Logic and Logical Philosophy
Volume 23 (2014), 57–68
DOI: 10.12775/LLP.2013.035

Jiří Raclavský

ON THE INTERACTION OF SEMANTICS AND DEDUCTION IN TRANSPARENT INTENSIONAL LOGIC (IS TICHÝ'S LOGIC A LOGIC?)

Abstract. It is sometimes objected that Tichý's logic is not a logic because it underestimates deduction, providing only logical analyses of expressions. I argue that this opinion is wrong. First of all, to detect valid arguments, which are formulated in a language, there needs to be logical analysis to ascertain which semantical entities (Tichý's so-called constructions) are involved. Entailment is defined as an extralinguistic affair relating those constructions. The validity of an argument, composed of propositional constructions, stems from the properties of the constructions. Such properties are displayed by the derivation rules of Tichý's system of deduction.

Keywords: logical analysis; deduction; entailment; Transparent Intensional Logic

1. Introduction

Tichý's logic or, more narrowly, *Transparent intensional logic*, is often thought to be difficult to classify, since it is not a logic in the usual sense of the word. By *logic* one usually understands a certain deduction system (or perhaps a class of them); only sometimes a logic is understood to be a certain analytic method or something similar. If a logic is the latter, there is a doubt, why it is not the former.

Some even say that in Tichý's logic the analytic component is dominant over the deductive component. Sometimes such remarks suggests

Received November 26, 2012. *Revised* October 3, 2013. *Published online* October 5, 2013

that Tichý's logic neglects something very important for any logic.[1] It subsequently seems that research carried out within the framework of Tichy's logic manage to miss out that which makes a logic a logic, namely, deduction: the investigation of what we can validly infer from what.

The aim of this paper is to show that this view of Tichý's logic is misleading. I will argue that Tichý's logic accommodates deduction to a much larger extent than it seems to some at first glance. In other words, Tichý's logic really is a logic.

The core of my justification is in fact simple and generally known. If the aim of logic is to detect valid arguments formulated in a language, it is essential first of all to determine what exactly the sentences of the argument mean. In other words, the *logical analysis* of (or a logical semantics for) language expressions is a necessary requirement for a logic. (Needless to say that this is in accordance with the view that the meanings of sentences, i.e. 'thoughts', are primary, and so entailments among the meanings of sentences are also primary. If this is clarified, and it is also clarified which expressions of this or that language express those 'thoughts', we are also ready to determine valid arguments in their verbal or linguistic formulation.)

It is Tichý's (unpublished) opinion that 'if we know, what we are talking about (i.e. what the meanings of our expressions are), we will also know what entails what' (cited in [9, p. 55]). This opinion is stronger and admittedly more controversial than that of the preceding paragraph. But I will attempt to show that it is very much the point.

In the rest of the paper I will gradually proceed from questions related to logical analysis to questions concerning deduction — '2. Relationship of logical analysis to entailment', '3. Relationship of entailment to deduction'. Some completion will be provided by the sections '4. Derivation systems' and '5. Concluding remarks'.

2. The relationship of logical analysis to entailment

First, I will introduce the basic notions of Tichý's semantics, though without going into details. I will not, for instance, explain how and why exactly Tichý models the meanings of expressions in a particular

[1] I have met such an opinion several times not only in personal communication but even in print. For instance, the supremacy of analysis over deduction in Tichý's logic was expressed by Karel Šebela [8].

hyperintensional way; I will simply recapitulate that it is so. The first part of this section focuses on notions related to logical analysis, the second part on the notion of entailment.

According to Tichý, *extensions* or *intensions* (which are functions from possible worlds w and moments of time t) are *denotata* of expressions. For instance, sentences denote *propositions*, i.e. total or partial intensions having truth-values (T or F) as their functional values.

