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A SPURIOUS CONFUSION IN TEMPORAL LOGIC

Abstract. R. L. Epstein and E. Buitrago-Díaz aspire to present a vitally new approach to temporal logic, an approach based on the idea of absolute truth-values. They claim the existing approaches are *confused* and *incoherent*, and contain a significant number of *nonsenses*. The alleged problems are generated by truth-values being relativized to positions in time. The fundamental incoherence consists in some confusion between propositions and their schemata. Epstein and Buitrago-Díaz have formulas be simply true or false and *describe* fixed areas of time. I endeavour to show that all objections Epstein and Buitrago-Díaz raise to existing temporal logic are misunderstandings. The calculus they present is easily reconstructable in existing calculi, so there is no new approach here. However, the calculus is correct and may be of some interest in logic.

Keywords: temporal logic; time; truth-value

Introduction

Since Aristotle's Sea Fight Tomorrow problem, there have always been made efforts to account for time in logic. There are two basic ways to do so: to use concepts of past, present and future, and to use concepts of earlier, simulataneously and after. The concepts of the former kind are called A-series and of the latter one B-series, after John M. E. McTaggart. The A-series usually involve tenses, while the B-series allow to keep sentences tenseless, usually involving time-variables or indicators instead. According to Richard L. Epstein methods of temporal logic so far suffer from *confusions*, are *incoherent* and contain a significant number of *nonsenses*. The alleged incoherence steams from having truth-values relative to time. In collaboration with Esperanza Buitrago-Díaz Epstein claims

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to develop a new approach to temporal logic. The alleged approach is to be free from the incoherences, as it keeps truth-values as absolute as they are in classical propositional calculus.

In this paper I endeavour to show that R. L. Epstein is wrong, when accusing temporal logic of confusion, incoherence and nonsense. Temporal logic is perfectly coherent and, if there is any confusion in it, it is one created by R. Epstein. Furthermore, what Epstein and Buitrago-Díaz present is not so new approach. All they deliver, may be easier and more clear achieved in existing temporal logic. Nevertheless, the logic Epstein and Buitrago-Díazhave constructed is a new calculus. A new calculus, not a new approach. The calculus is formally correct, but philosophically not so helpful, since it applies complicated tools to achieve rather modest effects. In Epstein works there seems to be also a number of terminological confusions, which I put aside.

1. Richard L. Epstein's General Position

According to Richard L. Epstein and Esperanza Buitrago-Díaz the existing way of constructing temporal logic causes "a confusion of scheme versus proposition, which makes any system built in that way incoherent" [2, p. 1], since "the methodology of that approach is incoherent" [3, p. 2] (Epstein appears to spell "scheme" in singular and "schema" in plural. I prefer to spell "schema" in singular and "schemata" in plural or "scheme" in singular and "schemes" in plural).

Epstein and Buitrago-Díaz define a proposition as "a part of speech which we agree to view as being true or false but nor both", furthermore, "a proposition is true or false, not both true and false, nor true sometimes and false another" [2, p. 2], whereas schemata are "true at some times and false at others" [3, p. 2] and "awaiting references for the names and a designated time in order to become a proposition" [2, p. 2]. The alleged problem arises with respect to all formal modal logics but not others [2, p. 1]. The sentence "Germany is bombing Britain" seems to be a schema, as it may be said true on August 8th 1940 and false on September 1st 1939. The sentence "Germany is bombing Britain on August 8th 1940" is simply true or simply false and so a proposition. However, the sentence "if Germany is bombing Britain, then Germany is bombing Britain" could be considered a schema, since it is "awaiting references for designated time", as well as a proposition, since it is not "true at some times and false at others". In my view Epstein's positions burdens two general errors (apart from a number of particular).

First, Epstein's position is fully based on a bit vague distinction between propositions and schemata. Consider an atomic formula "$p(x)$" of first order logic. Let the formula be satisfied by an object a, but not by another object b. Clearly, "$p(x)$" is a schema and "$\exists x\colon p(x)$" is a proposition in Epstein's vocabulary, since the truth-value of "$p(x)$" varies from object to object, whereas the truth-value of "$\exists x\colon p(x)$" is fixed. However, the formula "$p(x) \to p(x)$" should be considered as a schema, although its truth-value cannot vary from object to object. And yet, to Epstein using such formulas is confusing. I am going to try to show that Epstein continuously misregards the formulas, that are *always* true or *always* false, as the formulas, that are *simply* true or *simply* false. In my view, if there is any confusion in temporal logic, it is one created by Epstein.

Second, Epstein continuously misregards formal tools to be used as positions in philosophical debates. It is debatable whether truth-values are absolute or relative. And what it would actually mean, as there are many vitally different kinds of relativism. It is of serious interest in epistemology and philosophy of language. Epstein clearly inclines towards absolutism. I do not object. On the other hand there are artificial languages with artificial formulas and artificial truth-values. Whether the values are absolute or relative is a matter of arbitrary decision made by those constructing the calculi. The philosophical question is to what degree the calculus is reliable, not what tools are to be used to construct it.

