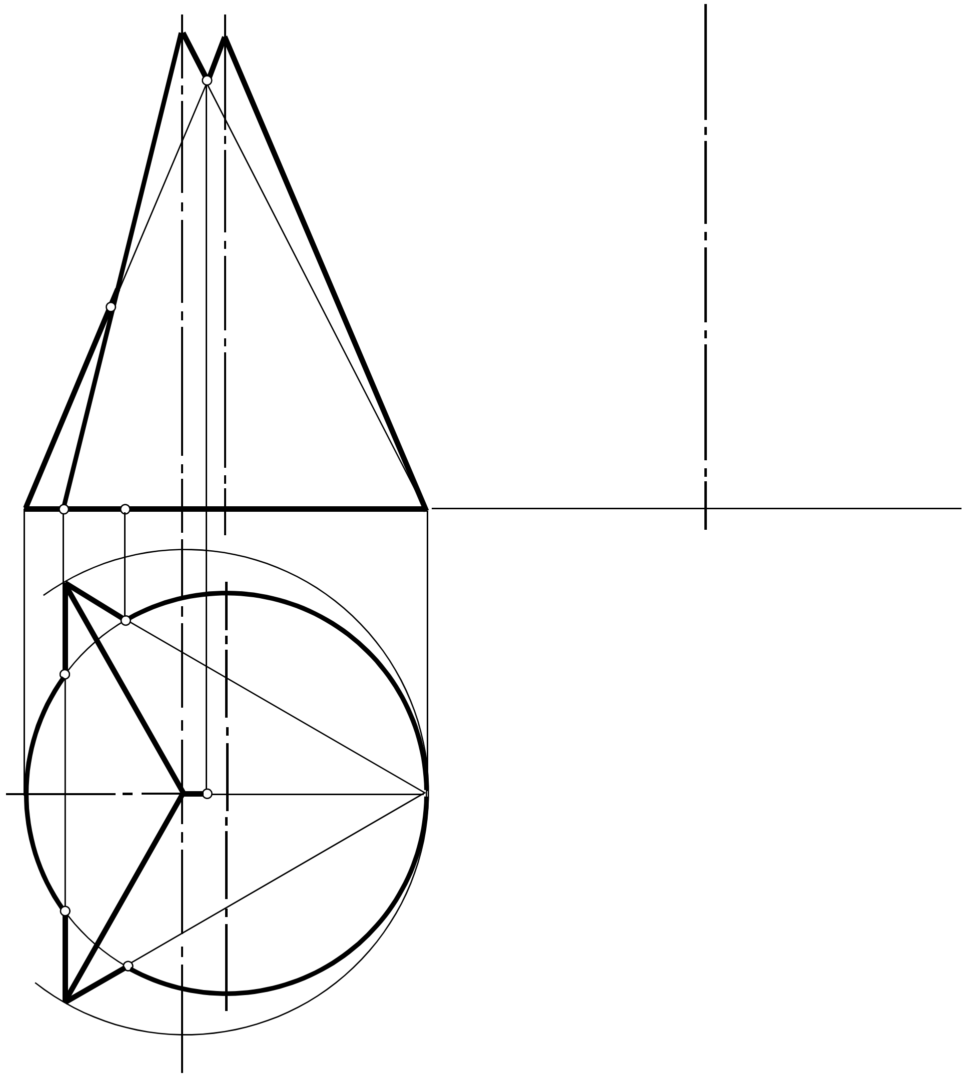


Fig. 6.8. Complete the horizontal and frontal projections and add a profile projection for the composite solid consisted of a right circular cone and a right triangular pyramid

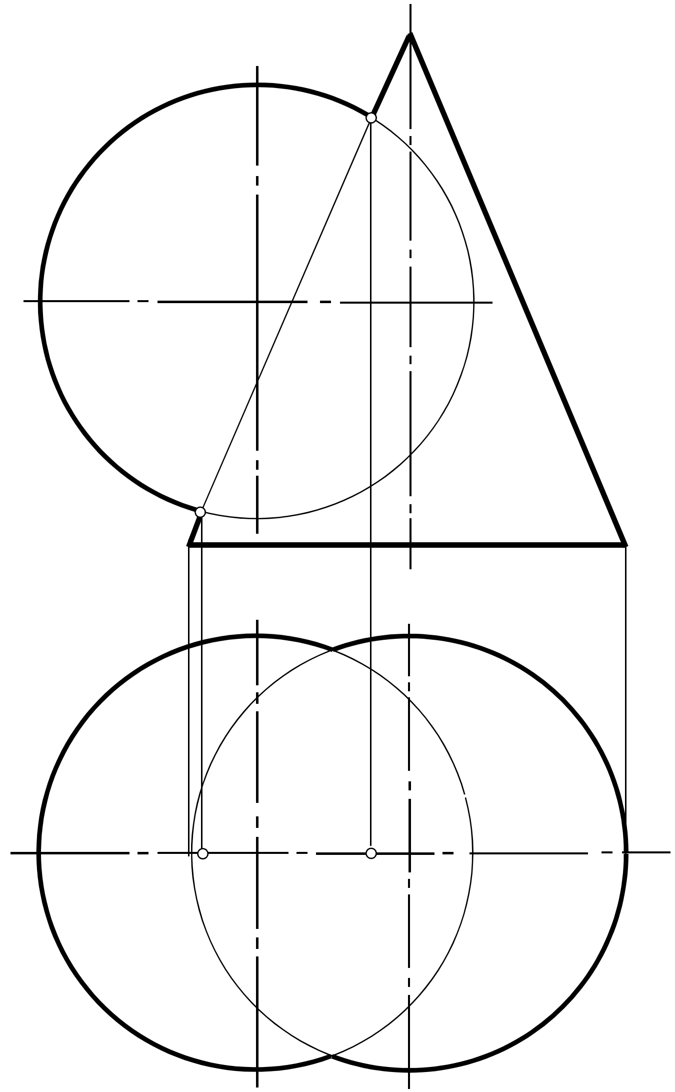


Fig. 6.9. Complete the composite solid projections consisted of a sphere and a right circular cone

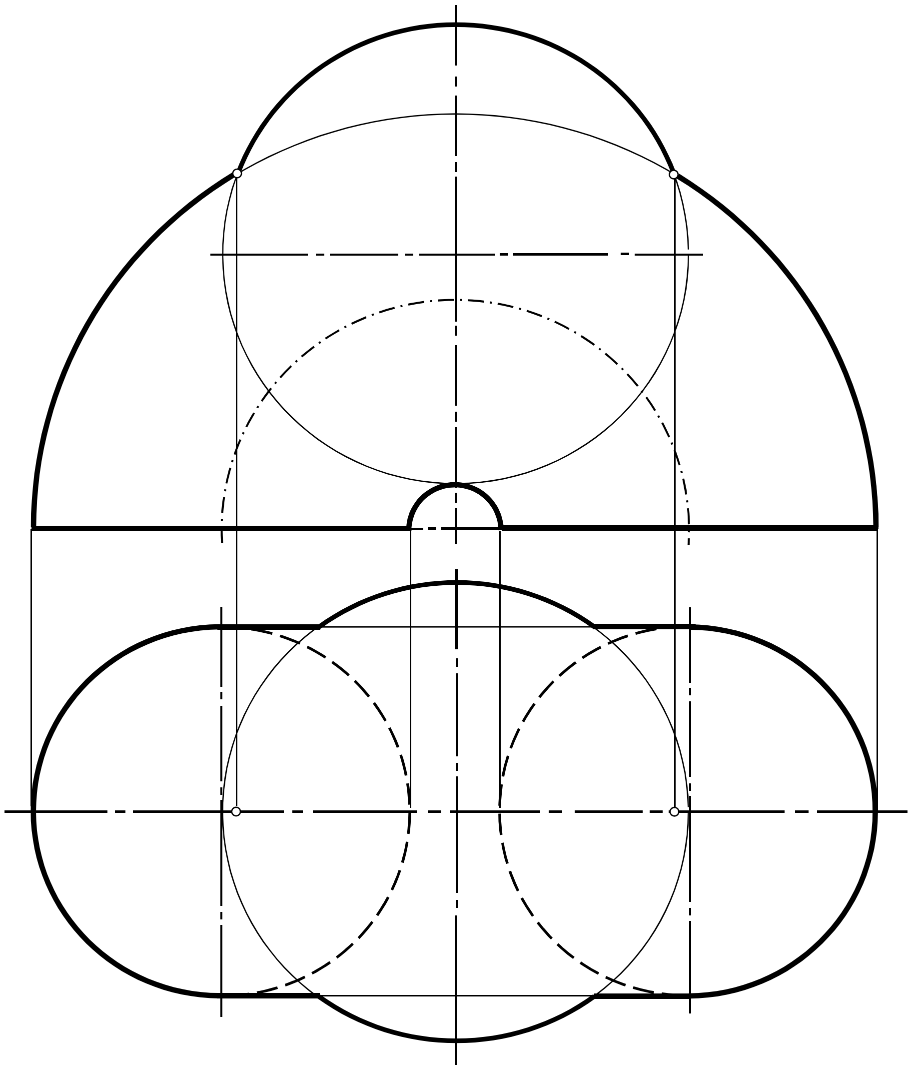


Fig. 6.10. Complete the composite solid projections

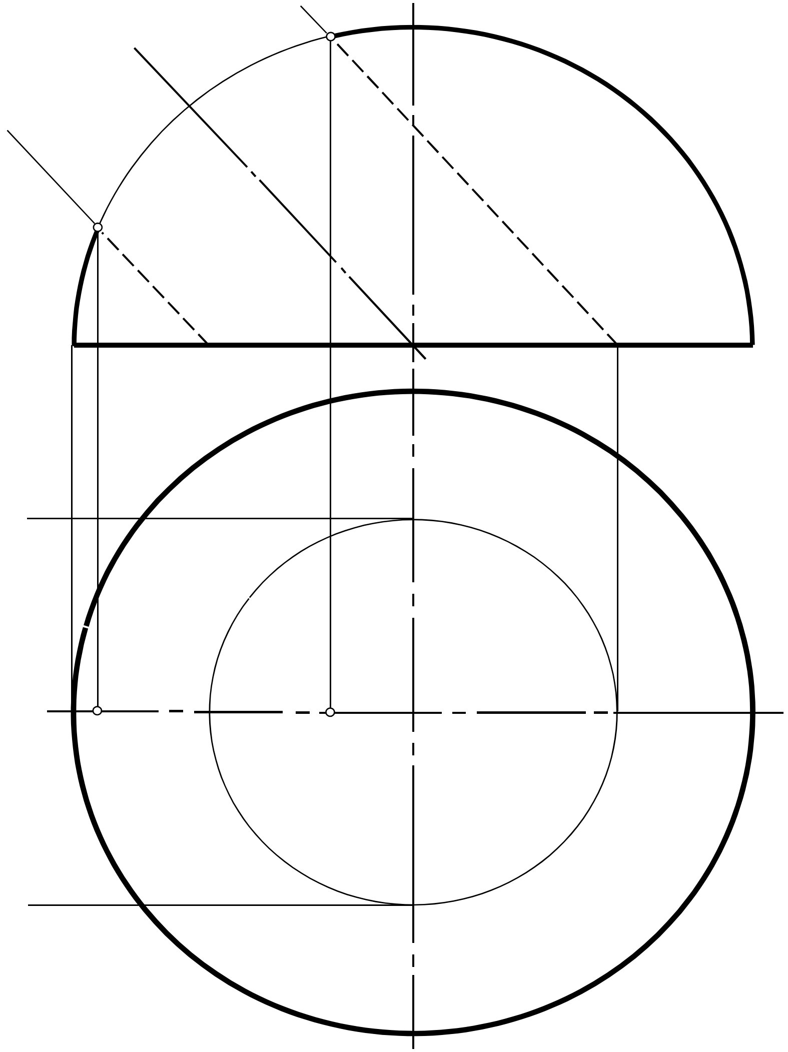


Fig. 6.11. Complete the composite solid projections

6.2.2. Method of the auxiliary concentric spheres

This method is widely used for solution of problems on intersection lines construction for revolution surfaces with intersecting axes. It works by particular properties of a sphere:

– possibility to set any axis through its centre;

– possibility to be a coaxial surface to any revolution surfaces.

