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IS THE ONTOLOGICAL PROOF FOR GOD'S EXISTENCE AN ONTOLOGICAL PROOF FOR GOD'S EXISTENCE?

Abstract. Two questions concerning Anselm of Canterbury's theistic argument provided in *Proslogion* Ch. 2 are asked and answered: is the argument valid? under what conditions could it be sound? In order to answer the questions the argument is formalized as a first-order theory called AP2. The argument turns out to be valid, although it contains a hidden premise. The argument is also claimed not to be ontological one, but rather an *a posteriori* argument. One of the premises is found to be false, so the argument is claimed not to be sound and to fail to prove its conclusion.

Keywords: *Ratio Anselmi*, ontological argument, theistic argument

Introduction

If the length and the intensity of the debate caused by an intellectual idea was the measure of the value of the idea, Anselm of Canterbury's famous argument for God's existence should be considered as an outstanding stroke of genius. The argument in question, called *Ratio Anselmi* or the ontological proof for God's existence, is presented in Anselm's work *Proslogion* [1, p. 93–122] (written in 1077–1078 and first entitled *Fides quaerens intellectum*). The objective of the present paper is to answer two questions concerning the famous argument. The two questions we are about to answer, are: is the argument valid (deductive), does the conclusion follow from the premises? what kind of theory (ontology) can guarantee the argument to be sound, in

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what kind of view of world are the premises true? Answering the questions listed, we will formulate a view concerning the nature of the argument itself.

In order to achieve our objective, we formalize and construe Anselm's argument as a first-order theory (without modal connectives). Although there is a lot of formalizations of *Ratio Anselmi*, they usually involve modal logic, which is itself philosophically highly disputable. Our first-order theory is called AP2, which abbreviates the phrase "Anselm's *Proslogion*, Chapter 2".

The essential idea of formalization *Ratio Anselmi* as a first-order theory was already presented in our lecture and discussed at 51st International Conference for History of Logic, Kraków, on October 27th, 2005, and published in polish as our paper [3]. The present paper is based on [3], however, it is not a translation of the previous one—it contains some new thoughts, some others have been refuted under discussion.

1. Ontological Arguments

Ontological arguments, in wide meaning, are arguments for the conclusion that God exists, from premises which are supposed to derive *a priori*, without any observation of the world being involved. Ontological arguments, in strict meaning, are supposed to derive the existence of God from a concept of God, a definition of God alone. In *Proslogion* Anselm is usually claimed to derive the existence of God from the concept of a *being than which no greater can be conceived*.

Anselm's argument has been widely discussed from the very beginning the present time. The most famous criticisms of it were submitted by Gaunilo of Marmoutiers (XIth century), Thomas Aquinas (XIIIth century) and Immanuel Kant (XVIII–XIXth century). On the other hand, Anselm's argument has been often defended. In the XIIIth century a semi-defence, was provided by Iohannes Duns Scotus. In the XVIIth century René Descartes defended a family of in a sense similar—but non identical—arguments. It is often said, Kant's criticism concerns with Decartes's reasoning rather than Anselm's one. An *a priori* argument for God's existence was defended and essentially developed by Gottfried Wilhelm Leibniz in the early XVIIIth century. In more recent times many ontological arguments have been presented and so much discussed, mostly based on modal logic, e.g. by Kurt Gödel, Charles Hartshorne, Norman Malcolm, Alvin Plantinga and others. The arguments mentioned bear many interesting connections to the genuine argument of Anselm. Our aim, however, is to analyse only the genuine Anselm's reasoning, provided at the beginning of his book *Proslogion*.

2. A first-order theory

There is a lot of logical analyses of *Ratio Anselmi*, coming from many briliant logicians. The analyses usually make profound use of modal logic. However, we recommend to analyse the *Proslogion* argument as a first-order theory. We think, there are two important reasons to do so.

First, as long as possible, one should use only classical logic, i.e.: truth-functional propositional calculus, first-order predicate calculus and identity theory. Modal logic is a very strong and brilliant mathematical tool, but it is itself philosophically highly disputable. It is a well known fact, that in many analyses, whether or not there exists God, it often depends on which one of the enormous number of modal calculi has been chosen for the analysis. Any philosophical analyse based on modal logic, bears the whole philosophical debate on modal logic itself. And the modal suppositions, those necessarily accepted for the analysis of *Ratio Anselmi*, are usually of the sort of high disputability. So, always and especially in cases of modalities, as long as possible one should use exclusively the standard logic, and if *Ratio Anselmi* can be analysed in standard logic, it should by analysed that way.

