RUCH FILOZOFICZNY
LXXVI 2020 2

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DOI: http://dx.doi.org/10.12775/RF.2020.029

The Forgotten Jerzy Łoś's Contribution to Philosophical Logic. Logics with Realization Operator R

Jerzy Maria Łoś was born on 22 March 1920 in Lvov and died on 1 July 1998 in Warsaw. He studied medicine, chemistry and philosophy at Jan Kazimierz University in Lvov until his education was interrupted by the Second World War. During the occupation, he held a white-collar position in a sugar mill in Lublin. After the war, Łoś returned to his philosophical studies at Maria Curie-Skłodowska University in Lublin where he defended his master's thesis entitled *Analiza metodologiczna kanonów Milla* (*Methodological analysis of Mill's cannons*) written under the supervision of Jerzy Słupecki. In a modified version, the work was published in the form of an article in 1947.[1] At that time, he worked as Słupecki's assistant at the Department of Mathematical Logic, the University of Wrocław. He continued his scientific work in Toruń and Warsaw, fo-

[1] Jerzy Łoś, "Podstawy analizy metodologicznej kanonów Milla" ("Foundations of the methodological analysis of Mill's canons"), *Annales Universitatis Mariae Curie-Skłodowska* 2.5. F (1947).

cusing on foundations of mathematics, algebra, and applied mathematics in economics and computer science.

Jerzy Łoś is well-known due to his numerous achievements in the world of logic and mathematics. His scientific interest is mostly centered around algebra and mathematical logic. However, in the early stages of his activity, in the late 40s, Łoś was involved in discovering and describing the first systems of philosophical logic: temporal and epistemic logic. His early research resulted in two articles: the above mentioned *Analiza metodologiczna kanonów Milla* (*Foundations of the methodological analysis of Mill's canons*)[2] and *Logiki wielowartościowe a formalizacja funkcji intensjonalnych* (*Many-valued logics and the formalization of intensional functions*).[3] In the first paper, Łoś described the first system of temporal logic. In the second one, the author formalized the first system of epistemic logic in history, i.e. logic of knowledge. The Iron Curtain limited the promotion of Łoś's achievements among Western scientific circles. For this reason, Łoś was never awarded the well-deserved title of the temporal and epistemic logic pioneer and his name is scarcely mentioned in the works concerning these areas. In the past and the present year, the 70th anniversary of the both publications is marked; hence, it is also the jubilee of the discovery of the first systems of temporal and epistemic logic. It is a great opportunity to bring back early ideas by Łoś to the wider public and to recollect his path-finding role in both epistemic and temporal logic.[4]

It should be highlighted here that publications by Łoś are the first works from the area of epistemic and temporal logic. We can state, therefore, that Jerzy Łoś initiated those disciplines, which – unfortunately – is rarely mentioned in the literature. It is not the case, however, that Łoś remained completely anonymous in the world society of logical philosophy. There are some works, though not many, in which the Polish logician and his formal system are mentioned. Substantially, it can be said that Łoś's name appears only in early, also pioneering, works on philosophical logic. Owing to reviews in foreign journals, main ideas included in Łoś's publications became accessible to wider public outside the country. What we mean here are two reviews: the first one, authored by

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[2] Ibidem.

[3] Jerzy Łoś, "Logiki wielowartościowe a formalizacja funkcji intensjonalnych" ("Many-valued logics and the formalization of intensional functions"), *Kwartalnik Filozoficzny* XVII 1–2 (1948).

[4] On this occasion, to commemorate the anniversary of Jerzy Łoś's death, on 24–27 September took place the 9th edition of the conference "Non-Classical Logic. Theory and Applications. The origins of philosophical logic: 70 years of Jaśkowski's and Łoś's contributions". The organizers were the Department of Logic of the Nicolaus Copernicus University together with the Department of Logic and Methodology of the University of Lodz. Source: http://ncl.umk.pl/LNK18/lnk18.html.