Note: if a plane of revolution surface axes is not parallel to the projection plane, the circles of their intersection are projected as ellipses and this makes the solution of the problem more complicated. For this reason, the method of auxiliary spheres should be used if:

– intersecting surfaces are surfaces of revolution;

– surfaces axes intersect and an intersection point can be taken for the centre of auxiliary spheres;

– both surfaces have a common plane of symmetry that is a level plane.

The beginning and the end points are specified as intersection points of the surfaces outlines (their level plane sections) as it has been mentioned above. Determining of the other points requires setting auxiliary spheres as mediators. Note: not all spheres may be used for the problem solution. The limits of the auxiliary spheres' usage should be considered.

The maximum radius of a cutting sphere is equal to the distance from the centre O to the farthest intersection point of the level outlines. The minimum cutting sphere is a sphere, which contacts one surface in other words it is inscribed into it, and intersect another. Therefore, it is necessary to inscribe spheres as the minimum ones into both given surfaces and select the appropriate one. Radii for all given spheres should be in the following range: ***Rmax*** **>** ***Raux*** **>** ***Rmin***.

When limits are clear, it is necessary to draw an auxiliary sphere of an arbitrary radius in the given range from the intersection point of the axes (point *O*'). This sphere is simultaneously coaxial to both surfaces and intersect with them in circular sections. As theirs common axes are parallel to the projection plane, they are projected in linear projection (line segment). As it has been mentioned, planes intersect in a line. Therefore, the taken auxiliary sphere provides us with two planes which are two circles, however, as these circles projects in lines their line of intersection projects in a point (literally in two competing points). As these two points are a result of intersection, they are common for both surfaces the section by a sphere were taken for. Line of intersection is a set of such common points. The rest of points should be determined by the same way with other auxiliary spheres. The other projection of points is drawn by their belonging with the help of the cutting plane method as usual. At the end point should be joined in the taken order in relevance with the visibility.

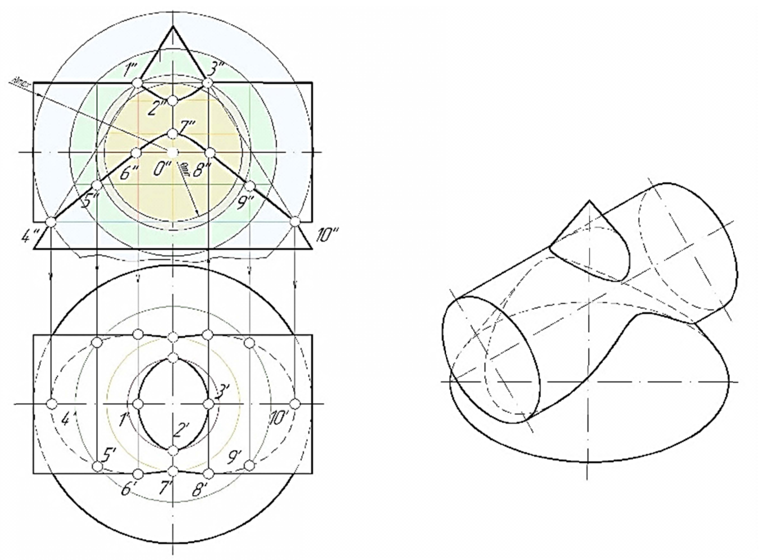


Fig. 6.12. Determining of an intersection line of a straight circular cone with a straight circular cylinder by the Method of auxiliary concentric spheres

Concentric cutting spheres method. Monge's Theorem. Practice

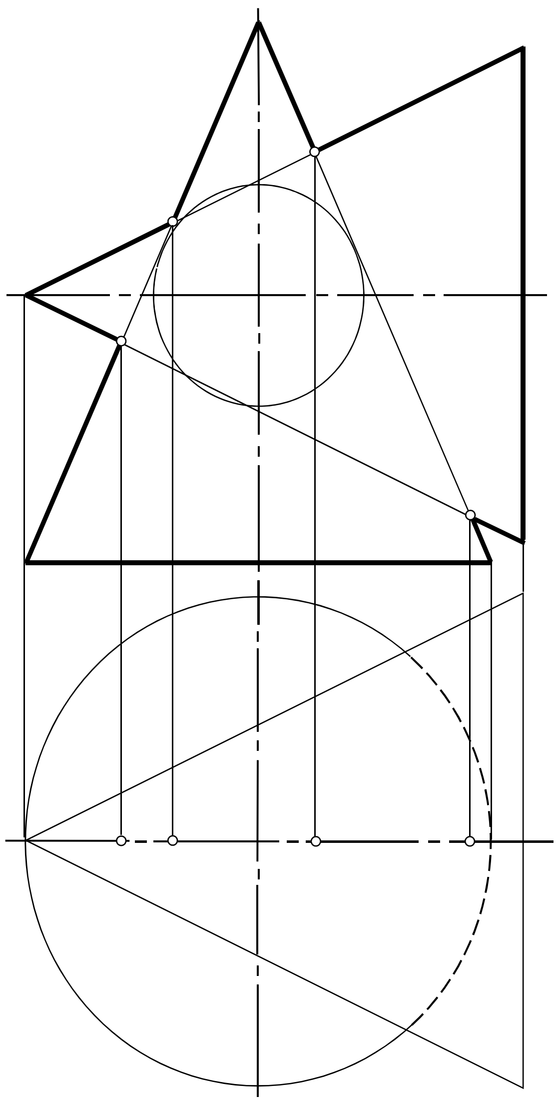


Fig. 6.13. Complete the projections of two intersecting cones

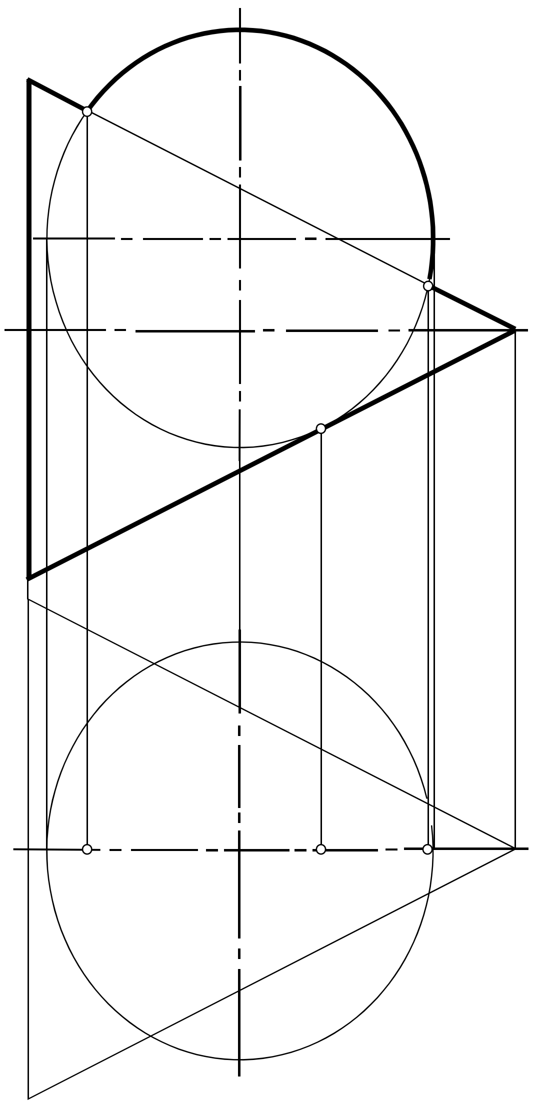


Fig. 6.14. Complete the projections of intersecting cone and sphere

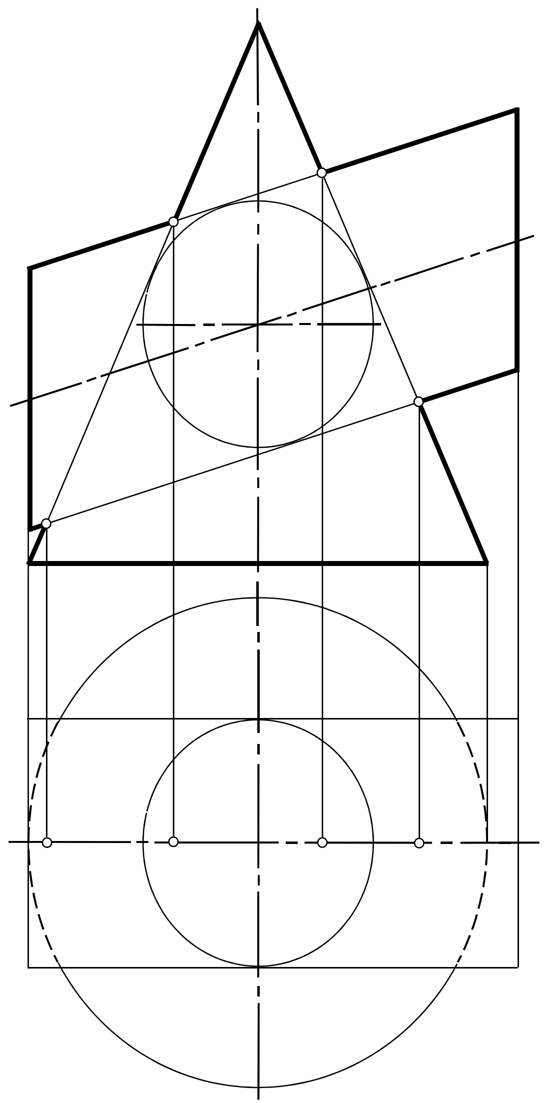


Fig. 6.15. Complete the projections of intersecting cone and cylinder (Monge's theorem)

6.2.3. Method of the auxiliary eccentric spheres

There are three graphical conditions for applying the method of auxiliary eccentric spheres:

– intersecting revolution surfaces of fourth order (open or closed torus) or surfaces of elliptical cylinder;

– there is a common plane of symmetry for both surfaces, and it is a level plane;

– axes of surfaces intersect or cross.

The essence of this method is similar to the previous one – concentric spheres method. The difference is that all auxiliary eccentric spheres have different centres. Therefore, a centre for all spheres should be specifying before their drawing.

The beginning and the end points are points of level outlines intersection, and they limit the field of solution.

As centres of spheres are varied, they are unknown, but we know that coaxial surfaces intersect in circles that makes possible to set firstly a circular section for one of the given surfaces. It is possible by taking an imaginary cutting plane (α) that cuts a circular section. It gives the centre or the middle point of the section, as the section is a circle. A perpendicular through the middle of the section is a radius of the required auxiliary sphere. The centre of this sphere is an intersection point of the perpendicular with the axis of the second surface. Having the centre enables to draw an auxiliary sphere from the found centre with radius that equals to distance from the centre to the limit points of the taken section (on the outline). The next part of the solving algorithm is the same as for the Method of auxiliary concentric sphere. The auxiliary sphere intersects with both surfaces in circles, one of them is known as it was taken before, and the second one is specified by the sphere intersection. Having two projections of circular sections gives their intersection line that is projected in one point (two competing points), that is a part of the required intersection line. The other projection of points is solved by their belonging and cutting plane method. Points are connected in the taken order.

