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THE LOGICAL WAY OF BEING TRUE: Truth values and the ontological foundation of logic

Abstract. In this paper I reject the normative interpretation of logic and give reasons for a realistic account based on the ontological treatment of logical values.

Keywords: reasoning; normativity; values; truth values; foundations of logic; logical worlds

1. Introduction. Subject-matter of logic and the problem of its foundations

1.1. This paper offers a kind of an extended comment to the famous statement by Gottlob Frege of logic as *the science of the most general laws of being true* (die Wissenschaft der allgemeinsten Gesetze des Wahrseins) [8, p. 39]. The main problem that should be examined in this connection is not only what logic is about, but rather the way in which logic is what it is about. While it is often claimed that the way is essentially *normative*, exactly this claim will be questioned in the subsequent exposition.

1.2. Since Aristotle logic has been standardly defined as the science of correct reasoning and valid argument, or *demonstration* [2, 24a]. According to such a view, “[l]ogic has the important function of saying what follows from what” [13, p. 3]. Although this understanding of the subject-matter of logic seems to suit well the basic “teaching needs” and

certainly possesses some explanatory power, it calls for further, mainly *foundational*, clarification.

1.3. Here is a small sample of questions open to the foundational discussion. What is reasoning, and how should it be represented for a logical reconstruction? Should it be treated as a process, and if yes, what kind of process, and how can the bearer of this process be explicated? On what grounds can reasoning be qualified as correct or incorrect, and if correct reasoning should obey logical rules, what is the basis for these rules? To provide answers to these questions various foundational strategies have been elaborated.

1.4. According to *psychologistic approach*, logical rules essentially reflect the process of sound human thinking. That is to say, logical laws are nothing else but “laws of thought” which ultimately prescribe how we should think if we wish to think correctly.

1.5. *Linguistic approach* treats logical rules as rules for handling language expressions. Being so understood, the laws of logic represent certain regularities which correspond to structural features of a given linguistic system.

1.6. By the *transcendental approach*, logical rules represent fundamental *a priori* structures of consciousness by means of which concepts and intuitions are synthesized to acquire knowledge of the world as it is given in the process of apperception.

1.7. Obvious differences notwithstanding, there is something the three strategies have in common. Namely, they all are of an explicitly *anti-realistic* character and interlink logic with an *activity* of some agent, whether it be the cognitive or linguistic activity (practice) of a human being or the conscious activity of a transcendental subject in the Kantian sense. Reasoning is represented (reconstructed) here as a certain process — either mental, or cognitive, or linguistic — governed by specific logical rules resting upon the corresponding activity just mentioned.

2. The conception of logic as a normative discipline

2.1. Grounding on an anti-realist understanding, one usually arrives at the conception of logic as a *normative discipline*. According to this conception, logic is considered to be a discipline which provides *norms*

for thought or reasoning and tells us how we *ought to* think or reason if we want to think or reason correctly.

2.2. Kant's characterization of logic as "a canon of the understanding and the reason" fits well into the normative paradigm: "In Logic we do not want to know how the understanding is and thinks, and how it has hitherto proceeded in thinking, but how it ought to proceed in thinking. Its business is to teach us the correct use of reason, that is, the use which is consistent with itself" [11, p. 14].

2.3. Edmund Husserl in his "Logical Investigations" also stressed the role of logic "as a normative and, in particular, as a practical discipline" [10, Ch. 1].

2.4. Most prominently, the idea of logic as a normative science was put forward by Charles Peirce who defined logic as "the theory of self-controlled, or deliberate, thought" [20, p. 62]. He explained further that "logic, as a true normative science, supposes the question of what is to be aimed at to be already answered before it could itself have been called into being" [21, 577].

2.5. Peirce's doctrine of logic is based on his wider conception of a normative science and on his general *Classification of Sciences*. In accordance with this classification, see, e.g., [20, p. 60–61], all sciences are either (A) *Science of Discovery*, or (B) *Science of Review*, or (C) *Practical Science*. *Science of Discovery*, in its turn, is either (I) *Mathematics*, or (II) *Philosophy*, or (III) *Idioscopy* (special sciences). *Philosophy* then can be divided into (a) *Phenomenology*, (b) *Normative Science*, and (c) *Metaphysics*. And finally, *Normative Science* has three divisions: (i) *Esthetics*, (ii) *Ethics* and (iii) *Logic*.