Any object is constructed by infinitely many non-identical but *equivalent* constructions. *Constructions* are abstract, and also extra-linguistic, structured procedures. They have an algorithmic character; they are not set-theoretical objects (though they usually construct set-theoretical objects). Constructions are specified by *which* objects they construct and *how* they do so. Basic kinds of constructions can be understood as objectual correlates of λ terms. These kinds are: variables (which are of form) x (the corresponding λ terms are variables as letters), trivializations ${}^0X$ ('constants'; X is any object or construction), compositions $[CC_1 \ldots C_n]$ ('applications'; C or C_i is any construction), closures λxC ('λ abstractions'). Constructions always construct an object of a particular type (e.g. of the type of propositions); *cf.* Tichý's theory of types.

Constructions can be aptly considered to be explicantia of language *meanings* which are *expressed* by language expressions. Thus, expressions mean constructions, which are their *logical analyses*.[2] I maintain that semantic notions are relative to a language.[3] Thus, an expression e expresses in a language l the construction c which constructs the denotatum of e in l. The semantic scheme is this (see [5, p. 63]):

expression e
| e expresses in l:
construction c i.e. the meaning of e in language l, the logical analysis of e
| the construction c constructs, the expression e denotes in l:
intension/non-intension i.e. the denotatum of the expression e in l

Tichý's *semantics* is *hyperintensional* in the sense that its individuation of meanings is finer than that of intensional semantics, where the meanings of expressions are simply intensions or extensions. The con-

[2] To provide an explicans of meaning of certain expression in a given language does not amount to providing its translation to some (possibly formal) language, i.e. offering merely another expression; *cf.* [6].

[3] For my explication of language and semantic notions, see [5, ch. IV.5].

temporary literature states many reasons for individuating meanings hyperintensionally.[4]

Logical analyses of sentences are essential for an examination of the validity of arguments made from them. A correct argument formulated in language is such that its conclusion follows from its premises. This is, however, a language-relative affair.

The common definition of entailment (viz. a class of sentences s_1, …, s_n entails a sentence (formula) s iff …) is inadequate. It is so because of an unwarranted ignorance of the fact that expressions, formulas being no exception, are language relative in the sense that they have one meaning in one language ('notation'), while having another meaning (or no meaning at all) in another language. Thus a sentence (formula) can be entailed by some class of sentences (formulas) in one language, yet not in another.

Entailment between sentences is dependent on entailment between entities which are meant by those sentences in this or that language. Therefore, a definition of language-entailment should rather be

> a class of sentences s_1, …, s_n *entails in* language l a sentence s iff what (viz. a class of constructions) is expressed by s_1, …, s_n in l entails* what (viz. a construction) is expressed by s in l.
>
> [5, p. 264]

And, of course,

> a language argument a is *valid in* language l iff (the class of) a's premises s_1, …, s_n entail, in l, a's conclusion s.

The definition just given employs the concept of entailment* between constructions, which calls for clarification. Since entailment is a topic for a specialized study, here I can only list the most essential features.

We could firstly define entailment between propositions and entailment between propositional constructions will be defined as dependent on it. Though I omit this here, there will still remain a link to the former notion thanks to the truth of constructions which is dependent on the truth of propositions:

> a construction c^k is *true* in* w at t iff there exists a truth-value o such that o is the value of that which (viz. a proposition) is

[4] For this and related topics see especially the first chapter of [5] or [1].

constructed by c^k in w at t and o is identical with the truth-value T. *cf.* [5, pp. 348–351]

Thus

a class of constructions $c_1^k, \ldots, c_n^k$ *entails** a construction c^k iff for every w and t it holds that if $c_1^k, \ldots, c_n^k$ are true* in w at t, then c^k is also true* in w at t. [5, p. 160]

To conclude: logical analyses (i.e. constructions) of expressions are closely and indubitably related to entailment; *if we know what sentences mean (i.e. which constructions) we are able to determine what entails what.* Now there is a question — how does this relate to deduction?

3. The relationship of entailment to deduction

Tichý's deduction system[5] is not very well known.

