Logic and Logical Philosophy
Volume 32 (2023), 363–381
DOI: 10.12775/LLP.2023.007

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Logical Constants and the Sorites Paradox

Abstract. Logical form is thought to be discovered by keeping fixed the logical constants and allowing the non-logical content in the sentence to vary. The problem of logical constants is the problem of defining what counts as a logical constant. In this paper, I will argue that the concept 'logical constant' is vague. I demonstrate the vagueness of logical constancy by providing a sorites argument, thereby showing the sorites-susceptibility of the concept. Many prior papers in the literature on logical constants hint at this vagueness, but do not explore how theories of vagueness apply to logical constants. In the second half of this paper, I do just this. I consider approaches to logical constants that resemble nihilism about vagueness and more recent theories that relativize truth to precisifications. Finally, I argue that approaches that accept the potential indeterminate status of putative logical constants are preferable to nihilism or relativism about logical constancy.

Keywords: logical constants; vagueness; sorites; nihilism about vagueness; supervaluationism

1. Introduction

In *Wissenschaftslehre* (WL), Bernard Bolzano defined logically analytic propositions in the following way. Uniform substitutions of the non-logical ideas of a logically analytic proposition will always result in a true proposition.[1] Despite the idiosyncracies of Bolzano's logical system, his definition of logical analyticity pops up repeatedly after him. It appears, for example, in the work of Alfred Tarski who defines logical truth as truth-preservation under uniform substitutions of non-logical words.

[1] Note that the term translated as "logically analytic proposition" matches what others call "logical truth."

Special Issue: Perspectives on Logical Form. Guest Editor: Pavel Arazim

Published online June 2, 2023

A potential problem arises with these definitions — the problem of logical constants. How do we demarcate the logical constants from the non-logical words? Bolzano notes this exact problem in WL §148. He claims that logical analyticity is ambiguous and open to interpretation.

> To be sure, this distinction has its ambiguity, because the domain of concepts belonging to logic is not so sharply demarcated that no dispute could ever arise over it. (Bolzano, 1973, §148)

The problem of logical constants is a problem of vagueness, and many philosophers who attempt to provide a solution to the problem fall back on the common approaches to vagueness. In this paper, I first set out to provide a sorites argument for logical constancy. Sorites-susceptibility is often taken as a sufficient condition for vagueness. So, creating such an argument is a strong reason to accept the vagueness of logical constancy. Secondly, I evaluate some of the approaches to logical constancy in the literature in light of the common complaints with their equivalent theories of vagueness. In particular, I consider approaches that treat everything as a potential logical constant or nothing as a logical constant. These approaches fail to capture the important features of logicality and should be avoided if alternative solutions are available. Finally, I argue that supervaluationism and degree theories are acceptable alternative solutions.

2. Some Notes about Vagueness

Before I begin, a few notes need to be made about vagueness. Consider a quintessential example of a sorites argument:

P1. A single grain of sand does not make a heap.
P2. If n grains of sand do not make a heap, then $n + 1$ grains of sand do not make a heap.
C. $1,000,000$ grains of sand do not make a heap.

Sorites arguments start with a base premise (P1) that is intuitively true. The inductive premise (P2) is a universal generalization of conditionals. Each conditional represents such a small step that if the antecedent is true, then intuitively the consequent is as well. Since the inductive premise is made up of a large number of intuitively true conditionals, the inductive premise also seems true. The conclusion of a sorites argument, however, is clearly false. But, this creates a problem. Sorites arguments are classically valid and they have intuitively true premises, but an intuitively false conclusion. Of course, the premises of a valid argument cannot be true when the conclusion is false. So, we are left with effectively three groups of options.

1. Reject a premise
2. Reject classical logic
3. Accept the conclusion

What makes the above argument a paradox is that the choice between 1, 2, and 3 is particularly difficult — it engenders seemingly intractable disagreement. This is not the case for every predicate we throw into the argument. For example, consider the following:

P1. 1 is a number less than $1,000$.
P2. If n is a number less than $1,000$, then $n+1$ is a number less than $1,000$.
C. $1,000$ is a number less than $1,000$.

In this argument, P2 is obviously false. Instantiating the variable n with the value 999 will result in a false conditional. There is no difficulty in explaining what went wrong with this argument, and so the predicate 'is a number less than $1,000$' is not sorites susceptible — it doesn't create a paradox.