In classical propositional calculus two absolute values are usually involved: 1 (simply true) and 0 (simply false). Usually, not necessarily. The second power of the classical matrix involves four truth-values: $\langle 1,1\rangle, \langle 1,0\rangle, \langle 0,1\rangle$ and $\langle 0,0\rangle$, and yet it is adequate for classical propositional calculus as well. Using the classical two-valued matrix does not oblige anyone to any position in epistemological debate on the nature of truth-values. Among logical tools there are many-valued and modal calculi. Many-valued calculi involve values other than classical. Modal calculi involve values relative to objects of one kind or another. Those simply serve describing and examining calculi. Nothing more, nothing less. Let so v be the operation of interpretation (evaluation) in propositional calculi. In classical propositional calculus $v(\varphi)$ is usually considered to be truth or falsehood, whereas in modal logic $v(\varphi)$ is usually considered to be the set of all the positions the formula φ is true at. That is why in classical propositional calculus it is said that $v(\varphi) = 1$ or $v(\varphi) = 0$, i.e. simply true or simply false respectively. In many-valued calculi other values are available. In modal logic it is said that $x \in v(\varphi)$ or $x \notin v(\varphi)$ instead, i.e. true relative to x or false relative to x respectively. Other words, in classical propositional calculus $v(\varphi) \in \{1, 0\}$, whereas in modal logic $v(\varphi) \in \wp(\Omega)$ — which, by the way, shows many bridges between modal and many-valued logics available. Such a position might be disagreed, but it is by all means coherent.

As I have mentioned, Epstein seems to incline towards absolute truth-values. So have done Bernard Bolzano, Kazimierz Twardowski and Alfred Tarski. Arthur N. Prior held the opposite position. As overwhelming majority of scholars before Bolzano, Prior considered truth-values to vary according to time [6]. In my view they are all legitimate to hold their philosophical positions. Now, Epstein blames Prior for incoherence. Well, Prior blames Epstein for the same in return (actually, Prior generously couples an accusation of determinism). In my view they are both wrong, as both positions are perfectly coherent (I do not say: equally true, but: equally coherent).

Epstein assumes all truth-values to be absolute. Then he proceeds to analysis of some logical calculi and, whenever he finds relative truth-values, he lays an accusation of incoherence in method. However, there is no incoherence in the method, as there is no assumption of truth-values being absolute in it. Quite the reverse, the method in question rests on the assumption that truth-values are relative to positions in time, to possible states of affair etc. The second error Epstein makes is usually called *petitio principii*.

2. Temporal Logic on Trial

Consider Priorean tense logic. The usual language of classical propositional calculus with sentence letters and connectives of, say, negation "$\neg$", conjunction "$\wedge$", disjunction "$\vee$", conditional "$\rightarrow$" and equivalence "$\equiv$" (obeying this order in absence of parentheses), may be completed by the connectives "F", "G", "P", "H", such that $\ulcorner \mathsf{F}\varphi \urcorner$, $\ulcorner \mathsf{G}\varphi \urcorner$, $\ulcorner \mathsf{P}\varphi \urcorner$, $\ulcorner \mathsf{H}\varphi \urcorner$ are formulas, provided so is φ. The just mentioned formulas are to be read: it will be the case that φ, it will always be the case that φ, it has been the case that φ, it has always been the case that φ. Call such language of tense logic $\mathbb{F}_{\mathsf{t}}$.

Epstein claims that the connectives "F", "G", "P", "H" convert schemata into propositions. If, for instance, "p" stands for the schema "Germany is bombing Britain" true at some times and false at others, then "$\mathsf{P}p$" stands for the proposition "it has been the case that Germany is bombing Britain" considered by Epstein simply true. Since the connectives $\mathsf{F}, \mathsf{G}, \mathsf{P}, \mathsf{H}$ can be nested, they seem to form propositions out of schemata as well as of propositions. The connective "P" forms the proposition "$\mathsf{PG}p$" out of the proposition "$\mathsf{G}p$" and the proposition "$\mathsf{P}p$" out of the schema "p". In Epstein's view "we have an endemic ambiguity of scheme vs. proposition in this formal logic" [2, pp. 3–5, 18].

Whether expressions like "Germany is bombing Britain" are schemata of statements possessing absolute truth-values, or they are complete statements varying truth-values according to time, is a matter of philosophical position. As there are no schematic letters in the vernacular, Epstein is philosophizing no less then Prior (the point has been addressed in the section 1). With respect to the natural language both accounts are equally tentative thus far.

With respect to the artificial language of modal logic Epstein is simply wrong. He continuously misregards the formulas, which are *always* true or false, as the formulas, which are *simply* true or false. It is actually elementary bookish knowledge that in modal logic, described by means of the possible-worlds semantics, the truth-values are relative to points (positions) of a fixed kind. Call the set of all the points Ω. In strictly modal logic the points mean possible states of affair, in temporal logic positions in time, in epistemic logic rational agents etc. Generally, no formula of the object language of modal logic is simply true or false, since there is nothing like simple truth or falsehood in the conceptual framework of modal logic. The relative or absolute nature of truth-values is the chief difference between modal and classical logic, whereas the number of values is the chief difference between classical and many-valued logic.

Let then $\mathfrak{M} = \langle \Omega, <, v \rangle$ be a model with a non-empty set Ω of positions in time, a relation $<$ of temporal succession in Ω, and $v \colon \mathbb{F}_{\mathsf{t}} \longrightarrow \wp(\Omega)$. Normally, when describing truth-conditions in modal calculi, one always accounts for the relativization of truth-values, e.g.:

$x \in v(\neg\varphi)$ if and only if $x \notin v(\varphi)$,

$x \in v(\varphi \wedge \psi)$ if and only if $x \in v(\varphi)$ and $x \in v(\psi)$,

$x \in v(\mathsf{F}\varphi)$ if and only if $y \in v(\varphi)$ for some y such that $x < y$,

$x \in v(\mathsf{G}\varphi)$ if and only if $y \in v(\varphi)$ for every y such that $x < y$,

$$x \in v(\mathsf{P}\varphi) \text{ if and only if } y \in v(\varphi) \text{ for some } y \text{ such that } y < x,$$
$$x \in v(\mathsf{H}\varphi) \text{ if and only if } y \in v(\varphi) \text{ for every } y \text{ such that } y < x$$

and similarly. It is easily to observe the occurrence of a free variable "x" on both sides of the copula "if and only if". That makes truth-values of all formulas in the object languages of typical modal calculi equally relative.

