Logic and Logical Philosophy
Volume 2 (1994), 55–68

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ON CAUSALITY

Ingarden's Analysis vs. Jaśkowski's Logic

CONTENTS

Received September 20, 1994

1. Introduction

Considering the growing need for formal counterparts of causal nexus (AI is desperately looking for a good one!) and thus trying to construct appropriate relations within a formal framework one faces the problem that the notion of "causal connection" is by no means explained with sufficient precision. How to overcome the resulting difficulties? There are at least three ways traced in the literature:

1. on the basis of more or less clearly formulated intuitions one defines formal functors and calles them "causal functors", "causal connectives", or the like; subsequently however, these nominal definitions should be justified (by showing their adequacy to notions of causality functioning in real language), since otherwise they are not justified at all;

2. starting from the use of causal terminology in some specified realm of natural language (say, in a given empirical science) one constructs manifolds of connectives which formal properties vary to some extend, thus covering all possible intentions of the causal notions as they are used in the considered context,

   hence, the constructed manifold shall contain all the metamathematical counterparts appropriate as formalizations of causal nexus in the considered realm and one has to figure them out subsequently;

3. starting from well-founded ontological assumptions concerning the real world, one designs all possible (i.e. consistent with the ontological settings about the structure of the world) kinds of causal connections and distinguishes then the cases of practical relevance, i.e. the kinds of causal nexus to be found in the real world.

All positions have their advocates in causal logic or artificial intelligence, and each of them join distinguished representatives of Polish philosophy: Jan Łukasiewicz was one of the very first authors who contributed to the "nominal account" (cf. [6]). The second group could be referred to as "mainstream considerations". One of its pioneers was Stanisław Jaśkowski ([4]). The third variant, for reasons which shall become clear in the next section, is somewhat underrepresentated in the literature. It could be called "formalontological" causal analysis. One of the leading figures in such an approach to causal reasoning is Roman Ingarden, the author of the most subtle analysis *"The Controversy on the Existence of the World"* ([3]).

2. Ingarden's Ontology of Causality

The latest volume of Ingarden's *opus magnum* on ontology, unfinished by the author, as a whole deals with the causal structure of the real world. As Roman Ingarden puts it

> The aim of formal ontology is nothing but to give an overview on these possibilities [of causal structures of the world — M. U.] The number of possibilities established purely formally can be reduced only by taking into account material ontology, and only then metaphysics or natural sciences might decide which one of these various cases is indeed realized. ([3], 390)

The starting point of his analysis is the following "rough definition" of causal nexus:

> Precisely, a causal connection between some given P and S takes place then and only then, if:
>
> 1. P and S differs from one another individually;
> 2. it is true that P implies S, but S does not imply P in the same way;
> 3. both P and S have the form of events or processes (possibly of phases of processes);
> 4. S takes place simultaneously with the occurrence of P;
> 5. both P and S are real (actual). ([2], 95)

The above explication is one of the rare attempts to be found in the literature what tries to give a complete and universal description of causal nexus. Nevertheless, according to Ingarden, this definition is seriously defective: it doesn't specify the kind of S's (the "effect's") being conditioned by P (the "cause"). Subsequently, this gap is bridged in what he meant to be comments on the definition. (In fact, these comments grew up into a whole additional volume of the "Controversy".)

Ingarden starts with a classification of events. He takes an event to be the "coming into existence of a state of affairs". He differentiates types of events by various criterias. The most interesting criterion for our purposes is the classification of events according to the realms of their causes and effects. Given a specific event $E(t)$, among the possible events (i.e. which take place in the same world) we take as its possible causes all the possible events which are not later then $E(t)$ and which are no simultaneous effects. On the other hand, possible effects are all non-earlier possible events not being simultaneous causes. (The notion of *world* refers to a strongly isolated system. The centerpoint of Ingardens analyses is however the concept of a relatively isolated system — or of relatively izolated systems. Those systems remains in an equilibrium for some time and then they are forced by a certain cause to switch into another state. For that reason there has to be a second system to which belongs the force entering the first one. Both systems are separated by an "isolator" which delimits them for a time but then gets strained at some moment.)

The *realm of causes* (RC) is explained to contain all possible causes of an event and respectively is the *realm of effects* (RE). What is more, both realm possess two *contents*, namely the inner content as well as the outer one. The inner content of the RC of some event $E(t)$ consists in all possible causes inside the object existing in time on which $E(t)$ takes place. Correspondingly, the outer content is formed by the possible causes taking place in the rest of the world.

