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Logical Forms: Validity and Variety of Formalizations

Abstract. Formalizations in first-order logic are standardly used to represent logical forms of sentences and to show the validity of ordinary-language arguments. Since every sentence admits of a variety of formalizations, a challenge arises: why should one valid formalization suffice to show validity even if there are other, invalid, formalizations? This paper suggests an explanation with reference to criteria of adequacy which ensure that formalizations are related in a hierarchy of more or less specific formalizations. This proposal is then compared with stronger criteria and assumptions, especially the idea that sentences essentially have just one logical form.

Keywords: logical form; formalization; validity; invalidity; substitution

1. Introduction

This paper uses a somewhat unusual strategy to shed light on the idea of logical form. I take what we may call the "standard practice" of formalizing ordinary language sentences and inferences as a starting point and reconstruct elements of a theory of logical form which is implicit in this practice. Underlying this strategy is the view that the goal of formalizing sentences and inferences is to use formulas to represent logical forms of these sentences or inferences.[1] In philosophy, a paradigmatic application of the standard practice of formalizing is assessing the validity of ordinary-language arguments with the help of a logical formalism, i.e., a formal language for which the notion of validity is defined. Doing this

[1] There are alternatives to this view, e.g., logical forms may be identified with formulas (see, e.g., Sainsbury, 2001, 35) or attributed to, e.g., propositions. The arguments I present in this paper can be adapted to such views.

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involves several steps: reconstructing an argument from a given text; representing it in a standard form (I will call such arguments "inferences"); formalizing the inference by assigning some formulas to its premises and the conclusion; and finally attempting a proof of validity.

As I will argue in Section 2, this standard practice of showing validity rests on the assumption that, although for any one inference a variety of formalizations is available in a given logical formalism, these formalizations are related in ways which explain why one formalization suffices to show validity. After some preliminaries about adequate formalizations (Section 3), such relations between formalizations are first discussed from the perspective of the standard practice of formalizing (Section 4). Section 5 then explores some alternatives that rely on stronger requirements or assumptions and promise to underwrite the idea that there is something like *the* logical form of a sentence.

2. A challenge to the standard explication of validity

Showing validity with the help of formalizations presupposes that an inference is indeed formally valid if there is a valid formalization of this inference. To make this more precise, we need to distinguish two meanings of "formal" (Brun, 2004, 24). First, "formal" contrasts with "informal": an inference can be assessed as valid according to a formalism or according to informal standards. Second, "formal" refers to validity in virtue of a logical form, in contrast to "material" (or "analytic") validity in virtue of the meaning of non-logical terms. Since formalizations are a means of dealing with validity in virtue of logical form, the idea guiding the standard practice can be framed more exactly as: an inference is informally valid in virtue of a logical form if there is a formalization of this inference which is valid in some given logical formalism.

By focusing on formal as opposed to material validity, I assume neither precise criteria for distinguishing logical from non-logical terms nor a clear-cut informal distinction between formal and material validity. It suffices that there is, first, an informal notion of valid inferences as inferences with a conclusion that must be true if their premises are true, independent of how the world actually is; and, second, that there is some informal distinction between formal and material validity, however vague. Formal validity may be explained informally as validity which is independent of the meaning of the non-logical expressions in the inference. Which expressions count as non-logical and which inferences and sentences as valid in virtue of a logical form can then be explicated (in Carnap's sense) with the help of formalizations, relative to a logical system. In this paper, I focus on classical first-order logic including zero-order logic ('propositional logic') but without functions, "FOL" for short. Often, I will simplify by dropping the relativization to FOL and use "i-valid" as abbreviating "informally valid in virtue of a FOL-form", "formalization" for "FOL-formalization", "f-valid" for "valid in FOL", and similarly for "i-invalid" and other related expressions.

With this background, we can say that the standard practice rests on the following explications of *i-valid* and *i-invalid*:[2]

(SV) An inference I (a sentence S) is *i-valid* iff there is at least one f-valid adequate formalization of I (of S).

(SIV) An inference I (a sentence S) is *i-invalid* iff all adequate formalizations of I (of S) are f-invalid.

The explications of *i-(in)valid* for sentences included in (SV) and (SIV) make it possible to simplify by focusing on formalizations of sentences. Inferences are then treated as finite sequences of sentences $\langle P_1, ..., P_n, C\rangle$ $(n > 0)$, which can be formalized as instances of $\phi_1 \wedge ... \wedge \phi_n \rightarrow \psi$ and evaluated with the help of the FOL-metatheorem that an inference $\phi_1, ..., \phi_n; \psi$ is f-valid iff $\phi_1 \wedge ... \wedge \phi_n \rightarrow \psi$ is f-valid.

(SV) and (SIV) involve a well-known asymmetry (Cheyne, 2012) since every inference admits of a variety of formalizations which need not all have the same validity-status. Trivially, all valid inferences have also f-invalid formalizations with distinct sentence letters for every premise and the conclusion. And (1), e.g.. can be formalized by the f-valid (1.1) or as an instance of Affirming the Consequent (1.2):

(1) If all tigers are stripy, there are no tigers which are not stripy.
There are no tigers which are not stripy.
Hence: All tigers are stripy.