Rosanna Keefe notes three features of vague predicates, "they admit borderline cases, they lack (or at least apparently lack) sharp boundaries and they are susceptible to sorites paradoxes" (Keefe, 2000, p. 6). Note that vague predicates need only *appear* to lack sharp boundaries. Some theories of vagueness, the brands of epistemicism in particular, hold that vague words have precise meanings. Those meanings are just concealed from us. Epistemicists, however, still believe that vague words are sorites susceptible. Noted epistemicist Roy Sorensen writes, "There is a universally accepted sufficient condition for vagueness that is used to show other predicates are vague: sorites embeddability" (Sorensen, 1990, p. 3). According to epistemicists, the difficulty in choosing between 1, 2, and 3 comes from our epistemic failings. No such failings emerge in the case of 'is a number less than $1,000$,' but they do in the case of 'make a heap.'

David Enoch (2007) claims that vague words have two important properties: tolerance and utility. Tolerant concepts fail to draw a border between when they apply and when they don't, and are thus sorites susceptible. Concepts that exhibit utility, on the other hand, draw a border, and are therefore useful. Tolerance and utility conflict with one another. If a predicate fails to draw a line, then it is tolerant but not useful. If it does draw a line, it is useful but not tolerant. The task for philosophers of vagueness is to find a way to adjudicate between the perceived tolerance and utility of vague words.

3. Precise Definitions

There are a few options for definitions of logical constancy that avoid sorites susceptibility. Logic is plausibly thought to be topic neutral. The rules of logic should apply regardless of whether we are reasoning about chess or metaphysics. One potential way of capturing the topic neutrality of logic is by permuting the objects of the universe and seeing what stays the same. The stuff that stays the same, then, is not influenced by the specific structures in the world. Tarski claimed that logical constants are the terms that are invariant "under all possible one-one transformations of the world onto itself" (Tarski, 1986, p. 149). Permutation invariance as a definition of logical constancy avoids sorites susceptibility. A potential constant is either provably invariant or provably not, and so at least one instantiation of the inductive premise of a sorites argument for logical constancy will be false. Logical constancy as permutation invariance is like the predicate 'is a number less than $1,000$.'[2]

Though permutation invariance provides a precise definition of logical constancy, it does so at the cost of adopting a controversial stance on logicality. The understanding of logic that comes out of Tarski's work is inextricably linked with mathematics.

> It turns out that the only properties of classes (of individuals) which are logical are properties concerning the number of elements in these classes. (Tarski, 1986, p. 151)

This result has garnered the complaint that permutation invariance overgenerates logical constants. Solomon Feferman (1999) has attempted to resolve this problem of overgeneration by tweaking the permutation invariance account of logical constancy. Notably, not all philosophers find the tight link between logic and mathematics problematic. Such disagreement is indicative of slightly different conceptions of logicality.

[2] Note that there are a number of different accounts of permutation invariance that capture slightly different sets of constants (see, e.g., Bonnay, 2008; Feferman, 1999; McGee, 1996; Sher, 1991).

Permutation invariance also has a problem with undergeneration. The S4 necessity operator, for example, is not counted as logical, even when invariance is applied to both the objects in the domain and to worlds. This is because the S4 necessity operator requires a certain structure among worlds to be maintained. Catarina Dutihl Novaes (2014) notes that the S4 necessity operator has a strong claim to logical constancy on the grounds that it may be the best way to understand logical necessity. It is possible to expand permutation invariance by adding these additional structures into the account. For example, the S4 necessity operator will be invariant under permutations of worlds that preserve the accessibility relation. Novaes argues that it is, then, not permutation invariance that is doing the work, but instead the added fixed structures. In addition, if we are going to add these structures, then it becomes more difficult to determine which structures should be preserved and which should not — a problem noted by van Benthem.

> Of course, the systematic question then becomes how to motivate (a minimum of) such additional structure independently.
>
> (van Benthem, 1989, p. 334)

The problems of under and overgeneration that plague permutation invariance and the accompanying debate are indicative of vagueness. Modal operators and set theoretic operators are borderline cases of logical constants. Determining whether or not permutation invariance is correct and thus logical constancy is not vague requires us to first wade through the borderline cases, determining what kinds of structure should be preserved and what kinds should be jettisoned. There is, I will now argue, a clearer example of undergeneration of logical constancy.[3]

The word ‘and’ is clearly a logical constant. Sure enough, with some minor finagling, permutation invariance captures truth-functional conjunction. The word ‘and,’ however, is not so simple that it’s meaning is clearly represented by the classical truth-table. Dorothy Edgington (1992) argues that, when dealing with degrees of truth, the best semantics for the standard suite of logical connectives is not degree-functional. Consider two balls, a and b that are incredibly similar in color. Suppose

[3] In addition to the possibility of counterexamples to proposed necessary and sufficient conditions for logical constancy, there is a dialectic problem with the proposal of such conditions. Warmbrōd (1999) argues that most attempts so far to offer necessary and sufficient conditions, including permutation invariance, fail to consider how the proposed property is linked to logicality.

