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Tomasz Jarmużek and Francesco Paoli

Relating Logic and Relating Semantics. History, Philosophical Applications and Some of Technical Problems

Abstract. Here, we discuss historical, philosophical and technical problems associated with relating logic and relating semantics. To do so, we proceed in three steps. First, Section 1 is devoted to providing an introduction to both relating logic and relating semantics. Second, we address the history of relating semantics and some of the main research directions and their philosophical applications. Third, we discuss some technical problems related to relating semantics, particularly whether the direct incorporation of the relation into the language of relating logic is needed. The starting point for our considerations presented here is the 1st Workshop On Relating Logic and the selected papers for this issue.

Keywords: relating logic; relating semantics; deontic logic; logic of variable inclusion; history of relating logic; epistemic logic; incorporating relation

1. Introduction

Relating Logic (henceforth, RL) is a logic of relating connectives –just as Modal Logic is a logic of modal operators. The basic idea behind a relating connectives is that the logical value of a given complex proposition is the result of two things:

(i) the logical values of the main components of this complex proposition; supplemented with
(ii) a valuation of the relation between these components.

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The latter element is a formal representation of an intensional relation that emerges from the connection of several simpler propositions into one more complex proposition.

More formally, let $A_1, \ldots, A_n$ be propositions with some fixed logical values and let c be an n-ary relating connective. Then the logical value of complex sentence $\mathsf{c}(A_1, \ldots, A_n)$ depends not only on the logical values of $A_1, \ldots A_n$, but additionally on the value of the connection between $A_1, \ldots A_n$. It therefore depends on an additional valuation of pairs (n-tuples) that is the part of the overall process of evaluation of the logical values of complex propositions built with relating connectives. This way we can form logical systems to deal with reasoning about non-logical relationships.

Often when we replace the parameters of classically valid arguments with real sentences and the classical connectives with certain natural language connectives, bizarre inferences result, such as the one below:

$$\frac{\begin{array}{c}\text{Ann has not died or Mark is in despair.}\\ \text{Mark is not in despair or Ann is calling for a doctor.}\end{array}}{\text{Ann has not died or Ann is calling for a doctor.}} \tag{a}$$

The problem arises because, when we construct everyday arguments, we consider not only the logical values of the sentences but also expect certain non-logical relationships to hold between them, such as a causal relationship in the case above. It is worth noting that the schema of inference (a) is valid from the classical point of view.

Further examples of such relationships conveyed by arguments expressed in natural language are analytic, temporal, content, preference and connexive relationships. The following is an example with a temporal precedence:

$$\frac{\begin{array}{c}\text{Jan Łukasiewicz was born in 1878 and}\\ \text{in the 1920s he discovered many-valued logics.}\end{array}}{\begin{array}{c}\text{Jan Łukasiewicz discovered many-valued logics in the 1920s}\\ \text{and he was born in 1878.}\end{array}} \tag{b}$$

The schema of inference (b) is also valid from the classical point of view. However, it is clear that the temporal aspect plays the important role in it and (b) should be recognized as invalid. The case of (c) is even more complex:

$$\frac{\text{If Mark loves Ann and Ann loves Mark, then Mark and Ann love each other.}}{\text{If Mark loves Ann, then Mark and Ann love each other or if Ann loves Mark, then Mark and Ann love each other.}} \tag{c}$$

In (c), you may not like the behavior of implication and conjunction versus disjunction, since two things make that Mark and Ann love each other, not at least one of them.

The problems appear because, in the contexts of making the arguments, we consider not only the logical values of sentences, but we also expect some relationships to hold between the content of the sentences involved in inferences (a), (b), and (c). For example, in the case of (b), we expect that Jan Łukasiewicz firstly was born in 1878 and then he discovered many-valued logics in the 1920s, not the other way round. So, what happens, happens in some temporal order which is asymmetric.

Let us formalize the arguments in the propositional language. As we said, they are all valid from the classical point of view.

$$\frac{\neg p \vee q \qquad \neg q \vee r}{\neg p \vee r} \tag{a1}$$

$$\frac{p \wedge q}{q \wedge p} \tag{b1}$$

$$\frac{p \wedge q \to r}{(p \to r) \vee (q \to r)} \tag{c1}$$

A formal treatment of these cases needs more than the standard formal apparatus of disjunction, conjunction and implication for handing extensional phenomena; it needs machinery to make sense of intensional phenomena too.[1] We assume models $\langle v, R \rangle$, where v is a binary valuation of variables and R is a binary relation defined on a set of formulas. Let us assume additionally that we have some recursive definitions of truth conditions in these models for all connectives we use.