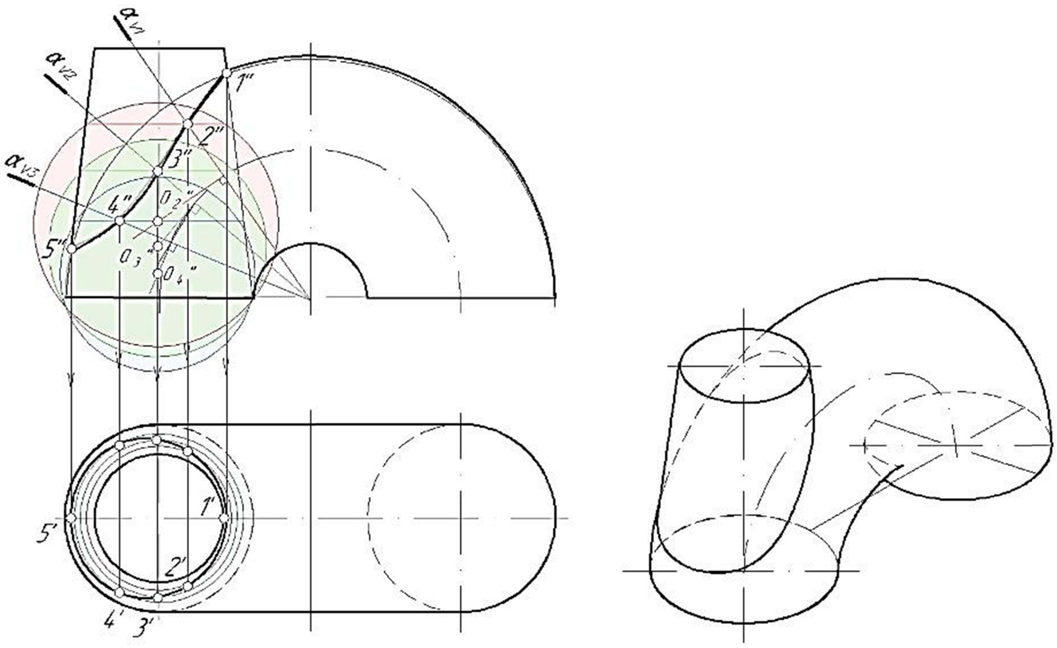


Fig. 6.16. Determining of an intersection line of a straight truncated circular cone with half of torus by the Method of auxiliary eccentric spheres

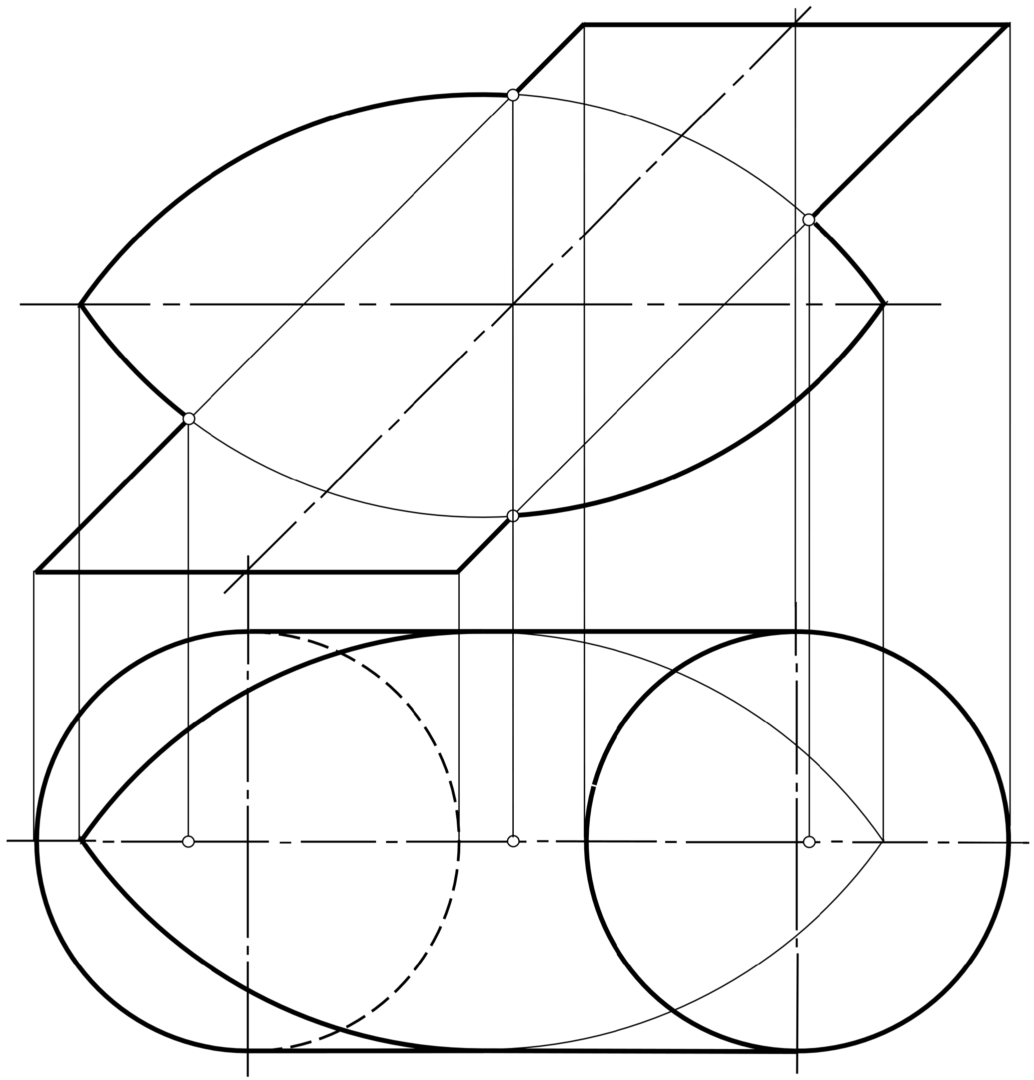


Fig. 6.17. Complete the projections of intersecting oblique elliptical cylinder and spindle (lemon) torus

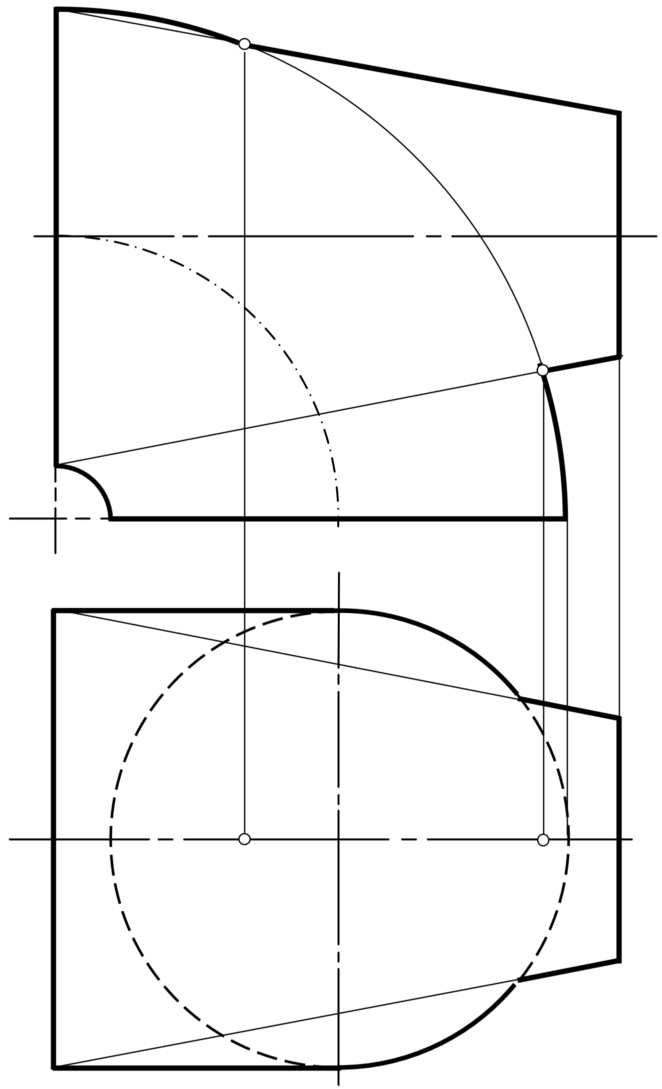


Fig. 6.18. Complete the projections of intersecting frustum cone and part of the ring torus

7. TRANSFORMATION METHODS

7.1. The auxiliary plane method

The auxiliary plane method is also known as **the change of reference-line method** or **the Method for replacement of planes**, and implies the substitution of a principal projection plane with a new one. The original does not change its location. A new projection plane should be taken in such a way that projection objects would change their property (e. g. general position objects are supposed to become particular ones) (fig. 7.1).

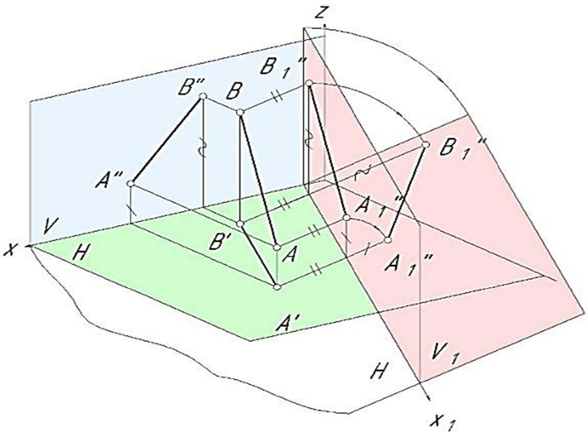


Fig. 7.1. A 3-dimentional model of a plane replacement

Four principal problems solved by the auxiliary plane method

1. Transform a line of general position into a line of level. Such transformation enables to determine the true size of the line-segment and its inclination angles to the projection planes (fig. 7.2, *a*).

2. Transform a level line into a projecting line, i. e. set it perpendicular to the projection plane and project the line as a point in this plane (fig. 7.2, *b*).