2.6. *Esthetics*, conceived in a Peircean sense as the most general normative discipline, is the science of *ideals* as such, or "of that which is objectively admirable without any ulterior reason" [20, p. 62]. *Ethics* deals with a restriction of these ideals to the realm of the "self-controlled, or deliberate conduct", and logic specifies them further to the activity of rational thought. All in all, normative science is "the science of the laws of conformity of things to ends", see [19, p. 30].

2.7. Laws of this kind are typically represented by *norms*. The latter constitute binding principles, patterns and standards expressed by rules,

prescriptions or directives designed to guide, control, regulate a certain domain *in accordance with some values*.

2.8. Most abstractly, a value can be explicated as something that is ascribed to something else through a special procedure of *evaluation*. The normative interpretation implies a division of values into two main categories — the positive values, such as “right”, “proper”, “acceptable”, “desirable”, “admirable”, etc., and the negative ones which are just the opposite to the former. By hypostatizing these categories one obtains two most general, and in fact paradigmatic, values: Good and Bad.

2.9. Being normatively interpreted, logical values also fall within the above division, i.e., as C. Peirce put it: “[I]f the distinction [between] Good and Bad Logic is a special case [of the distinction between] Good and Bad Morals, by the same token the distinction of Good and Bad Morals is a special case of the distinction [between] esthetic Goodness and Badness” (see in [19, p. 30]).

2.10. Under the normative understanding, logical rules are supposed to represent logical norms, whereas logical norms regulate logical activity which is carried out in accordance with logical values. Logical values separate thus *logical goodness* from *logical badness*, and logical systems (systems of logical rules or laws) appear to be normative systems.

2.11. Clearly, normative science should concern oneself, in one way or another, with normative systems. Yet, such a concern may be twofold. The main task of a particular normative science could be either to investigate *existing* normative systems of certain sort, or to design (establish) *new* normative systems.

2.12. As to the normative role of logic in this respect both possible solutions look rather unsatisfactory. If the task of logic as a normative science is to investigate the existing systems of rules and principles actually used in real practice of reasoning and demonstration, then, first, logic turns out to be an empirical discipline, and second, the problem of justification of such systems themselves remains unsolved anyway. If logic is somehow entitled to design (establish) new systems of rules, then, first, it is totally

unclear *on what grounds* logic is supposed to obtain such an entitlement, and second, the same problem of justification of such systems still arises.

3. Logical values and/as logical entities

3.1. The normative treatment of logic faces serious difficulties giving rise to what can be dubbed a *foundational shift* and *anthropologization of logic*.

3.2. The question about the subject-matter of a science concerns the *nature* of the science and addresses its *basic* aspects. It is a foundational question. On the other hand, a discipline can have various *applications* focused on its *design* assignments (cf. distinction between descriptive science and design sciences drawn by Ilkka Niiniluoto in [18]). Careful differentiation between these aspects may undermine the whole conception of normative science by ascribing the normative dimension only to *art*, but not to science.

3.3. Here are some objections to this conception as articulated by George Sabine: “The only really normative discipline, it is said, is an art rather than a science. In so far as logic and ethics furnish a technique for thinking and acting, they are arts; in so far as they are sciences, they are descriptive, as all sciences must be. The term ‘normative science’ is self-contradictory. A science, it is said, is purely cognitive and cannot, in its capacity as a science, lay down a rule for action; it deals only with facts and generalizations of fact” [23, p. 434].

3.4. Thus, emphasizing too much the possible normative role of logical rules may result in a misleading confusion between basic (foundational) aspects of logic and its applied (design) aspects. By committing this confusion, one runs into the *foundational shift* as to the nature of logic.