I will introduce here, though in a simplified manner, its basic notions. I will show then how entailment relates to deduction. (Both entailment and deduction are understood in an objectual sense; the way to proceed from the objectual level to the linguistic level was shown in the preceding section.) A *match* M is a pair

$$X : C,$$

where C is a construction and X is the trivialization of an object of type ξ or a variable ranging over objects of type ξ. We will say that a match is *satisfied* by a valuation, which means that the construction C constructs on that valuation the very same object as the construction X. A *sequent*

$$\Phi \Rightarrow M$$

has two members; Φ is a class of matches and M is a match. A sequent is *valid* if every valuation which satisfies all the members of Φ also satisfies M. A *derivation rule* (in Tichý's earlier terminology, an *inferential rule*), is a validity-preserving operation on sequents.

It is of the form

$$\Phi_1 \Rightarrow M_1; \Phi_2 \Rightarrow M_2; \ldots; \Phi_n \Rightarrow M_n \models \Phi \Rightarrow M.$$

[5] *Cf.* especially [11], which condenses the material covering deduction from [10]. An elaboration of substitutability of variables can be found in [12]. Material relevant to deduction can be found also in [3].

Its final sequent $\Phi \Rightarrow M$ is valid when all the sequents $\Phi_1 \Rightarrow M_1$, $\Phi_2 \Rightarrow M_2$, …, $\Phi_n \Rightarrow M_n$ are valid.

Let me explain what happens here. Matches can be understood as certain identity statements — a particular match thus puts the identity relation between an object O and the result of constructing of certain construction C, which is that object O.[6] Now let p or p_i be a variable for propositions, C^π or C_i^π be a construction of a proposition. Sequents such as

$$\{p_1 \colon C_1^\pi, p_2 \colon C_2^\pi, \ldots, p_n \colon C_n^\pi\} \Rightarrow p \colon C^\pi$$

can be *construed* as certain implications holding between the conjunctive connection of these constructions (matches) and the final propositional construction (match).[7] Arguments understood in an objectual way (i.e. not their verbal, linguistic formulations) can be *represented* just by the sequents of form $\{p_1 \colon C_1^\pi, p_2 \colon C_2^\pi, \ldots, p_n \colon C_n^\pi\} \Rightarrow p \colon C^\pi$. If the sequent is in fact a logically true implication, we can *view* a valid argument as a rule

$$\models \{p_1 \colon C_1^\pi, p_2 \colon C_2^\pi, \ldots, p_n \colon C_n^\pi\} \Rightarrow p \colon C^\pi.$$

(From a more common viewpoint, the validity of the argument is *measured* by that rule.)

Thus, properties of propositional constructions (or: which properties are possessed by the propositional constructions) determine the entailment (which classes of constructions entail* which constructions), i.e. determine the transfer of validity by means of the respective rule of derivation.[8] So this is how I understand the relationship of deduction and entailment.

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[6] I mean constructions of the form $[{}^0O\,{}^0\!=[{}^0\Gamma^\xi C]]$ (alternatively: $\lambda w \lambda t[{}^0O\,{}^0\!=[{}^0\Gamma^\xi C]]$), where ${}^0\Gamma^\xi$ constructs the partial function which maps constructions to the ξ-objects (if any) constructed by them (remark: a use of so-called double execution, *cf.* [13], would be more appropriate here). Trivializations of well-known logical functions are written in an infix way.

[7] Since implication (material conditional) operates on truth-values, we have to find a way from construction-matches to propositions, or rather their values. It is not technically complicated to implement it but it is omitted here.

[8] Notice also that sequents or derivations concerning propositional constructions (*cf.* the above example) are only a special case of what can be treated by Tichý's system of deduction. In Tichý's system one can also work with sequents concerning, e.g., classes of numbers, etc. Tichý thus substantially expands the field for deduction (already noticed by Jan Štěpán in [2, p. 106]).