Secondly, we think *Ratio Anselmi* does not need any modal analysis. The common use of modal logic for the analysis in question begins with Leibniz and has been established later by Hartshorne's idea, that there are actually two proofs in *Proslogion*: one proof in the chapter 2 [1, p. 101–102] and the other in the the chapter 3 [1, p. 102–103]. Hartshorne claims, all the historical criticism of *Ratio Anselmi* applies to the chapter 2, but not to the chapter 3. One should so analyse the proof contained in the chapter 3, and the best way to do it is by use of modal logic.

We find the idea just sketched false. There is only one theistic argument in *Proslogion* 2–3, and the chapter 3 continues the chapter 2. More accurately, the chapter 2 is a kind of lemma for the chapter 3. In the chapter 2 it is derived that God exists. And in the chapter 3 it is derived that if God exists at all, God exists necessarily. God's existing necessarily essentially involves modalities and—perhaps—justifies some use of modal logic, but that is the case of the chapter 3, not the chapter 2. We claim, most logical analyses do not concern with the famous statement from the chapter 2: "*Existit ergo procul dubio id quo maius cogitari non valet et in intellectu, et in re*" [1, p. 102], but with much less known statement from the chapter 3: "*Quod utique sic vere est, ut nec cogitari possit non esse* [1, p. 102], which can be thought to be deeply modal. One can easily notice, that what is common to the majority of logical analyses of *Ratio Anselmi* since Leibniz and Hartshorne, is so called *logic of perfection*. There are always involved two claims, expressed in that way or another:

- either God exists necessarily or it is impossible for God to exist;
- it is possible that God exist.

The first claim is often thought to be justified, broadly saying, *a priori*, but the second one is always problematic. It was problematic for Leibniz and Hartshorne, and it is highly problematic for the recents analyses of the ontological argument. It was not, however, problematic for Anselm in the chapter 3 of his *Proslogion*, because in the chapter 2 he had already proved that God exists, and so that God possibly exists (for *ab esse ad posse valet consequentia*). All he was about to do in the chapter 3 was to strengthen the claim to necessity. We have been discussing the problem more widely in our earlier paper [4].

Actually in the chapter 2 there appears a kind of modal concept: the concept of nothing greater being *conceivable*. However we recommend to hope, one can represent this kind of modal involvement in first-order logic. For it is usual to interpret quantifiers modally, i.e., 'for all' means 'for all that is conceivable' and 'there exists' means 'there is conceivable'. We hope to show, that the modal involvement of the chapter 2 is not as vital as that of the chapter 3.

So, we think, the proof for God's existence in the chapter 2 should be analysed in non modal logic, although the modality should be—perhaps—somehow involved for the chapter 3, strengthening the result of the vital the chapter 2. As it was said, in the present paper we are interested in the *Proslogion*, the chapter 2, exclusively, and so we are going to build a non modal theory for the analysis of the argument in question.

3. The language of AP2

AP2 is a first-order theory, so the language of AP2 is a standard first-order language. In the vocabulary, beside logical symbols of the first-order classical logic, individual variables and parentheses we have four specific predicates.

Variables. We use the letters: 'u', 'v', 'w', 'x', 'y', 'z' as individual variables. Individual variables of AP2 represent two non empty sets: a set of persons (rational agents, their intellects) and a set of concepts. For simplicity AP2 is a one-sort theory with one domain, and the atomic formulas containing a name of a person instead of a concept, or on the opposite, are claimed to be false. We think that the simplification cannot cause any misunderstanding.

We suppose, all concepts have references (objects of reference), however, the references of some concepts may exist in reality and the references of some other concepts may not exist in reality. We also suppose, persons can understand concepts, and ascribe existence to their references (or decline the references of existence). Generally, whether or not the reference of a concept exists in reality is independent from whether or not a person, who understands the concept, ascribes existence to its reference.