Henry Hiż,[5] was published in *Mathematical Reviews* and the other, which was written by Roman Suszko,[6] appeared in the *Journal of Symbolic Logic*. They were both prepared in just a few years' time after Łoś's publications, so the American scientists could familiarize with the achievements by Słupecki's assistant almost immediately (as for those times). Therefore, the system of temporal logic was presented in the book entitled *Formal logic* by Arthur Prior.[7] Nicholas Rescher also acknowledges Łoś's contribution by writing:

> The founder of assertion logic is the Polish logician Jerzy Łoś. In his important 1948 paper, Łoś developed what he called a logic of 'belief' or 'assertion' upon the following basis. An L-operator was introduced with 'Lxp' to mean 'the man x believes (or: is committed to) the proposition p'.[8]

We can be sure that Rescher regarded Łoś as the founder of epistemic logic[9] and considered his article as important. Unfortunately, this opinion was not influential in the world and the knowledge about Łoś's early work on philosophical logic was forgotten for some reason. In the influential contemporary literature, while Jerzy Łoś's contribution to the topic is completely forgotten, the precedence is given to later works – very often inspired by the Polish logician's ideas. Lack of such information in historical books and papers is especially striking as such sources should inherently provide reliable knowledge. An example of such a work is a book on the history of temporal logic, *Temporal Logic: from Ancient Ideas to Artificial Intelligence*.[10] Despite its historical character, this book does not even contain a footnote about Łoś's pioneer work. It is quite similar in the case of an introductory textbook to philosophical logic, *The Blackwell Guide to Philosophical Logic*. A single reference to Łoś's papers can be found in neither the part devoted to temporal logic[11] nor that on epistemic logic.[12] Łoś is also absent in the both bibliographies.

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[5] Henry Hiż, "Review of: J. Los, Podstawy analizy metodologicznej kanonów Milla", *Journal of Symbolic Logic* 16, I (1951): 58–59.

[6] Roman Suszko, "Review: Jerzy Los, Many-Valued Logics and the Formalization of Intensional Functions", *Journal of Symbolic Logic* 14, I (1949): 64–65.

[7] Arthur Prior, *Formal Logic* (Oxford: Clarendon Press, 1955), 313.

[8] Nicholas Rescher, "Topological Logic", in: *Topics in Philosophical Logic*, ed. Nicholas Rescher (Dodrecht-Holland: Reidel Publishing, 1968), 262–263.

[9] Back then, the term 'assertion logic' was used for logics covering propositional attitudes, i.e. epistemic modalities.

[10] Peter Øhrstrøm, Per F.V. Hasle, *Temporal Logic: From Ancient Ideas to Artificial Intelligence* (Netherlands: Springer, 1995).

[11] Yde Venema, "Temporal Logic", in: *The Blackwell Guide to Philosophical Logic*, ed. Lou Goble, (Massachusetts, Oxford: Blackwell Publishers, 2001), 203–223.

[12] John-Jules Meyer, "Epistemic Logic", in: *The Blackwell Guide to Philosophical Logic*, ed. Lou Goble (Massachusetts, Oxford: Blackwell Publishers, 2001), 183–202.

In the influential *Stanford Encyclopedia of Philosophy*, in the respective entries, the attitude towards Jerzy Łoś's discoveries is far from reliable. In the entry, *Temporal Logic*,[13] Łoś's name appears neither in the text, nor in the bibliography. On the other hand, in the introduction to the *Epistemic Logic* entry[14] one can find Jerzy Łoś among many names of prominent logicians and philosophers involved in the area of epistemic logic. It is the only mention of him, however, because in the further parts of the entry there is no discussion on his logical system of knowledge and, what is more, Łoś's paper is not included in the bibliography.

The above-mentioned examples are just a few of the numerous cases. There is no doubt that the pioneering nature and significance of Łoś's work in the area of philosophical logic has been almost completely forgotten outside Poland. It should be stressed that temporal and epistemic logic are not marginal branches of logic interesting only to few enthusiasts – both disciplines are important in philosophy, computer science and studies on Artificial Intelligence, which makes Łoś's achievements even greater.

The main idea behind the both works by Łoś was to introduce a special operator which – from the grammatical point of view – binds some name with a sentence. Such an operator nowadays is often referred to as an "R-operator" and it enables expressing lots of contents that can be expressed neither in a pure sentential language nor in a quantifier language. The elastic nature of the R-operator comes from the fact that the name that is bound by it can be interpreted in plenty of ways. Originally, in the work by Łoś, those names were interpreted as time moments – in temporal understanding from the first article and as agents of knowledge – in the case of the article devoted to epistemic logic. Now, we will recall the logic of Łoś given in both of his articles.

In the first article published in 1947, natural sciences and their philosophy seem to be his main inspiration. Łoś belived that the nature of logic is mathematical in such a sense that the objects of its concern exist beyond time. His ambition, however, was to provide logic which would serve as a tool for reasoning in natural sciences, for example in physics, in which time is an essential variable. The alphabet of Łoś's logic consists of propositional letters p, q, r,…; quantifiers $\forall, \exists$; classical connectives $\sim, \wedge, \vee, \rightarrow, \leftrightarrow$ of negation, conjunction, disjunction, implication and equivalence, respectively; individual variables x, y, z,…; brackets), (; and specific symbols, U and δ. In the system, quantification

[13] Valentin Goranko, Antony Galton, "Temporal Logic", *The Stanford Encyclopedia of Philosophy*, access 12.09.2020, https://plato.stanford.edu/archives/win2015/entries/logic-temporal.