(For completeness, let us also introduce Tichý's construal of derivability. A sequent is *derivable* from a class of sequents *according to* a *derivation rule*. A finite sequence of sequents is called a *derivation with respect to class R of derivation rules*, written $\vdash_R \Phi \Rightarrow M$ if every item of that sequence, i.e. a *derivation step*, is derivable from the preceding steps according to some derivation rule from R. Note that this guarantees the relevance of that movement on the derived step. Observe also that Tichý construed *inference* as a sequence of (valid) arguments, i.e. a sequence of logical truths. Tichý criticized so-called inferences from assumptions, which are not logical truths; *cf.* [13, chapter Inference] or [15]).

Let us have a closer look at an important property of certain derivation rules. Bi-directional derivation rules

$$\models x\colon C_1 \Leftrightarrow x\colon C_2$$

(which is a shortcut for $\models \{x\colon C_1\} \Rightarrow x\colon C_2$ and $\models \{x\colon C_2\} \Rightarrow x\colon C_1$) elucidate which object is constructed by the construction C_1 (and C_2). Thus, they also elucidate which particular construction it is.

In some cases, one of the two constructions C_1 and C_2 is significantly simple. Consider the 'transformation of disjunction to implication' as an example

$$\models f\colon \lambda o_1 o_2[o_1 \ {}^0\!\vee\ o_2] \Leftrightarrow f\colon \lambda o_1 o_2[[{}^0\neg\, o_1]\ {}^0\!\rightarrow o_2],$$

(where f is a variable for binary truth-function, o_i a variable for truth-values).[9] I consider this sort of rule to be a *definition* as it satisfies many intuitions concerning definitions ([5], 287–290). The definition says that the equivalent of $\lambda o_1 o_2[o_1 \ {}^0\!\vee\ o_2]$, i.e. ${}^0\!\vee$, is the construction $\lambda o_1 o_2[[{}^0\neg\, o_1]\ {}^0\!\rightarrow\ o_2]$; in other words, the definition elucidates which object, which truth-function, is constructed by the construction ${}^0\!\vee$. The definition does not 'create' a 'new' construction ${}^0\!\vee$; the construction ${}^0\!\vee$ is already there before the definition. The definition only makes clear which object is constructed by ${}^0\!\vee$ and in this way it makes also clear how ${}^0\!\vee$ relates to the (equivalent) construction $\lambda o_1 o_2[[{}^0\neg\, o_1]\ {}^0\!\rightarrow o_2]$. To put the point a bit differently, the validity of the sequent (definition) is given by which particular objects are constructed by ${}^0\!\vee$, ${}^0\neg$, ${}^0\!\rightarrow$.

So-called inferential semantics (and also the theory of implicit definition or of 'defining' theory) is based on the intuition that the meaning

[9] The construction $\lambda o_1 o_2[o_1 \ {}^0\!\vee\ o_2]$ is η-reducible to ${}^0\!\vee$. Thanks to another rule, the definition can be stated in the form $\models o\colon [o_1 \ {}^0\!\vee\ o_2] \Leftrightarrow o\colon [[{}^0\neg\, o_1]\ {}^0\!\rightarrow o_2]$.

of an unknown but just introduced operator will be set by showing its inferential relations. As A. N. Prior remarked, during his discussion of the operator 'tonk', the meaning of an operator was already there before its introduction; the inferential rules only show exactly what meaning it has. Tichý would surely subscribe to such a view as it is fully in the spirit of his approach. Setting now definitions aside, there are many other derivation rules which show (practically manifest) properties of objects. For instance, one of the properties of implication — viz. that it returns the truth-value T for $\langle \mathrm{T}, \mathrm{T} \rangle$ — is exhibited by the rule

$$\Phi \cup \{{}^0\mathrm{T}\colon o_1\} \Rightarrow {}^0\mathrm{T}\colon o_2 \models \Phi \Rightarrow {}^0\mathrm{T}\colon [o_1\ {}^0\!\!\to o_2].$$ [10]

Let us summarize what has appeared repeatedly in this section: *if we know what the meaning of a certain expression is, i.e. which construction it is (so we also know which object is constructed by it), we are also able to determine valid arguments, which can be understood as valid inference rules.*

4. Derivation systems

Still, some might object even now that Tichý's logic is not a logic in the proper sense, referring here to calculi, completeness and similar things. In order to consider this matter from the viewpoint of Tichý's logic, let us consider a class of constructions *CS* and a class of derivation rules *R*. By a *derivation system* I will understand the pair

$$\langle CS, R \rangle.$$ [11]

For the sake of illustration we will utilize the example of propositional logic (PL).