By M, n, and o Ingarden denotes the fact that a given realm or content is maximal (i.e. containing all the possible events which are possible for it to contain at all), partial (containing some, but not all), and empty, respectively. Every event is then characterized by four contents: the outer RC and RE and the inner RC and RE. Thereby we get $3^4 = 81$ types of events. (Scrutinizing subsequently the possibility of the occurrence of particular types of events in the real world, Ingarden arrives at theses concerning the causal structure of the world, although in general they do not concern properties of individual causal connections.)

Let $E(t)$ be an event taking place in the world at t. An additional upper index k points out that $E^k(t)$ is an individual event. The characteristics of the four contents are added as lower indexes where the outer contents of RC and RE, standing first, are separated by a slash from the inner ones. For instance,

$$E^k(t)_{Mn/Mo}$$

denotes an individual event taking place at t with maximal outer realm of causes, partial outer realm of effects, maximal inner realm of causes and empty inner realm of effects. In other words, we are faced with an event causally depending from everything which do no changes at all inside the object on which it takes place and partial changes in the rest of the world.

At the moment it is not clear at all, whether all 81 types of events theoretically available are possible in the real world, i.e. whether they are consistent with the previously established postulates about the causal nexus and with the assumptions about the form of the world. In particular, it seems to be problematic to assume, all events taking place in the world are of one and the same type.

First of all, Ingarden excludes (according to his intuitions and to the postulates resulting therefrom) all causeless events. There are 9 event-types among the 81 for which both contents of RC are empty:

$$\begin{array}{lll} E_{oM/oM}, & E_{oM/on}, & E_{oM/oo} \\ E_{on/oM}, & E_{on/on}, & E_{on/oo} \\ E_{oo/oM}, & E_{oo/on}, & E_{oo/oo} \end{array}$$

Additionally, there shall be excluded (without an explanation as evident as the one before) all event-types with both outer contents empty. Thereby the number of cases is reduced by 15 to 66. All the remaining event-types are to be analyzed. That means, one has to investigate the consistency of their postulated existence with the assumptions made on the form of causal nexus and of the world.

One can easily imagine that this means quite a lot of work. And what is more, Ingarden quickly runs into big trouble when scrutinizing causal connection's internal temporal dependencies. And last not least: the outcome of all these efforts is not very impressive. I mean, the intellectual achievement is doubtlessly most admirable. However, the impact of the result is low: He gets a handful of types of events which are consistent with his ontological postulates on the structure of the real world, and consequently some variety of possible causal structures which may occur in the world. Keeping in mind his attitude to formal ontology, the real existence of such event-types and types of causal connections can be shown only within the "positive sciences". So what we get is at the utmost some hints for searching for real causality.

3. Jaśkowski-systems

3.1. Jaśkowski's original construction

We leave Ingarden for a while and try to follow another way. One of the great Polish logicians, Stanisław Jaśkowski, gave a formalization of Ingarden's concept of cause as an "aside product" of his own considerations on causal logic. Stanisław Jaśkowski's investigations on causal logic were stimulated by one of his teachers, Stanisław Leśniewski, who asked for the representability of causal connectives within an extensional framework. (In order to solve this problem Jaśkowski was constrained to assume a rather exotic concept of extensionality.)

Jaśkowski defined various causal connectives at the basis of two non--classical systems: Q_f and Q^*. The former one is called "calculus of factors" whilst the latter one is called "calculus of chronological factor succession". The crucial point in each of the definitions is the use of so-called "dependent functorial variables". These objects were introduced into the literature by Heyting (cf. [1]). Jaśkowski gave the following explanation:

> Suppose that the truth of a sentence $\mathfrak{P}$ depends on certain factors which cannot be determined strictly: for instance, a person is to toss a coin, and the sentence $\mathfrak{P}$ means "during the game heads will turn up more times than tails will".
>
> For a certain sequence of random events the sentence $\mathfrak{P}$ will prove true, whereas for some other sequence it will prove false. Thus the sentence $\mathfrak{P}$ may be assumed to be a function that takes on the values: truth and falsehood, according to the values of the variables that stand for the random events. Since the functional relationship is not revealed by the notation, a sentence of this kind may be represented by the dependent sentential variables introduced by Heyting [...], in a way similar to that in which in mathematics the functions of the variable x are often represented by the letter y. ([5], 148)

