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CAN CONTRADICTIONS BE ASSERTED?

Abstract. In a universal logic containing naive semantics the semantic antinomies will be provable. Although being provable they are not assertible because of some pragmatic constraints on assertion I will argue for. Furthermore, since it is not acceptable that the thesis of dialethism is a dialethia itself, what it would be according to naive semantics and the prefered logical systems of dialethism, a corresponding restriction on proof theory is necessary.

1. Truth without assertibility?

Strong paraconsistency (dialethism) claims that some contradictions are true. Theories which entail contradictions might be correct. So some contradictions have to be true, since they are provable. Take, for example, the Liar in naive semantics

(λ) λ is false.

by familiar reasoning (valid in at least some paraconsistent logics) we arrive at the conclusions that (λ) is both true and false, its truth being derivable by *reductio* from the assumption of its being false.

Now, something that is true should be assertible by any speaker towards an audience. Furthermore, being provable (λ) fulfills even the strictest condition that a semantics of assertibility could insist on. The semantic battle between truth and assertibility does not apply to (λ). However, there might be another battle to be fought. I will explore some *pragmatic* constraints on assertibility that might be strong enought to make (λ), although provable, not assertible.

2. Is dialethism a dialethia itself?

What about the truth of "A is true" in case A is a dialethia? In a many-valued logic it is possible that statements/sentences concerning the value of other statements/sentences are bivalent: "A is true" is true only if A is true; in case A is undecided "A is true" is simply false (cf. [1, p. 82–84]).

What if A is dialethia? Graham Priest claims that in this case "A is true" is a dialethia itself (see [5, 238ff])!

The difficulty arises because of convention (T), which — neglecting paraconsistently the difference between object and metalanguage — we can state as:

(T) $$A \text{ is true} \iff A$$

By contraposition we arrive at:

(T′) $$\neg A \iff \neg(A \text{ is true})$$

If A is a dialethia we have $0 \in v(A)$ so by the definition of negation[1] $1 \in v(\neg A)$. And with Modus Ponens applied to (T′) we get:

$$1 \in v(\neg(A \text{ is true}))$$

and once again by the definition of negation:

$$0 \in v(A \text{ is true}))$$

Furthermore it is true that A is true, therefore:

$$0 \in v(A \text{ is true}) \text{ and } 1 \in v(A \text{ is true}).$$

"A is true" now is a dialethia itself. We would have the following truth table:

A	A is true
1	1
0	0
0, 1	0, 1

and similiar for "A is false".

[1] Here and in the following remarks I mean negation in the sytem LP [5], being an extension of classical negation.

This, however, would have paradoxical consequences. The main thesis of (strong) paraconsistency is that some contradictions are true. Now, this should have the form "$(\exists A)A$ is true" with respect to at least some contradiction/dialetheia "A". Since "A is true" is according to the truth table a dialethia itself, the thesis of dialethism will be a dialethia. The main thesis of (strong) paraconsistency would be antinomic! Priest in his "Concluding Self-referential Postscript" accepts this result — even feels happier about it (?). To me this consequence seems absurd.

3. Pragmatic constraints on assertibility

A minimal condition on a philosophical thesis should be that it claims to be true only. Otherwise the triviality which should be avoided in case of theories containing contradictions would reappear. The reason is: an antinomy asserts nothing (in a pragmatic sense of "assertion" to be specified). There might be statements which are true and false at the same time, but there can be no reason to assert them in a theoretical debate (i.e, reasons besides training one's vocal chords or being on stage etc.), since nothing is excluded by claiming them to be true. No possible state of the world (no piece of information) is rejected because of their assertion. The act of asserting something can only fail with respect to a dialethia, since the felicity conditions of assertion contain, at least: (i) that we claim something to be the case by citing reasons or giving a justification and (ii) see this information as worthy of being uttered, because the acceptance of this information would make a difference in the following discourse or action. But in case of a dialethia this only can misfire. The aim of assertion is truth, and nothing but the truth: only the "true only" excludes its opposite and, thereby, commits itself. In asserting something we commit ourselves against some opponent. Usually we do not consider this, since usually we do not argue in antinomic contexts. In non-antinomic context it is sufficient to show the falsity of the claim of the opponent to infer to the truth of our claim. In the case of a dialethia we are not able to argue for it in this ordinary fashion, since in its case the success or the failure of our argumentation is irrelevant to justify the claim made. The dialethia is validated in any case. Any justification given by us plays only in the hands of the opponent. Any argument that I will present to make the case of a dialithia "A" will be an argument for "$\neg A$" (i.e. the claim of my opponent). Therefore, we cannot assert contradictions/dialethias with the knowledge of them being contradictory, since then asserting them is irrelevant or not preferable to the assertion of their negation. Contradictions and dialethias, at least if we know them to be such things, violate the conversation maxim of relevance. We will be unable to commit ourselves.[2]