(1.1) $\forall x(Tx \rightarrow Sx) \rightarrow \neg\exists x(Tx \wedge \neg Sx); \neg\exists x(Tx \wedge \neg Sx)$
$\Rightarrow \forall x(Tx \rightarrow Sx)$

(1.2) $p \rightarrow q; q \not\Rightarrow p$
Tx: x is a tiger; Sx: x is stripy; p: All tigers are stripy
q: There are no tigers which are not stripy

[2] There are stronger notions of invalidity, I will not discuss, e.g., "strong logical incorrectness" (Peregrin and Svoboda, 2017) and "super-invalidity" (Cheyne, 2012).

The asymmetry is that for establishing i-validity, one formalization suffices (e.g., 1.1), but for establishing i-invalidity, one f-invalid formalization (e.g., 1.2) is not enough; rather all formalizations must be shown to be f-invalid. This invites a challenge to (SV) if we assume, as I will do in this paper, that we are not dealing with formalizations which represent different readings of an ambiguous sentence: why should just one f-valid formalization be decisive? What ensures that other, possibly f-invalid, formalizations do not speak against i-validity?

To answer this challenge, we must argue that the formalizations of a sentence are not just a motley collection but related in ways which are (i) validity-preserving or (ii) not validity-challenging in the following sense: if Φ is an f-valid formalization of S, any other formalization Ψ of S is (i) f-valid or (ii) related to Φ in a way which explains why the f-invalidity of Ψ does not count against the validity of S.

Before we can discuss candidates for such relations, we must introduce another element of the standard practice of formalizing. As explicit in (SV) and (SIV), only *adequate* formalizations are relevant to questions of validity. Which formalizations should count as adequate raises many issues which cannot all be discussed appropriately here. Sect. 3 just briefly rehearses the basic points necessary for the subsequent discussion.

3. Some basic points on the adequacy of formalizations

This section sketches a few basic points of the approach to formalization from (Brun, 2004, 2014); some alternatives will be mentioned in Sect. 5.

As central necessary conditions of adequacy, two types of correctness criteria are widely adopted implicitly and sometimes explicitly defended. Consider the following two ways of arguing that (2.1) and (2.2) are incorrect formalizations of (2):

(2)	Beth was amused, because Alf ate asparagus ice cream.	
(2.1)	$p \wedge q$	p: Beth was amused
(2.2)	$p \rightarrow q$	q: Alf ate asparagus ice cream

One way to show that (2.1) and (2.2) are incorrect is to look at a situation in which (2) is false, but “Beth was amused” and “Alf ate asparagus ice cream” are true. If we interpret p and q correspondingly as true, FOL-semantics rules that $p \wedge q$ and $q \rightarrow q$ are true, although they should come out as false, which shows that the formalizations cannot be correct.

Another way to argue that (2.1) and (2.2) are incorrect is to use them in a formalization of (3). This yields the f-valid (3.1) and (3.2). But (3) is invalid, and this shows that (2.1) and (2.2) cannot be correct:

(3)	Beth was amused; Alf ate asparagus ice cream. Therefore: Beth was amused, because Alf ate asparagus ice cream.	
(3.1)	$p; q \Rightarrow p \wedge q$	p: Beth was amused
(3.2)	$p; q \Rightarrow p \rightarrow q$	q: Alf ate asparagus ice cream

The following criteria capture these ideas:

(TC) A formalization Φ of a sentence S is correct iff Φ has the same truth conditions as S if Φ is interpreted corresponding to S.

(VC) A formalization Φ of a sentence S is correct iff for every inference I containing S as a premise or the conclusion: I is informally valid if there is an f-valid formalization Ψ of I which contains Φ as a formalization of S.

For present purposes, (TC) and (VC) suffice. But of course, these criteria are in need of more precise formulation and involve assumptions and consequences that need to be scrutinized. (VC), e.g., comes with a threat of circularity and with a pull towards holism since it directly rests on validity-verdicts and involves formalizations of other sentences than S, which are presupposed to be correct. Here, I simply assume that such points can be dealt with adequately (for more precise formulations and further discussion, see, e.g., Brun 2004, ch. 11; Peregrin and Svoboda 2017, 69–71).

Importantly, correctness can only be assessed if formalizations include not only a formula but also a correspondence scheme, which relates every non-logical symbol occurring in the formula to exactly one natural language expression (hence a *formalization* Φ can be defined as an ordered pair $\langle \phi, \kappa \rangle$ with formula ϕ and correspondence scheme κ).[3] When formalizing (4), e.g., one could make the mistake of putting the negation in the wrong place. That this happened in (4.2) but not in (4.1) cannot be read off the formulas alone, but only be determined with the help of the correspondence schemes:

[3] Correspondence schemes are exclusively used in assessing the adequacy of formalizations. They do not turn formulas into meaningful expressions and they are not part of the formal language and hence cannot play any role in proofs of f-validity.