What Epstein attempts to do, in order to avoid the universal relativity of the truth-values, is to introduce the term "$\mathfrak{n}$", standing for the present time, i.e. now. Using that term Epstein pretends to be able to rephrase the above presented truth-conditions as follows:

$$v(\mathsf{F}\varphi) = 1 \text{ if and only if } y \in v(\varphi) \text{ for some } y \text{ such that } \mathfrak{n} < y, \quad (1)$$
$$v(\mathsf{G}\varphi) = 1 \text{ if and only if } y \in v(\varphi) \text{ for every } y \text{ such that } \mathfrak{n} < y, \quad (2)$$
$$v(\mathsf{P}\varphi) = 1 \text{ if and only if } y \in v(\varphi) \text{ for some } y \text{ such that } y < \mathfrak{n}, \quad (3)$$
$$v(\mathsf{H}\varphi) = 1 \text{ if and only if } y \in v(\varphi) \text{ for every } y \text{ such that } y < \mathfrak{n}. \quad (4)$$

That is the alleged confusion Epstein reproaches modal logic with [2, pp. 4–5]. However, this is clearly some misunderstanding, as Epstein ignores that the term "$\mathfrak{n}$" is token-reflexive, and hence, $y < \mathfrak{n}$ means simply that y is earlier than the position the formula in question is true at. Since there is no occurence of the term "$\mathfrak{n}$" on the left side of the copula "if and only if", Epstein's version of the truth-conditions is much closer to some incoherence than the traditional one.

Consider an Epstein like semiformal example. In Epstein's account for traditional temporal logic the sentence "it has been the case that Germany is bombing Britain" is simply true if and only if there exists a date x, earlier than now, such that the sentence "Germany is bombing Britain" is true at x. Epstein claims that "it has been the case that Germany is bombing Britain" is a proposition possessing an absolute truth-value, whereas "Germany is bombing Britain" is a schema whose truth-value varies from time to time. If it had been the case, Epstein would have immediately fall in contradiction, as it would have been enough to find a date x, the schema "Germany is bombing Britain" being true at, and a date y, the schema "Germany is bombing Britain" being false at, to prove that both it has been the case that Germany is bombing Britain and it has not been the case that Germany is bombing Britain. Analogical misunderstanding concerns Epstein's view of alethic modal logic [2, pp. 16–17].

The same confusion between propositions and schemata — Epstein claims — is created in positional calculi with time-variables. The language $\mathbb{F}_{\mathrm{R}}$ of the classical propositional calculus is to be completed by the connective "$\mathcal{R}$" of temporal realization, the set $\mathbb{I}$ of schematic or variable terms referring to positions in time, classical quantifiers, particular "$\exists$" and universal "$\forall$", ranging over the time-variables in $\mathbb{I}$, and the accurate object language counterpart "$\prec$" of the metalingual relation $<$ of temporal succession. Sometimes the parametrical term "$\mathfrak{n}$" is also included. Inscription $\ulcorner \mathcal{R}_\alpha \varphi \urcorner$ is a formula, provided φ is a formula and α belongs to $\mathbb{I}$. The inscription is to be read: at the time α it is the case that φ, whereas $\ulcorner \alpha \prec \beta \urcorner$ is to be read: α is earlier than β.

Epstein clearly voices an objection to nested occurences of the positional connective "$\mathcal{R}$". The sentence "$\mathcal{R}_a p$" is supposed to be a proposition and simply true or false. Whereas the sentence "p" would be a schema and change its truth-value from time to time. Consider now the formula "$\mathcal{R}_a \mathcal{R}_b p$". The outer token of the connective "$\mathcal{R}$" seems to range over a proposition, whereas the inner one clearly ranges over a schema [2, pp. 26–27]. That position is a misunderstanding no less than the former one. Again, in modal logic typically it is truth-values that is relative to positions of a kind. That concerns also formulas built by use of the connective "$\mathcal{R}$":

$$x \in v(\mathcal{R}_\alpha \varphi) \text{ if and only if } d(\alpha) \in v(\varphi), \tag{5}$$

$d(\alpha)$ being the position referred to by α (the unique referent of the term α). It can be immediately observed that the varable "x", ranging over positions in time, occurs on the left side of the copula "if and only if" exclusively. However it means nothing similar to Epstein's definitions (1)–(4). The definition (5) might be completed by an indifferent component, e.g.:

$$x \in v(\mathcal{R}_\alpha \varphi) \text{ if and only if } d(\alpha) \in v(\varphi) \text{ and } x = x,$$
$$x \in v(\mathcal{R}_\alpha \varphi) \text{ if and only if } d(\alpha) \in v(\varphi) \text{ or } x \neq x.$$

Peculiarity of the connective "$\mathcal{R}$" (usually) consists of its lack of token-reflexiveness. A formula $\ulcorner \mathcal{R}_\alpha \varphi \urcorner$ is true either in all positions or in no position of a fixed model: $v(\mathcal{R}_\alpha \varphi) = \Omega$ or $v(\mathcal{R}_\alpha \varphi) = \varnothing$. Nevertheless, although the formula is not token-reflexive, its truth-values are exactly as relative as truth-values of other modal formulas. Again, Epstein continuously misregards the formulas, which happen to be *always* true or false, as the formulas, which are *simply* true or false.