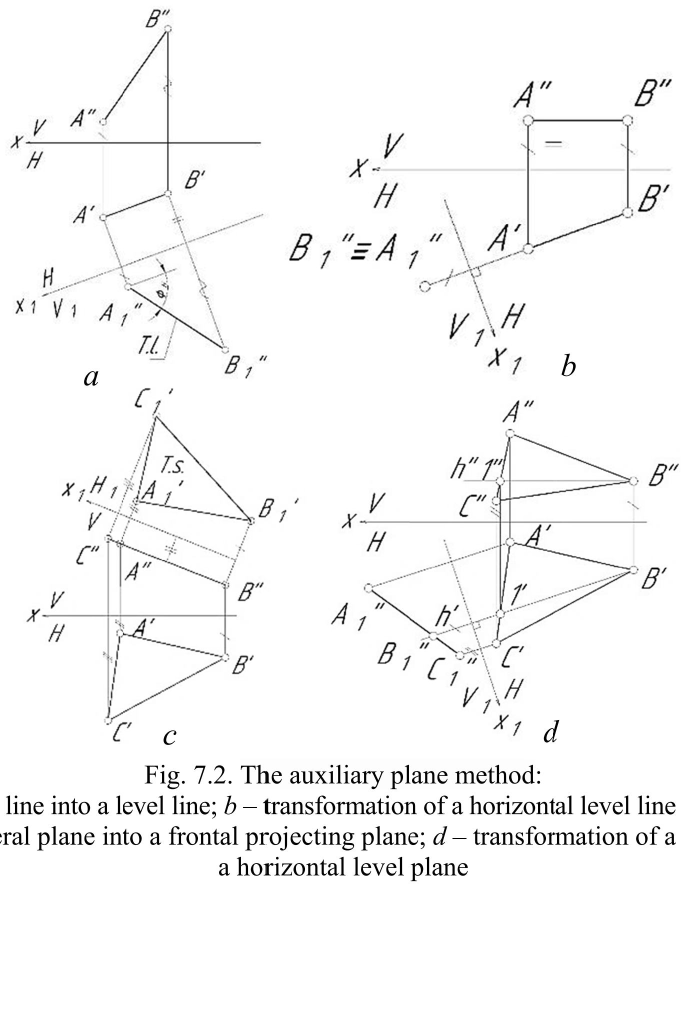


Fig. 7.2. The auxiliary plane method:
a – transformation a general line into a level line; *b* – transformation of a horizontal level line into a frontal projecting line; *c* – transformation of a general plane into a frontal projecting plane; *d* – transformation of a frontal-projecting plane into a horizontal level plane

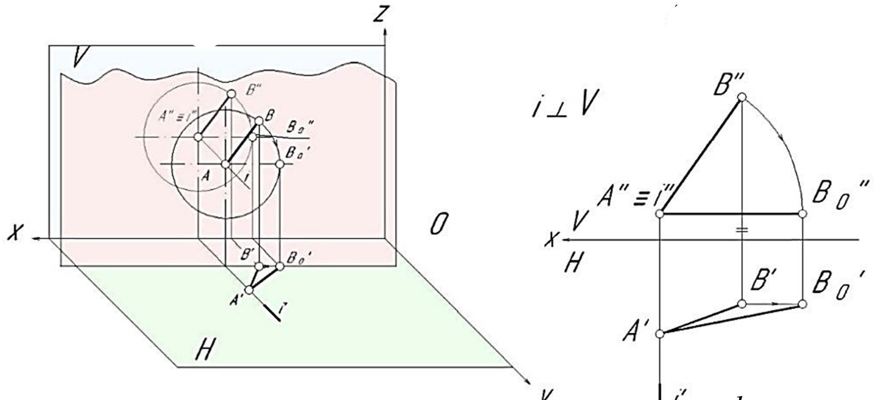
3. Transform a general plane into a projecting one (fig. 7.2, *c*), i. e. positioned perpendicular to one of the projection planes.

4. Transform a projecting plane into a level plane (fig. 7.2, *d*).

In order to transform the general object (a line or a plane) into projecting one, it is necessary to make two replacements, i. e. solve both problems, the first and the second ones, successively. Firstly we transform a general object into a level and then it is transformed into a projecting line.

7.2. Method for rotation around projecting axis

According to this method, the object has to be rotated around some projecting axis for changing its property from general into particular. In other words, the object changes its position and planes of projection are static, in contrast with the previous method. In such method, a general line can be transformed into a level line by its rotating to the parallel to axis location round set projecting axis.



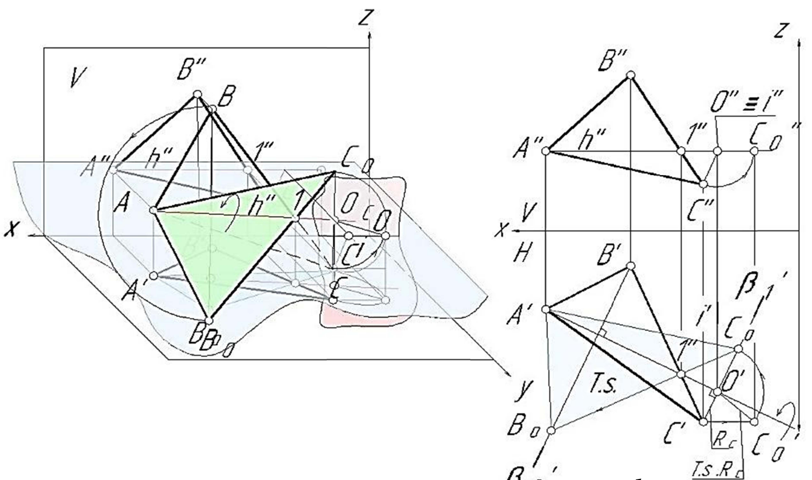
a *b*

Fig. 7.3. Determining of the line true length by the rotation around projecting axis method: *a* – a 3-dimantional model; *b* – a drawing of it

7.3. Method for rotation around main line (frontal or horizontal)

For solving a problem by this method, the main line (f or h) obviously should be drawn firstly. This level line is used as an axis line of rotation. Then, the traces of auxiliary rotation planes for all plane points are specified as perpendiculars to the level line. One point is fixed as it belongs to a level line, and it rotates round itself. Points of intersection of those perpendiculars with the main (level) line are centres of rotation. Radii of their rotation are general lines, and their true length should be solved by any known method.

These two methods are also called **methods of revolution about a line axis**.



a *b*

Fig. 7.4. Determining of a plane true size by the rotation around its main line (horizontal) method: *a* – a 3-dimantional model; *b* – a drawing of it

For more information, see [1, p. 29–34].

The auxiliary plane method. Practice

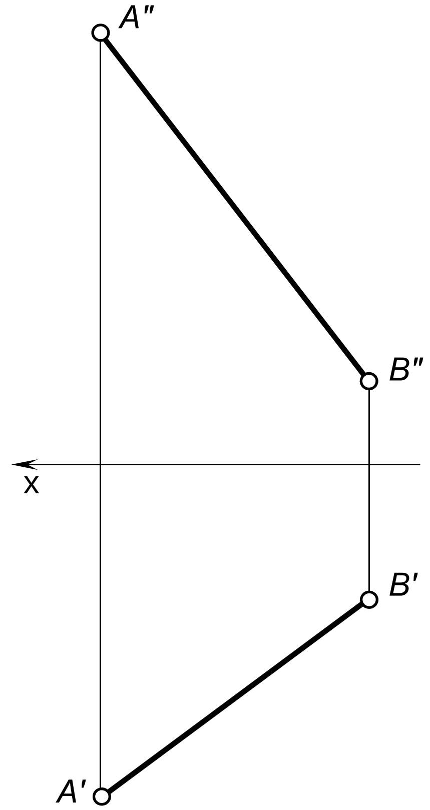


Fig. 7.5. Determine the true length of the general line AB(AB'', AB')

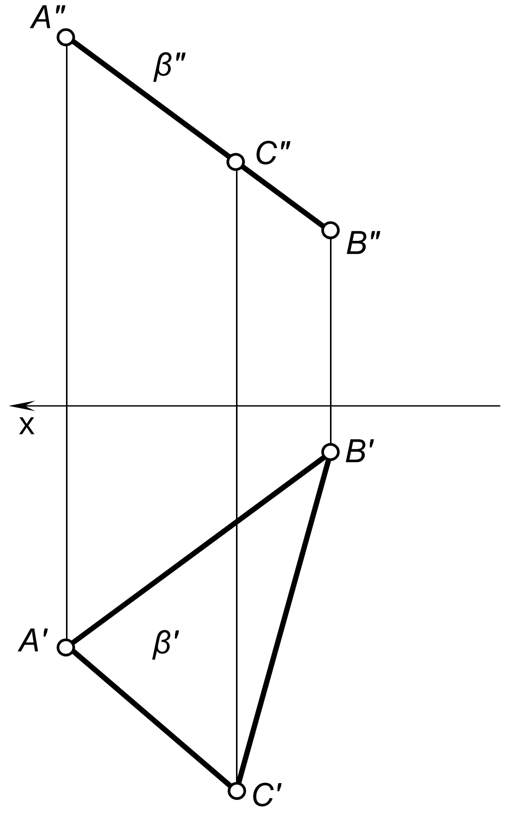


Fig. 7.6. Determine the true size of the frontal-projecting plane β(ΔABC)

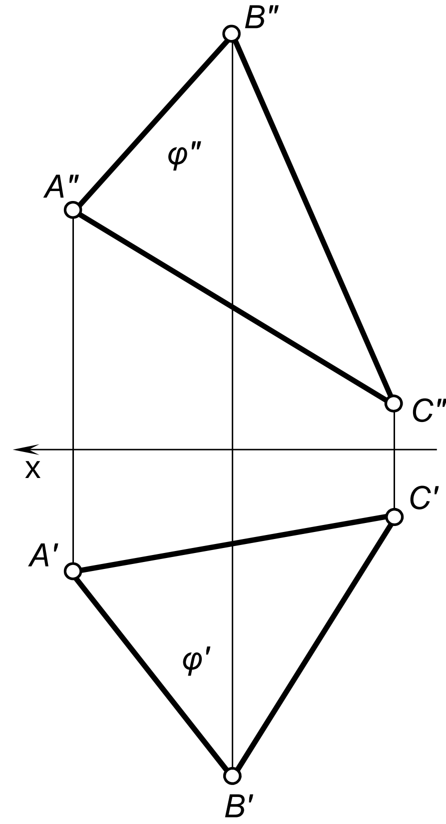


Fig. 7.7. Determine the true size of the general plane φ(ΔABC)

Method for rotation around the projecting axis. Practice

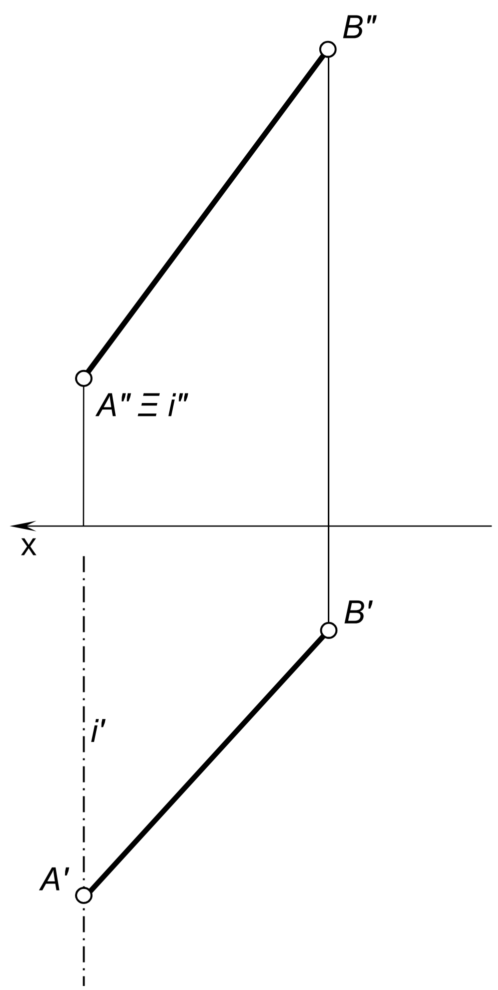


Fig. 7.8. Determine the true length of the general line AB(AB", AB') by rotating around frontal-projecting axis i(i", i') to horizontal location