3.5. For another thing, there are sciences, like psychology, sociology, history, etc., which are essentially focused on human beings and human activity as their very subject-matter. In certain sense, these sciences may be deemed “anthropological” to the extent that the regularities they study just would not exist if there were no humans. By restricting the subject-matter of logic to reasoning, thought or language structures conceived as a kind of human activity, one perpetrates the *anthropologization of logic* inextricably connecting the latter with this activity.

The “anthropological treatment” covers then not only the subject-matter of logic, but also the logical laws and logical values.

3.6. To see the fallaciousness of such a treatment, imagine that mankind ceases to exist, and there is nobody left who can reason, think or create language structures. Would the laws of logic (logical regularities) then continue to hold?

3.7. A suitable analogy with mathematics may also be illuminating in this respect. Mathematics can be seen as a tool for accomplishing calculations. However, would it be justified on this ground to claim the *nature* of mathematics to be the science of correct calculations? To characterize it as a theory of self-controlled, or deliberate, calculations, a discipline which provides norms for calculations? Mathematics certainly tells us how we ought to calculate if we want to calculate correctly. Does it mean, however, that the *basic task* of mathematics is to separate correct (“good”) calculations from the wrong (“bad”) ones?

3.8. By and large, “anthropological” (anti-realistic) approaches to the foundations of logic have serious drawbacks. Psychologistic strategy turns logic into a branch of psychology. Linguistic strategy relativizes logic with respect to the given languages (linguistic frameworks). Transcendentalistic strategy is hardly compatible with the fact of existence of many (non-classical) logical systems (see [24, pp. 10–11]).

3.9. If we seek to construe logic as a fully objective discipline avoiding unjustified dependence on any “anthropological background”, it could be reasonable to take a look at the *ontological* (realistic) strategy of grounding logical rules.

3.10. To achieve this goal it is necessary to abandon the normative treatment of logical values and logical systems.

3.11. Logic can find its ontological foundations in the notion of logical value interpreted non-normatively as a special kind of object. The natural idea of such logical objects was set forth by Frege in [6] and [7], who introduced two classical truth values — *the True* and *the False* — which can be ascribed to sentences as their possible denotations.

3.12. As Nuel Belnap explains in [3, p. 306]: “Truth values were put in play by Frege to be the *denotations* of sentences, in contrast with their *senses*. If I may use ‘*T*’ and ‘*F*’ as names of the two classical truth

values, then the story is that the denotation of ‘snow is white’ is T or F according as snow is or is not white. What a happy ideal!”

3.13. Truth values are mainly employed in a semantic evaluation of sentences. Despite this principal role of logical values, ontologically they can be considered specific entities on their own, primitive abstract objects constituting a certain domain serving as the realm of logical investigation.

4. Logic as the science of logical entities

4.1. Jan Łukasiewicz, following the guiding idea by Frege, formulated a realistic view on the subject-matter of logic:

“All true propositions denote one and the same object, namely truth, and all false propositions denote one and the same object, namely falsehood. I consider truth and falsehood to be *singular* objects. . . Ontologically, truth has its analogue in being, and falsehood, in non-being. The objects denoted by propositions are called *logical values*. Truth is the positive, and falsehood is the negative logical value. [. . .] Logic is the science of objects of a special kind, namely a science of *logical values*.” [15, p. 90]

4.2. This may seem to be a rather limited view, since truth values are not necessarily the only entities that may lie within the scope of interest of a logical theory. Even if truth values may be regarded as the basic (most fundamental) logical entities, the logical realm as such should be much wider. Logic can more generally be defined then as the science of logical entities. Clearly, this science is not empirical but *a priori*, since logical entities are of a purely abstract character. But what are these entities?

4.3. By answering the latter question, the fundamental division of entities into objects and functions (suggested by Frege, see, e.g., [17]) may prove useful. Applying this division to the realm of logical entities one obtains a distinction between *logical objects* and *logical functions*. Logical objects are nothing else but truth values, and logical functions are functions ranged over the logical objects.

4.4. Logical entities can naturally be organized into *logical structures*. A logical structure is determined by a collection of truth values serving as

a carrier set of the structure, and additional machinery for structuring this set.