Predicates. The four non logical predicates are vital for the language AP2. There is one unary predicate 'E', two binary predicates: 'A' and 'B', and one predicate on four terms 'C'. The atomic formulas should be read as follows:

$\mathsf{A}ux$ – the person u understands the concept x;

$\mathsf{B}ux$ – the person u understands the concept x and thinks that the reference of x exists in reality.

$xu\mathsf{C}yv$ – the reference of the concept x understood by the person u is conceived as something greater than the reference of the concept y understood by the person v;

$\mathsf{E}x$ – the reference of the concept x exists really.

Usually we accept simpler readings of the predicates, equivalent to the readings listed.

The formula '$\mathsf{A}ux$' expresses Anselm's idea, that the reference of the concept x exists in the understanding of the person u. The formula '$\mathsf{B}ux$' expresses his idea, that the person u understands the concept x and thinks that its reference exists in reality (never mind, whether or not it in fact exists). And the formula '$\mathsf{E}x$' expresses the idea, that the reference of the concept x, in fact, exists in reality [1, p. 101].

Logical symbols and formulas. We use standard logical symbols. In AP2 there are truth-functional connectives: '$\neg$', '$\wedge$', '$\vee$', '$\rightarrow$' and '$\equiv$', where the bounding force gets weaker (longer) from left to right, and the quantifiers '$\forall$' and '$\exists$', where the scope, unless given by parentheses is the shortest formula.

Any n-place predicate with n individual variables is a formula. If α is any individual variable, ϕ and ψ are any formulas, then $\ulcorner\neg\phi\urcorner$, $\ulcorner(\phi \wedge \psi)\urcorner$,

$\ulcorner(\phi \vee \psi)\urcorner$, $\ulcorner(\phi \rightarrow \psi)\urcorner$, $\ulcorner(\phi \equiv \psi)\urcorner$, $\ulcorner\forall\alpha\ \phi\urcorner$ and $\ulcorner\exists\alpha\ \phi\urcorner$ are also formulas and there are no more formulas. The language thus constructed we call the language AP2. Having defined the language, we are now about to discuss the set of theorems of the AP2 theory.

4. "Proslogion" Chapter 2 formalized

As it was already more than once mentioned, AP2 is a first-order theory, so it is an extension of classical first-order calculus. Any substitution of a logical theorem of the standard first-order calculus, with formulas of the language AP2 is a theorem of the theory AP2. Consequently, all the derivable inferential rules of the first-order logic are derivable rules of the theory AP2. We now concentrate in the specific axioms of the calculus AP2.

We begin with examining *Ratio Anselmi* as it is explicitly formulated in *Proslogion* the chapter 2. In the language provided we formalize Anselm's conclusion and its three premises. The premises are axioms of AP2 and we want to examine, whether or not the premises are consistent and whether or not the conclusion is derivable from the premises, i.e., whether or not the explicitly formulated axioms are strong enough to make the conclusion a theorem.

The conclusion. Anselm's conclusion is: a being than which no greater can be conceived exists in reality ("*Existit ergo procul dubio aliquid quo maius cogitari non valet, et in intellectu et in re*" [1, p. 102]). We formalize the conclusion as:

$$\exists_x (\mathsf{E}x \wedge \neg\exists_{yuv}\, yu\mathsf{C}xv) \tag{C}$$

The formula (C) should be read as follows: "there exists a concept x such that the reference of the concept x exists in reality and there is no such a concept y nor such persons u, v, that the reference of the concept y understood by the person u is conceived as something greater than the reference of the concept x understood by the person v".

Anselm explicitly formulates three premises, claiming to derive (C) from them. In AP2 the premises of Anselm are specific axioms.

The Axiom of the Fool. According to the Axiom of the Fool, one can think of a being than which no greater can be conceived, one can understand the concept of such a being ("*Convincitur ergo insipiens esse vel in intellectu* *aliquid quo nihil maius cogitari potest*” [1, p. 101]). One can so understand the concept of such a being, without necessarily accepting the existence of the being in question and without necessarily real existence of it. This is our first axiom:

$$\exists_x \neg\exists_{yuv}\, yu\mathrm{C}xv \tag{F}$$

The axiom (F) should be read in the following way: “there exists such a concept x, that there is no such a concept y nor such intellects u, w, that the reference of the concept y understood by the person u is conceived as something greater than the reference of the concept x understood by the person v”.