It is easy to observe that if we interpret the connective $\vee$, present in (a1), in the models $\langle v, R \rangle$ in the following way: $\langle v, R \rangle \models A \vee B$ iff

[1] For an overview on possible relating models see [Jarmużek, 2021; Jarmużek and Klonowski, 2021].

$\langle v, R\rangle \models A$ or $\langle v, R\rangle \models B$, and $R(A, B)$, then inference (a1) is not valid.[2] However, if we assume that R is transitive and closed under negation in the following sense: $R(A, B) \Rightarrow R(A, \neg B)$, then (a1) is valid. The above suggests that the non-transitivity of relations between particular propositions occurring in (a) may be needed.

In addition, if we interpret the connective $\wedge$ present in (b1) in the models $\langle v, R\rangle$ in the following way: $\langle v, R\rangle \models A \wedge B$ iff $\langle v, R\rangle \models A$ and $\langle v, R\rangle \models B$, and $R(A, B)$, then inference (b1) is not valid. However, if we assume that R is symmetric, then (b1) is valid. So, in the argument (b) the non-symmetry of relation between the propositions is expected, because the proper temporal precedence is even asymmetric.

Finally, if we interpret the connective $\to$ present in (c1) in the models $\langle v, R\rangle$ in the following way: $\langle v, R\rangle \models A \to B$ iff $\langle v, R\rangle \not\models A$ or $\langle v, R\rangle \models B$, and $R(A, B)$, then inference (c1) is not valid. But, if we assume that R fulfills in the models the condition:

$$R(A \wedge B, C) \Rightarrow R(A, C) \text{ and } R(B, C) \tag{$*$}$$

for all formulas A, B, C, then (c1) is valid, assuming that $\vee$ is classically interpreted. Thus, ($*$) establishes the special relationship between the conjunction, the implication and the disjunction which is probably assumed in (c).

Although the simplest model for a relating logic is a pair: $\langle v, R\rangle$, the situation may get more complicated. When we have more connectives in a language, some of them may be extensional, and some of them relating, requiring an additonal evaluation of being related [see Jarmużek and Klonowski, manuscript,s]. We can also use multi-relating models to represent more types of non-logical relations between sentences. In addition, the valuation of relationships between sentences may not be binary but many-valued or more subtly graded. Furthermore, we can mix relating semantics with possible world semantics, equipping all worlds with additional valuations of complex sentences [see Jarmużek, 2021]. For example, somebody could expect that the inference (c1) is modal in its nature. So that the implication is a strict implication.

In fact, we already may say that structure $\langle v, R\rangle$ is a one-world model, indeed. The world is a valuation of variables it contains. But, naturally, we would like to also have models with more worlds and a non-empty

[2] Preserving here and in the remaining cases, of course, the classical meaning of negation.

accessibility relations to interpret inferences like inference (c1) in a modal way. So we could have more general semantic structures at our disposal.

Let us recall the notion of a *possible world model*. A possible world model is a structure $\langle W, Q, v\rangle$ where:

- W is a non-empty set of possible worlds
- $Q \subseteq W \times W$ is an accessibility relation
- $v\colon PV \times W \longrightarrow \{0,1\}$ is a valuation of propositional variables PV at possible worlds.

We add to the model a family of binary relations $\{R_w\}_{w\in W}$ indexed by worlds $w \in W$. We will therefore consider the models:[3]

$$\langle W, Q, \{R_w\}_{w\in W}, v\rangle. \tag{$**$}$$

In the models ($**$) for any possible world w there exists a relating relation. Let us assume again that we have some recursive definitions of truth conditions in these models for all connectives we use. Now we can interpret the implication as strict one in the following way. Let $\mathfrak{M} = \langle W, Q, \{R_w\}_{w\in W}, v\rangle$ and $w \in W$. $\mathfrak{M}, w \models A \to B$ iff:

$$\text{for all } u \in W(Q(w,u) \Rightarrow \mathfrak{M}, u \not\models A \text{ or } \mathfrak{M}, u \models B, \text{ and } R_u(A,B)).$$

It easy to observe that under such an interpretation of $\to$ the inference (c1) is not valid. However, if we assume that the relations in a model satisfy the following condition:

$$R_w(A \wedge B, C) \Rightarrow R_w(A, C) \text{ and } R_w(B, C)$$

for all formulas A, B, C and worlds $w \in W$, then (c1) is valid. So, it can suggest that specific relations between the connectives in case of (c) may be needed. And if you accept them, then your reasoning is correct.