Epstein generously calls temporal and modal calculi “nonsenses”. He calls the formulas like “$\mathsf{F}p$”, “$\mathsf{G}p$”, “$\mathsf{P}p$” and “$\mathsf{H}p$” nonsense. Since the atomic formula “p” is supposed to stand in the present tense, it is simply true or simply false. Prefixing it by tense connectives is nonsense. For example, imagine Winston Churchill, uttering the sentence “Germany is bombing Britain” on August 8$^{\text{th}}$ 1940. Epstein claims that the sentence is simply true, and means — qua uttered on that day — that Germany is bombing Britain on August 8$^{\text{th}}$ 1940. Hence, prefixing it by tense connectives is nonsense [2, p. 3]. Analogically Epstein claims that prefixing propositions by modal connectives is nonsense, to say anything is possible “right here and now” is nonsense as well, “it is true or it is false now and there is no possibility about that [2, p. 15]. Nested modal connectives are to be nonsense as well [2, p. 17]. Epstein summarizes: “We talk lots of nonsense and confusion. We often reason badly. As logicians we try to bring clarity to our reasoning. We have a responsibility not to add to our confusion” [2, p. 29]. I do not actually understand why those would be nonsenses. Epstein gives no justification, no clues but his arbitrary verdicts.

Generally, Epstein uses concepts of syntactic categories as if they were indisputable dogmas. Contrarily, there is no necessity of applying those concepts at all. If there is a recursive definition of the set of formulas accepted, there is no base for any accusation of proposition versus schema confusion of any kind. Let me also mention on passing that there exist positional calculi, which account for the difference between token-reflexive quasi-formulas and complete formulas posessing absolute truth-values [4]. Hence, Epstein’s attempt is not an absolute novelty.

3. The Calculus $\mathsf{TL}_{\mathsf{PC}}$

Richard Epstein and Esperanza Buitrago-Díaz submit a calculus called $\mathsf{TL}_{\mathsf{PC}}$, appearing to create a new approach in temporal logic, free of confusions and based on coherent methodology [3]. The alphabet of classical propositional calculus is supplemented by four connectives: “$\wedge_{\mathsf{bb}}$”, “$\wedge_{\mathsf{be}}$”, “$\wedge_{\mathsf{eb}}$”, “$\wedge_{\mathsf{ee}}$” of temporal conjunction to the effect that “$\ulcorner(\varphi \wedge_{\mathsf{bb}} \psi)\urcorner$”, “$\ulcorner(\varphi \wedge_{\mathsf{be}} \psi)\urcorner$”, “$\ulcorner(\varphi \wedge_{\mathsf{eb}} \psi)\urcorner$” and “$\ulcorner(\varphi \wedge_{\mathsf{ee}} \psi)\urcorner$” are formulas, provided so are φ and ψ. Those conditions define the language $\mathbb{F}_{\mathrm{EBD}}$. All the temporal connectives are supposed to be counterparts of the vernacular phrase “and then”:

$\ulcorner \varphi \wedge_{\mathsf{bb}} \psi \urcorner$ is true if and only if φ is true and ψ is true, and the period φ describes begins before the period ψ describes begins,
$\ulcorner \varphi \wedge_{\mathsf{be}} \psi \urcorner$ is true if and only if φ is true and ψ is true, and the period φ describes begins before the period ψ describes ends,
$\ulcorner \varphi \wedge_{\mathsf{eb}} \psi \urcorner$ is true if and only if φ is true and ψ is true, and the period φ describes ends before the period ψ describes begins,
$\ulcorner \varphi \wedge_{\mathsf{ee}} \psi \urcorner$ is true if and only if φ is true and ψ is true, and the period φ describes ends before the period ψ describes ends.

The truth-value assignment is meant to be classical. Furthermore every formula is meant to describe some period of time, e.g. with respect to the battle of Vienna the sentence "Sobieski beat Kara Mustafa on September 12th 1683" is true and describes September 12th 1683, while the sentence "Kara Mustafa beat Sobieski on September 12th 1683" is false, but describes the same September 12th 1683 [3, pp. 4–7]. A temporal frame

$$\mathfrak{F} = \langle \Omega, < \rangle$$

is a non-empty set of time instants and $<$ is a linear ordering of Ω. It is assumed that there is a beginning point $\mathfrak{b}_\Omega$ and the ending point $\mathfrak{e}_\Omega$ of time, but they may be considered limits in infinity [3, p. 8]. Of course, $x \leq y$ if and only if $x < y$ or $x = y$. An interval is a set $I \subseteq \Omega$ such that

$$I = \{x \in \Omega\colon a < x \text{ and } x < b\} \text{ or}$$
$$I = \{x \in \Omega\colon a \leq x \text{ and } x < b, \text{ and } a \neq \mathfrak{b}_\Omega\} \text{ or}$$
$$I = \{x \in \Omega\colon a < x \text{ and } x \leq b, \text{ and } b \neq \mathfrak{e}_\Omega\} \text{ or}$$
$$I = \{x \in \Omega\colon a \leq x \text{ and } x \leq b, \text{ and } a \neq \mathfrak{b}_\Omega, \text{ and } b \neq \mathfrak{e}_\Omega\},$$

for some $a, b \in \Omega$. The instant a is the beginning point $\mathfrak{b}_I$ of I, the instant b is the ending point $\mathfrak{e}_I$ of I, and $\mathfrak{b}_I, \mathfrak{e}_I$ are ending points of I. Intervals of Ω may be open or closed both on the left and right. Furthermore any finite union $A \subseteq \Omega$ of intervals $I_1, I_2, \ldots, I_n$, i.e.