The axiom (F) claims accurately, there exists a concept in question. So, we identify the existence of a concept with its ability to be understood: for any concept there exists a person, which understand the concept. Whether do so, or to introduce extra axioms for concepts being understood, is only a matter of taste. One could support the theory AP2 with extra axioms for the concepts’ being understandable, but all our formal results would be then identical, and calculi would be more complicated.

The Axiom of the Wise Man. According to the Axiom of the Wise Man, it can be thought, that that than which no greater can be conceived exists in reality (“*Si enim vel in solo intellectu est, potest cogitari esse et in re,* [. . .]” [1, p. 101]). Our formal expression for the axiom is:

$$\forall_x(\neg\exists_{yuv}\, yu\mathrm{C}xv \rightarrow \exists_u\, \mathrm{B}ux) \tag{M}$$

The axiom (M) should be read as follows: “for any concept x, if there is no such a concept y nor such persons u, v, that the reference of the concept y understood by the person u is conceived as something greater than the reference of the concept x understood by the person v, then there exists such an intellect w, that the intellect w understands the concept x and w thinks that the reference of the concept x really exists”.

The Axiom of the Hierarchy of Being. Anselm supposes one more thing, which can be called the Axiom of the Hierarchy of Being. Anselm claims, to think of any object, thinking that the object in question exists in reality, is to think of something greater than to think of the same object, thinking the object exist only in the understanding. There must be supposed a hierarchy of being such that, a concept of any thing, attached with the thought that the thing exists in reality, is a concept of something greater than the same concept without the thought of the things existence in reality.

The premise is expressed in the phrase 'but that is something greater', added to the Axiom of the Wise Man ("*Si enim vel in solo intellectu est, potest cogitari esse et in re, quod maius est*" [1, p. 101]). So, we accept as an axiom of AP2 the formula:

$$\mathrm{B}ux \wedge \mathrm{A}vx \wedge \neg\mathrm{B}vx \rightarrow xu\mathrm{C}xv \tag{H}$$

The axiom (H) should be read: "if, for any person u understands any concept x and a thinks that the reference of x exists in reality and for any person v understands the concept x without the thought that the reference of x exists in reality, then the reference of the concept x understood by the person u is conceived as something greater than the reference of the concept x understood by the person v".

5. The hidden premise

To answer the first question, that has been asked at the beginning of the paper, we have to examine, whether or not the formula (C) is derivable form the formulas (F), (M) and (H). If not, we should ask, whether the argument is wrong (*non sequitur*) or it is entymemathic—there are hidden premises.

5.1. Non sequitur?

The conclusion (C) does not follow from the premises (F), (M) and (H). One can know that just by observing, that the predicate 'E'—which is essential for the conclusion (C)—does not appear in any premise. However, one can prove that result as well.

THEOREM 1. *The formula* (C) *is not derivable from the formulas* (F), (M) *and* (H) *in first-order logic.*

PROOF. We prove this theorem by an interpretation. Let $\mathfrak{N}$ be the set of natural numbers, including 0, and let individual variables represent elements of $\mathfrak{N}$. Let also '+' and '<' be symbols in the Peano Arithmetic PA. We interpret the language of AP2 as follows:

AP2	PA
Aux	$u = x$
Bux	$u = x$
$yuCxv$	$y + u + v < x$
Ex	$x \neq 0$

In such an interpretation the formulas (F), (M) and (H):

$$\exists_x \neg\exists_{yuv}\; y + u + v < x$$
$$\forall_x(\neg\exists_{yuv}\; y + u + v < x \to \exists_w\, w = x)$$
$$\forall_{xuv}(u = x \wedge v = x \wedge v \neq x \to x + u + v < x)$$

are true formulas of PA, and the formula (C):

$$\exists_x\,(x \neq 0 \wedge \neg\exists_{yuv}\; y + u + v < x)$$

is a false formula of PA. □

From Theorem 1 it follows immediately that the three axioms: (F), (M) and (H) are consistent.

5.2. The Axiom of the Intelligibility of Being

The result achieved—i.e. the lack of entailment together with the consistency of the premises—makes us justified to search for the hidden premises of Anselm's argument. One should examine, how the set of premises should be completed to gain the entailment in question, preserving the consistency of premises. The premise to be found, should be founded in Anselm's philosophy, so that it be probable, it was accepted by Anselm as an obvious claim.