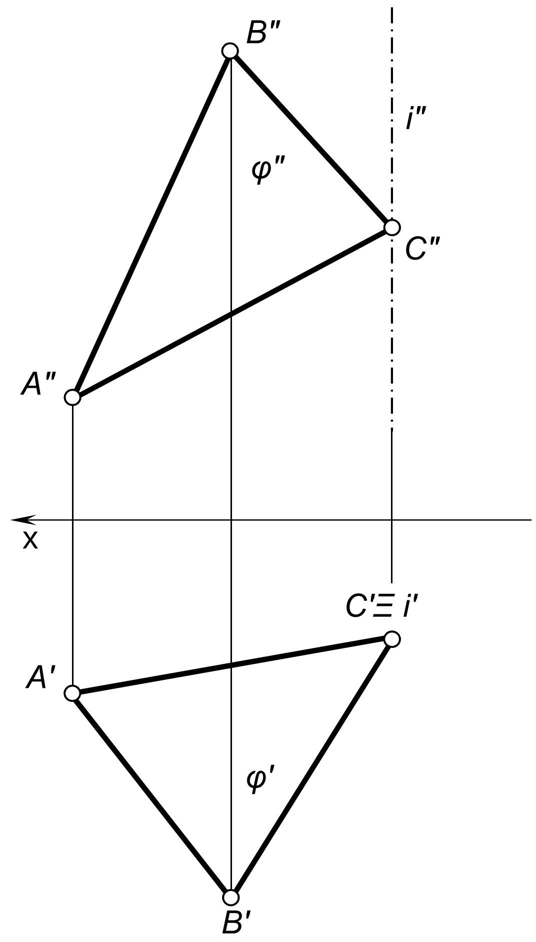


Fig. 7.9. Determine the angle of inclination for general plane φ(ΔABC) and horizontal projection plane by rotating around horizontal-projecting axis i(i", i')

Method for rotation around the main line (frontal or horizontal). Practice

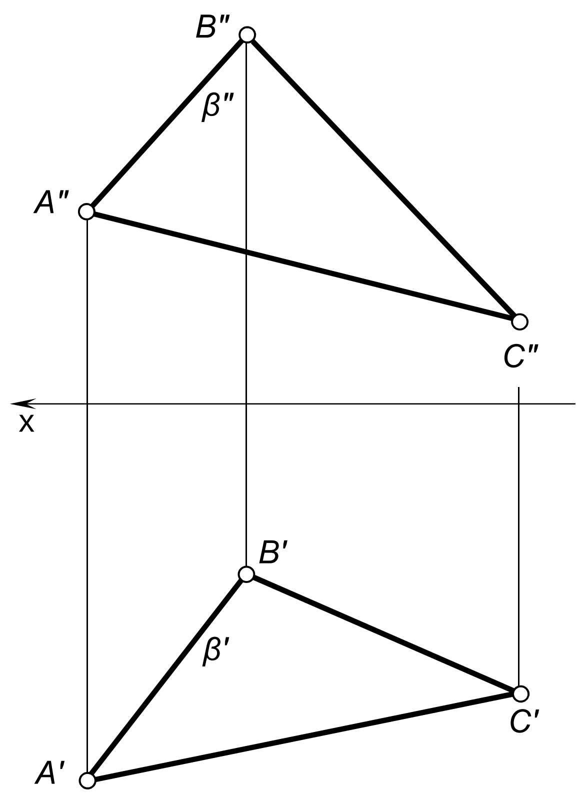


Fig. 7.10. Determine the true size of the general plane β(ΔABC) by rotating around the horizontal main line till coinciding with the horizontal projection plane

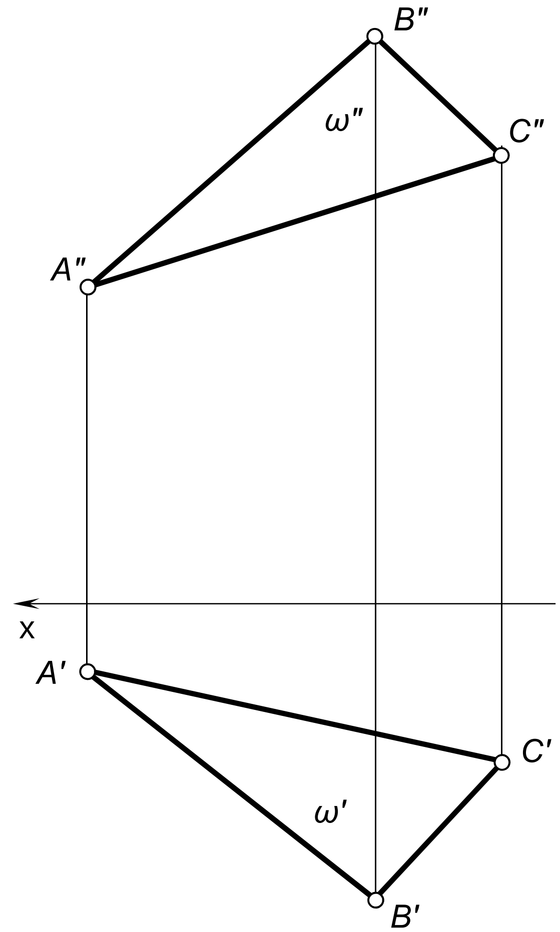


Fig. 7.11. Determine the true size of the general plane ω(ΔABC) by rotating around the frontal main line till coinciding with the frontal projection plane

Solids sectioning by a general plane (transformation methods). Practice

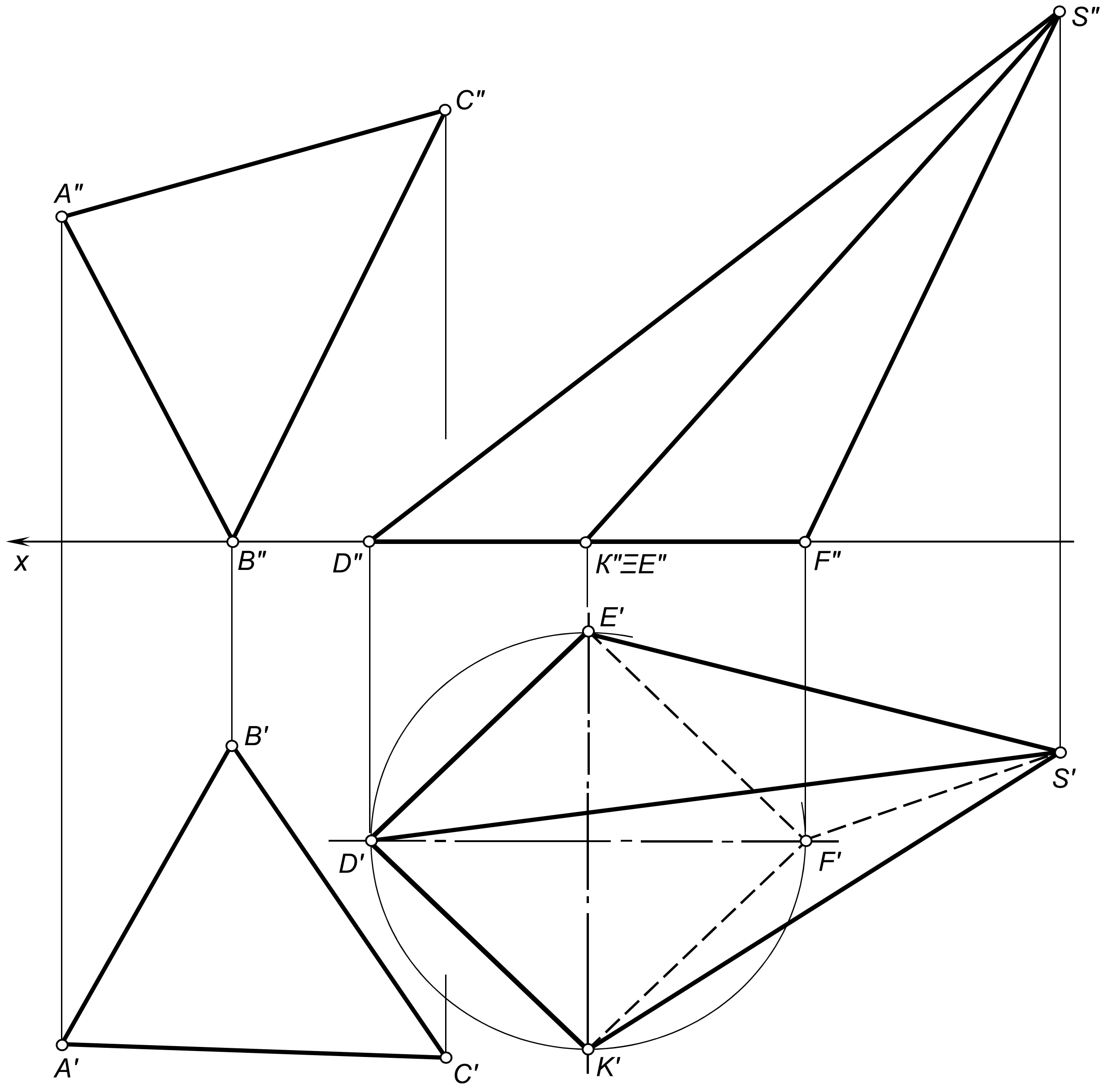


Fig. 7.12. Obtain an intersection line of the oblique square pyramid SDEFK with the general plane β(ΔABC). Determine the true shape of a section taking a horizontal main line for β(ΔABC). Use the auxiliary plane method (Change of reference-line method). It is also required to determine the true shape of the β(ΔABC) plane by the rotation method