Considering such pragmatic constraints asserting something is more than having a true belief accompanied by some justification. Because of these pragmatic constraints the action of asserting something fails in case of dialethias or known contradictions. Notwithstanding that we have proved some contradictions we are not able to assert them. Therefore, dialethism as a philosophical position cannot be a dialethia itself on pains of being not assertible, i.e. being no contender in the debate at all.

Even the validity of the law of contradiction in some paraconsistent logics does not commit the adherent of such a system to a dialethic thesis of dialethism itself: from

$$\models (\neg(A \wedge \neg A)) \tag{1}$$

we get by the definition of validity the statement:

$$\mathsf{T}(\neg(A \wedge \neg A)) \tag{2}$$

From this we get by the definition of negation:

$$\mathsf{F}(A \wedge \neg A) \tag{3}$$

The argument of the Aristotelian (who claims that there are no true contradictions) should continue:

$$\neg\mathsf{T}(A \wedge \neg A) \tag{4}$$

and

[2] It will not help here to take the informational content of a sentence to be not those worlds or sentences it excludes but the information "it carries", i.e. the sentences it implies (cf. [7, p. 118]), since a dialethia *by its very definition* and by the usual reasoning as applied, for example to the Liar, implies its own negation, and, therefore, implies everything its negation implies. A dialethia and its negation have the same content even in the light of a criterion of "positive information value", like Priest's. A dialethia implies ist own negation, and so, by transitivity, all that the negation implies, and vice versa: the share their set of consequences. Since Priest sometimes defends dialethism by denying that a rule of reasoning just employed by his critics is not valid in "the paraconsistent logic]" — which seems to mean different logics on different occasions of this kind of argument — , it should be mentioned that on some occasions he takes validity of Modus Ponens and transitivity as conditions on any logic of a conditional.

$$(5) \qquad \forall A(\neg \mathsf{T}(A \wedge \neg A))$$

since A has been chosen arbitrarily. So by the duality of quantifiers:

$$(6) \qquad \neg(\exists A)\mathsf{T}(A \wedge \neg A)$$

which could be considered to be the thesis of the Aristotelian. The step from (3) to (4), however, is not paraconsistently valid, since one cannot conclude paraconsistently from the falsity of a statement that it is not true (as well). The negation of the thesis of the dialethist (i.e., (6)), therefore, cannot be derived from the acceptance of the law of excluded contradictions.

In my opinion the dialethist should say: antinomies are true, but they are not assertible. Concerning an operator or predicate of assertion "ASSERT" it would not be valid:

$$(*) \qquad \text{ASSERT}(\mathsf{T}(A)) \supset \text{ASSERT}(A)$$

Even if we can assert the truth of a dialethia A (since, for example, we have a proof of A), this should not imply that we can assert A itself. The reason for this — given above — is that antinomies have no cognitive content. For metalogical reasons I might (should) be motivated to be a dialethist and to assert that some contradictions are true, but I have no inclination to assert any antinomy simpliciter. I believe that the Liar is true, but I am not the Liar myself, I hope. What would I assert by it?[3]

4. Proof-theoretical adjustments

An argumentation like the one of §3 could point to a pragmatic solution of the problem posed by Priest's "self-referential postscript". On the other hand we have the problem that our proof-theory and our semantics result in the thesis of dialethism being a dialethia itself. If we adopt the principle that dialethias should not be asserted that would make it unassertible. Since we want and have to assert it we have to adjust our proof-theory. The argumentation