[3] The models ($**$) are special cases of models: $\langle W, Q, \{\{\{v_i\}_{i\in I}\}_w\}_{w\in W}, v\rangle$ in which, instead of families of valuations $\{\{\{v_i\}_{i\in I}\}_w\}_{w\in W}$, we add to the model families of binary relations $\{\{\{R_i\}_{i\in I}\}_w\}_{w\in W}$ indexed by worlds $w \in W$, but with one relation per a world, so just $\{R_w\}_{w\in W}$. The structures $\langle W, Q, \{\{\{v_i\}_{i\in I}\}_w\}_{w\in W}, v\rangle$ are very general, especially since all valuations in the models can be many-valued. It is worth noting that generally a relation is just a characteristic function, hence a special case of valuation. Moreover, in these models it is assumed that there may be more valuations in a world, e.g., each functor may have its own valuation. Then we call such semantics multi-relating. Such models and appropriate terminology were introduced in the article [Jarmużek, 2021].

Any models that mix relations with possible worlds can be named *modalized models* [see Jarmużek and Malinowski, 2019b; Jarmużek and Klonowski, 2020]. Last, but not least, any semantics may be treated as relating one, when we assume that in case of complex sentences a relationship is represented by a universal relation.

The solution that relating semantics offers seems to be quite natural, since when two (or more) propositions in natural language are connected by a connective, some sort of emergence occurs. In fact, the key feature of intensionality is that adding a new connective results in the emergence of a new quality, which itself does not belong to the components of a given complex proposition built by means of the same connective. An additional valuation function determines precisely this quality. Talk of emergence is justified here, because the quality that arises as a result of the connections between the constituent propositions is not reducible to the properties of those propositions. Consequently, if the phenomenon of emergence is to be properly captured, we need additional valuations in a model. The key feature of relating semantics is that it enables us to treat non-logical relations between sentences seriously. Therefore, probably the semantics seems to have a lot of power of expression.[4]

2. The 1st Workshop on Relating Logic

The main aim of the *1st Workshop On Relating Logic* (1st WRL) was to create an international community of logicians, that explores the potential of RL and relating semantics.[5]

The scope of the workshop was determined by the topics of submissions. They included among others:

- applications of relating semantics,
- algebraic interpretation of relating logics,
- comparison of relating semantics with other formal semantics,

[4] It may sound strange to talk about *power of expression* in the context of semantics, because this term is usually used in reference to a syntax, to some formal language. In [Jarmużek, 2021] the term was intensionally refered to semantics, since the author thinks that also logical semantics can be compared in respect with which logical systems can be determined by those logical semantics. The term *power of determining* was also used alternatively.

[5] The 1st Workshop On Relating Logic took place in September 25–26, 2020; see `https://www.filozofia.umk.pl/en/department-of-logic/call-for-workshop-on-relating-logic/12`

- history of relating logics,
- modal extensions of relating logics,
- model theory of relating logics,
- philosophical logics defined by relating semantics,
- proof theory for relating logics,
- philosophical foundations of relating logics,
- other related topics (like dependence logic, set-assignment semantics etc.).

One of the main issues addressed in the 1st WRL was to determine the generalities of relating semantics. In that sense, and as a result of the meeting, we characterize, broadly, the concept of relating semantics in the following way.

Let us consider a language consisting of a countable set of propositional variables and with n propositional connectives, $c_1, \ldots, c_n$. Suppose that we have two non-empty domains of logical values and their sub-domains:

1. logical values for propositions DV_1 and the designated logical values D_1,
2. connection values for ordered tuples of propositions DV_2 and the designated connection values D_2.

A semantics for the language is a *relating semantics* iff at least for one connective c_i the valuation of all complex propositions of the form $c_i(\varphi_1, \ldots, \varphi_j)$, where j is the arity of c_i, in a world w requires not only valuations of pairs $(\varphi_1, w), \ldots, (\varphi_j, w)$ in DV_1, but also a valuation of j-tuples $((\varphi_1, \ldots, \varphi_j), w)$ in DV_2 [see Jarmużek, 2021; Jarmużek and Klonowski, 2021]. A valuation of j-tuples $((\varphi_1, \ldots, \varphi_j), w)$ in DV_2 can in a formal semantics represent various logical or non-logical relationships between $\varphi_1, \ldots, \varphi_j$ in a world w, for example:

- content relationships, for example, the relatedness relation,
- analytical relationships,
- causalities,
- temporal orderings,
- preference orderings,
- logical consequences of some logic,

among many others.

A function with a co-domain DV_2 is used to evaluate either a relationship between $\varphi_1, \ldots, \varphi_j$ or a relationship between some objects to which we refer by means of $\varphi_1, \ldots, \varphi_j$ — for example, events, facts or states of affairs — in the relating semantics.