[14] Vincent Hendricks, John Symons, "Epistemic Logic", *The Stanford Encyclopedia of Philosophy*, access 12.09.2020, https://plato.stanford.edu/archives/fall2015/entries/logic-epistemic/.

over propositional variables is allowed just as in case of individual variables which refer to points as well as intervals of time. The U operator expresses time realization: the Uxp formula means that the state p is realized in time x. δ is used in the language to form complex terms: δ (*x*, *y*) denotes the point in time that occurs after x in the interval of the length y. What is important is that the U operator cannot be iterated, i.e. expressions of the form are well formed formulas, but *UxUxp* are not. The following secondary expressions are introduced in the language given by the following definitions:

$$\varphi I \psi := \forall x(Ux\varphi \leftrightarrow Ux\psi)$$

$$x \varrho y := \forall p(Uxp \leftrightarrow Uyp)$$

$$x \pi y := \exists n(\delta(x,n) \varrho y)$$

$$x \pi n y := \delta(x,n) \pi y$$

The system consists of the following axioms:

1) $Ux{\sim}p \leftrightarrow {\sim}Uxp$,
2) $Ux(p \to q) \to (Uxp \to Uxq)$,
3) $Ux((p \to q) \to ((q \to r) \to (p \to r)))$,
4) $Ux(p \to ({\sim}p \to q))$,
5) $Ux(({\sim}p \to p) \to p)$,
6) $\forall x Uxp \to p$,
7) $\forall x \forall e \exists y(\delta(x,e) \varrho y)$,
8) $\forall x \forall e \exists y(\delta(y,e) \varrho x)$,
9) $\forall x \exists p \forall y(Uxp \leftrightarrow x \varrho y)$.

The first two axioms are the laws of distribution of classical connectives over the operator. This makes it "transparent" for classical logic. Axioms 3)–5) allow obtaining any classical tautology within the scope of *U* because the formulas that appear right after *Ux* in those axioms together constitute Łukasiewicz's formalization of classical propositional logic. By completeness of the Łukasiewicz's system, any tautology is derivable from those three formulas. 6) says that the formula that is realized at all moments is the theorem of the system. The idea behind the expression 7) is that for any point x and any interval e there is a latter point y that occurs after e since x. This intuitively means that time is forwards – infinite. 8) expresses the same idea about the past: time is backwards – infinite. The last one is the so-called "clock axiom" which says that any point in time can be described by some temporal function.

In the second article, the language is built from classical connectives ~,→,↔; propositional variables p, q, r,…; individual variables x, y, z,…;

the general quantifier $\forall$; and the epistemic L operator. A set of formulas is the smallest X set meeting conditions:

a) for any propositional variable A, $A \in X$,
b) if x is an individual constant and $A \in X$, then $L_x A \in X$,
c) if $A \in X$, then $\sim A \in X$,
d) if $A, B \in X$, then $A \leftrightarrow B$, $A \rightarrow B \in X$.

Łoś gives the seven following axioms for his system:

(1) $L_x p \leftrightarrow \sim L_x \sim p$,
(2) $L_x((p \rightarrow q) \rightarrow ((q \rightarrow r) \rightarrow (p \rightarrow r)))$,
(3) $L_x(p \rightarrow (\sim p \rightarrow q))$,
(4) $L_x((\sim p \rightarrow p) \rightarrow p)$,
(5) $L_x(p \rightarrow q) \rightarrow (L_x p \rightarrow L_x q)$,
(6) $\forall_x L_x p \rightarrow p$,
(7) $L_x L_x p \leftrightarrow L_x p$.