4.5. As an example of the logical structure consider a propositional *valuational system*. Such a system is a triple, $\langle \mathcal{V}, \mathcal{D}, \mathcal{F} \rangle$, where \mathcal{V} is a non-empty set of truth values with at least two elements, \mathcal{D} is a non-empty proper (designated) subset of \mathcal{V} , and $\mathcal{F} = \{f_1, \dots, f_n\}$ is a set of (truth-value) functions on \mathcal{V} . Now, assume an inductively defined propositional language built upon a non-empty set of atomic sentences and a set of propositional connectives. The functions from \mathcal{F} can be used to determine the connectives. It is possible to define a valuations function as a map from the set of atomic sentences into \mathcal{V} , and extend this function to the whole language. The elements from \mathcal{D} can be standardly used for defining the notions of the logical law and logical entailment.

4.6. Another kind of the logical structure is represented by a *truth-value lattice* (\mathcal{V}, \leq) defined on a partially ordered set of truth values \mathcal{V} (with at least two elements). Again, it is possible to introduce a valuations function from the set of atomic sentences into \mathcal{V} , and to define logical connectives through the truth-value lattice operations. Ordering relation \leq on \mathcal{V} , called “logical order”, is used for defining the relation of logical entailment as expressing an agreement with this order.

4.7. Any logical structure (valuational system, truth-value lattice, etc.) may be seen as a *semantic framework* of a certain logical system in the sense that the entailment relation defined in the former determines the consequence relation of the latter.

5. Diversity of logical worlds and logical systems

5.1. The basic carrier set of a given logical structure can be characterized as a particular *logical world* forming the ontological basis for this (and maybe some others) logical structure and corresponding logical systems. By this realistic approach the fundamental principles of one logic or another turn out to be just specific *ontological preconditions* imposed on the given logical world.

5.2. For example, the well-known *law of excluded middle* expresses simply the Fregean postulate of exactly two truth values (the bivalence principle). As such, this postulate is not so indisputable as it may seem at first glance. The notion of the logical world as a collection of logical objects

by no means predetermines the amount of these objects, i.e. the number of elements constituting the given “logical universe”.

5.3. It was Łukasiewicz, who, as early as 1918, challenged Frege’s view that there might be only two truth values and advanced idea of a *many-valued logic* [14]. “Many” means here “more than two”, and Łukasiewicz considers a possibility of a third logical value by stressing the point that some propositions, e.g., the ones about future contingents, are “neither true nor false but *indeterminate*” [16, p. 126].

5.4. According to Łukasiewicz, “we could say that ontologically there corresponds to these sentences neither being nor non-being but possibility. Indeterminate sentences, which ontologically have possibility as their correlate, take the third truth value” [16, p. 126].

5.5. Intuitively, various interpretations of the third truth value are possible: “undefined”, “nonsensical”, “paradoxical”, etc. As a result one can obtain, e.g., “logic of indeterminacy” [12, p. 332], “logic of nonsense” [4], “logic of paradox” [22], and other logical systems. Of course, adding more values extends further the set of possible logical systems.

5.6. This plurality of logical systems is due to various ontological preconditions that can be taken for a particular logical world. Acceptance or rejection of certain ontological preconditions generates new logical worlds, as well as corresponding logical systems. That is to say, the plurality of logical systems is a direct offshoot of the ontological diversity of logical worlds.

5.7. The situation here is analogous to the one in geometry. If at first constructing non-Euclidean geometries was considered just a sort of sophisticated intellectual exercise, then it became evident that each geometry of the kind correlates to a certain geometrical world, viz. geometrical space with its specific features. We have thus *Euclidean space*, *Riemannian space*, *Lobachevskian space*, etc.

5.8. Similarly one can speak about a set of “possible logical worlds” underlying various logical systems. This set evidently has an infinite cardinality, since it would hardly be possible to restrict in a reasonable way the ontological postulates that can be taken for a particular logical world. Let us characterize briefly some of these worlds.