The name ‘relating semantics’ seems to be justified since a valuation of j-tuples which receives a designated value induce j-ary relations among the formulas, which allow us to evaluate various relationships not expressible by means of extensional relationships.

If $DV_2 = \{1, 0\}$ and 1 is the designated value then an evaluation of relationship between $\varphi_1, \ldots, \varphi_j$ can be reduced to one j-ary relation over set of formulas. We usually call a relation over a given set of formulas a *relating relation* and use the symbol R to denote that relation. Finally, by ‘relating logic’ we mean any logic that is determined by some relating semantics [Jarmużek, 2021; Jarmużek and Klonowski, 2021; Jarmużek and Kaczkowski, 2014]. Some examples of relating logics are:

1. classical mono-relating logic [see Jarmużek and Klonowski, 2021; Klonowski, 2018, 2019; Jarmużek and Klonowski, submitted,m]
2. fragments of classical mono-relating logic:
   - relatedness logic [see Epstein, 1979, 1990; Paoli, 1993, 1996, 2007],
   - dependence logic [see Epstein, 1987, 1990],
   - Boolean connexive logic [see Jarmużek and Malinowski, 2019a,b; Malinowski and Palczewski, 2021; Klonowski, 2018, 2021],
   - Classical Propositional Logic (when R is assumed to be a universal relation).

During the 1st WRL, we identified the following problems as some of the most important for the understanding of RL:

1. problem α: axiomatization of logics defined by relating semantics (by given classes of valuations/relations)
2. problem β: relating semantics for logics defined as some set of formulas closed under some rules of inference
3. problem γ: defining philosophical logics by relating semantics (reduction of various logical connectives to relating connectives)
4. problem δ: relationships between relating semantics and other kinds of formal semantics (problem of reduction)
5. problem η: combining relating semantics with other kinds of formal semantics.

The articles selected for the first volume contain partial solutions to some of the problems listed above.

3. History, philosophical applications and technical problems

Sometimes, the history of a phenomenon enables us to understand its essence. For this reason, we consider that it is worth looking at the history of relating logic and relating semantics treating the technical issues as part of a larger whole. If we do so, we will discover that their history, while tortuous and complex, is also revealing of their nature and constraints. Such a history is honestly and meticulously reconstructed in the paper "History of relating logic. The origin and research directions" (by Mateusz Klonowski).

In that paper, the history of and the current research directions in relating logic are presented. For this purpose the Epstein's Programme is described. It postulates accounting for the content of sentences in logical research, which is a special case of relating logic. Moreover, the set-assignment semantics is discussed. Next, the Torunian Programme is introduced. Such an introduction to the Programme explores the particular approaches to various non-logical relationships in logical research, including those which are content-related (which is the essence of relatedness and dependence logics). The author also presents a general description of relating logic and semantics as well as the most prominent issues regarding the Torunian Programme, including some of its special cases and the results achieved to date.

This article points out at an interesting bond between New Zealand and Poland between 1976 and 2010. This bond is grounded in the transition from engaging with a specifically understood relationship to focusing on a general and arbitrary way of understanding and applying it, including combining relating semantics with other semantics (problem η). On this basis we can see the difference between relatedness semantics and relating semantics. The former indicates a similarity (this is what the word relatnedness is), e.g., in terms of content. The latter is open to any interpretations, arity of connectives and mixing with other familiar semantics.

The article argues that, apart from the person of R. Epstein, also D. Walton is an indispensable researcher for the development of relating semantics [see Walton, 1979a,b, 1982].

Another interesting feature of the discussed article is that it rightly presents set-assigment semantics as a special case of relating semantics.

In the paper "Pure variable inclusion logics" (by Francesco Paoli, Michele Pra Baldi, and Damian Szmuc) some technical issues are discussed. The aim of this article is to discuss pure variable inclusion logics, that is, logical systems where valid entailment require that the propositional variables occurring in the conclusion are included among those appearing in the premises, or vice versa. We can treat them as relating logics under one of two assumptions:

(a) either the systems are compact and then we can reduce the entailment to some relating implications with a relation defined by the inclusion of variables property
(b) or they are not compact, but we allow the connectives of infinite arity.

Another possibility would be to extend the notion of relating logic to consequence relations.

The subsystems of Classical Logic satisfying the requirements (the inclusion of variables) are studied and the authors assess the extent to which it is possible to characterise them by means of a single logical matrix. In addition, both of these companions to Classical Logic in terms of appropriate matrix bundles and as semilattice-based logics are semantically described by showing that the notion of consequence in these logics can be interpreted in terms of truth (or non-falsity) and meaningfulness (or meaninglessness) preservation. The Płonka sums of matrices are finally used to investigate the pure variable inclusion companions of an arbitrary finitary logic [see, e.g., Ledda, Paoli and Baldi, 2019; Paoli, 2007].