(4) Joe loves beer, but Mary does not.
(4.1) $p \wedge \neg q$ — p: Joe loves beer; q: Mary loves beer
(4.2) $p \wedge \neg q$ — p: Mary loves beer; q: Joe loves beer

In the following, we will often compare formalizations of the same sentence. To simplify this, it is convenient to use *compatible* correspondence schemes which (in contrast to 4) do not assign different ordinary language expressions to the same non-logical symbol. We can then define: two formalizations $\Phi = \langle \phi, \kappa \rangle$ and $\Psi = \langle \psi, \lambda \rangle$ are *f-equivalent* ($\Phi \Leftrightarrow \Psi$) iff $\phi \Leftrightarrow \psi$ and κ and λ are compatible.

Correctness, however, is not sufficient for adequacy because correctness is closed under f-equivalence, but f-equivalent formalizations need not be equally adequate. Otherwise we would have to accept, e.g., that every pair of informally equivalent sentences could be shown to be i-equivalent by a trivial proof of an instance of $\phi \Leftrightarrow \phi$:

(5) "All tigers are stripy" is equivalent to "There are no tigers which are not stripy".
(5.1) $\forall x(Tx \rightarrow Sx) \Leftrightarrow \forall x(Tx \rightarrow Sx)$

A similar result holds for all inferences which are materially valid according to informal standards. If we assume, e.g., that (6) is informally valid, then its premise will informally be equivalent to the conjunction of the premise and the conclusion, and therefore (6.1) is a correct formalization of (6), which could be used to 'show' that (6) is informally valid in virtue of a FOL-form—which is clearly mistaken, given that (6) is a paradigmatic example of a materially valid inference:

(6) Alf is a bachelor. Hence: Alf is not a carrot.
(6.1) $p \wedge \neg q \Rightarrow \neg q$ — p: Alf is a bachelor; q: Alf is a carrot.

In the literature, several options for additional criteria of adequacy are discussed (see e.g. Brun, 2004; Sainsbury, 2001). Two prominent ideas are requirements which appeal to relations between the syntactical structure of sentences and formulas (e.g. Peregrin and Svoboda, 2017, 72–3) and the idea that formalizations should be the result of a systematic procedure of formalizing (Brun, 2014), which includes the idea that formalizations of complex sentences can be built up compositionally from formalization of their parts (as paradigmatically in Montague, 1970).

In the context of this paper, criteria of adequacy must be addressed in connection with the challenge (raised in Sect. 2) that we must show that all adequate formalizations of a sentence stand in relations which are validity-preserving or explain why they are not validity-challenging. We can therefore work from two ends: introduce additional criteria of adequacy and show that they ensure that formalizations stand in validity-preserving or not validity-challenging relations, or identify such relations and use them as a basis for further criteria of adequacy (in addition to the correctness-criteria). In what follows, I will pursue the second strategy.

4. Relations between formalizations

This section discusses three relations between formalizations which can be defended as validity-preserving or not validity-challenging, beginning with two obvious candidates, equivalence and notational variance.

4.1. Equivalence

Equivalence is clearly validity-preserving: if two adequate formalizations are f-equivalent they lead to the same validity-verdict. But what is the general connection between equivalence and adequacy of formalizations? In Sect. 3, we saw that two formalizations can be f-equivalent yet only one of them be adequate. And it is also clear that not all adequate formalizations of the same sentence are f-equivalent, e.g., if one consists of a single sentence letter whereas another uses quantifiers (as in 1 above). So equivalence cannot be used as a basis for a further criterion of adequacy since neither of the following holds:

(E1) If Φ is an adequate formalization of sentence S and $\Phi \Leftrightarrow \Psi$, then Ψ is an adequate formalization of S.

(E2) If Φ and Ψ are adequate formalizations of sentence S, then $\Phi \Leftrightarrow \Psi$.

Despite these negative results, one might think that at least in some cases we can appeal to equivalence to show that two formalizations stand in a validity-preserving relation. After all, there seem to be examples of equally adequate f-equivalent formalizations, e.g., if there are f-equivalent options to set the scope of a quantifier (7) or to formalize sentences expressing an exclusive alternative (schemes 8 and 9):

(7)	Dogs bark and cats meow.	
(7.1)	$\forall x(Fx \rightarrow Gx) \wedge \forall x(Hx \rightarrow Ix)$	Fx: x is a dog; Gx: x barks
(7.2)	$\forall x((Fx \rightarrow Gx) \wedge (Hx \rightarrow Ix))$	Hx: x is a cat; Ix: x meows

(8) $(\phi \land \neg\psi) \lor (\neg\phi \land \psi)$
(9) $(\phi \rightarrow \neg\psi) \land (\neg\phi \rightarrow \psi)$

It is, however, not clear that equivalence really plays a role in answering the challenge of showing that adequate formalizations stand in validity-preserving or non-validity-challenging relations, because many criteria of adequacy that go beyond correctness speak against accepting such examples of equivalent formalizations as equally adequate. One may argue, e.g., that a compositional procedure of formalizing should produce (7.1), not (7.2) (cf. Epstein 1994, 182). Additionally, there is the suspicion that the equivalence of formalizations in (7) and (8)–(9) tells us nothing about logical forms since they are mere artefacts of the formal language. Maybe the possibility of setting brackets differently only shows that the syntax is not fully logically transparent (see Wittgenstein, 1989, 5.461). And if a specific operator for *exclusive or* were available, formalizations which instead use instances of (8) or (9) could be deemed inadequate (Brun, 2004, 238–9, 248–9).