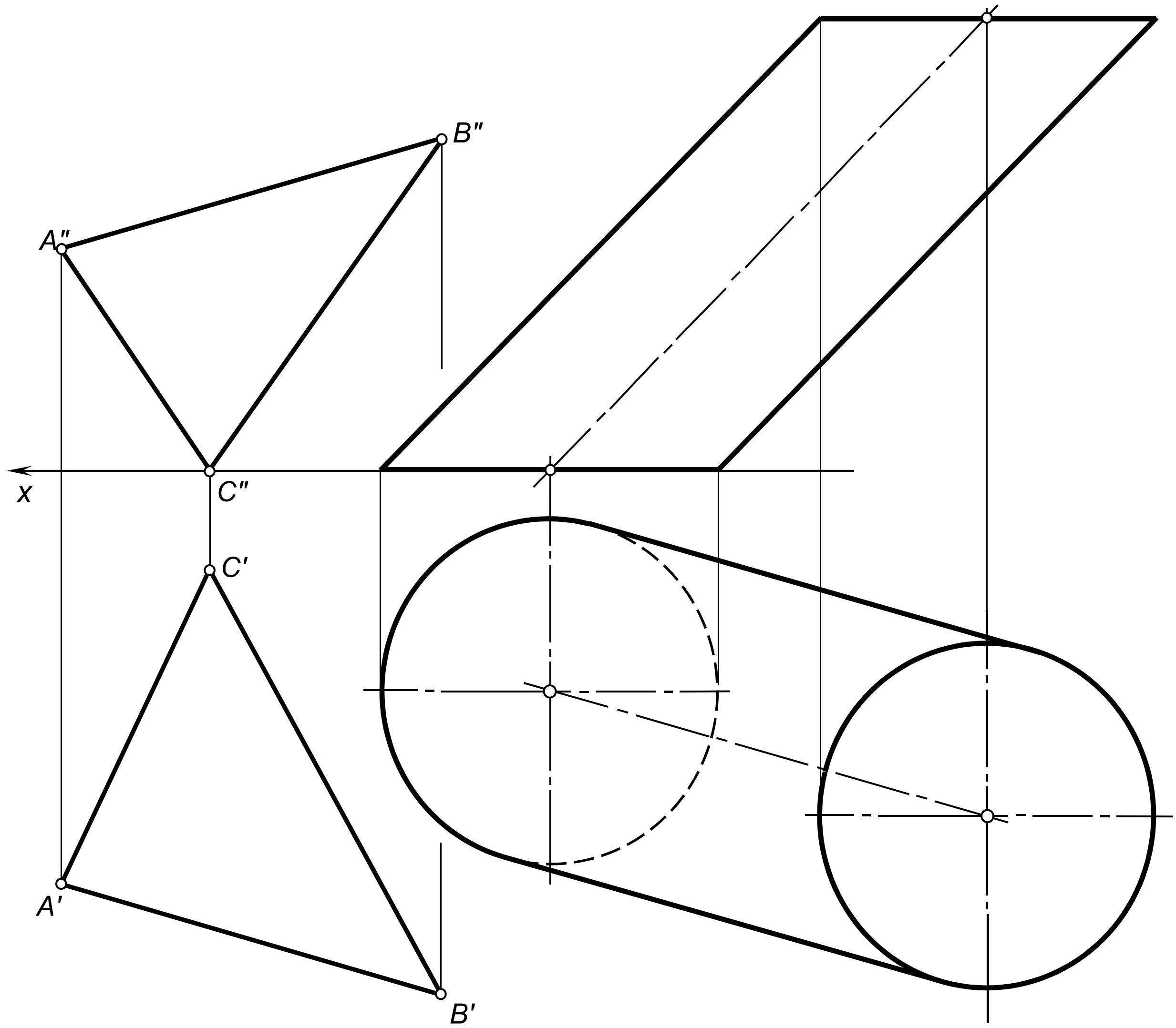


Fig. 7.13. Obtain an intersection line of the oblique circular cylinder with the general position plane β(ΔABC). Determine the true shape of a section taking a horizontal principal line for β(ΔABC). Use the Projection plane replacement method (Change of reference-line method). It is also required to determine the true shape of the ABC triangle by the rotation method

8. SURFACE DEVELOPMENT. METHOD OF TRIANGULATION

The development of the side surface of the pyramid according to the true size of its edges is carried out using the following graphical algorithm.

1. Specify the true size for all pyramid edges by any method (Method of rotation around projecting line, method of replacement planes of projection etc.) and for its base. (If the base is the plane of the level, then its true size is given in one of the projections).

2. Draw the faces of the pyramid successively in the free field of the drawing according to their true sizes with compass by some additional thin arcs, so that they have a circular vertex S and adjoin each other.

3. Finish the drawing in accordance with GOSTs – fold lines have to be drawn by Long dashed double dotted thin line.

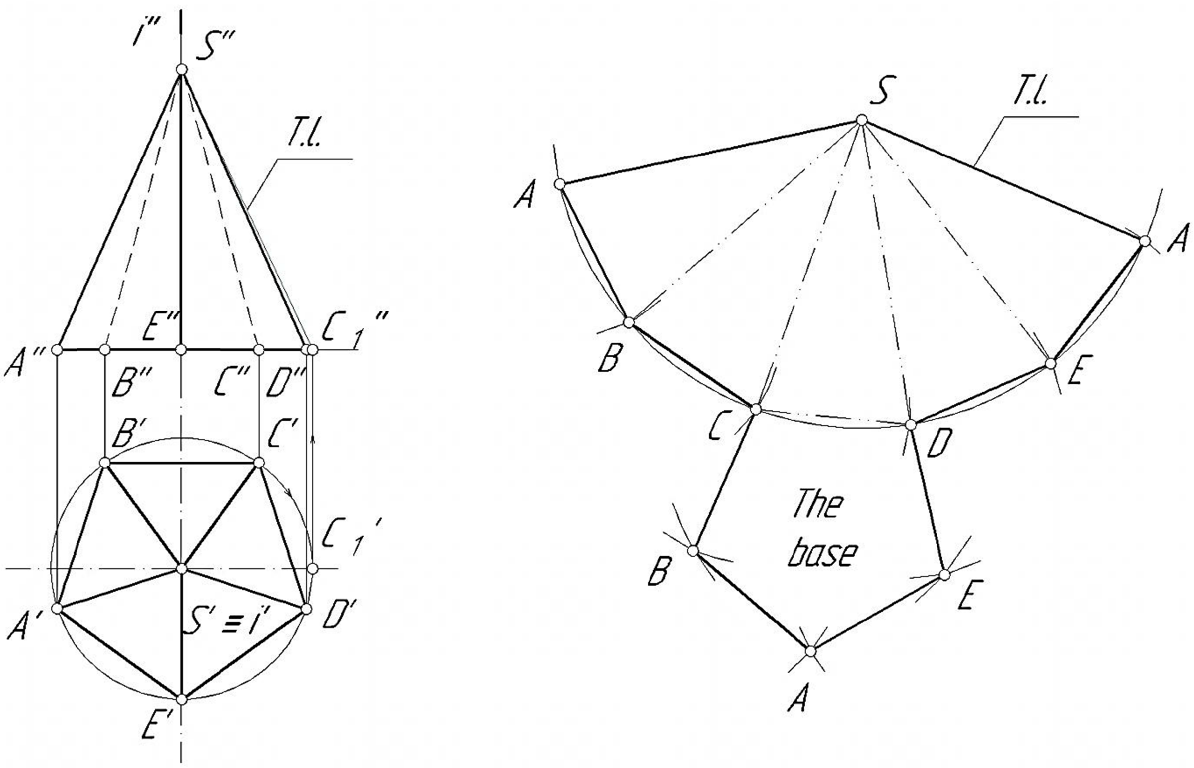


Fig. 8.1. The development of a triangular pyramid surface

Surface development. Practice

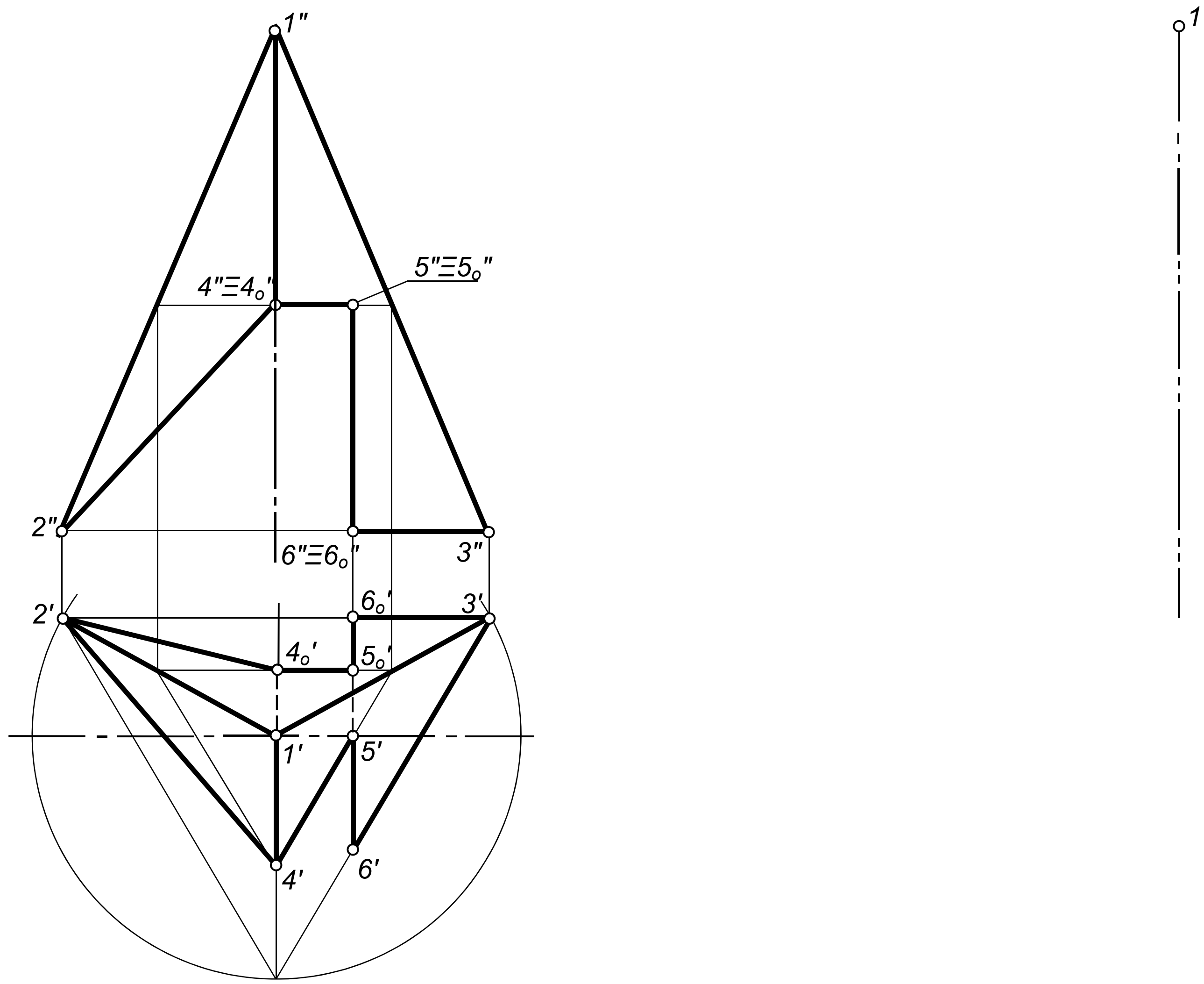


Fig. 8.2. Draw a surface development for a triangular pyramid with frontal-projecting, horizontal and profile level sections.
Put edge 13 on the folding line (provide the minimum perimeter of the development)

9. AXONOMETRIC PROJECTION

9.1. Orthogonal isometric projection

It is also known as an isometric axonometry. By this type of the axonometric projections, angles among axis lines are equal to 120°. The distortion coefficients are: $K_x^2 + K_y^2 + K_z^2 = 2$.

Therefore, $K_x = K_y = K_z = 0.82$[1] [4] (0.816 [9]). In drawing practice, the projected unit length segments are rounded to the nearest whole number for $K_x = K_y = K_z = 1$ simplifying work with axonometric and equal to which corresponds to the obtained representation enlarged by a factor 1.22 [4] (1.225 [9]).

A circle in axonometric projection is generally projected in an ellipse. When constructing an ellipse, it is necessary to know the direction of its axes and their dimensions. There are several methods for drawing a circle in isometric projection. Here is the one of them (see fig. 9.1). The biggest axis of ellipse is *AB* = 1.22d the smallest axis is *CE* = 0.71d where **d** is the diameter of the given circle (fig. 9.1).

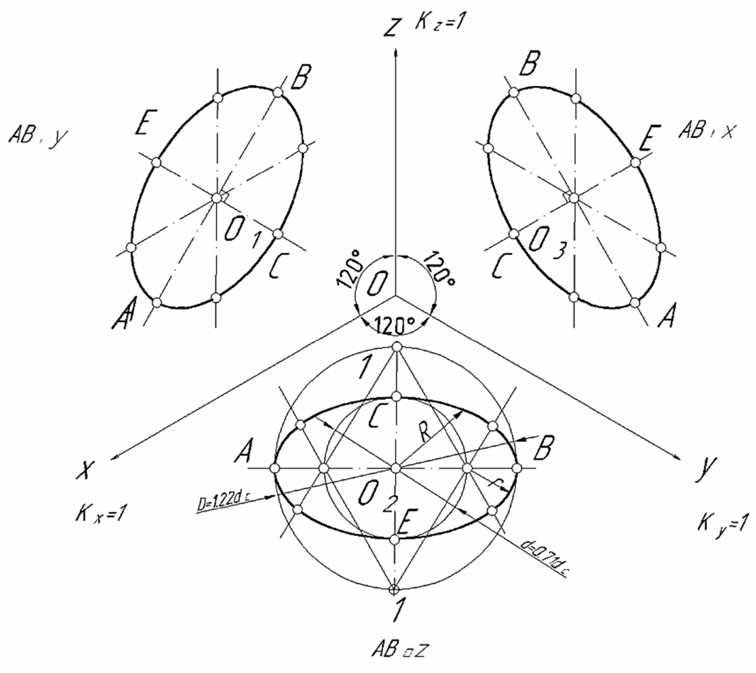


Fig. 9.1. An isometric projection of a circle

Note: a minor axis of an ellipse is always perpendicular to the major one. When a circle projection is constructed (a circle lies in one of the co-ordinate planes), the minor axis of the ellipse is directed parallel to the axonometric axis which does not participate in the formation of the plane.