[3] The felicity conditions of assertions have been investigated, of course, in the theory of speech acts (cf. [3, §3]; cf. also Priest's "teleological" conception of truth [7, p. 77ff]). Sainsbury bases his rejection of paraconsistency partly on the non-assertibility of contradictions (cf. [9, chap. 6]). To repeat: I consider it possible that I can happen to believe a contradiction or, may be, even a dialethia (cf. Priest in [7, p. 119ff]), but I deny that I can assert a contradiction (disagreeing with Priest, [7, p. 117ff]), since I take assertion to be more closely tied to relevant linguistic and practical interaction than mere belief.

in §3 saves us from asserting dialethias which are, nevertheless, true. To *assert* dialethism we have to ensure that dialethism is no dialethia at all.

How could we block the derivation in §2 if we consider convention (T) as beyond doubt? Modus Ponens seems even more beyond doubt.[4] The truth conditions of negation cannot be altered without being deviant with respect to extensional operators (i.e., being deviant on to large a scale). The cullprit has to be contraposition.

Contraposition in convention (T) gives us that $\mathsf{T}(A)$ with respect to an antinomy A is true as well as false. This holds independently of "$\mathsf{T}(\ )$" being an operator or a predicate.[5]

Without contraposition in convention (T) we could infer nothing from the falsehood of A (i.e. the truth of $\neg A$) concerning the truth value of 'A is true". And this is how it should be in a paraconsistent semantics, since the truth values "true" and "false" should be independent enough from each other to allow for paraconsistent evaluations.

What about contraposition in the proof-theory of strong paraconsistency?

> In **LP semantics** contraposition is valid as a consequence relation, and can be derived in Bloesch's Tableau Proof System [2].

PROOF.

1.	$\mathsf{T}(A \supset B)$				
2.	$\mathsf{T}'(\neg B \supset \neg A)$				
3.	$\mathsf{F}'(\neg B)$	$\mathsf{T}' \supset$,2			
4.	$T'(\neg A)$	$\mathsf{T}' \supset$,2			
5.	$\mathsf{T}'B$	$\mathsf{F}'\neg$,3			
6.	$\mathsf{F}'A$	$\mathsf{T}'\neg$,4			
7a.	$\mathsf{F}A$	$\mathsf{T} \supset$,1	7b.	$\mathsf{T}B$	$\mathsf{T} \supset$,1
8a.	#	7a,6	8b.	#	7b,5

□

Both branches are closed, the consequence relation, therefore, is **LP**-valid. The corresponding conditional is valid in **LP** anyway, since **LP** in an

[4] Modus Ponens fails in Priestd's Logic **LP** [5]. That should be a good reason to look for another system, since Modus Ponens is surely our most entrenched intuition concerning a conditional. I consider here a second system **SKP**. **SKP** is a sequential calculus, based on the calculus given by Priest in [6], supplied with quantification, identity and the modal semantics of entailment given in [7]; for **SKP** cf. [4, chap. 4.2].

[5] "$v(A, 1)$" on the other hand expresses that a statement is related by *its name* to an evaluation function. Into "$v(\ .\ ')$" we are not allowed to substitute using conventions of type (T), since this would be substitution into quotation.

extension of classical propositional logic. It is difficult to see how to block the validity of contraposition in **LP**, but, in my opinion at least, because of its invalidation of Modus Ponens **LP** is doomed anyway.[6]

What about our second system **SKP**? Contraposition is a rule of this calculus, (R4). Why is this rule sound? It is sound because of a corresponding stipulation in the semantics of **SKP**.[7] The truth condition of entailment (symbolized "⇒") in **SKP** is