also that, $v(\text{a is red}) = 0.5$ and $v(\text{b is red}) = 0.49$.[4] Using the traditional degree-functional semantics for conjunctions and negations, v(a is red and b is not red) $= \min(v(\text{a is red}), v(\text{b is not red})) = 0.5$ — a middling truth.[5] This is clearly incorrect. '*a* is red and *b* is not red' should have a very low degree of truth. Because they are so similar in color, if *a* is red, then *b* should be close to red as well. The problem with the degree-functional semantics is that they do not consider the dependence between the conjuncts. There are structural properties about the world such as the similarities between the colors of objects that inform the truth-values of complex sentences involving those objects. The closer *a* and *b* are in color the more v(a is red and b is not red) will approach 0. Edgington argues that the degrees of truth of the conjuncts do not determine the degree of truth of a conjunction. Instead, they put limits on the degree of truth of the conjunction. The value of a conjunction must be between 0 and the value of its least true conjunct. Where the value is, in that range, will depend on structural features of the world outside of just the truth-values of the conjuncts.

At issue here is whether or not Edgington's non-degree-functional 'and' is a logical constant despite the fact that it is not permutation invariant. To resolve this issue, consider Edgington's goal. She is not proposing some new word that sounds the same as 'and.' Instead, she is attempting to uncover the actual meaning of the word 'and' as it appears in real instances of reasoning in contexts of uncertainty and vagueness. We form arguments in these contexts and in those arguments 'and' often plays some role. Some of these arguments are valid and some are not, even when we allow for dependence relations to alter the truth-values of our operators. For example, conjunction elimination is still valid with Edgington's 'and' (see Edgington, 1992, p. 197).

'And' is such a quintessential example of a logical constant, that if the ordinary meaning of 'and' is not permutation invariant, then permutation invariance is not a necessary condition for logical constancy. Though some logicians may be unconcerned with capturing the logical structure of natural language reasoning, as a whole the discipline of logic has spent enormous amounts of time and energy concerned with the logical forms and semantics of natural language sentences. If part of the

[4] Edgington uses the function v as a valuation function that takes sentences as inputs and outputs degrees of truth.

[5] $v(\text{b is not red}) = 1 - v(\text{b is red}) = 0.51$.

project of logical research is to understand natural language reasoning and such reasoning uses contants that are not permutation invariant, then permutation invariance is not necessary for logical constancy.

4. Base Step

William Lycan demonstrates the imprecision in the line between analytic and logical by constructing a proto-sorites series of inferences. His series starts with the paradigmatic analytic inference *x is a bachelor* $\longrightarrow$ *x is unmarried*, and it ends with the paradigmatic logical inference *P and Q* $\longrightarrow$ *P*. In between these Lycan lists nineteen other inferences, including geometric, mathematical, and modal inferences. Lycan states his results as follows:

> But the general moral is clear: in no such good healthy list of examples can any great obtrusive break be seen, between the merely lexical and the genuinely "structural". Logicalness as opposed to lexicalness seems to be a matter of degree or at best of grade. (Lycan, 1989, p. 393)

Lycan's sorites series gives us some initial evidence for the vagueness of the concept 'logical constant.' It does not, however, give us conclusive evidence. Lycan claims that we cannot find an obvious break in the series, but his series is not so fine-grained that a reasonable person couldn't accept some slightly unsatisfying cutoff between the analytical entries in the series and the logical entries. What makes sorites arguments so paradoxical is their fine-grainedness. In this section, I will lay out the base step of a sorites argument that can be infinitely fine-grained.

In the previous section, I discussed Edgington's semantics for conjunctions. We will use Edgington's 'and' as a jumping off point for a sorites argument. The central idea is that certain structural features about the world like color similarities can play a role in the determination of truth-values for logical constants.

Consider how color similarity played a role in Edgington's example. The more similar a and b are in color the closer the conjunction gets to 0. If we remove the negation from the second conjunct and evaluate the sentence 'a is red and b is red,' we will find the reverse. The more similar a and b are with respect to color the closer the truth-value is to the minimum value of the conjuncts.

Of course, other structural features of the world will play roles in determining the truth-values of conjunctions. Examples can easily be generated by considering other ways in which objects can be similar. Consider, for example, 'a is rich and b is not rich.' If a and b have around the same amount of money, then the truth-value of this conjunction will be close to 0.

To construct the sentential operators for a sorites series, we will capture the influence of these structural features with a function, d. Let $\wedge_E$ be Edgington's conjunction, and let d be defined as follows:

$$d(\phi, \psi) = \frac{v(\phi \wedge_E \psi)}{\min(v(\phi), v(\psi))} \quad \text{if } \min(v(\phi), v(\psi)) > 0$$

When $\min(v(\phi), v(\psi))$ is 0, let the value of $d(\phi, \psi)$ be evaluated in the closest possible world where $\min(v(\phi), v(\psi)) > 0$. If there is no world where $\min(v(\phi), v(\psi)) > 0$, then $d(\phi, \psi) = 0$.