4.2. Notational Variance

Notational variance is a second validity-preserving relation accepted in the standard practice of formalizing since notational variants should differ exclusively in notation, but not with respect to their adequacy or the validity-verdicts they yield, i.e., notational variants are expected to have the following properties:

(NV-A) If Φ is an adequate formalization of sentence S, then all notational variants of Φ are adequate formalizations of S.

(NV-V) If Φ is an adequate formalization of sentence S, then all notational variants of Φ lead to the same i-validity-verdict as Φ.

Candidates of relations which meet these conditions are relettering of bound variables (10.1–10.2) and uniform substitution of elementary non-logical symbols for predicates, sentences or individual-terms by elementary symbols of the same category (10.1 and 10.3):

(10)	Ijon Tichy has visited all planets.	
(10.1)	$\forall x(Px \rightarrow Vax)$	Px: x is a planet; Vxy: x has visited y
(10.2)	$\forall \boldsymbol{y}(P\boldsymbol{y} \rightarrow Va\boldsymbol{y})$	a: Ijon Tichy
(10.3)	$\forall x(\boldsymbol{F}x \rightarrow V\boldsymbol{b}x)$	Fx: x is a planet; b: Ijon Tichy

For notational variants resulting from correct relettering of bound variables, (NV-V) holds since such variants are well-known to be f-equivalent (see e.g. Kleene, 1974, §33). And (NV-A) straightforwardly holds according to all criteria of adequacy discussed in the literature.

The other forms of notational variance need further explication. An obvious starting point is the idea that two formalizations $\langle\phi,\kappa\rangle$ and $\langle\psi,\lambda\rangle$ are notational variants if ψ results from ϕ by isomorphic (hence bijective) substitution of predicate-letters by predicate-letters of the same arity (incl. permutation of argument places), sentences-letters by sentences-letters and individual-constants by individual-constants. (NV-V) would then follow from the more general result that f-validity is closed under uniform substitution (Kleene, 1974, §34). However, implementing this idea requires additional work, which cannot be undertaken here. The reason is that notational variance of formalizations depends not only on the formulas involved but also on the correspondence schemes. (10.4), e.g., should count as a notational variant of (10.1), although (10.5) violates (NV-A) in relation to (10.1):

(10.4)	$\forall x(Px \to V\boldsymbol{xa})$	Px: x is a planet; Vxy: $\boldsymbol{y}$ has visited $\boldsymbol{x}$
(10.5)	$\forall x(Px \to Vxa)$	Px: x is a planet; Vxy: x has visited y

(10.4) also illustrates that isomorphic substitution permits permutation of argument places of predicates.

Even without an exact definition of *isomorphic substitution of non-logical symbols by symbols of the same category in formalizations*, it is quite clear that (NV-A) holds for such substitutions according all criteria proposed in the literature. And in the present context, we also do not need to address the complications mentioned in the preceding paragraph since any such definition must guarantee validity-preservation (NV-V).

4.3. More and less specific formalizations

We now turn to the theoretically most important validity-preserving relation between formalizations. The standard practice of formalizing routinely admits for the same sentence more and less specific formalizations, which represent a more or less 'detailed' or 'fine-grained' analysis of the sentence's logical form. Writing $\Phi > \Psi$ for "Φ is more specific than Ψ", we have, e.g., (11.4) > (11.2) > (11.1), (11.4) > (11.3) > (11.1) and (11.4) > (11.1), but neither (11.2) > (11.3) nor (11.3) > (11.2):

(11)	Jerry is faster than Tom.	
(11.1)	p	p: Jerry is faster than Tom.
(11.2)	Fa	Fx: x is faster than Tom; a: Jerry
(11.3)	Gb	Gx: Jerry is faster than x; b: Tom
(11.4)	Hab	Hxy: x is faster than y

More or less specific formalizations are the basis for Quine's (1996, 160) well-known maxim of shallow analysis "*expose no more logical structure than seems useful* for the deduction or other inquiry at hand" and for the well-established strategy of formalizing 'step-by-step' (e.g. Baker-Plummer et al., 2011, chs 11.3–4), which can be motivated by considerations of compositionality. In the classic example ascribed to De Morgan, one may start with the relatively unambitious formalizations:

(12)	Every horse is an animal.	
(13)	Every head of a horse is a head of an animal.	
(12.1)	$\forall x(Fx \to Gx)$	Fx: x is a horse; Gx: x is an animal
(13.1)	$\forall x(Hx \to Jx)$	Hx: x is a head of a horse
		Jx: x is a head of an animal

However, since (13.1) does not permit to show that the inference from (12) to (13) is informally valid, one has reason to seek a more specific formalization of (13) by analysing "x is a head of a horse" and "x is a head of an animal" and substitute the results (13.a) and (13.b) for Hx and Jx in (13.1) to get (13.2), which permits the desired proof:

(13.a)	$\exists y(Fy \land Kxy)$	Kxy: x is a head of y
(13.b)	$\exists y(Gy \land Kxy)$	
(13.2)	$\forall x(\exists y(Fy \land Kxy) \to \exists y(Gy \land Kxy))$	

Surprisingly, relations between more and less specific formalizations have not been investigated much, although doing this paves the way for investigating some philosophically pertinent questions about formalizations and logical forms (in what follows, I mainly draw on ideas from Castañeda, 1975, ch. 3.8; see also Brun, 2004, ch. 13.4). To begin with, we need to elaborate the idea of more or less specific formalizations a bit.