9.2. Orthogonal dimetric projection

Dimetric axonometry is used when a view of the object to be represented is of main importance and for square surfaces if their edges are located on axes. For the orthogonal dimetric projection, two factors are equal and the third one is a half of one of them: $K_x = K_z = 0.94$[1], $K_y = 0.47$[1]. After their simplification they are: $K_x = K_z = 1$[1,2] and $K_y = 0.5$[1,2]. The axis Z is vertical, the axis x has an angle 7°10' [1] (7° [2]) with the horizontal level and the axis y has angle 41°25' (42° [2]). These angles can be specified in the following way (fig. 9.2, *a*):

Circles by the orthogonal dimetric projection method are ellipses. Two of them are the same and the third one is different. The biggest axis AB for all three ellipses is 1.06d. For circles in the horizontal and profile projection planes, the smallest axis CE has the same direction with axes x and z, respectively, and is equal to 0.35d. For a circle in the frontal plane of projection it is 0.95d and the biggest axis AB is perpendicular to the axis y. The biggest axis and the smallest one are mutually perpendicular (fig. 9.2, *b*).

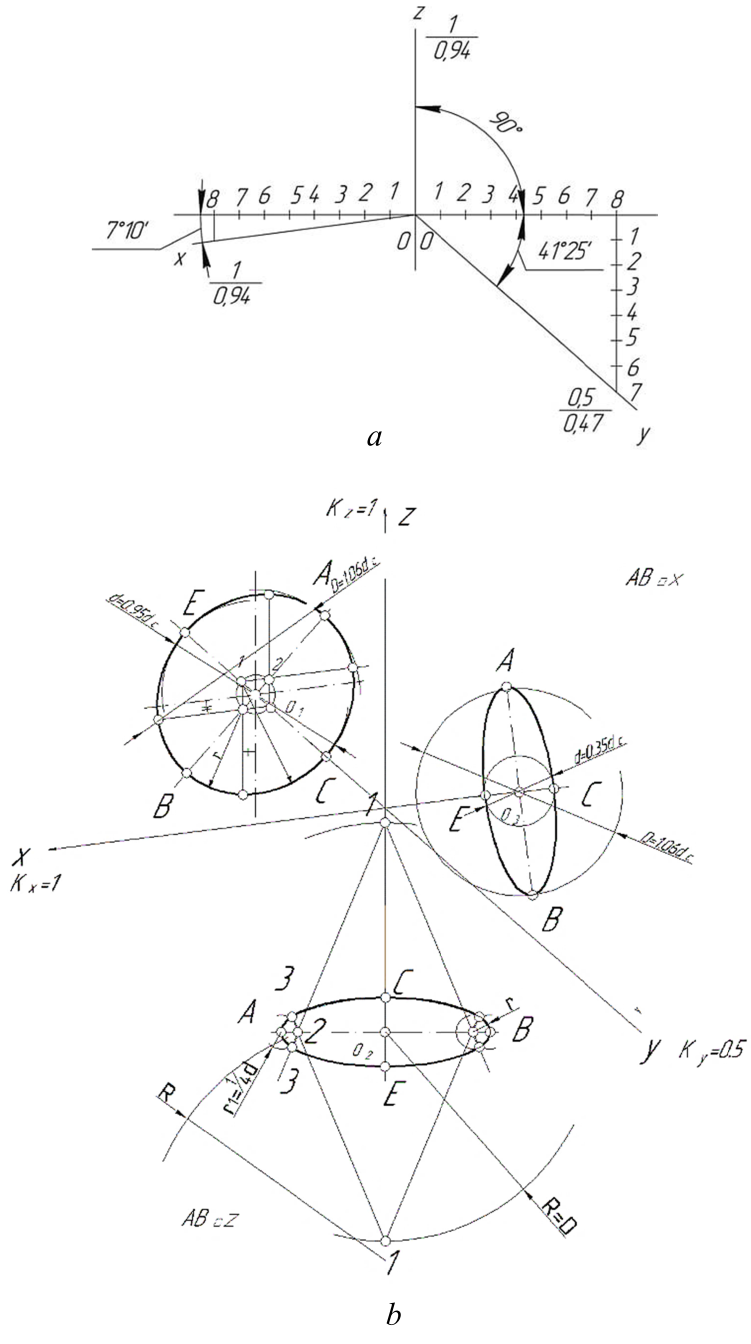


Fig .9.2. Orthogonal dimetric projection:
a – angles between axes for an orthogonal dimetric projection; *b* – an orthogonal dimetric projection of a circle

9.3. Oblique frontal isometric projection

Oblique frontal isometric projection is also known as a **Cavalier axonometry.** This projection method is similar to the cabinet axonometry except that the scales on the three projected axes are identical and equal to 1. In both other projections, circles are distorted in ellipses with the major axis AB that is equal to 1.3d and the minor CE that is equal to 0.54d. The major axis has an inclination in 22°3’ [4], (7° [9]) to the axes z and x for the projection of a profile level circle and for the projection of a horizontal level circle, respectively (fig. 9.3).

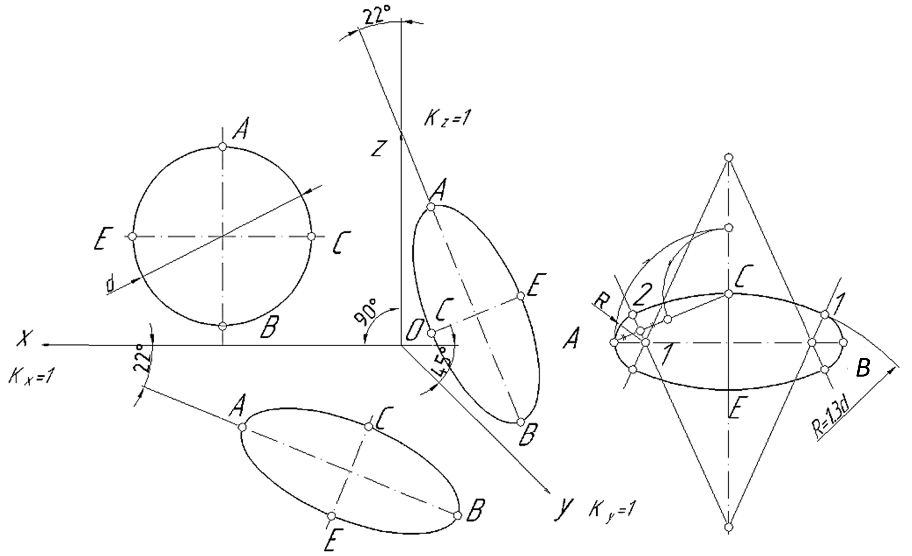


Fig. 9.3. An oblique frontal isometrci projection of a circle (cavalier axonometry)

9.4. Oblique frontal dimetric projection

Oblique frontal dimetric projection is also known as **a cabinet axonometry**. As for all dimetric projection distortions, coefficients are: $K_x = K_z = 1$ and $K_y = 0.5$ (their precise value have been mentioned above). In this type of oblique axonometry, the projection plane is normally vertical and the projection of the third coordinate axis is chosen by convention at 45° to the remaining projected orthogonal axes. In such axonometry the frontal plane of projection is not distorted and saves its true representation (a circle is a circle). (fig. 9.4). In both other projections, circles are distorted in ellipses with the major axis AB that is equal to 1.07d and the minor CE that is equal to 0.33d. The major axis has an inclination in $7^{o}14'^{1}$ (7^{o2}) to the axes z and x for the projection of a profile level circle and for the projection of a horizontal level circle, respectively.

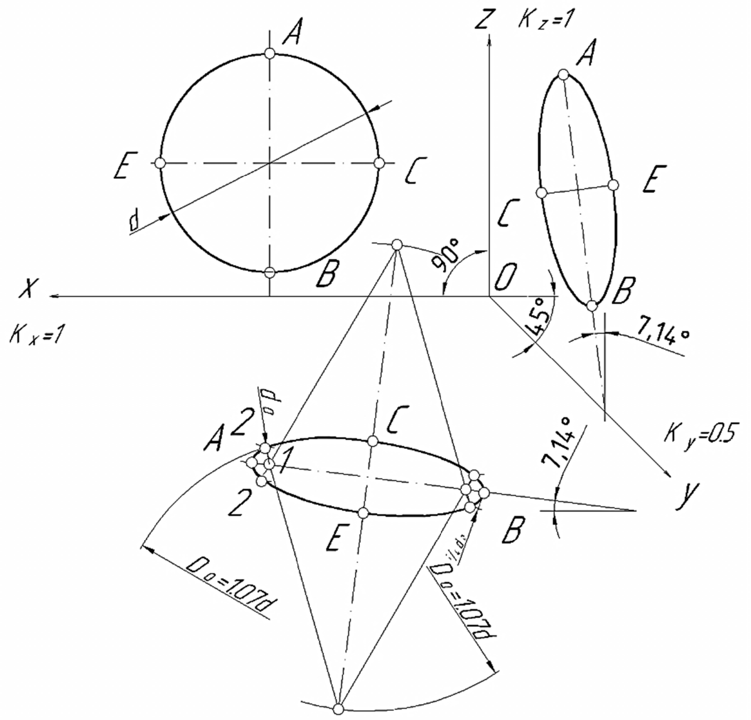


Fig. 9.4. An oblique frontal dimetric projection of a circle (cabinet axonometry)

For more information see standards [4; 9].