The solution is provided by an observation that the premises (F), (M) and (H) entail the following formula:

$$\exists_x\,[\forall_u\,(Aux \to Bux) \wedge \neg\exists_{yuv}\, yuCxv] \qquad (\star)$$

which should be read: "there exists a concept x such that for any person, if this person understands x then it understands x and thinks that the reference of x exists in reality and there is no concept y nor persons u, v that such the reference of y understood by u is conceived as something greater than the reference of x understood by v".

The formula ($\star$) is closely related—although vitally different—to the formula (C). According to the formula ($\star$) it is not possible to think it, that than which no greater can be conceived does not exist in reality, having understood the concept in question. One could say, the formula ($\star$) is an epistemic version of rather ontological (C). We will show the proof for ($\star$) later, as Theorem 4, because it is a part of another, here more essential, proof of the formula (C).

In order to gain a theory, which includes (C) as a theorem, one need to find a derivation from ($\star$) to (C). To have that guaranteed, one can accept another axiom of AP2:

$$\forall_x [\forall_u (\mathsf{A}ux \to \mathsf{B}ux) \to \mathsf{E}x] \tag{I}$$

which may be called the *Axiom of Intelligibility of Being*, and should be read: "for any concept x, if for any person u, u thinks that the reference of x exists in reality, provided u understands x, than the reference of x exists in reality".

Generally, the formula (I) should be weaken, for it claims, the reference of any concept exists, provided the concept is understood by nobody. So, the Axiom of the Intelligibility might have been formulated:

$$\forall_x [\forall_u (\mathsf{A}ux \to \mathsf{B}ux) \to (\mathsf{E}x \vee \neg\exists_u \mathsf{A}ux)] \tag{wI}$$

But we have for simplicity excluded such concepts, so we can accept the axiom (I). Actually, having our formalization, one can immediately get the more complicated one, including the concept non being understood, with perfectly identical formal results.

6. Anselm's Argument in AP2

The theory AP2 can now be defined. It is a first-order theory, in the language AP2, with all the specific axioms: (F), (M), (H) and (I). Having construed the theory AP2, we are about to examine some formal properties of the theory and prove Anselm's conclusion (C) to be a theorem of AP2.

6.1. Consistency of AP2

As we have said, AP2 is the first-order theory with the specific axioms: (F), (M), (H) and (I). In the proof of Theorem 1 we have already shown the three first of those axioms to be consistent. However, we now need to prove the consistency result for the whole theory AP2.

Theorem 2. *The theory* AP2 *is consistent.*

Proof. We prove this theorem by an interpretation. Like in the proof of Theorem 1, let $\mathfrak{N}$ be the set of natural numbers, including '0', and let individual variables represent elements of $\mathfrak{N}$. Let also '+' and '<' be symbols in the Peano Arithmetic PA. We interpret the language of AP2 as follows:

AP2	PA
$\mathsf{A}ux$	$u = x$
$\mathsf{B}ux$	$u = x$
$yu\mathsf{C}xv$	$y + u + v < x$
$\mathsf{E}x$	$x = x$

In such an interpretation all the formulas (F), (M), (H) and (I):

$$\exists_x \neg\exists_{yuv}\ y + u + v < x$$
$$\neg\exists_{yuv}\ y + u + v < x \to \exists_w\, w = x$$
$$u = x \wedge v = x \wedge v \neq x \to y + u + v < x$$
$$\forall_u\,(u = x \to u = x) \to x = x$$

are true formulas of PA. □

The consistency result is vital for the analysis being developed. For if the set of Anselm's premises had not been consistent, any statement could have been derived from them (any formula of the language AP2 would have been a theorem of the theory AP2). In such a case it would have not been interesting, the teistic conclusion to be derivable from the premises, for an antitheistic conclusion would have been equally derivable from the premises in question.