In a first step Jaśkowski explains, for purely technical reasons, an intermediate calculus Q, which language contains independent sentential variables (i.e. individual variables) and dependent sentential variables (i.e. quasi-functional variables with unfixed number of arguments). The set FOR_Q of formulae of Q contains nothing but the set of all dependent variables $p, q, r, \ldots$ and the following chains of signs $\neg H$, $H \wedge G$, $\forall x : H$ (where x is an independent variable) only if it contains H and G. Subsequently the class of Q-tautologies is established by means of a translation t from FOR_Q to FOR_1, the language of first-order predicate calculus PC_1:

Let $H \in FOR_Q$ and let's assume, that H contains exactly all independent variables $x_1, \ldots, x_n$.

$$t(H) \stackrel{\text{df}}{=} H[p/P(x_1, \ldots, x_n), \ldots, q/Q(x_1, \ldots, x_n), \ldots]$$

As usual one can define a logical calculus Q as $Q \stackrel{\text{df}}{=} \langle FOR_Q, Cn_Q \rangle$ with a consequence relation $Cn_Q : 2^{FOR_Q} \longrightarrow 2^{FOR_Q}$

$$H \in Cn_Q(X) \divideontimes \forall F \in X\ (t(F) \in PC_1 \ \Rightarrow\ t(H) \in PC_1)$$

or, as it is stated in Jaśkowski's original paper [4] as a set of tautologies $Q = Cn_Q(\emptyset)$.

The next step is the definition of the causal system Q_f. The set of formulae of Q_f is generated as usual by a denumerable set of (dependent) variables AT by means of one one-argument connective $\neg$ and two two-argument connectives $\wedge$ and $\Box_f$. Formulae of the form $\Box_f(H)G$ are to be read as "G is true for all values of factors of H". In the calculus Q_f, it is not possible to state precisely, what these "factors of H" are — it can merely be indicated: "factors of H" are those individual variables of dependent variables of H, which assignment determines the value of H. (Although those variables do not occur explicitly in Q_f.) The precise meaning of this concept emerges together with its semantical explanation.

3.2. Multidimensional first-order frames

The construction of Q_f allows a far reaching generalization: in fact, on any regular modal logic one can set up causal calculi in the style of Jaśkowski. Moreover, it is possible to combine this technique with the powerful tool of relational semantics for other non-classical logic. Thereby one obtains, e.g., intuitionistic or discussive causal calculi (cf. [9]). Unfortunately, to demonstrate this it needs a rather complex conceptual machinery. In order to cut down the technical subtleties and to make thereby the presentation more transparent, we confine ourselves to normal modal logic.[1] For any normal system S the class of all of its first-order frames $\mathcal{K}_S$ is adequate for S.

[1] For the general case and for technical details see [9]

Definition 1. *Let $\mathcal{F}_i = \langle W_i, Q_i, R_i, \Pi_i \rangle$; $i \leqslant n$ be $\mathcal{K}_S$-frames. The structure $\mathcal{F}_1 \times \ldots \times \mathcal{F}_n = \langle \mathcal{W}, \mathcal{Q}_1, \ldots, \mathcal{Q}_n, \mathcal{R}_1, \ldots, \mathcal{R}_n, \mathcal{P} \rangle$ is called* ***n-dimensional product*** *of $\mathcal{F}_i$; $i \leqslant n$ $\Leftrightarrow$*

1° $\mathcal{W} = W_1 \times \ldots \times W_n$;

2° $\forall i \leqslant n : \mathcal{R}_i = id_1 \times \ldots \times id_{i-1} \times R_i \times id_{i+1} \times \ldots \times id_n$;

3° $\mathcal{P} = \Pi_1 \times \ldots \times \Pi_n$. $\boxed{DEF}$

The ordered pair $\langle \mathcal{F}^n, v \rangle$ is denoted by $\mathcal{M}^n$ and called *n-dimensional model*, with the valuation $v : AT \longrightarrow \mathcal{P}$. It extends as usual to FOR. Thus we define the *truth of a formula in a point of the model*.