First, we take the value of Edgington's conjunction for ϕ and ψ, and then divide that by the minimum value of the two. The result is a value between 0 and 1 that represents how close the value of the Edgington conjunction was to the constraints set by the truth-values of the conjuncts. Remember that Edgington believes that the value of a conjunction is between 0 and the minimum value of its conjuncts. d merely represents how close the conjunction is to the minimum value of its conjuncts. When the minimum value of the conjuncts is actually 0, we go to a world where this is not the case so that we can evaluate the degree to which the stuctural features like color similarity would influence the truth-value of the conjunction.

Let $\wedge_n$[6] be a schema for sentential operators, defined as follows:

$$v(\phi \wedge_n \psi) = \begin{cases} (\min(v(\phi), v(\psi)) \cdot \frac{1000-n}{1000}) + (d(\phi, \psi) \cdot \frac{n}{1000}) & \text{if } < 1 \\ 1 & \text{otherwise} \end{cases}$$

When $n = 0$, $v(\phi \wedge_n \psi) = \min(v(\phi), v(\psi))$. When $n = 1000$, $v(\phi \wedge_n \psi) = d(\phi, \psi)$. If $0 < n < 1000$, the truth of the sentence is influenced by both the min function and d function in a proportion determined by n.

We cannot rule out $\wedge_1$ as a logical constant merely on the grounds that it is not permutation invariant. A little non-numerical structure in our logical constants is not taboo, as Edgington's conjunction shows

[6] Note that I use the $\wedge$ symbol for these sentential operators because they have a role for min to play. As such, they are loosely linked with the degree-functional semantics for conjunctions. This is not to indicate that I think these sentential operators are possible meanings for the word 'and.'

us. Edgington's conjunction uses the min function to determine one of the constraints on the truth-value of the whole conjunction. Where the value of the conjunction falls within the range between 0 and the minimum value of the conjuncts is determined by other structural features of the world like the similarities between objects. $\wedge_1$ rearranges the two functions that play roles in Edgington's conjunction. Rather than using min to constrain the truth-value, min contributes to it. The structural features of the world that picked out a value between min and 0 now pick out a value between 1 and 0, which also contributes to the value of the whole sentence. The same parts of the world that determine truth-values for Edgington's conjunction determine truth-values for $\wedge_1$. Notably, however, the structural features have a much lower influence on the truth-value of $\wedge_1$, than they do for $\wedge_E$. If $\wedge_E$ counts as a logical constant, as I argued in the previous section that it should, then $\wedge_1$ should count as one too.

Some may be hesitant to include $\wedge_1$ on the grounds that it is too gerrymandered. Note, however, that how gerrymandered a sentential operator is will depend on our linguistic practices. To a hypothetical linguistic community that uses $\wedge_1$, it may not seem gerrymandered at all. In addition, 'gerrymandered' is clearly a vague concept. So, if the goal is to escape a sorites argument by appealing to the gerrymandered nature of the example, then one would only be introducing a different dimension on which logical constancy is vague.[7] Similarly, $\wedge_1$, and its related sentential operators cannot be excluded from logical constancy on the grounds of usefulness in a logical system or the likelihood that an actual linguistic community will use the operator. Doing so will again produce additional dimensions of vagueness on which sorites series can be run.

5. Inductive Step

For the sorites argument itself, we simply start at 1 and progress to 1000.

P1. $\wedge_1$ is a logical constant.

[7] Iacona (2018) considers a similar source of vagueness in his discussion of the vagueness of logical forms. For Iacona, some formula ϕ is the logical form of a sentence s iff it is an adequate formalization of s. There are countless formulae that are logically equivalent to ϕ, but not all of them are adequate formalizations. Some are too gerrymandered. Iacona claims that there is no precise line between adequate and inadequate formalizations.

P2. If $\wedge_n$ is a logical constant, then so is $\wedge_{n+1}$
C. $\wedge_{1000}$ is a logical constant.

A sorites argument becomes paradoxical when both premises are plausibly true and the conclusion is plausibly false. If one of the premises can be rejected immediately, then the paradox does not arise. I argued for P1 in the previous section. Here, I will provide a justification for P2. From $\wedge_1$ and Edgington's conjunction, we can conclude that some level of influence from various structural features of the world is fine. If some level of influence is consistent with logical constancy, then increasing that influence by a negligible amount will not affect the status of the operator. In fact, in a linguistic community that uses $\wedge_1$, some members of the community may unwittingly be using $\wedge_2$ or even $\wedge_{10}$ without any outwardly noticeable effects. The argument that applied to $\wedge_1$ applies pretty much just as well to $\wedge_2$. Note also that the increment in P2 can be arbitrarily small. So, we could have instances of the conditional like 'If $\wedge_1$ is a logical constant, then so is $\wedge_{1.0000000001}$.'