6.2. Ratio Anselmi formalized

We now want to analyse the Anselm's reasoning itself. We will prove that the formula (C) is a theorem of AP2. We will build a genuine first-order proof. Let α be any individual variable and let ϕ, ψ, χ be any formulas. We will use the following derivable rules of the first-order classical logic:

$$\frac{\vdash \phi \to \psi}{\vdash \exists_\alpha\, \phi \to \psi} \quad \text{provided that } \alpha \text{ is not a free variable in } \psi \tag{1}$$

$$\frac{\vdash \phi \to \psi}{\vdash \phi \to \forall_\alpha\, \psi} \quad \text{provided that } \alpha \text{ is not a free variable in } \phi \tag{2}$$

$$\frac{\vdash \phi \to \psi \qquad \vdash \exists_\alpha \phi}{\vdash \exists_\alpha (\psi \wedge \phi)} \tag{3}$$

$$\frac{\vdash \forall_\alpha (\phi \to \psi) \qquad \vdash \exists_\alpha (\phi \wedge \chi)}{\vdash \exists_\alpha (\psi \wedge \chi)} \tag{4}$$

$$\frac{\vdash \phi \to (\psi \to \chi) \qquad \vdash \phi \to \psi}{\vdash \phi \to \chi} \tag{5}$$

Furthermore, by virtue of the rule of commutation Com we can change the sequence of the antecedents of any implication, and by virtue of the rule of extensionality ex we can interchange arguments of any equivalence theorem. We also use of modus ponens MP. The theorems of classical first-order logic we assign as $\vdash_{\mathsf{Log}}$, and an application of an inference rule we mark with the sign '×'.

Theorem 3. *The formula* (C) *is a theorem of the theory* AP2.

Proof. First, we formulate the logical axioms of AP2: any correct uniform substitution of any theorem of Log (the first-order classical logic), with any formulas of AP2 is a theorem of the theory AP2. We have also the following specific axioms of AP2: (F), (M), (H) and (I). The formula (C) can be derived from the axioms listed in the following way:

1. $xuCxv \to \exists_{yuv}\, yuCxv$ — by: $\vdash_{\mathsf{Log}}\ \phi(\alpha/\beta) \to \exists_\alpha\, \phi$
2. $Bux \wedge Avx \wedge \neg Bvx \to \exists_{yuv}\, yuCxv$ — by (H), 1 and $(\phi \to \psi), (\psi \to \chi) \vdash_{\mathsf{Log}} (\phi \to \chi)$
3. $(Bux \wedge Avx \wedge \neg Bvx \to \exists_{yuv}\, yuCxv) \to$ $\to [Bux \to (Avx \to (\neg Bvx \to \exists_{yuv}\, yuCxv))]$ — by $\vdash_{\mathsf{Log}}\ (\phi_1 \wedge \phi_2 \wedge \phi_3 \to \psi) \to (\phi_1 \to (\phi_2 \to (\phi_3 \to \psi)))$
4. $Bux \to [Avx \to (\neg Bvx \to \exists_{yuv}\, yuCxv)]$ — by 2, 3 × mp
5. $(\neg Bvx \to \exists_{yuv}\, yuCxv) \equiv (\neg \exists_{yuv}\, yuCxv \to Bvx)$ — by $\vdash_{\mathsf{Log}}\ (\neg\phi \to \psi) \equiv (\neg\psi \to \phi)$
6. $Bux \to [Avx \to (\neg \exists_{yuv}\, yuCxv \to Bvx)]$ — by 5, 4 × ex

7. $\exists_u \, \mathrm{B}ux \rightarrow [\mathrm{A}vx \rightarrow (\neg\exists_{yuv} \, yu\mathrm{C}xv \rightarrow \mathrm{B}vx)]$ — by 6 × (1)
8. $\neg\exists_{yuv} \, yu\mathrm{C}xv \rightarrow [\exists_u \, \mathrm{B}ux \rightarrow (\mathrm{A}vx \rightarrow \mathrm{B}vx)]$ — by 7 × com
9. $\neg\exists_{yuv} \, yu\mathrm{C}xv \rightarrow (\mathrm{A}vx \rightarrow \mathrm{B}vx)$ — by 8, (M) × (5)
10. $\neg\exists_{yuv} \, yu\mathrm{C}xv \rightarrow \forall_v \, (\mathrm{A}vx \rightarrow \mathrm{B}vx)$ — 9 × (2)
11. $\exists_x \, [\forall_v \, (\mathrm{A}vx \rightarrow \mathrm{B}vx) \wedge \neg\exists_{yuv} \, yu\mathrm{C}xv]$ — by 10, (F) × (3)
12. $\exists_x \, (\mathrm{E}x \wedge \neg\exists_{yuv} \, yu\mathrm{C}xv)$ — by 11, (I) × (4)

The row 12 contains the formula (C), which is our formalization of Anselm's theistic conclusion. □

As we have already mentioned on the page 298 a part of the proof presented provides a derivation of the formula (⋆) from the explicitly formulated Anselm's premises. Indeed, notice that the rows 1–11 of the above proof give the derivation of (⋆) from (F), (M) and (H).