(S⇒)

$$\begin{aligned} 1 \in v(w, A \Rightarrow B) \leftrightarrow\ & (\forall w' \in W)(w'Rw \ \&\ 1 \in v(w', A) \rightarrow 1 \in v(w', B)) \ \&\\ & (\forall w' \in W)(w'Rw \ \&\ 0 \in v(w', B) \rightarrow 0 \in v(w', A)),\\ 0 \in v(w, A \Rightarrow B) \leftrightarrow\ & (\exists w' \in W)(w'Rw \ \&\ 1 \in v(w', A) \ \&\ 0 \in v(w', B)\,. \end{aligned}$$

The second conjunct in clause (i) guarantees the validity of contraposition. If we drop it we can construct a countermodel:

$$\begin{array}{ccc} @ & \Longrightarrow & w \\ A & & A \\ B & & B \\ & & \neg B \end{array}$$

The actual world @ sees only itself and the inconsistent world w. Such worlds are not only allowed here, but are needed as models of dialethias — in fact, if some contradictions are provable, all worlds are inconsistent. Here w is inconsistent with respect to B. In this model $A \Rightarrow B$ is true, since all A worlds which are accessible from @ are B worlds. However, there is a $\neg B$ world which is not at the same time a $\neg A$ world. So $(\neg B \Rightarrow \neg A)$ is not true for @, but false. In the modfied (S⇒) contraposition is therefore *only* false for @, so it is not valid in the modified semantics. The solution to our problem, therefore, seems to be to drop (R4) in **SKP** and to modify its semantics of entailment accordingly. Since as paraconsistent logicians we

[6] Priest says on one occasion ([8, p. 255]) that he now rejects the contraposition of convention (T) as employed in [5]. In [7] this is mentioned merely as being an *option*. In some recent articles, however, Priest refutes critics by appealing to the invalidity of their reasoning in the logic **LP**, which also is said to be *the* logic of paraconsistency in [8]. Now, Priest could believe both these things consistently, if in convention (T) we have a biconditional different from the **LP**-biconditional, but then we should know more about the logic of *this* biconditonal.

[7] Cf. Note 6 concerning **SKP**. For the following semantics of entailment and the models build around the actual world @, cf. [7].

cannot presuppose contrapostion for the quasi-metalinguistically used "→" either, we arrive at a more complex truth condition:
(S⇒′)

$$1 \in v(w, A \Rightarrow B) \rightarrow (\forall w' \in W)(w'Rw \;\&\; 1 \in v(w', A) \rightarrow 1 \in v(w', B)),$$

$$1 \notin v(w, A \Rightarrow B) \rightarrow \neg(\forall w' \in W)(w'Rw \;\&\; 1 \in v(w', A) \rightarrow 1 \in v(w', B)),$$

$$(\forall w' \in W)(w'Rw \;\&\; 1 \in v(w', A) \rightarrow 1 \in v(w', B)) \rightarrow 1 \in v(w, A \Rightarrow B),$$

$$\neg(\forall w' \in W)(w'Rw \;\&\; 1 \in v(w', A) \rightarrow 1 \in v(w', B)) \rightarrow 1 \notin v(w, A \Rightarrow B),$$

$$0 \in v(w, A \Rightarrow B) \rightarrow (\exists w' \in W)(w'Rw \;\&\; 1 \in v(w', A) \;\&\; 0 \in v(w', B),$$

$$0 \notin v(w, A \Rightarrow B) \rightarrow \neg(\exists w' \in W)(w'Rw \;\&\; 1 \in v(w', A) \;\&\; 0 \in v(w', B),$$

$$(\exists w' \in W)(w'Rw \;\&\; 1 \in v(w', A) \;\&\; 0 \in v(w', B) \rightarrow 0 \in v(w, A \Rightarrow B),$$

$$\neg(\exists w' \in W)(w'Rw \;\&\; 1 \in v(w', A) \;\&\; 0 \in v(w', B) \rightarrow 0 \notin v(w, A \Rightarrow B).$$

By droping (R4), though, we lose some means of indirect argumentation. The equivalents to classical Negation Introduction and Modus Tollens are no longer valid. Our paraconsistent logic become even more weaker. But not all means of indirect reasoning or reducio are blocked, for example we still have[8]:

(DR2) $A \vDash (\neg A) \;\rightarrow\; \vDash (\neg A)$

PROOF.