Definition 2. *H* ***is true in the point*** *$\tilde{x}$* ***of the model*** *$\mathcal{M}$ (symbolically: $\mathcal{M}^n \models H[x]$) iff*

1° $\mathcal{M}^n \models p[\tilde{x}] \quad \Leftrightarrow \quad \tilde{x} \in v(p), \quad p \in AT$;
2° $\mathcal{M}^n \models \neg H[\tilde{x}] \quad \Leftrightarrow \quad \mathcal{M}^n \not\models H[\tilde{x}]$;
3° $\mathcal{M}^n \models H \wedge F[\tilde{x}] \quad \Leftrightarrow \quad \mathcal{M}^n \models H[\tilde{x}] \ \wedge \ \mathcal{M}^n \models F[\tilde{x}]$. $\boxed{DEF}$

In order to interpret the case of the non-classical operator $\Box_f$ (which is the only interesting one) some abbreviations are helpful. For all $i \leqslant n$ we use diamonds $\Diamond_i$ and boxes $\Box_i$ thus defined:

$$\mathcal{M}^n \models \Diamond_i H[\tilde{x}] \Leftrightarrow \exists \tilde{y} \in \mathcal{W} : \ \tilde{x} \, \mathcal{R}_i \, \tilde{x} \ \wedge \ \mathcal{M}^n \models H[\tilde{y}]$$

and

$$\Box_i H \stackrel{\text{df}}{=} \neg \Diamond_i \neg H.$$

Let $k \leqslant n$. The symbol $f^n(k, H)$ is defined as $\Diamond_1 \ldots \Diamond_n (\Diamond_k H \wedge \Diamond_i \neg H)$ and to be read as "k has influence on the truth-value of H" or else as "k is a factor of H". For $\kappa = \{k_1, \ldots, k_m\} \subseteq \{1, \ldots, n\}$ let

$$\begin{aligned} f^n(\kappa, H) &\stackrel{\text{df}}{=} f^n(k_1, H) \wedge \ldots \wedge f^n(k_m, H), \\ \Box_\kappa H &\stackrel{\text{df}}{=} \Box_{k_1} H \wedge \ldots \wedge \Box_{k_m} H. \end{aligned}$$

We are now in a position to explain the concept of the "set of factors of a formula in a point of a model": $\kappa \subseteq \{1, \ldots, n\}$ is the *set of factors of H in the point $\tilde{x}$ of the model $\mathcal{M}^n$*, if and only if

$$\mathcal{M}^n \models f^n(\kappa, H)[\tilde{x}].$$

Let H, G, n and κ be as before.

$$\Box^n_{f(H)} G \stackrel{\text{df}}{=} G \wedge \bigwedge_{\kappa \subseteq \{1 \ldots n\}} \Big(f^n(\kappa, H) \rightarrow \Box_\kappa G \Big).$$

The formula $\Box^n_{f(H)}G$ is to be read as "G is true and it is necessary for any set of functors of H".

Definition 3. (continuation of Definition 1.) *Let $H, F \in FOR_f$ and let $\tilde{x} \in \mathcal{W}$. We define* ***the truth of $\Box_{f(F)}H$ in a point $\tilde{x}$ of the model $\mathcal{M}$:***

4° $\mathcal{M}^n \models \Box_{f(H)}G[\tilde{x}] \ast \mathcal{M}^n \models \Box^n_{f(H)}G[\tilde{x}]$.[2] $\boxed{DEF}$

The above definitions make clear what is meant by a formula from FOR_f true in a point of a model. As usual one can now define the truth of a formula in a model, in a frame, and, finally, in a class of frames.

For every normal modal system S and for every natural n the acceptance relation explained in definitions 2 and 3 determines as usual a consequence operation $\mathcal{J}^n_S$ in FOR_f. Their intersection $\mathcal{J}_S(X) \overset{\text{df}}{=} \bigcap_{n \in \omega} \mathcal{J}^n_S(X)$ is obviously a consequence operation, too. Since the well-known modal system of **S5** Lewis is normal, it determines a consequence operation $\mathcal{J}_{\mathbf{S5}}$.

Theorem 1. ([7]) $\mathcal{J}_{\mathbf{S5}}(\emptyset) = Q_f$ $\boxed{THM}$

The construction of the inference relation $\mathcal{J}_S$ is based on a generalization of Jaśkowski's ideas. This fact, together with the result established in theorem 1, motivates us to name the set of formulae $\mathcal{J}_S(\emptyset)$ for any normal $\mathcal{J}$ the *Jaśkowski-system designated by S*.

3.3. Further causal systems

Q as well as Q_f are auxiliary calculi only. The main causal system created by Jaśkowski was his "calculus of the chronological succession of factors" Q^*. We proceed with a reformulation of the original definition (see [4]) in multi-modal terminology and subsequently indicate how to generalize this construction. The language of Q^* arises from L_f by adding two further two-argument non-classical connectives $\Box_e$ and $\Box_d$. In order to interpret FOR^* in points of n-dimensional models we have to explain the interpretations of those additional connectives.