The problem η: combining relating semantics with other kinds of formal semantics is considered in two subsequent papers.

In the article "Alternative semantics for normative reasoning with an application to regret and responsibility" (by Daniela Glavaničová and Matteo Pascucci) a fine-grained analysis of notions of regret and responsibility (such as agent-regret and individual responsibility) in terms of a language of multi-modal logic is provided. This language undergoes a detailed semantic analysis via two sorts of models:

(i) relating models, which are equipped with a relation of propositional pertinence,

(ii) synonymy models, which are equipped with a relation of propositional synonymy.

There, the authors select a class of strictly relating models and show that each synonymy model can be transformed into an equivalent strictly relating model. They also define an axiomatic system that captures the notion of validity in the class of all models, called by them strictly relating models. This article has successfully used a combination of relating semantics and possible world semantics.

Another article that incorporates relating semantics into possible world semantics is "Relating semantics for epistemic logic" (by Alessandro Giordani). The ambitious aim of this paper is to explore the advantages deriving from the application of relating semantics in epistemic logic. The author discusses a few versions of relating semantics and the ways in which they can be differently exploited for studying modal and epistemic operators. Furthermore, he presents several standard frameworks which are suitable for modeling knowledge and related notions. Also relating semantics for such frameworks are set out. This latter point may be the greatest advantage of the article as it simplifies the semantics for epistemic logic (although in a different way than proposed in [Jarmużek, 2021].

Also the logic of knowledge based on justification logic is studied by using tools of relating semantics, and this shows how relating semantics helps us to provide an elegant solution to some problems related to the standard interpretation of the explicit epistemic operators.

The last technical problem is addressed in the article "Incorporating the relation into the language? A survey of approaches in relating logic" (by Luis Estrada-González, Alessandro Giordani, Tomasz Jarmużek, Mateusz Klonowski, Igor Sedlár, and Andrew Tedder). In it, the authors discuss the problem of whether the relation between formulas in the relating model can be directly introduced into the language of relating logic and some stances on that problem are presented. The authors also address other questions in the vicinity, such as which kind of connective would be the incorporated relation, or whether the direct incorporation of the relation into the language of relating logic is really needed.[6]

[6] This paper collects and expands upon the views presented and discussed during the meeting "Do we really need relation R to be directly incorporated into the language of relating logic?". The meeting took place on February 26, 2021 and the recording

Accordingly, the main question addressed in this paper is:

Q1. Can R be incorporated into the relating language?

Although such a move can seem feasible, it remains to be seen what are the necessary and sufficient conditions to do so, and by means of what techniques and procedures. One can consider whether such a connective *can* or even *must* be introduced into the syntax for some reason.

Thus, there are other related questions in the vicinity that are considered:

Q2. What kind of functor is the incorporated R?

Q3. Can the functor counterpart of R be iterated or nested?

Q4. The direct incorporation of R into the language of relating logic is really needed?

At the beginning of the article a positive answer to the question on the possibility of introducing the relations by means of special connectives is explored. The point of view expressed there is mainly endorsed by view of Alessandro Giordani. The next section is devoted to the question on the need of incorporating R into the language, and the views there correspond to Igor Sedlár and Andrew Tedder. Although no strong claim about the need is done there, it is argued that it may make the presentation of some logics simpler and more elegant.

Finally, another approach to these questions is presented, mainly due to Tomasz Jarmużek and Mateusz Klonowski. It is argued there that, in many cases, it is possible to incorporate the relation R into the language. Nonetheless, in all those cases there is no need to do so. Moreover, it is argued that there are other cases where such incorporation is not possible at all, and the question then is what consequences does this have for metalogical studies. In many cases, adding a new functor to a language will lead to the adoption of a substantially different language than the one that was supposed to be the subject of a given consideration. Of course, while we do not need to incorporate the relation directly, we sometimes can do it. The question is which kind of functor is the counterpart of relating relation? The article also provides a semiotic analysis of an answer to this question.

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is available here https://vc.umk.pl/playback/presentation/2.0/playback.html?meetingId=8389f9831f8d30a45207636ebb03baff0ecbb59f-1614329449410.

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Tomasz Jarmużek
Department of Logic
Nicolaus Copernicus University in Toruń
Poland
jarmuzek@umk.pl

Francesco Paoli
Department of Pedagogy, Psychology, Philosophy,
University of Cagliari,
Italy
paoli@unica.it