$$A = I_1 \cup I_2 \cup \ldots \cup I_n,$$

has ending points:

$$\mathfrak{b}_A = \min\{\mathfrak{b}_{I_1}, \mathfrak{b}_{I_2}, \ldots, \mathfrak{b}_{I_n}\},$$
$$\mathfrak{e}_A = \max\{\mathfrak{e}_{I_1}, \mathfrak{e}_{I_2}, \ldots, \mathfrak{e}_{I_n}\},$$

which are simply the smallest beginning and the biggest ending point in the union. There are some relations on intervals or finite unions of intervals defined:

$A <_{\mathsf{bb}} B$ if and only if $\mathfrak{b}_A < \mathfrak{b}_B$,
$A <_{\mathsf{be}} B$ if and only if $\mathfrak{b}_A < \mathfrak{e}_B$,
$A <_{\mathsf{eb}} B$ if and only if $\mathfrak{e}_A < \mathfrak{b}_B$,
$A <_{\mathsf{ee}} B$ if and only if $\mathfrak{e}_A < \mathfrak{e}_B$,
$A =_{\mathsf{bb}} B$ if and only if neither$A <_{\mathsf{bb}} B$, nor $B <_{\mathsf{bb}} A$,
$A =_{\mathsf{ee}} B$ if and only if neither$A <_{\mathsf{ee}} B$, nor $B <_{\mathsf{ee}} A$,
$A =_{\mathsf{be}} B$ if and only if neither$A <_{\mathsf{be}} B$, nor $B <_{\mathsf{eb}} A$,

A, B being either intervals or finite unions of intervals. A model based on the frame $\mathfrak{F}$ is a system

$$\mathfrak{M} = \langle \Omega, <, v, \mathfrak{t} \rangle,$$

$\langle \Omega, < \rangle$ being the frame $\mathfrak{F}$, the function v from formulas to $\{1, 0\}$ is the interpretation and $\mathfrak{t}$ is a function from formulas to areas of time those formulas are meant to describe. The $\mathfrak{t}(\varphi)$ is any interval, if φ is an atomic formula. If $\varphi_1, \varphi_2, \ldots, \varphi_n$ are all atomic formulas contained in φ, then simply

$$\mathfrak{t}(\varphi) = \mathfrak{t}(\varphi_1) \cup \mathfrak{t}(\varphi_2) \cup \ldots \cup \mathfrak{t}(\varphi_n).$$

Furthermore $\mathfrak{b}_\varphi = \mathfrak{b}_{\mathfrak{t}(\varphi)}$ as well as $\mathfrak{e}_\varphi = \mathfrak{e}_{\mathfrak{t}(\varphi)}$. With respect to the classical connectives the interpretation v is exactly classical, e.g.

$v(\neg\varphi) = 1$ if and only if $v(\varphi) = 0$,
$v(\varphi \wedge \psi) = 1$ if and only if $v(\varphi) = 1$ and $v(\psi) = 1$,

while

$v(\varphi \wedge_{\mathsf{bb}} \psi) = 1$ if and only if $v(\varphi \wedge \psi) = 1$ and $\mathfrak{t}(\varphi) <_{\mathsf{bb}} \mathfrak{t}(\psi)$,
$v(\varphi \wedge_{\mathsf{be}} \psi) = 1$ if and only if $v(\varphi \wedge \psi) = 1$ and $\mathfrak{t}(\varphi) <_{\mathsf{be}} \mathfrak{t}(\psi)$,
$v(\varphi \wedge_{\mathsf{eb}} \psi) = 1$ if and only if $v(\varphi \wedge \psi) = 1$ and $\mathfrak{t}(\varphi) <_{\mathsf{eb}} \mathfrak{t}(\psi)$,
$v(\varphi \wedge_{\mathsf{ee}} \psi) = 1$ if and only if $v(\varphi \wedge \psi) = 1$ and $\mathfrak{t}(\varphi) <_{\mathsf{ee}} \mathfrak{t}(\psi)$.

Of course, v is a function, so $v(\varphi) = 1$ if and only if $v(\varphi) \neq 0$. The concepts of validity, consequence etc. are classical [3, pp. 5, 12, 16]. A number of derivative connectives are defined:

$$\ulcorner \varphi \prec_{\mathsf{bb}} \psi \urcorner \overset{\text{df}}{=} \ulcorner (\varphi \rightarrow \varphi) \wedge_{\mathsf{bb}} (\psi \rightarrow \psi) \urcorner,$$

$$\ulcorner \varphi \prec_{\mathsf{bb}} \psi \urcorner \overset{\text{df}}{=} \ulcorner (\varphi \rightarrow \varphi) \wedge_{\mathsf{bb}} (\psi \rightarrow \psi) \urcorner,$$

$$\ulcorner\varphi \prec_{\mathsf{bb}} \psi\urcorner \stackrel{\mathrm{df}}{=} \ulcorner(\varphi \to \varphi) \wedge_{\mathsf{bb}} (\psi \to \psi)\urcorner,$$

$$\ulcorner\varphi \prec_{\mathsf{bb}} \psi\urcorner \stackrel{\mathrm{df}}{=} \ulcorner(\varphi \to \varphi) \wedge_{\mathsf{bb}} (\psi \to \psi)\urcorner,$$

and

$$\ulcorner\varphi \approx_{\mathsf{bb}} \psi\urcorner \stackrel{\mathrm{df}}{=} \ulcorner\neg(\varphi \prec_{\mathsf{bb}} \psi) \wedge \neg(\psi \prec_{\mathsf{bb}} \varphi)\urcorner,$$

$$\ulcorner\varphi \approx_{\mathsf{ee}} \psi\urcorner \stackrel{\mathrm{df}}{=} \ulcorner\neg(\varphi \prec_{\mathsf{ee}} \psi) \wedge \neg(\psi \prec_{\mathsf{ee}} \varphi)\urcorner,$$