The next requirement for a successful sorites argument is a plausibly false conclusion. $\wedge_{1000}$ is the same function as d. With $\wedge_{1000}$, we have a function concerning a variety of different structural features of the world, which are traditionally thought to be outside of the realm of logic. Remember that these structural features could be color similarity relationships or they could concern the amounts of money people have, or any number of other relations between objects. It is plausible that d is not a logical relation, since most of these similarity relationships do not fall within the traditional purvey of logic.[8] There is an important difference between $\wedge_{999}$ and $\wedge_{1000}$. The latter is the same function as d, but the former has a more substantial role for min to play. This may indicate that the last step of the sorites series is where a sharp border can be drawn. Note, however, that the value of $\phi \wedge_{999} \psi$ is mostly determined by the kinds of structural features of the world that render $\wedge_{1000}$ plausibly non-logical. So, if $\wedge_{1000}$ is non-logical, it is unclear why allowing the truth-values of the conjuncts some influence would make the operator logical. Consider a sentence like 'a is red $\wedge_{999}$ b is red.' where a and b are incredibly similar with respect to their colors. Because

[8] Note that the point here is not that such relations are certainly beyond the scope of logic. Some views of vagueness I will cover will hold that the conclusion of sorites arguments are true, and hence d is a logical function. The point, instead, is that metaphysical relations are intuitively non-logical.

a and b are similar in color, $v(a \text{ is red} \wedge_E b \text{ is red})$ will be close to $\min(v(a \text{ is red}), v(b \text{ is red}))$. So, $d(a \text{ is red}, b \text{ is red})$ will be close to 1, and consequently $v(a \text{ is red} \wedge_{999} b \text{ is red})$ will also be close to 1. The truth-values of the conjuncts have almost no effect on the truth-value of the whole sentence. Instead, the similarity in color between a and b explains the high truth-value for the sentence. Color similarities are not traditionally thought to be logical properties. So, plausibly $\wedge_{999}$, like $\wedge_{1000}$, is not a logical constant.

One of the defining features of logic is its topic neutrality. Of course, the arguments here have indicated that logic is not wholly topic neutral. That being said, it is still a feature that our account of logicality should countenance. $\wedge_{999}$ is too influenced by particular structures in the world to be counted as topic neutral enough for logical constancy.

We have now a completely fine-grained sorites argument for logical constancy. The premises are plausibly true and the conclusion is plausibly false, but the argument is classically valid. So, we are faced with the three options described at the beginning of this paper. Note, however, that P1 does not have to be undeniably true nor does C have to be undeniably false to generate the paradox. P1's truth and C's falsity are, I take it, controversial. The point here is that they are believable enough to make their rejection unpalatable. And so, logical constancy is sorites-susceptible, meeting the sufficient condition for vagueness.

6. All or Nothing

Some philosophers treat vague words as defective. By failing to draw a sharp border, vague words either include all of the members of a sorites series in their extensions or none of them.

As an example of such a view, Peter Unger (1979) famously argues against the existence of ordinary objects by appealing to sorites arguments. Consider the following argument about clouds:

P1. $500,000,000$g of water under proper conditions make a cloud.
P2. If n grams of water under proper conditions make a cloud, then $n-1$ grams of water under proper conditions make a cloud.
C. 1g of water under proper conditions makes a cloud.

Unger takes this argument as a *reductio* of the existence of clouds. Suppose that clouds exist, then we can run this argument to show that any amount of water is sufficient for making a cloud. It is not the case that any amount of water can make a cloud, and so clouds do not exist. We can also run the sorites argument the other direction, starting with 1g not making a cloud and conclude that no amount of water is sufficient. Unger treats this second sorites argument as a direct argument for the conclusion that clouds do not exist. He accepts the sorites arguments and follows them through to their conclusions. Since sorites arguments can be made for any ordinary objects, Unger concludes that ordinary objects do not exist. In Enoch's terms, nihlists like Unger are giving up utility in favor of tolerance. Vague words are useless because they fail to draw borders.

The same reasoning can be applied to logical constants. Since the concept 'logical constant' fails to draw a sharp border, there are no logical constants. M. J. Cresswell hints at a view like this in his discussion of adverbs.

> If we try to mark off a class of entailments which depend only on the 'logical words' of English we are faced with the invidious task of deciding what these are. [...] The decision as to what should count as a logical word in English has to be made by fiat and I cannot myself see any good reason for making it at all. (Cresswell, 1974, p. 470)

When no words are kept constant through substitutions, no sentence is a logical truth. If everything in the sentence can vary, then each sentence is bound to have some substitution instance that comes out as false.