Intuitively, more specific formalizations can be explained as structure preserving refinements: if we move from a more general to a more specific formalization, we represent the logical structure represented by the more general formalization plus some additional structure. That more specific formalizations instantiate the structure of more general formalizations then underwrites i-validity preservation:

(MS-V) If a sentence S is i-valid according to an adequate formalization Φ of S, then S is i-valid according to all adequate formalizations Ψ of S which are more specific than Φ.

To unpack these ideas, I first focus on relations between formulas. Refinement and structure preservation motivate counting ϕ as more specific than ψ under two conditions: if ϕ results from ψ through uniform substitution for elementary non-logical symbols for sentences, predicates or individuals-terms by (i) more complex expressions of the same category or (ii) other symbols of the same category which already occur in ψ. If we treat sentence letters as predicate letters of arity 0, (i) can be specified as uniform substitution of predicate letters of arity n by formulas with n free variables which contain a predicate of arity greater than n (11.1–4), a sentence connective or a quantifier (13.1–2). (ii) is illustrated by:

(14) $(p \wedge q) \vee (\boldsymbol{p} \wedge s) > (p \wedge q) \vee (\boldsymbol{r} \wedge s)$

For an explication of *more specific*, we can again turn to the relation of substitution. The basic idea is that more specific formulas are substitution instances of more general formulas but not the other way around; and notational variants, which are substitution instances of each other, are equally specific. To spell out this idea, two complications must be accounted for. First, substitution for predicate letters must observe certain restrictions concerning variables, which can make relettering of variables necessary. This can be accounted for by referring to notational variants in a definition of *more specific*. Second, some substitutions cannot be interpreted as refinements of the logical structure, namely those which reduce the arity of a predicate. To exclude this, we need a restriction to argument-conserving substitutions. This suggests definitions along the following lines: a formula ϕ is *more specific* than a formula ψ ($\phi > \psi$) iff ϕ is a notational variant of a uniform argument-conserving substitution instance of ψ, but ψ is not a notational variant of a uniform argument-conserving substitution instance of ϕ; two formulas ϕ and ψ are *equally specific* iff they are notational variants of each other.[4] Given such definitions, one can argue that (MS-V) holds in FOL because f-validity is closed under uniform substitution.[5] Extending the definition of *more*

[4] See Schurz (1995) for formal explications of *argument-conserving substitution* ("ϵ-substitution") and *at least as general* (the converse of *at least as specific*).

[5] It is important to note that closure of f-validity under substitution does not hold in all logics (see Punčochář, 2023; Schurz, 2001).

specific from formulas to formalizations would again need additional work to account for the complications introduced by correspondence schemes just as in the case of notational variants (Sect. 4.2). In the following, I simplify by assuming that we compare formalizations with the same correspondence scheme and that for any two formalizations $\Phi = \langle \phi, \kappa \rangle, \Psi = \langle \psi, \kappa \rangle$: $\Phi > \Psi$ iff $\phi > \psi$.

We can now use the relation *more specific* to make a postulate of hierarchical structure explicit, which is fundamental for the standard practice of formalizing and showing validity:

(PHS) If Φ and Ψ are two adequate formalizations of some sentence S, then (i) Φ and Ψ are notational variants, or (ii) $\Psi > \Phi$, or (iii) $\Phi > \Psi$, or (iv) there is an adequate formalization X of S, with $X > \Phi$ and $X > \Psi$.

Intuitively, we can picture (PHS) as claiming that the process of specification is allowed to branch, provided that the branches can be 'reconnected' by further specification. Although in (11), e.g., neither (11.2) $>$ (11.3) nor (11.3) $>$ (11.2), (PHS) is met, because we can introduce a fourth formalization, (11.4), by substituting $[Fx/Hxb, Gx/Hax]$.

(PHS) is not a consequence of the criteria of adequacy mentioned in Sect. 3. There are other reasons to accept (PHS). One can argue, e.g., that (PHS) captures an aspect of the idea that logical analysis should be systematic, namely that it is compositional insofar as more specific formalizations can be seen as the result of formalizing constituents of more general formalizations (Brun, 2014).

In the present context, the decisive point is that (PHS) ensures that adequate formalizations stand in a relation that is not validity-challenging. Thus, (PHS) is a crucial element in answering the explanatory challenge raised in Sect. 2. If Φ is an f-valid adequate formalization of a sentence S, then Φ suffices to show the i-validity of S, because (PHS) guarantees that no adequate formalization Ψ of S counts against the i-validity of S, even if Ψ is not a notational variant of Φ: if $\Psi > \Phi$, then Ψ is f-valid as well; if $\Phi > \Psi$, then Ψ is either f-valid or its f-invalidity just indicates that showing the i-validity of S requires a more specific formalization, e.g., Φ; if there is an adequate formalization X of S and $X > \Phi$, $X > \Psi$, then X is f-valid since Φ is and Ψ is either f-valid or its f-invalidity just indicates that showing the i-validity of S requires a more specific formalization, e.g., X.