$$\ulcorner\varphi \approx_{\mathsf{be}} \psi\urcorner \stackrel{\mathrm{df}}{=} \ulcorner\neg(\varphi \prec_{\mathsf{be}} \psi) \wedge \neg(\psi \prec_{\mathsf{eb}} \varphi)\urcorner,$$

with the intention that

$v(\varphi \prec_{\mathsf{bb}} \psi) = 1$ if and only if $\mathfrak{t}(\varphi) <_{\mathsf{bb}} \mathfrak{t}(\psi)$,
$v(\varphi \prec_{\mathsf{be}} \psi) = 1$ if and only if $\mathfrak{t}(\varphi) <_{\mathsf{be}} \mathfrak{t}(\psi)$,
$v(\varphi \prec_{\mathsf{eb}} \psi) = 1$ if and only if $\mathfrak{t}(\varphi) <_{\mathsf{eb}} \mathfrak{t}(\psi)$,
$v(\varphi \prec_{\mathsf{ee}} \psi) = 1$ if and only if $\mathfrak{t}(\varphi) <_{\mathsf{ee}} \mathfrak{t}(\psi)$

and

$v(\varphi \approx_{\mathsf{bb}} \psi) = 1$ if and only if $\mathfrak{b}_\varphi = \mathfrak{b}_\psi$,
$v(\varphi \approx_{\mathsf{ee}} \psi) = 1$ if and only if $\mathfrak{e}_\varphi = \mathfrak{e}_\psi$,
$v(\varphi \approx_{\mathsf{be}} \psi) = 1$ if and only if $\mathfrak{b}_\varphi = \mathfrak{e}_\psi$.

Those defined connectives appear in axioms [3, p. 20]. Notice that the defined connectives are not truth-value sensitive. There are also two auxiliary symbols introduced: "$\bigwedge$" and "$\bigvee$" being generalized conjunction and disjunction respectively. Let γ be an atomic formula, $\varphi(\gamma)$ a formula containing γ, and ψ any formula. Then

$$\bigwedge_{\gamma \text{ in } \psi} \varphi(\gamma)$$

is the conjunction of the form $\ulcorner\varphi(\gamma_1) \wedge \varphi(\gamma_2) \wedge \ldots \wedge \varphi(\gamma_n)\urcorner$, with γ_1, γ_2, $\ldots$, γ_n being all the atomic formulas appearing in ψ, associated to the left. And

$$\bigvee_{\gamma \text{ in } \psi} \phi(\gamma)$$

is the disjunction of the form $\ulcorner\varphi(\gamma_1) \vee \varphi(\gamma_2) \vee \ldots \vee \varphi(\gamma_n)\urcorner$, with γ_1, γ_2, $\ldots$, γ_n being all the atomic formulas appearing in ψ, associated to the left. The calculus is invariantly axiomatized based on classical propositional calculus to the effect that every substitution of a classical tautology is a theorem and the *Modus Ponens* is the unique primitive rule of inference. There are also specific axiom schemata accepted in an impressive number of 46! Here is a sample of the axiomatics:

$$\neg(\varphi \prec_{\mathsf{bb}} \psi), \tag{A1}$$

$$\neg(\varphi \prec_{\mathsf{ee}} \psi), \tag{A2}$$

$$(\varphi \approx_{\mathsf{bb}} \psi) \wedge (\psi \approx_{\mathsf{bb}} \chi) \rightarrow (\varphi \approx_{\mathsf{bb}} \chi), \tag{A3}$$

$$(\varphi \approx_{\mathsf{bb}} \psi) \wedge (\psi \approx_{\mathsf{be}} \chi) \rightarrow (\varphi \approx_{\mathsf{be}} \chi), \tag{A5}$$

$$(\varphi \prec_{\mathsf{bb}} \psi) \rightarrow \neg(\psi \prec_{\mathsf{bb}} \phi). \tag{A27}$$

and

$$(\varphi \wedge_{\mathsf{bb}} \psi) \equiv ((\varphi \wedge \psi) \wedge (\varphi \prec_{\mathsf{bb}} \psi)), \tag{A39}$$

$$(\varphi \wedge_{\mathsf{ee}} \psi) \equiv ((\varphi \wedge \psi) \wedge (\varphi \prec_{\mathsf{ee}} \psi)), \tag{A40}$$

$$(\varphi \wedge_{\mathsf{be}} \psi) \equiv ((\varphi \wedge \psi) \wedge (\varphi \prec_{\mathsf{be}} \psi)), \tag{A41}$$

$$(\varphi \wedge_{\mathsf{eb}} \psi) \equiv ((\varphi \wedge \psi) \wedge (\varphi \prec_{\mathsf{eb}} \psi)) \tag{A42}$$

and

$$(\varphi \prec_{\mathsf{bb}} \psi) \equiv \bigvee_{\gamma \text{ in } \varphi} \Big(\bigwedge_{\chi \text{ in } \psi} \gamma \prec_{\mathsf{bb}} \chi \Big), \tag{A43}$$

and even

$$\begin{aligned}
&(\varphi \prec_{\mathsf{eb}} \psi) \equiv \\
&\bigvee_{\gamma \text{ in } \varphi} \bigvee_{\chi \text{ in } \psi} \Big(\bigwedge_{\gamma' \text{ in } \varphi} \Big((\gamma' \prec_{\mathsf{ee}} \gamma) \vee (\neg(\gamma \prec_{\mathsf{ee}} \gamma') \wedge \neg(\gamma' \prec_{\mathsf{ee}} \gamma)) \Big) \wedge \\
&\wedge \bigwedge_{\chi' \text{ in } \psi} \Big((\chi \prec_{\mathsf{bb}} \chi') \vee (\neg(\chi \prec_{\mathsf{bb}} \chi') \wedge \neg(\chi' \prec_{\mathsf{bb}} \chi)) \Big) \wedge \\
&\wedge (\gamma \prec_{\mathsf{eb}} \chi) \Big).
\end{aligned} \tag{A45}$$

Notice that all the axioms 1–38 and 43–46 concern the defined terms of temporal relations with no indication to truth-values. Only the axioms 39–42 make vital use of the truth values of temporal formulas, and they do it in a rather modest way, which means the calculus is quite close to classical [3, pp. 20–23]. The system has been examined matalogically and found adequate. The proofs are routine, but quite comprehensive. There are also some confusions as concerns completeness and strong completeness, but I will not discuss it in detail [3, pp. 23, 30].