The other option that also accepts sorites arguments at face value is to treat everything as a logical constant. Do the *modus ponens* to Unger's *modus tollens*. We might choose to keep everything fixed. When nothing can be substituted, true sentences become logical truths. This is because they are true on all zero substitutions of non-logical words. They become vacuous logical truths. Of course, such a view is absurdly extreme. Saying that all words are logical constants *simpliciter* is not a reasonable position. A more reasonable approach that will be covered in the next section states that all words *can* be logical constants when relativized to contexts. This less extreme view, however, suffers from similar problems.

All or nothing approaches to vagueness are not popular. They are, after all, extreme error theories that contend that we are systematically wrong when we think that our words are properly meaningful. Since most of the words we use are vague, if vague words admit everything or nothing into their extensions, then we are mistaken about the meanings of just about every word we use. Of course, proponents of all or nothing views of vagueness offer explanations for our mistakes and paraphrases for ordinary language. Despite these explanations they have many bullets to bite.

Potentially the worst problem for all or nothing views of vagueness is the role that we play in creating languages. Error theories that claim that we are wrong in our observation of the world have their pros and cons, but unlike our observations of the outside world, we play a pivotal role in shaping our languages. So, an error theory that we are systematically wrong about the existence of clear cases of our predicates is a particularly difficult sell.

7. Precisifications

Vague words are a lot like ambiguous words. The word ‘bank,’ for example, picks out two different extensions — financial institutions on the one hand and the sides of rivers on the other. Vague words also pick out multiple different extensions. Unlike ambiguous words, vague words pick out a sometimes uncountable number of different but incredibly similar extensions. These individual extensions are called precisifications.

A precisification of a word is a precise extension for that word. A precisification for a language is an interpretation of the language such that every word has a precise extension. Precisifications are used in a variety of ways by theories of vagueness. The most famous of these is supervaluationism. Supervaluationists say that a sentence is true if it is true on all admissible precisifications and false if it is false on all of them. It is true, for example, that Jeff Bezos is rich because it is true on every way that ’rich’ can be precisified. It is false that the author of this paper is rich because I am not rich on any admissible way of precisifying the term. Finally, it is neither true nor false — it is indeterminate — that Bernie Sanders is rich. On some precisifications of ‘rich’ he is and on others he is not.

Enoch defends the use of precisifications on the grounds that they help to alleviate the tension between tolerance and utility. Vague words are tolerant and not useful. Precisified words are useful but not tolerant. The tension between the two properties emerges when we equivocate between vague words and their precisified counterparts.

In recent years, theories that use precisifications have moved away from defining truth as truth on all precisifications. In particular, Diana

Raffman (2014) relativizes classical truth to precisifications and Nicholas J. J. Smith (2008) relativizes degrees of truth to precisifications. Theories like Raffman's and Smith's ostensibly agree with the all or nothing approach to vagueness. There is something wrong with vague words. But, instead of throwing them out, they opt to make use of the many nearby acceptable meanings of vague words.

This relativistic approach to vagueness is also one of the most common in the literature on logical constants. Tarski, at one point, hinted at such a relativistic view of logic. In discussing concepts like logical consequence and tautologicality, Tarski says that they are:

> relative concepts which must, on each occasion, be related to a definite, although in greater or less degree arbitrary, division of terms into logical and extra-logical. (Tarski, 1956, p. 420)

John Etchemendy, in a rebuke of Tarski's more precise understanding of logical consequence, says the following:

> Any language, regardless of its expressive devices, gives rise to a consequence relation, a relation that supports inferences from sentences in the language to other sentences in the language. The study of this relation is the study of the logic of that language.
> (Etchemendy, 2008, p. 282)

In different *languages*, different words count as logical constants. Here, languages act like precisifications. Within Kripke's language of modal operators the epistemic operators do not need to act as logical constants. So, individual words can be logical constants in one context and not constants in another.

There is a problem with the use of precisifications. Consider the supervaluationist. If all precisifications are used, then it's not true of anyone that they are rich, since the cutoff for 'rich' could be put unreachably high. Similarly, its not false of anyone that they are rich, since the cutoff could be placed arbitrarily low. Intuitively, 'some people are rich and some people are not rich' is true, but making use of all precisifications renders it indeterminate. So, supervaluationists draw a line between admissible and inadmissible precifications. For a sentence to be true, it must be true on all admissible precisifications. Of course, what counts as an admissible precisification is vague. So, we will need precisifications of 'admissible precisification.' These too must be admissible, and the end result is the phenomenon of higher-order vagueness — when the method we use to analyze vagueness is itself vague.