Classical propositional logic (CPL) operates on a certain *area* of objects, namely two truth-values and n-ary total truth-functions. But the *subject matter* of CPL ('aboutness') consists of certain constructions of objects from the objectual area. Once more: the subject matter of CPL is not made of the objects, but rather certain constructions of those objects.

[10] Many such rules were established by Tichý and Oddie in their paper [3, p. 214].

[11] This is in fact only a rudimentary form of a derivation system (they were first introduced in [4]). The present construal, elaborated together with Petr Kuchyňka [7], is a bit more complex.

In our case, the constructions comprise i. variables for truth-values (o, o_1, $\ldots$, o_n, i.e. familiar ps and qs), ii. trivializations of the truth-functions ($^0\neg$, $\ldots$, $^0\vee$, $^0\rightarrow$, $\ldots$), and also iii. compositions of constructions from i. and ii., e.g. the construction $[o_1\ {}^0\!\rightarrow o_2]$. Note that CPL is not about constructions of the form of closure, e.g. $\lambda o_1 o_2[[^0\neg\, o_1]\ {}^0\!\rightarrow o_2]$; neither is CPL about variables, say f, for the truth-functions; it is not about various constructions built from such constructions and constructions from i.–iii. Now I will show that Tichý's system of deduction can treat them and hence it is capable of treating all constructions which can be considered to be in the subject matter of PL.

For CPL, there exist a number of calculi - in our construal, certain derivation systems.[12] As so-called axioms, one can choose some 'tautological' constructions, e.g. $[o_1\ {}^0\!\rightarrow [o_2\ {}^0\!\rightarrow o_1]]$. (Alternatively, axioms can be understood as categorical rules.) They form a class $\mathrm{ACS}_{\mathrm{CPL}}$, which is a subclass of $\mathrm{CS}_{\mathrm{CPL}}$. It is obvious that with help of derivation rules from $\mathrm{R}_{\mathrm{CPL}}$, e.g. $\models \Phi \cup \{o\colon [o_1\ {}^0\!\rightarrow o_2], o\colon o_1\} \Rightarrow o\colon o_2$ (i.e. modus ponens), one can reach (by deriving) all 'tautological' constructions from the difference of classes $\mathrm{CS}_{\mathrm{CPL}}$ and $\mathrm{ACS}_{\mathrm{CPL}}$, which amounts to the completeness of this particular $\mathrm{DS}_{\mathrm{CPL}}$.

(It is well-known that calculi for PL work exclusively with some 'connectives'. It means that the calculi operate only within a part of $\mathrm{DS}_{\mathrm{CPL}}$ which has been considered above because the calculi do not allow, for instance, constructions containing $^0\vee$ or they 'introduce' them by mean of definitions, which means — as I take it — that they utilize rules such as $\models o\colon [o_1\ {}^0\!\vee o_2] \Leftrightarrow o\colon [[^0\neg\, o_1]\ {}^0\!\rightarrow o_2]$.)

Yet this is only a fragment of what PL is from my viewpoint. If we look on this occasion at Tichý's papers on deduction we notice that Tichý considered not only constructions of various kinds but mainly a lot of different rules. Thus particular derivation systems have to be extracted or selected from Tichý's writings. We can then study derivation in, say, 'quantified' PL. In derivation systems for such a logic the class of constructions CS is similar to that for CPL, but containing certain constructions of kind closure and also their compositions with $^0\forall$ or $^0\exists$. For instance, $[^0\forall\lambda o_1 o_2[[^0\neg[o_1\ {}^0\!\vee o_2]]\ {}^0\!\leftrightarrow [[^0\neg\, o_1]\ {}^0\!\wedge [^0\neg\, o_2]]]]$, which is in fact De Morgan's law, is surely something PL can express. For other