If, however, it coud happen that Ψ is f-invalid and Φ and Ψ do not meet (PHS), we would face a dilemma. If there is no further f-valid formalization Ω of S such that $\Omega > \Psi$, then it is hard to see why the f-invalidity of Ψ should not count against the i-validity of S. If there is such a formalization Ω, we can explain why the f-invalidity of Ψ does not count against the i-validity of S (Ψ is not specific enough to show that S is i-valid), but without (PHS) we cannot be sure that no other formalization Ψ' exists, which raises exactly the same challenge as Ψ.

A cornerstone of this argumentation is that being less specific is not a validity-challenging relation. More precisely, if Φ and Ψ are two formalizations of a sentence S, $\Phi > \Psi$, Φ is f-valid and Ψ is f-invalid, then we can always explain why Ψ does not challenge the i-validity of S established by Φ. In essence, the explanation is what motivated the explication of *more-specific*: Ψ represents less structure of S than Φ, but not a different structure, since Φ is a *structure preserving* refinement of Ψ, which is secured by the requiring that Φ results from Ψ by an argument-preserving uniform substitution. That Ψ is f-invalid gives us therefore no reason to think that S has no i-valid logical structure since all the logical structure represented by Ψ is represented by the f-valid Φ as well.[6] Actually, we should expect that some less specific formalizations do not permit to show that S is i-valid even if more specific ones do. When we move from more to less specific formalizations, we lose information about the logical structure of S and sooner or later reach a minimally specific formalization, which consists of just one sentence letter, merely represents that S is a sentence and, of course, is f-invalid.

The upshot of the discussion in this section is that the standard practice of showing validity can rely on (SV), because this practice implicitly presupposes that formalizations meet (PHS). The hierarchical structure described in (ii)–(iv) can thus be interpreted as a further, albeit negative criterion of adequacy. We can, e.g., argue that

(13.3) $\quad \forall x \forall y(Fy \wedge Kxy \rightarrow Gy \wedge Kxy)$

is not an adequate formalization of (13) if (13.1) is, because in contrast to (13.2), (13.3) is not more specific than (13.1) and no formalization can be more specific than both (13.2) and (13.3). Similarly in example (7): (PHS) tells us that (7.1) and (7.2) are not both adequate.

[6] This holds in FOL, but not in logics in which substitution is not generally f-validity-preserving (see again Punčochář, 2023; Schurz, 2001).

5. Adding more constraints?

So far, I have argued that the standard practice of showing validity rests on a bundle of related principles (which I will refer to as "the standard account"). It adopts (SV) as an explication of the i-validity of ordinary-language inferences and explains why one formalization suffices to show i-validity mainly by (PHS). However, the standard account also includes an asymmetry between validity- and invalidity-verdicts that seems unwelcome. It surely would be advantageous if formalizations could be used to show not only i-validity, but also i-invalidity. In this section, I investigate some ideas of how the the asymmetry between validity- and invalidity-verdicts could be eliminated by adopting more constraints on formalizations.

First, however, we must briefly consider a more radical alternative. Why not insist that, strictly speaking, we can only show *relative to an adequate formalization* Φ *of* I that an inference I is i-valid or i-invalid? One could then discard (PHS) and there would no longer be an asymmetry between validity- and invalidity-verdicts. This move would be desperate, though, because the questions about the relation between different verdicts of i-(in)validity would just re-emerge on the informal side. What to make of an inference that is i-valid relative to a formalization Φ, but i-invalid relative to another, Ψ? The answer would depend, again, on how Φ and Ψ are related. If, e.g., the formula in Φ is $p \to q$, but $r \wedge q \to q$ in Ψ, the most reasonable conclusion is that Φ is less specific than Ψ and therefore can be ignored, given that Ψ is adequate.

5.1. Unanimous, ambitious and complete formalizations

A good starting point for investigating more restrictive accounts of formalization is a principle of "unanimous" formalization:

(UV) The adequate formalizations of a sentence S are either all f-valid or all f-invalid.[7]

Implementing (UV) is attractive since it enables us to replace (SIV) by (UIV), which not only eliminates the asymmetry of showing validity and invalidity, but immediately explains why one formalization suffices to show i-(in)validity:

[7] Note that (UV) implies neither (E1) nor (E2).

(UIV) An inference I is *i-invalid* iff there is at least one f-invalid adequate formalization of I.

Following Sagi (2020, 301–2), we can interpret (UV) as a "formalization meta-constraint", as the requirement that criteria of adequate formalization ensure (UV). Of the many options for tightening criteria of adequacy, I will focus on a particularly straightforward one, strengthening (VC). Peregrin and Svoboda's (2017, ch. 5) theory of formalization goes in this direction by combining (VC) as a necessary criterion with comparative criteria of adequacy, which include ambitiousness:

(AMB) A formalization Φ of a sentence S is the more adequate, the more informally valid inferences which are in the intended scope of FOL and contain S as a premise or conclusion can be shown to be i-valid with the help of an f-valid formalization Ψ of I in which S is formalized by Φ.

The restriction "in the intended scope of FOL" is essential. Without it, (AMB) would imply that FOL suffices to show the validity of all kinds of informally valid inferences, taking over the job of, e.g., modal or deontic logic (Peregrin and Svoboda, 2017, 64-5).