1.	$A \vDash (\neg A)$	Assumption
2.	$\neg A \vDash (\neg A)$	Axiom 1
3.	$A \vee \neg A \vDash (\neg A)$	R1, 1, 2
4.	$\emptyset \vDash (A \vee \neg A)$	Axiom 7
5.	$\emptyset \vDash (\neg A)$	R9, 3, 4 □

And:

(DR3) $\vDash A \Rightarrow \neg A \;\rightarrow\; \vDash (\neg A)$

PROOF. (SKPT7) (i.e., $\emptyset \vDash A \Rightarrow A$), R3, Axiom 7, R5. □

In cases where we can established by a non indirect derivation that the truth of A entails the truth of $\neg A$ we have a substitute of Negation Introduction. Reasoning now affords more assumptions concerning entailments, standing in for contrapositions.

[8] For the format of proofs cf. [4, chap. 4.2].

Dropping rule (R4) gives us the calculus **SKP**$^-$, which yields a proper subset of the theorems of **SKP**. The metalogical results of soundness and non-triviality arrived for the latter are passed on.

One remaining alternative to giving up contraposition would be to claim (cf. [7, p. 109]) that the conditional in convention (T) is no entailment (i.e, is not the conditional of our paraconsistent calculus). In this case we would have to investigate the logic and semantic of still another conditional (besides paraconsistent entailment and the sequence relation "$\models$"). The whole approach seems to be *ad hoc*.[9]

5. Some afterthoughts about the Liar

Suppose you accept my proposal. Dialethist can claim with a clear conscience that some contradictions are true, but they will not assert any antinomy simpliciter. This seems to work well with respect to a lot of antinomies. I might even assert that the Liar is true, since that is what we prove in naive semantics. But what about the corresponding claim that the Liar is false? If I claim that the Liar is a dialethia I claim that it is true and false. By conjunction elimination I should arrive at

(1) The Liar is false.

Now, this sentence speaks about the Liar, and it says just what the Liar says, i.e.

(2) $$(1) \iff (\lambda)$$

where

(λ) λ is false.

We seem to face a dilemma: either we assert a dialethia itself, what we should not do according to my argumentation in §3, or if we block the procedure just outlined we face a problem of expressibility: some semantic fact about the Liar (its being false) would become ineffable. And ineffability of semantic facts is unacceptable since it bereaves dialethism of its main argument against hierarchy conceptions of semantics which are ineffable themselves.

[9] Priest refers on the page mentioned to Stalnakers conditional (of counterfactuals). Stalnakers conditional, however, is tied not only to a similarity relation on possible worlds, which has no place in our semantics yet, but also to further restrictions on logic. Transitivity, for example, which holds for entailment, fails for Stalnakers conditional; cf. [10].

Two ad hoc solutions could be: either to weaken the ban on asserting dialethias (allowing some exceptions to a default rule) or to invent some other description of the Liar. If the Liar is the dialethia paraconsistent logicians are most concerned with I could assert

(3) "The dialethia that paraconsistent logicians are most concerned with" is false.

If we treat "is false" as a predicate we are still allowed to substitute different *names* of an expression for each other (i.e. substitute λ for "the dialethia that paraconsistent logicians are most concerned with"), but if we now assert of (3) that it is false

(4) "The dialethia that paraconsistent logicians are most concerned with is false" is true.

We have inside the scope of the truth predicate a name of (3) and into this we cannot substitute λ for "the dialethia that paraconsistent logicians are most concerned with" (this being substitution into quotation marks). This blocks

$$(*) \qquad (4) \iff (\lambda)$$

So we can assert that it is true that the Liar is false and, therefore, that the Liar is false, i.e. we could assert our semantic fact concerning the falsity of the Liar. So some assertions about dialethias are dialethias themselves, like (1) is. Some are not, like (4). And that is all we need for consistently asserting the thesis of dialethism. If only we can express the thesis of dialethism in some way — and in some way with respect to any single dialethia like the Liar — Priest's problem of self-application is solved.

Since we already have seen that the thesis of the Aristotelian cannot be proved, we are saved. Surely more needs to be said about this.

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