The relativistic accounts do not escape this worry. If all precisifications are allowed in, then the extensions of vague predicates explode. The predicate 'is a cloud' could have an extension identical to the predicate 'is a baseball.'[9] But taking this position would effectively divorce the evaluation of truth from the actual meanings of our words. This is why Raffman restricts her account to only consider admissible precisifications. What the relativist is doing is no different from the nihilist. Whereas the nihilist jumps off the vagueness train at the first stop — rejecting the usefulness of the vague word itself — the full relativist gets off at the second stop — rejecting the usefulness of 'admissible precisification.' The result, however, is the same. Utility is completely lost.

Despite the problems with overextending the set of precisifications, many views do exactly this. Etchemendy considers any study of consequence relations between sentences within a language to be logic. This is effectively allowing every precisification that still treats logical constants as words. This view is like the extreme all or nothing view from the previous section. The difference is that Etchemendy does not treat every word as a logical constant *simpliciter*, rather, they are logical constants relative to languages.

The issue with the all or nothing view is still present, though. A central element of our pre-theoretic understanding of logic is that it is topic-neutral. Of course, it isn't wholly topic-neutral, but that does not remove this important feature from our conception of logic. To allow for inferences about chess moves or about electrons to count as logical inferences on the grounds that elements of chess notation or features of electrons can be treated as logical constants is to remove the topic-neutrality of logic. Etchemendy appears to be confusing the logic of x with the field of logic. In chemistry we may study the logic of electrons — what inferences can be made from facts about electrons. In Logic, as a field of study, we study the nature of formal inferences divorced (to some extent) from the specific features of the world.

A better approach than relativizing logicality to different precisifications is to embrace the vagueness of the concept and allow some words to be indeterminately logical. This can be done by accepting the supervaluationist treatment of vagueness. Andrea Iacona hints at such an approach.

[9] Note that this is because we can make sorites series from 'is a cloud' to 'is a baseball.' For example, start with a mass of water molecules in the sky and slowly remove and replace the atoms with ones that would compose a baseball.

Definition 17. A quantifier expression e is logical if and only if, for every sentence s in which e occurs and for every pair of interpretations i and i' such that i' differs from i in the domain assigned to e, s has the same logical form in i and i'. (Iacona, 2016)

Here Iacona is concerned with determining which quantifiers are logical based on how their logical form changes with changes in their domains. In particular, Iacona wants to separate precise quantifiers like 'all' from vague quantifiers like 'most.' The end result is that precise quantifiers are logical and vague quantifiers are not. Note that the vagueness of the potential logical constants themselves is a separate issue from the vagueness of logical constancy in general.[10]

Of course, Iacona's position here is not directly supervaluationist. Definition 17 only covers the logicality of quantifiers, but it shows the potential for such an account extended beyond quantifiers. A word is a logical constant if it is a logical constant on all admissible precisifications of logical constancy. There are indeterminate logical constants — perhaps the modal operators or the epistemic operators. There are clear logical constants like those of first order logic. There are also clear non-constants such as the predicate 'is cheese.'

The quote from Lycan earlier in this paper indicates another option. We could treat logicality as coming in degrees. Ian Hacking gives a clear explication of how we might approach the vagueness of logical constancy from a degree-theoretic perspective. Hacking defines logical constants as "a constant that can be introduced by operational rules like those of Gentzen" (Hacking, 1979). The caveat is that such rules must "be conservative with respect to the basic facts of deducibility." He contends, however, that logic can be expanded beyond this core, and doing so is a matter of degree. The more liberal a constant is with the facts of deducibility, the less logical, but of course, such liberalness can come in degrees.

Pavel Arazim who is somewhat amenable to a degree-theoretic approach to logical constancy notes one potential problem such a view may

[10] Potentially, a sorites argument could be generated for logical constancy that starts with a slightly vague logical constant and ends with an incredibly vague concept. Somewhere along the way, the concepts in the series become too vague to be logical constants. The increases in vagueness in the series would be measured by changes in the sizes of the border areas of the vague concepts. Of course, such a sorites series would be difficult to motivate because many would reject the possibility of vague logical constants.

need to overcome. "One may well find pairs of expressions A and B such that A is more logical from one point of view while B from another one" (Arazim, 2017). An ordering by degrees of logicality may work for the $\wedge_n$ operators, because they are arranged from most to least logical. It will be much harder to quantify logicality when comparing, say, epistemic operators to deontic operators. This is a general problem with degree theories of vagueness. If we are already having trouble sorting things into binary options, why would things be easier to sort into an infinite number of incredibly precise categories? These difficulties were Smith's impetus to accept a relativism about degrees of truth. But, if we are going to make that move anyway, it may be easier to simply stick with supervaluationism.