From the perspective taken in this paper, the main effect of (AMB) is that less specific formalizations will often count as less adequate. But (AMB) does not motivate an alternative explanation of why one formalization suffices to show validity, which is still provided by notational variance and (PHS). (AMB) also does not (and is not claimed to) do away with the asymmetry. As a comparative criterion, (AMB) leaves open whether we can, given an f-invalid formalization of a sentence, find a more ambitious f-valid formalization. To definitely exclude that a more ambitious formalization could be found, we could require adequate formalizations to be maximally ambitious. This motivates a 'counterpart' of (VC) which requires completeness (e.g. Epstein, 1994, 167):

(COM) A formalization Φ of a sentence S is adequate only if it is complete; it is *complete* iff for every informally valid inference I which is in the intended scope of FOL and contains S as a premise or conclusion, there is an f-valid formalization Ψ of I which contains Φ as a formalization of S.

(COM) has far-reaching consequences. It invalidates Quine's maxim of shallow analysis and drastically reduces the number of adequate formalizations of a sentence. Less specific formalizations are inadequate if they are not equivalent, particularly most routine formalizations by a single sentence letter. In return, (COM) permits to replace (SIV) by (UIV).

However, (COM) not only 'inherits' the challenges to (VC), it also comes with its own problems. One is that the reference to the intended scope of FOL presupposes an informal difference between validity in general and validity in virtue of FOL. But "valid in virtue of FOL" ("i-valid") is a technical term, introduced by (SV). It is therefore at least doubtful whether we informally have a firm grip on i-validity.[8] A major reason for this is that formalizing is also a creative activity and unexpected ways of analysis can be invented, as exemplified in Davidson's (1980) analysis of action sentences:

(15) Tom beats up Jerry in the attic.

(15.1) $\exists x(Fabx \land Gcx)$ $Fxyz$: z is a beating up of y by x
Gxy: y happens at location x
a: Tom; b: Jerry; c: in the attic

In the present context, another point is decisive. Adding (COM) to the standard account has the consequence that showing validity requires complete formalizations, but this is unwelcome and unmotivated since the much less restrictive (PHS) already guarantees everything we need to show validity. There is no reason to require a complete formalization if validity can be shown by a less ambitious one. The next section therefore investigates an alternative to unanimous formalization.

5.2. Maximally specific formalizations

The basic obstacle to showing invalidity by means of an f-invalid formalization is the difficulty of ruling out that more specific f-valid formalizations exist. Hence, a plausible strategy to secure an invalidity-verdict is to show that there are no more specific formalizations. To implement this idea, we can first define:

(XS) A formalization Φ of a sentence S is *maximally specific* iff it is adequate and every formalization $\Psi > \Phi$ of S is inadequate.

Using this notion, we can express the assumption that the adequate formalizations of a sentence cannot be ever more specific and that the

[8] Defenders of (COM) might think that (COM) is so compelling that we should give up the standard account and any distinction between different kinds of validity (see e.g., Baumgartner and Lampert's (2008) "Tractarian view").

maximally specific formalization of a sentence is uniquely determined up to notational variance:[9]

(X1) For every sentence S, there is a formalization Φ of S such that Φ is maximally specific and for all adequate formalizations Ψ of S, $\Phi \geq \Psi$ (i.e., Ψ is either less specific or a notational variant of Φ).

(SIV) can now be replaced by (XIV), which is, given (X1), equivalent:

(XIV) An inference I is *i-invalid* iff there is an f-invalid maximally specific formalization Φ of I.

With (XIV), we no longer have an asymmetry between validity- and invalidity-verdicts since i-invalidity can now be shown by using one maximally specific formalization (e.g. Halbach, 2013, 165). The question remains how we can establish maximal specificity. If we first focus on particular examples, it often appears quite clear that a given formalization is maximally specific. It seems, e.g., plausible that no adequate formalization more specific than (11.4) can be found for (11), as long as we insist on separating formalization in FOL from semantic analysis and are confident that formalizations like (11.5) are instances of the latter:

(11)	Jerry is faster than Tom.	
(11.4)	Hab	Hxy: x is faster than y; a: Jerry; b: Tom
(11.5)	$\exists x \exists y(Max \wedge Mby \wedge x > y)$	Mxy: x can move at speed y

However, showing that a formalization is maximally specific boils down to showing a negative existential, which is made particularly hard by the creative aspects of formalizing. Without knowing about Davidson's ideas and (15.1), (15.2) may well seem to be a maximally specific formalization of (15) just as (11.4) does in relation to (11):[10]

(15.2)	$Habc$	$Hxyz$: x beats up y at location z a: Tom; b: Jerry, c: in the attic

The upshot is that an i-invalidity-verdict based on a formalization Φ can be at most as reliable as the judgement that Φ is maximally specific, which in turn must be based on an informal judgement about the scope of FOL. We are thus essentially back to a problem diagnosed for (COM),

[9] (X1) follows from (PHS) and the assumption that for every adequate formalization Φ of S, there is a formalization Ψ of S which is maximally specific and $\Psi \geq \Phi$.