The rules of proving for the system are: modus ponens, the substitution rule, rules for the quantifiers and the extensionality rule for extensional contexts (those that do not occur within the scope of the L operator). The first axiom says that an agent believes that p if and only if he does not believe not p. The next three axioms are, just as in the previous case, Łukasiewicz's axioms for classical propositional logic. They allow obtaining any tautology within the scope of the epistemic operator. The fifth axiom is an analogon of the K axiom in modal logic. It says that if an agent knows that p implies q, then if he knows p, then he also knows q. This means that an agent is deductively closed. The next formula expresses a somewhat controversial claim that if everybody believes that p, then p is true. The last axioms express the idea that belief is equivalent to iterated belief, meaning that agent believes that he believes that p if an only if he believes that p. It is easy to see that the notion of belief formalized by Łoś is highly idealized. It presupposes both logical omniscience of the agent as well as infallibility of the whole group of agents, which by the way is recognized by Łoś himself:

> the above axiomatics has been built in such a way to make its investigations as easy as possible. Because of that it has probably run away from the intuitions…[15, 16]

Fortunately, the parts of Łoś's legacy described above have been recently rediscovered and creatively continued by the Polish logical

[15] Author's translation.
[16] Łoś, "Logiki wielowartościowe a formalizacja funkcji intensjonalnych" ("Many-valued logics and the formalization of intensional functions").

community gathered around centers in Toruń (Nicolaus Copernicus University in Toruń) and Lublin (The John Paul II Catholic University of Lublin). The effects of those collaboration come down to three books by

- Tkaczyk,[17] in which the systems of positional logics weaker than the system MR are presented for an analysis of temporal modalities;
- Jarmużek,[18] in which Jerzy Łoś's logic is used for a reconstruction of The Master Argument by Diodorus Cronus;
- Jarmużek, Tkaczyk,[19] in which the minimal positional logic MR is introduced and deeply analyzed, together with historical remarks and possibilities of extensions of the system;

and a series of articles. Those articles can be divided into two categories: historical articles recalling Łoś's ideas, i.e. Lechniak,[20] Jarmużek, Tkaczyk,[21] and innovative articles about positional logic, i.e. Jarmużek, Pietruszczak[22] in which the minimal system of positional logic MR was first formulated together with its complete semantics, Jarmużek[23] in which those ideas are continued, Tkaczyk[24] in which a system weaker than MR is introduced and studied, and Karczewska[25] in which a very interesting metalogical property of any proper MR-extension is proven. This whole research has been inspired by Jerzy Łoś's ideas and continued till now.

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[17] Marcin Tkaczyk, *Logika czasu empirycznego. Funktor realizacji czasowej w językach teorii fizykalnych* (Lublin: Wydawnictwo Katolickiego Uniwersytetu Lubelskiego, 2009).

[18] Tomasz Jarmużek, *Jutrzejsza bitwa morska. Rozumowanie Diodora Kronosa* (*Tomorrow Sea-Fight: Diodorus Cronus' Argument*) (Toruń: Wydawnictwo Naukowe UMK, 2013).

[19] Idem, Marcin Tkaczyk, *Normalne logiki pozycyjne* (Lublin: Wydawnictwo KUL, 2015a).

[20] Marek Lechniak, "Logika epistemiczna Jerzego Łosia a teoria racjonalnego zachowania" ("Epistemic logic of Jerzy Los and the theory of rational behavior"), *Roczniki Filozoficzne* XXXVI (1988): 77–89.

[21] Tomasz Jarmużek, Marcin Tkaczyk, "Jerzy Łoś and the Origin of Temporal Logic", in: *Handbook of the 5th World Congress and School on Universal Logic*, ed. Jean-Yves Beziau et al. (Istanbul: Publications of Turkish Logic Society, 2015b); Tomasz Jarmużek, Marcin Tkaczyk, "Jerzy Łoś Positional Calculus and the Origin of Temporal Logic", *Logic and Logical Philosophy* 28 (2019): 259–276.

[22] Tomasz Jarmużek, Andrzej Pietruszczak, "Completeness of Minimal Positional Calculus", *Logic and Logical Philosophy* 13 (2004): 147–162.

[23] Tomasz Jarmużek, "Minimal Logical Systems with R-operator: Their Metalogical Properties and Ways of Extensions", in: *Perspectives on Universal Logic*, ed. Jean-Yves Bezieau et al. (Italy: Polimetrica Publisher, 2007), 319–333.

[24] Marcin Tkaczyk, "Negation in weak positional calculi", *Logic and Logical Philosophy* 22, 1 (2013): 3–19.

[25] Anna Maria Karczewska, "Maximality of the Minimal R-Logic", *Logic and Logical Philosophy* 1–11 (2017): 193–203.

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Summary

The aim of the paper is to bring back Jerzy Łoś's great contribution to the area of epistemic and temporal logic. Although his mathematical achievements are widely recognized and appreciated around the world, his early works are still very little-known, despite their pioneering nature. The authors also give an account of current research and results inspired by Łoś's legacy.

Keywords: logic, epistemic, temporal, Jerzy Łoś, R operator