5.9. *The World of Parmenides-Hegel.* It consists of *the only* truth value. Truth and falsehood are here indistinguishable from one another (fuse into one truth value); being and non-being are identical. This world is described rather expressively by Hegel in his “The Science of Logic”:

“*Being, pure being*—without further determination. In its indeterminate immediacy it is equal only to itself and also not unequal with respect to another; it has no difference within it, nor any outwardly. [...] It is pure indeterminateness and emptiness. [...] Being, the indeterminate immediate is in fact *nothing*, and neither more nor less than nothing. [...] Nothing, pure nothingness; it is simple equality with itself, complete emptiness, complete absence of determination and content; lack of all distinction within. [...] Nothing is ... the same determination or rather absence of determination, and thus altogether the same as what pure being is.” [9, p. 59]

Clearly, this world cannot contain designated truth values, there is only one logical object and nothing from which it could be distinguished. Therefore it is impossible to define the notions of the logical law and logical entailment on the basis of this world. Properly speaking, no logic is possible in this world (except perhaps dialectical one), and we have here a kind of degenerated logical world.

5.10. *The World of Frege.* It comprises exactly two classical truth values—*truth* and *falsehood*. By now it is one of the best explored logical worlds. Here, there is one designated value—*truth*. This is the world of the laws of classical logic first formalized and codified by Frege in his “Begriffsschrift” (1879) and also by Alfred Whitehead and Bertrand Russell in their *Principia Mathematica* (1910–1913).

5.11. *The World of Brouwer-Heyting.* According to the constructive conception of truth adopted in intuitionistic logic of L. E. J. Brouwer and Arend Heyting, a proposition is considered true if and only if it is constructively proved. This world, like the world of Frege, has two truth values, but is distinct from Frege’s conception; truth obtains here an additional qualitative characteristic and is interpreted as *constructive truth*.

5.12. *The World of Łukasiewicz-Kleene.* As already noted, this world signifies an enlargement of the world of Frege by a third logical value interpreted as *neither truth nor falsehood* (indeterminacy, uncertainty). *Truth* remains here the only designated truth value. This world allows

several axiomatizations, such as the Łukasiewicz's three-valued logic or Kleene's three-valued logic.

5.13. *The World of Priest.* This world also consists of three logical values, but the third value obtains here another interpretation — *both true and false* (contradiction, absurdity, paradox). There are two designated values here — *truth* and *both truth and falsehood*. Based on this world, a new direction of the modern non-classical logic emerged, the so-called “paraconsistent logic” for which the classical principle *ex falso quodlibet* does not hold.

5.14. *The World of Dunn-Belnap.* Nuel Belnap, by developing some key ideas of J. Michael Dunn, proposed to interpret truth values as information “told to a computer”. By this interpretation, there are four truth values: *truth* (“told only truth”), *falsehood* (“told only falsehood”), *none* (“told neither truth nor falsehood”), and *both* (“told both truth and falsehood”). *Both* and *truth* are designated. The logic of this world is a *first-degree entailment system* of relevance logic developed by Anderson and Belnap, see [1, § 15].

5.15. The list of logical worlds can be further continued, including *the Jain World* (with seven truth values), *the World of Post* (with the infinite set of truth values), *the World of Zadeh* (with fuzzy truth values), and many others.

5.16. It is noteworthy that there is no one-to-one correspondence between logical worlds and logical systems. The same logical world may serve as a basis for several logical systems, depending, e.g., on the truth conditions for logical connectives that may vary. It is, however, plausible to suppose that any “full-fledged” logical system should be based on some logical world.

6. Conclusion

The main points of this paper can be summarized as follows.

6.1. When we speak of norms, we always speak of values, but the converse does not hold.

6.2. When we speak of normative systems, we always speak of systems of rules, but the converse does not hold.

- 6.3.** Any science has basic aspects (concerning its proper subject-matter) and design aspects (concerning its possible applications).
- 6.4.** No science is normative with respect to its basic aspects, and so is logic.
- 6.5.** Any science is more or less normative with respect to its design aspects, and there is nothing special about logic in this respect.
- 6.6.** The idea of logic as a normative science is a typical misconception when basic aspects of a science are confused with its design aspects.
- 6.7.** The ontological explanation of logic explicates it as an *a priori* science about specific logical entities — logical objects (truth values) and logical functions — organized in logical worlds and logical structures serving as ontological and semantical bases for logical systems.

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