[10] (15.1) > (15.2): $\exists x(Fabx \wedge Gcx) = Habc\,[Hyzw \,/\, \exists x(Fyzx \wedge Gwx)]$.

with the difference that it now affects only i-invalidity-, not i-validity-verdicts. Yet, arguing that a given formalization is maximally specific seems the best we can do to show invalidity within the standard account.

This way of arguing invalidity still rests on (PHS), but not on the general existence claim (X1). Defending this claim would need a general, not merely case-specific, demarcation between FOL-formalization and semantic analysis. Maybe the sceptical worries about such a demarcation can be overcome. If so, (X1) could be used to introduce a stronger variant of the standard account, which substantiates the idea that formalizations and logical forms exhibit a unity that gives sense to speaking of *the* logical form of a sentence: although every sentence has many adequate formalizations in FOL, every sentence has fundamentally just one FOL-form, represented by a maximally specific formalization; less specific formalizations can be explained as representing not all but merely some aspects of the sentence's logical form; and notational variants only indicate that the language of FOL permits us to symbolize the same logical form in different ways.[11]

However, combining (PHS) and (X1) leads to a tension since defending (X1) demands that, beyond a certain point, more specific formalizations can be rejected as expressing semantic analyses outside the scope of FOL, while defending (PHS) may call for seeking a more specific formalization. If, e.g., one holds that FOL-formalizations of the same sentence can differ in what is treated as a predicate and what as an individual-term, one may come up with formalizations like the following:[12]

(16)	Red is a colour.	
(16.1)	Fa	Fx: x is a colour; a: red
(16.2)	$\forall x(Gx \rightarrow Hx)$	Gx: x is red; Hx: x is coloured

For the sake of argument we can assume that (16.1) and (16.2) are adequate and do not draw on ambiguities in (16). Now, since neither (16.1) nor (16.2) is more specific than the other, (PHS) demands that there is

[11] (X1) is rarely adopted explicitly (e.g. Curtis, 1993, 52). But speaking of *the* logical form is widespread among logicians who admit, explicitly or in practice, more or less specific formalizations (e.g. Peregrin and Svoboda, 2017, 34). It is then often not clear whether this manner of speaking really expresses a commitment to (X1).

[12] von Savigny (1976, 44) uses (16) to make exactly this point (see also Quine, 1996, 97–9).

a formalization more specific than both. Here is a candidate:[13]

(16.3) $\forall x(Ixa \rightarrow \exists y(Jy \wedge Ixy))$

Ixy: x is part of the (discontinuous) object y
Jx: x is a (discontinuous) colour object

For defenders of (X1), this is a problem if they rely on a general demarcation between FOL-formalization and semantic analysis according to which (16.3) is a semantic analysis just as clearly as (11.5) is: if the argument that (11.4) is maximally specific is convincing, it is hard to see why (16.1) and (16.2) should not be maximally specific as well.

Of course, one might find the particular example (16) not very convincing. But the point exemplified in (16) is clear nonetheless. The aim of showing that two formalizations which seemingly violate (PHS) actually meet it potentially conflicts with the aim of demarcating FOL-formalization from semantic analysis in a general way, not just for specific cases. So the question is whether we should give more weight to (PHS) or to a general demarcation between FOL-formalization and semantic analysis. In the present context, we can argue: since (PHS) is needed to secure (SV), but maximally specific formalizations are not, (PHS) takes priority, and a general demarcation of FOL-formalization from semantic analysis should be accepted only if we can successfully argue that it will not lead to maximally specific formalizations violating (PHS). As long as we do not have such an argument, it is better to stick to the standard account and remain sceptical about the general assumption that every sentence has, up to notational variance, a unique maximally specific FOL-formalization and fundamentally just one logical form in FOL.

6. Conclusion

The standard practice of using formalizations to show the validity of ordinary-language inferences rests on an explication of validity according to which one valid formalization suffices to show validity (SV). This raises the challenge of explaining why other, adequate but invalid formalizations do not undermine the validity-verdict. As I have argued, the key to answering this challenge is criteria of adequacy that arguably guide, mostly implicitly, the standard practice of formalizing. Invalid

[13] $\forall x(Ixa \rightarrow \exists y(Jy \wedge Ixy)) = Fa\,[Fz\,/\,\forall x(Ixz \rightarrow \exists y(Jy \wedge Ixy))]$
$= \forall x(Gx \rightarrow Hx)\,[Gx\,/\,Ixa, Hx\,/\,\exists y(Jy \wedge Ixy)]$

formalizations are not a challenge to validity-verdicts, if all adequate formalizations of a sentence must be either notational variants of each other or related in a hierarchy of more or less specific formalizations. Invalid formalizations can then be deemed harmless because they are just too unspecific, i.e., do not provide an enough ‘detailed’ analysis of logical form. Alternative suggestions promote criteria which would ensure that all adequate formalizations of a sentence have the same validity status, that they provide a complete analysis of a sentence’s first-order logical form, or that only maximally specific formalizations are adequate. The latter requirement would also give a clear sense to talk about *the* (first-order) logical form of a sentence. However, these stricter criteria would ban routine formalizations, they are not needed to explain why a single formalization suffices to show validity, and it is not clear how to integrate them into the standard account of formalizing without tensions.

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