For $k = 1, \ldots, n$ let $c^n(k, H) \overset{\text{df}}{=} \Box_k \ldots \Box_n H \vee \Box_k \ldots \Box_n \neg H$. It is to be read as "the truth of H does not depend on $k, \ldots, n$". The signs $c^n(0, H)$ and $c^n(n+1, H)$ are treated as empty.

[2] The two-argumentarity of $\Box_f$ is thereby rather fictitious: $\Box_f$ is a one-place modal operator indexed by the factors of a further formula.

Furthermore, for $k = 0, \dots, n$ let $e^n(k, H) \overset{\text{df}}{=} \neg c^n(k, H) \wedge c^n(k+1, H)$, what can be read as "the truth of H doesn't depend on $k+1, \dots, n$, it depends however on $k, \dots, n$", or "k is the *efficient factor* of H". The set $k, k+1, \dots, n$ is called the *definitive set* of H.

The following definitions establish the meaning of the concepts "*efficient factor* of H *in the point* $\tilde{x}$ *of the model* $\mathcal{M}$" [$e(H, \mathcal{M}^n, \tilde{x})$, for short] and "*definitive set* of H *in the point* $\tilde{x}$ *of the model* $\mathcal{M}$" [$d(H, \mathcal{M}^n, \tilde{x})$]:

Let $k \leqslant n, \kappa = \{k, \dots, n\}$:

$$e(H, \mathcal{M}^n, \tilde{x}) = k \divideontimes \mathcal{M}^n \models e^n(k, H)[\tilde{x}]$$

as well as

$$d(H, \mathcal{M}^n, \tilde{x}) = \kappa \divideontimes \mathcal{M}^n \models e^n(k, H)[\tilde{x}].$$

Let n be any natural number. Then we set

$$\Box^n_{e(H)} G \overset{\text{df}}{=} \bigwedge_{0 \leqslant k \leqslant n} \Big(e^n(k, H) \rightarrow \Box_k G \Big)$$

and

$$\mathcal{M}^n \models \Box_{e(H)} G[\tilde{x}] \divideontimes \mathcal{M}^n \models \Box^n_{e(H)} G[\tilde{x}].$$

This explication seems to be in good accordance with Jaśkowski's intuitions: $\Box_{e(H)} G$ expresses the truth of G for all values of H's efficient factor — if $e^n(k, H)$ holds in a situation $\tilde{x}$, then G is true in all situations differing from $\tilde{x}$ only by the value of their k^{th} component, i.e. of the efficient factor of H under those circumstances. Furthermore, $\Box_{d(H)} G$ means that G is true for all values of H's definitive set. And finally, for H, G and n be as before, let

$$\Box^n_{d(H)} G \overset{\text{df}}{=} \bigwedge_{1 \leqslant k \leqslant n} \Big(e^n(k, H) \rightarrow \Box_k \dots \Box_n G \Big)$$

and

$$\mathcal{M}^n \models \Box_{d(H)} G[\tilde{x}] \divideontimes \mathcal{M}^n \models \Box^n_{d(H)} G[\tilde{x}].$$

Definition 2, together with the above settings establishes the acceptance of a formula of L^* in a point of a n-dimensional model.

The acceptance relation $\models$ defined in $\mathcal{K}_S^n$, for $n \in \omega$ establishes a consequence operation in FOR^* by

$$\mathfrak{J}_S^n(X) \stackrel{\mathrm{df}}{=} \{ H \in FOR^* \ : \ \mathcal{K}_S^n \models X \Rightarrow \mathcal{K}_S^n \models H \}$$

and

$$\mathfrak{J}_S(X) \stackrel{\mathrm{df}}{=} \bigcap_{n \in \omega} \mathfrak{J}_S^n(X).$$

We call $\mathfrak{J}_S(\emptyset)$ the *general Jaśkowski-system designated by S*. Clearly, by creating Q^* Jaśkowski tried to obtain more adequate counterparts of causal relations than it was possible in the Q_f-system. Thanks to the above generalization, chances for that increase considerably: On the one hand, all the constructions of what he called "causal connectives" can be restated in any general Jaśkowski-system. And on the other hand, the sets of $\mathfrak{J}_{\mathbf{S5}}$-theorems vary for different designating modal systems. Therefore in different general Jaśkowski-systems we obtain slightly varying properties of a given connective, or — if one likes to say so — whole classes of connectives result for each single connective defined in Q^*. Having once defined some causal connective as closely as possible to our intuitions, this allows "fine tuning" of its properties by choosing the appropriate designating system. Hence, by generalizing Jaśkowski's construction, we accomplish a large number of potential formal counterparts of causal relations.