Regardless of whether one accepts a degree theory or supervaluationism as applied to logical constants, both views are preferable to the all or nothing approach and the corresponding relativist approach. The value of accepting indeterminate cases is that we can do justice to the original intuitions about the meaning of 'logic.' Logic is topic-neutral and formal. It cannot be completely topic-neutral and completely formal and so a vague boundary is created between the logical and the non-logical. But, a vague boundary is still a boundary, and so ignoring it by going all or nothing or by relativizing ignores the word we are trying to analyze. Supervaluationism is an internally consistent and plausible theory that has the virtue of honoring the original, vague intensions of our words. Since we have viable options that do not require us to be completely misguided about the meaning of the word 'logic,' we ought to take one of those options.

Acknowledgments. I would like to thank all of the participants at the "What are Logical Forms (Good For)?" conference. Their questions and comments greatly improved the quality of this project.

References

Arazim, P., 2017, "Expressivist perspective on logicality", *Logica Universalis* 11 (4): 409–419. DOI: 10.1007/s11787-017-0171-6

Bolzano, B. and Berg, J., 1973, *Theory of Science*, edited, with an introd., by J. Berg, translated from the German by B. Terrell., D. Reidel Publishing Company, Dordrecht. DOI: 10.1007/978-94-010-2513-3

Bonnay, D., 2008, "Logicality and invariance", *Bulletin of Symbolic Logic*, 14: 29–68. DOI: 10.2178/bsl/1208358843

Cresswell, M.J., "Adverbs and events", *Synthese* 28: 455–481. DOI: 10.1007/978-94-009-5414-4_2

Edgington, D., 1992, "Validity, uncertainty and vagueness", *Analysis* 52 (4): 193–204. DOI: 10.4324/9781315235158-10

Enoch, D., 2007, "Epistemicism and nihilism about vagueness: What's the difference?", *Philosophical Studies* 133: 285–311. DOI: 10.1007/s11098-005-5337-z

Etchemendy, J., 2008, "Reflections on consequence", pages 262–299 in D. Patterson (ed.), *New Essays on Tarski and Philosophy*, Oxford University Press, Oxford. DOI: 10.1093/acprof:oso/9780199296309.003.0011

Feferman, S., 1999, "Logic, logics, and logicism", *Notre Dame Journal of Formal Logic* 40: 31–54. DOI: 10.1305/ndjfl/1039096304

Hacking, I., 1979, "What is logic?", *The Journal of Philosophy* 76 (6): 285–319.

Iacona, A., 2015, "Vagueness and quantification", *Journal of Philosophical Logic* 45: 579–602. DOI: 10.1007/s10992-015-9387-1

Iacona, A., 2018, *Logical Form: Between Logic and Natural Language*, Springer International Publishing. DOI: 10.1007/978-3-319-74154-3

Keefe, R., 2000, *Theories of Vagueness*, Cambridge University Press, Cambridge.

Lycan, W., 1989, "Logical constants and the glory of truth-conditional semantics", *Notre Dame Journal of Formal Logic* 30 (3): 390–400. DOI: 10.1007/978-94-011-0936-9_10

McGee, V., 1996, "Logical operations", *Journal of Philosophical Logic* 25: 567–580. DOI: 10.1007/BF00265253

Novaes, C.D., 2014, "The undergeneration of permutation invariance as a criterion for logicality", *Erkenntnis* 79 (1): 81–97. DOI: 10.1007/s10670-013-9469-9

Raffman, D., 2014, *Unruly Words: A Study of Vague Language*, Oxford University Press, Oxford. DOI: 10.1093/acprof:oso/9780199915101.001.0001

Sher, G., 1991, *The Bounds of Logic: A Generalized Viewpoint*, MIT Press, Cambridge, MA.

Smith, N. J. J., 2008, *Vagueness and Degrees of Truth*, Oxford University Press, Oxford. DOI: 10.1093/acprof:oso/9780199233007.001.0001

Sorensen, R., 2000, "Vagueness implies cognitivism", *American Philosophical Quarterly* 27 (1): 1–14.

Tarski, A., 1956, "On the concept of logical consequence", pages 409–420 in *Logic, Semantics, Metamathematics*, translated by J. H. Woodger, Clarendon Press, Oxford. DOI: 10.2307/2216869

Tarski, A., 1986, "What are logical notions?", Edited by J. Corcoran, *History and Philosophy of Logic* 7: 143–154. DOI: 10.1080/01445348608837096

Unger, P., 1979, "There are no ordinary things", *Synthese* 41 (2): 117–154.

van Benthem, J., 1989, "Logical constants across varying domains", *Notre Dame Journal of Formal Logic* 30 (3): 315–342. DOI: 10.1305/ndjfl/1093635152

Warmbrōd, K., 1999, "Logical constants", *Mind* 108: 503–538. DOI: 10.1093/mind/108.431.503

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