[12] Generally, DS is best given by the explicit determination of all primitive constructions of its CS. In the case of (objectually understood) calculi — though they are DSs — , a more fundamental strategy is to determine its composed 'tautological' constructions (i.e. constructions constructing the truth-value T on any valuation).

examples of PL: admitting variables for truth-functions we get PL of 'higher' order; allowing quantification over constructions of PL-objects we get even a real higher-order logic (in the sense of Tichý's ramified theory of types). And I haven't even mentioned the possibility to accept constructions of partial functions, as Tichý always did.

If we want to work within modal PL, we will admit into the appropriate derivation systems variables for propositions, possible worlds, moments of time ($p, \ldots, w, \ldots, t, \ldots$), and the appropriate quantifiers (having thus also operators of necessity and possibility). Then one of Tichý most fruitful contributions is a sophisticated treatment of substitutivity of variables such as o by means of constructions such as p_{wt} (this way it is possible to correct classical rules of, say, PL, in order to keep validity, which is generally lost in frameworks which adopt partiality). We can continue with the enrichment of CPL further and further. Tichý thus provided an extensive and at the same time unified framework of deduction. (The investigation of this framework is a task for the future.)

Since it relates to non-empirical matters, the example with PL does not well illustrate one important feature of derivation systems. The rules of system of deduction can be roughly classified in three groups: 1. 'basic' (if one does not know them, one understands nothing), 2. 'displaying' (displaying, for instance, particular properties of implication) and 3. 'content' (definitions). It is not only the rules of kind 2. that can be acquired from the analytical cognition of an object. Content rules are perhaps more interesting in that respect. Their exemplary use can be found in Tichý's and Oddie's study on the logic of ability, freedom and responsibility [3]. In that paper, the rules of kind 1 and 2 are introduced first and the derivation system DS is then gradually enriched by allowing other rules (especially content rules) which concern the notions of ability, freedom and responsibility.

5. Concluding remarks

In conformity with the current logical methodology, numerous axiomatic systems, or logics, are proposed; their role is twofold. Firstly, their task is to define implicitly some key notions (or objects). A particular system of modal logic, for instance, should specify the notion of the property "being a necessary proposition" (i.e. to define the meaning of the 'box').

Secondly, their task is to lay out a certain derivation system in which the deduction with that notion can take place.

We have already seen that the first task is superfluous from Tichý's viewpoint. Tichý even remarked that the very idea of logic presupposes that the entities for which an axiomatic system is proposed exist prior to that axiomatization (*cf.* [13, p. 277]).

As regards the second task, recall that those logics are proposed only for thematically narrow fields; for instance, deontic logics only investigate a few notions related to norms. But the research carried out within Tichý's logic concerns a rather great number of subjects. We can put them in categories such as 'logic of propositional attitudes', 'logic of subjunctive conditionals', 'temporal logic', etc. Observe also that the ambition of Tichý's logic is to work on a unified framework. It thus cannot happen that the results of, say, Tichý's 'temporal logic' would be incompatible with Tichý's logic of 'propositional attitudes'.

I do not claim that for all these 'sublogics' of Tichý's logic concrete derivation systems have been already built. Such things are task for the future. On the other hand, we note that Tichý offered a number of derivation rules which can be utilized in those particular 'logics'. Recall also that the discovery of rules goes hand in hand with an adequate analysis or, more precisely, with the proper explication of relevant intuitive notions.

Acknowledgements. I owe thanks to Petr Kuchyňka for discussions of the views presented in this paper and also reading and commenting on an early draft. I would also like to thank Jan Štěpán for comments. Thanks belong also to the anonymous reviewers.

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Jiří Raclavský
Department of Philosophy
Faculty of Arts
Masaryk University
Arne Nováka 1
602 00, Brno, the Czech Republic
`raclavsky@phil.muni.cz`