4. The Calculus $\mathsf{TL}_{\mathsf{PC}}$ in Positional Logic

The most traditional version of temporal conjunction is Georg H. von Wright's connective "T", such that $\ulcorner \varphi \mathsf{T} \psi \urcorner$ is a formula, provided so are φ and ψ, and is to be read: φ and then ψ. The connective is definable in Prior's tense calculi:

$$\ulcorner \varphi \mathsf{T} \psi \urcorner \overset{\text{df}}{=} \ulcorner \varphi \wedge \mathsf{F}\psi \urcorner,$$

so it does not actually express any temporal relation. The formula rather means that it is now the case that φ and it will be the case that ψ. In the sense it would be true that 2+2=4 and then 3+3=6 etc. No moment of time, no moment of change seems to be involved. One should admit that Epstein and Buitrago-Díaz's calculus is more accurate than von Wright's. However, all Epstein and Buitrago-Díaz deliver, may be achieved easier and better in existing positional logic or first order logic with time-variables. So, the calculus $\mathsf{TL}_{\mathsf{PC}}$ cannot be considered any development. It seems to be a proper part of existing positional calculi. First, consider instant semantics. Obviously

$$\ulcorner \mathcal{R}_\alpha \varphi \ \mathsf{T} \ \mathcal{R}_\beta \psi \urcorner \overset{\text{df}}{=} \ulcorner \mathcal{R}_\alpha \varphi \wedge \mathcal{R}_\beta \psi \wedge \alpha \prec \beta \urcorner$$

or, if one prefers to hide the time determiners,

$$\ulcorner \varphi \mathsf{T} \psi \urcorner \overset{\text{df}}{=} \ulcorner \exists \alpha, \beta \colon (\mathcal{R}_\alpha \varphi \wedge \mathcal{R}_\beta \psi \wedge \alpha \prec \beta) \urcorner.$$

However, it is not what Epstein and Buitrago-Díaz want to achieve. All one needs more to achieve Epstein and Buitrago-Díaz's calculus is to introduce intervals and finite unions of intervals in time continuum. Actually, a lot of work has been done in the field of interval or period talk in tense logic [1]. I am going to show a simple way to reconstruct Epstein and Buitrago-Díaz's ideas in the field of positional calculus, to the effect that the language $\mathbb{F}_{\mathrm{EBD}}$ is a proper part of the existing positional language $\mathbb{F}_{\mathrm{R}}$. Actually, even a weaker language of the calculus MR by Tomasz Jarmużek and Andrzej Pietruszczak [4], enriched with quantifiers, identity and temporal succession, would be sufficient to reconstruct fully the calculus $\mathsf{TL}_{\mathsf{PC}}$, but let me put it aside. Define some obvious auxiliary symbols. Of course,

$$\ulcorner \alpha \preccurlyeq \beta \urcorner \overset{\text{df}}{=} \ulcorner \alpha \prec \beta \vee \alpha = \beta \urcorner.$$

Identify intervals with ordered pairs of instants in the usual way. By means of positional logic symbols define a predicate "$\sqsubset$" and an operator "$\oplus$" to the effect that "$x \sqsubset A$" means the instant x belongs to A being an interval or a finite union of intervals, and $A_1 \oplus A_2 \oplus \cdots \oplus A_n$ is the union of intervals A_1, A_2, ..., A_n:

$$x \sqsubset_1 \langle a_1, b_1\rangle \equiv (a_1 \preccurlyeq x \wedge x \preccurlyeq b_1),$$
$$x \sqsubset_1 (a_1, b_1\rangle \equiv (a_1 \prec x \wedge x \preccurlyeq b_1),$$
$$x \sqsubset_1 \langle a_1, b_1) \equiv (a_1 \preccurlyeq x \wedge x \prec b_1),$$
$$x \sqsubset_1 (a_1, b_1) \equiv (a_1 \prec x \wedge x \prec b_1)$$

and

$$x \sqsubset_{n+1} (A \oplus B) \equiv (x \sqsubset_n A \vee x \sqsubset_1 B),$$

and finally

$$x \sqsubset A \equiv \exists n\colon x \sqsubset_n A.$$

For short, say also that

$$b \prec A \equiv \forall a\colon (a \sqsubset A \to b \prec a),$$
$$A \prec b \equiv \forall a\colon (a \sqsubset A \to a \prec b),$$

for any instants a, b and interval or finite union of intervals A.