But even when that logical work is done, almost all interesting problems remain open. Logic can make propositions — the choice of a specific causal connective as the formalization of the considered kind of causal nexus is not the business of pure logic alone.

4. Efficient Factors

Let us therefore take one more look on Ingarden's considerations — is there anything to do for causal logic? Recall Jaśkowski's definition of the efficient factor. The efficient factor of a formula H may be understood as the factor finally determining the value of H under the given circumstances. This very much resembles Ingarden's notion of *cause*:

> Wir haben bis jetzt hauptsächlich auf drei Punkte in der Bestimmung des Begriffs der Ursache Nachdruck gelegt: a) auf ihre Gleichzeitigkeit mit ihrer unmittelbaren Wirkung, b) darauf, dass sie nicht die volle

> hinreichende Bedingung ihrer Wirkung, sondern lediglich der letzte in Gestalt eines Ereignisses eintretende Faktor ist, der die bereits bestehenden Tatbestände (die oft so genannten "Umstände") zur vollen aktiven hinreichenden Bedingung der Wirkung ergänzt, und c) daß sie ihre Wirkung hervorbringt, also gewissermassen schöpferisch und jedenfalls aktiv ist. ([3], 76_{12-8})
>
> *Up to now we emphasized mainly three points in the definition of the concept of cause: a) that it is simultaneous with its immediate effect, b) that it is not the full sufficient condition of its effect, but merely the last event, which completes the already existing facts (the often so-called "circumstances") to the full active sufficient condition of the effect, and c) that it brings about its effect, thus being creative to a certain extent and in any case active.*

or, the following characterization:

> Die "Ursache" tritt eben dort und nur dort auf, wo es zu einem Übergang von einem Seinszustand in einen qualitativ anderen, von einem Vorgang in einen anderen, von einem Ereignis in ein anderes kommt. Das Neue und andere, das sich im Vollzug dieses Überganges realisiert, muss seinen "Grund", seine "Ursache" haben. Und diese Ursache ist nichts anderes als ein Ereignis (insbesondere der Abschluss eines Vorgangs), der einen bereits vorhandenen Bestand an zwar unentbehrlichen, aber nicht hinreichenden Bedingungen eines zu bedingenden Ereignisses zu dessen aktiver hinreichender Bedingung ergänzt. Dadurch hört der bisherige Zustand des in der Zeit verharrenden Gegenstandes auf zu sein, und die Ursache ruft zugleich einen neuen Gesamtzustand hervor, in dessen Rahmen sich ihre Wirkung befindet. ([3], 74^{18-24})
>
> *The "cause" appears exactly there where happens a transitions from one state of being into a qualitatively different one, from one process into another, from one event to an other. The new and the different what realizes during this transition must have its "reason", its "cause". And this cause is nothing but an event (particularly the end of a process) which makes the active sufficient condition out from a stock of already existing necessary but insufficient conditions of this event.* [...]

By the way, doesn't that look like a pre-image of the INUS-approach to causality?

Jaśkowski claimed that the efficient factor of a real event *is* its cause in the sense of Roman Ingarden. There are some interesting details in here.

Ingarden's concept of a cause is not subject to the construction of causal connectives in Jaśkowski-systems. It is not even expressible in the language L^* — it does not fit into any syntactical category. Nevertheless, Jaśkowski's remark is interesting; at least with respect to the subsequent construction of generalized Jaśkowski-systems. We will show that multi-dimensional Kripke--structures provide an adequate semantic framework for Q^*. On the other hand, despite of their name, those structures do not contain any temporal elements. Hence it comes out that Ingarden's concept of a cause can be formalized by means of a semantic framework without any temporal relations. Therefore the questions arises, whether the assumption of temporal orderings in the course of events is essential in Ingarden's definition, or whether the temporal ordering could be left out. Keeping in mind Ingarden's enormous difficulties concerning the problems of temporal succession in singular cause-effect-relations (cf. [3], 44 ff.) such a modification would be highly desirable. Jaśkowski's formalization, trying to be adequate to the philosophical prototype, achieves thereby a position from which one may pose relevant and possibly important questions about the original philosophical issue — a nice example of how logic and philosophy may interact.

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