Axonometric projections. Practice

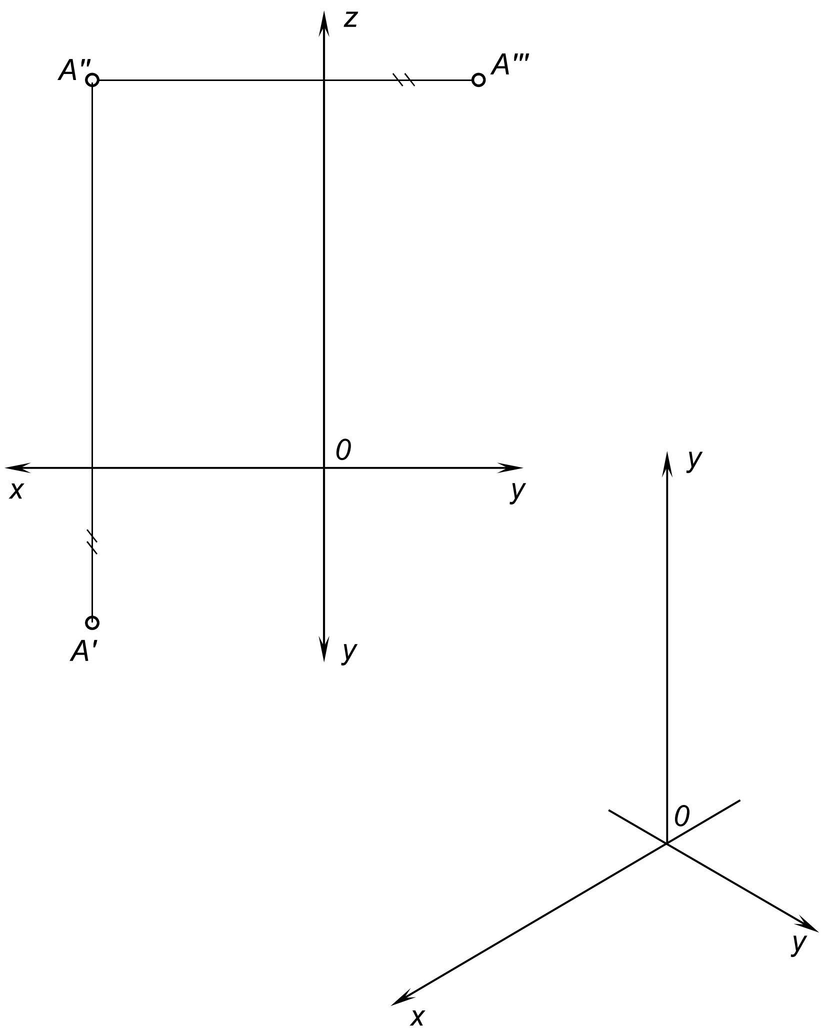


Fig. 9.5. Draw an isometric axonometry of the point A(A", A', A"')

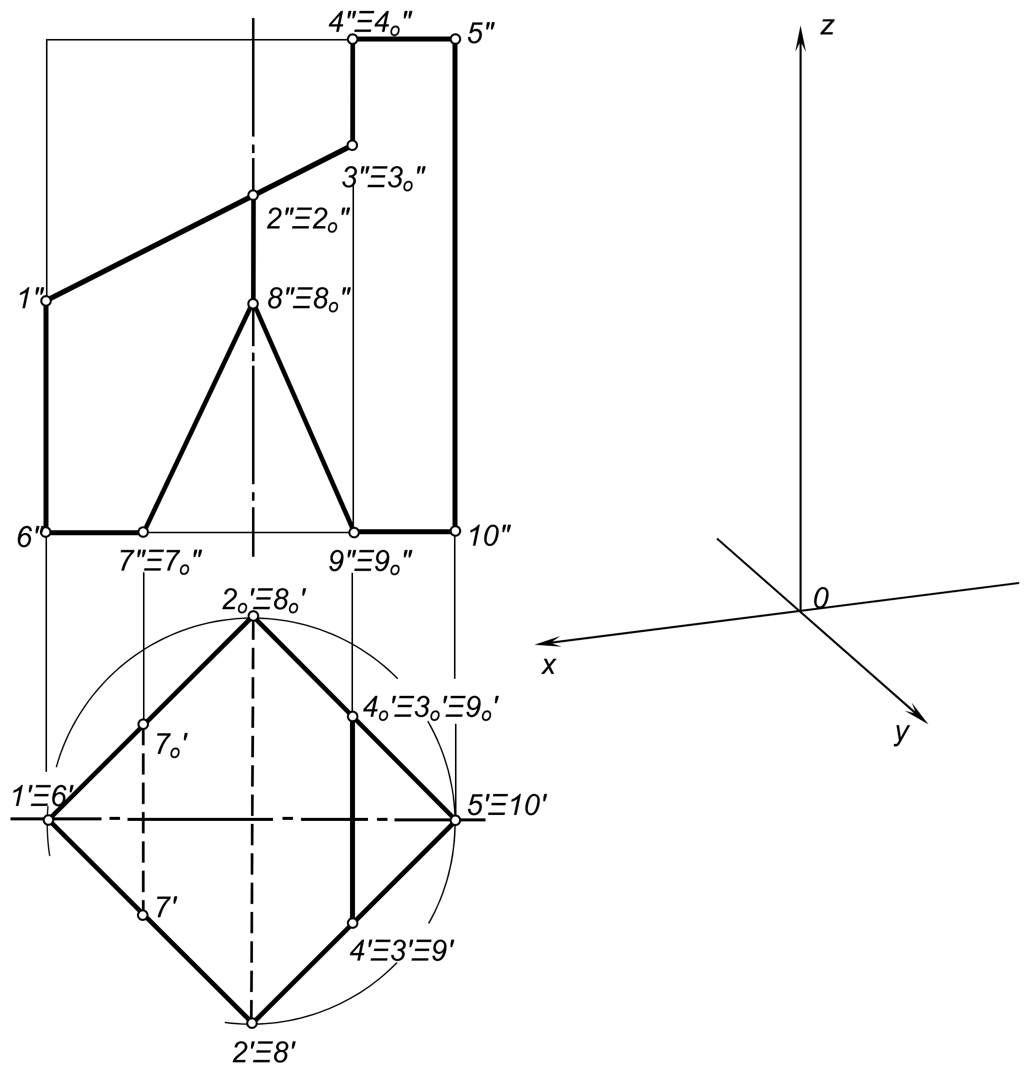


Fig. 9.6. Draw a dimetric axonometry for the prism

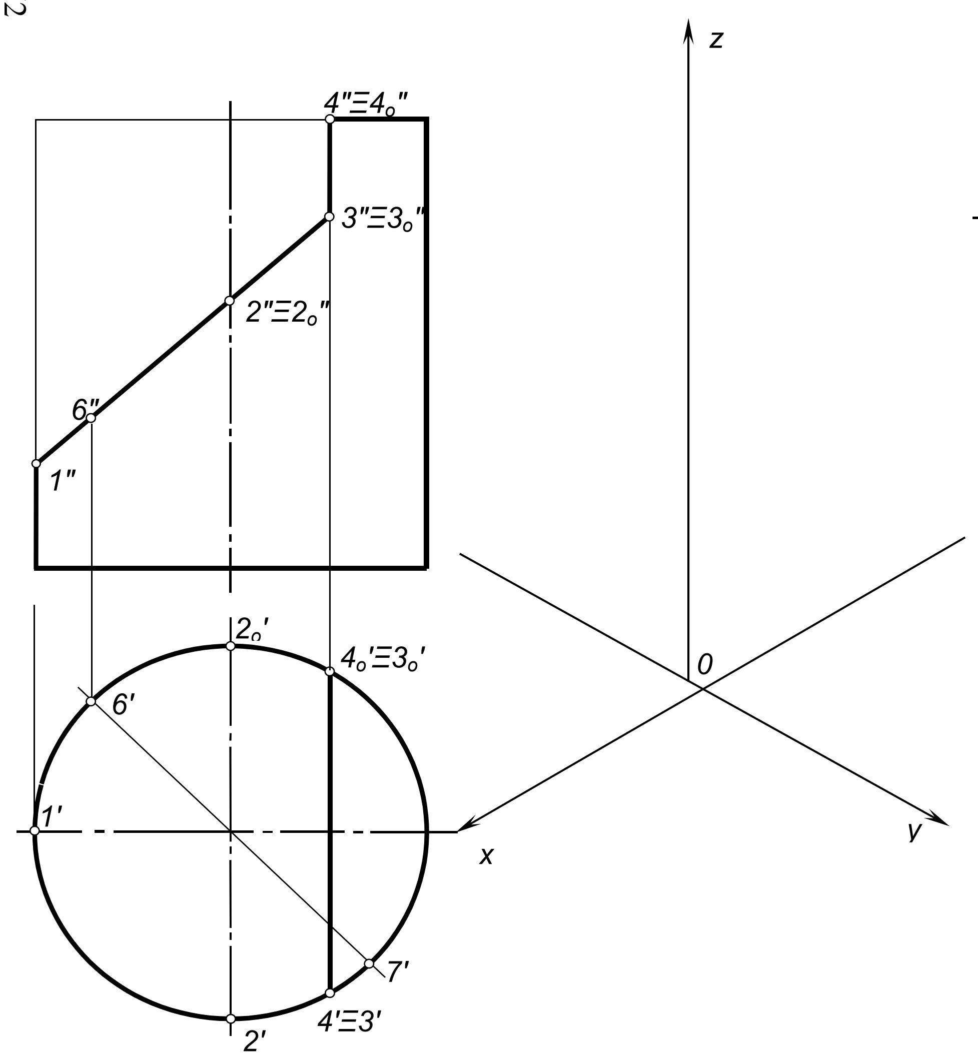


Fig. 9.7. Draw an isometric axonometry for the cylinder

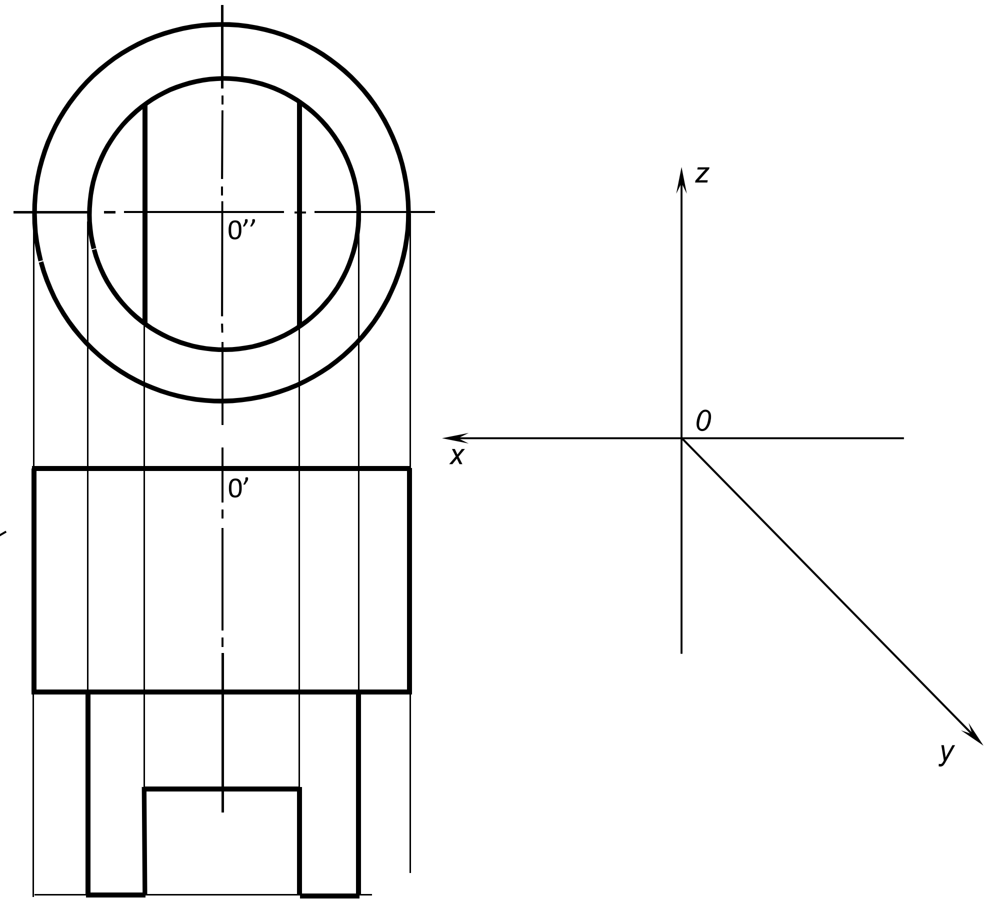


Fig. 9.8. Draw a cabinet axonometry (oblique dimetric) for the composite solid

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