Using those symbols, one can define a connective "$\mathcal{R}^\star$", which is to be similar to "$\mathcal{R}$" but range over intervals and finite unions of intervals rather than instants. So, $\ulcorner \mathcal{R}^\star_A \varphi \urcorner$ means that A holds in the period A. Actually, in positional logic there is a number of reliable versions of "$\mathcal{R}^\star$" which Epstein and Buitrago-Díaz's logic is blind to:

$$\ulcorner \mathcal{R}^\star_A \varphi \urcorner \overset{\text{df}}{=} \ulcorner \exists a\colon (a \sqsubset A \wedge \mathcal{R}_a \varphi)\urcorner, \qquad (\mathcal{R}^\star\text{a})$$

$$\ulcorner \mathcal{R}^\star_A \varphi \urcorner \overset{\text{df}}{=} \ulcorner \forall a\colon (a \sqsubset A \to \mathcal{R}_a \varphi)\urcorner \qquad (\mathcal{R}^\star\text{b})$$

or, as Epstein and Buitrago-Díaz seem to prefer,

$$\begin{aligned}\ulcorner \mathcal{R}^\star_A \varphi \urcorner \overset{\text{df}}{=} \ulcorner &\forall a\colon (a \sqsubset A \to \mathcal{R}_a \varphi)\\ &\wedge \forall c\colon (c \prec A \to \exists b\colon (c \preccurlyeq b \wedge b \prec A \wedge \neg\mathcal{R}_b \varphi))\\ &\wedge \forall c\colon (A \prec c \to \exists b\colon (A \prec b \wedge b \preccurlyeq c \wedge \neg\mathcal{R}_b \varphi))\urcorner.\end{aligned} \qquad (\mathcal{R}^\star\text{c})$$

The definition ($\mathcal{R}^\star$c) compared to ($\mathcal{R}^\star$b) forces to attribute formulas with maximal possible intervals. This is supposed to assure the moment of change, which is present in temporal conjunction in the language $\mathbb{F}_{\mathrm{EBD}}$. That is why the definition ($\mathcal{R}^{\star}$c) seems to be closest to Epstein and Buitrago-Díaz's intuitions. However, other versions of the definition, especially ($\mathcal{R}^{\star}$b) are quite reliable and really defendable as well. Introduce also a simple projection symbol "$\mathcal{T}$", reading time references:

$$\mathcal{T}(\mathcal{R}^{\star}_{A}\varphi) = A,$$
$$\mathcal{T}(\neg\varphi) = \mathcal{T}(\varphi),$$
$$\left.\begin{array}{l}\mathcal{T}(\varphi \wedge \psi)\\ \mathcal{T}(\varphi \vee \psi)\\ \mathcal{T}(\varphi \rightarrow \psi)\\ \mathcal{T}(\varphi \equiv \psi)\end{array}\right\} = (\mathcal{T}(\varphi) \oplus \mathcal{T}(\psi)),$$

Now, one is in a position do define all Epstein and Buitrago-Díaz's connectives:

$$\ulcorner\phi \wedge_{\mathsf{bb}} \psi\urcorner \overset{\mathrm{df}}{=} \ulcorner(\varphi \wedge \psi) \wedge (\exists a \sqsubset \mathcal{T}(\varphi)\colon \forall b \sqsubset \mathcal{T}(\psi)\colon a \prec b))\urcorner,$$
$$\ulcorner\phi \wedge_{\mathsf{be}} \psi\urcorner \overset{\mathrm{df}}{=} \ulcorner(\varphi \wedge \psi) \wedge (\exists a \sqsubset \mathcal{T}(\varphi)\colon \exists b \sqsubset \mathcal{T}(\psi)\colon a \prec b))\urcorner,$$
$$\ulcorner\phi \wedge_{\mathsf{eb}} \psi\urcorner \overset{\mathrm{df}}{=} \ulcorner(\varphi \wedge \psi) \wedge (\forall a \sqsubset \mathcal{T}(\varphi)\colon \forall b \sqsubset \mathcal{T}(\psi)\colon a \prec b))\urcorner,$$
$$\ulcorner\phi \wedge_{\mathsf{ee}} \psi\urcorner \overset{\mathrm{df}}{=} \ulcorner(\varphi \wedge \psi) \wedge (\exists a \sqsubset \mathcal{T}(\psi)\colon \forall b \sqsubset \mathcal{T}(\varphi)\colon a \prec b))\urcorner,$$

provided neither "a" nor "b" occur in φ or ψ. It seems obvious that Epstein and Buitrago-Díaz's calculus is simply a proper part of positional logic as well as the first order temporal language. Notice that the traditional calculus is also much mor efficient and much mor adjustable. First, one can easily switch between versions of the connective "$\mathcal{R}^{\star}$", like ($\mathcal{R}^{\star}$a), ($\mathcal{R}^{\star}$b), ($\mathcal{R}^{\star}$c) or similar. It is even more important that time references of formulas are automatically uncovered in positional logic. To achieve this, Epstein and Buitrago-Díaz involve some complicated process of formalization. For example, to formalize the sentence "it was raining and then the sun was shining, and then it was raining" Epstein and Buitrago-Díaz cannot simply use the reflex formula:

$$p \wedge_{\mathsf{eb}} q \wedge_{\mathsf{eb}} p,$$

since the letter "p" involves a time reference, which should be different in the two cases of rain. Instead, Epstein and Buitrago-Díaz involve some semi-formal mediator: "(it was raining)$_1$ and then (the sun was shining), and then (it was raining)$_2$", to be formalized by means of the formula:

$$p \wedge_{\mathsf{eb}} q \wedge_{\mathsf{eb}} r,$$

which is obviously very inefficient, as the connection between similar formulas remains completely covered up. Contrarily, in positional logic one can simply use the formula:

$$\mathcal{R}^{\star}_{A}(p) \wedge_{\mathsf{eb}} \mathcal{R}^{\star}_{B}(q) \wedge_{\mathsf{eb}} \mathcal{R}^{\star}_{\Gamma}(p),$$

which is most clear and efficient. There is much more advantages of existing temporal logic in comparison with Epstein and Buitrago-Díaz's proposal. Hence, all in all, the proposal seems neither new nor better, but certainly much more complicated, than existing calculi, and it is complicated in vain.

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