Theorem 4. *The formula* (⋆) *is derivable from the formulas* (F), (M) *and* (H) *in first-order logic.*

We have thus shown, Anselm's argument is a valid argument from consistent premises. There is no formal problems concerning the deduction, no logical mistakes. The only point of discussion—as regards the validity—is the hidden premise, the Axiom of the Intelligibility of Being, which has to be observed.

7. A Posteriori Argument

In our analysis—if it is acceptable—Anselm's argument is a deduction from four axioms. The set of axioms has been proved to be consistent, so it has a model, there is a view of world making the theory AP2 true. Generally, we claim, *Ratio Anselmi* to be a valid argument from consistent premises. Beside the standard first-order logic Anselm needs four premises (F), (M), (H) and (I). Those premises state:

- one can understand the concept of God;
- one can think that God really exists;
- to think of anything together with thinking that it really exists, is to think of something greater than to think of the same object without thinking that it exists;

- if one cannot understand a concept without thinking that the reference of the concept really exists, the reference of the concept in question must exist really.

We are now about to discuss in some more detail the view of the world supposed in *Ratio Anselmi*. We now ask of the soundness of Anselm's argument, we ask what the world would have to be like to make all the premises of *Ratio Anselmi* true.

We want to suggest, *Ratio Anselmi* does not seem to be an ontological (*a priori*) argument. We claim that in Anselm's argument the conclusion that God exists is not derived from the concept of God as a being than which no greater can be conceived. The conclusion that God exists is derived from four statements concerning the structure of the world and its relationship to the human understanding. God exists here not because he is a being than which no greater can be conceived. God—i.e. the being than which no greater can be conceived—exists, because the world is such as there must exist a being than which no greater can be conceived.

In other words, we suggest that Anselm's argument is no more *a priori* than those of Aquinas's, and other argument, e.g. entropic ones. For instance, in Aquinas's argument *ex motu* there is a concept of God, who is said to be a first mover, put in motion by no other. One can observe a close analogy between Aquina's words: "*Ergo necesse est devenire ad aliquod primum movens, quod a nullo movetur, et hoc omnes intelligunt Deum*" [2, pp. I, 1, 3] and the words of Anselm: "*Sic ergo vere est aliquid quo maius cogitari non potest, ut nec cogitari possit non esse. Et hoc es tu, domine deus noster*" [1, p. 103] or "*Et quidem credimus te* [*domine deus*] *esse aliquid quo nihil maius cogitari possit*" [1, p. 101]. In both cases—Anselm's and Aquinas's—there is a concept of God. Of course, one can hardly discuss the problem of God without having any concept of him. However, there is no derivation of God's existence from the concept, *Ratio Anselmi* is not a piece of reasoning of a kind

> God is a being that has all perfections;
> existence is a perfection;
> ―――――――――――――――――
> God has existence;

like the famous "*si deus est deus, deus est*" by Bonaventure or some similar pieces of reasoning of Descartes. The criticism of Gaunilo, Aquinas or Kant concern such arguments. For, according to the criticism, one could define any predicate P by a predicate Q and the existence condition:

$$Px \stackrel{\text{df}}{\longleftrightarrow} Qx \wedge x \text{ exists}$$

and then easily prove the existence of P. We think, our formalization shows that Anselm's argument is not an argument of the kind, and so such criticism fails to concern *Ratio Anselmi*.

We want to emphasise, Anselm's Argument would have been an argument of that kind, it would have been a derivation of Gods existence from a concept of God, only if Anselm had claimed that the negation of his theistic conclusion, i.e. the statement:

for every being x
there can be conceived something greater than x

to be self-inconsistent. And Anselm has not done it. He has claimed that statement to be inconsistent with some views concerning the world and the human understanding, formalized in the present paper. In *Ratio Anselmi* there is derivation of the existence of God—understood with the concept being discussed—from some suppositions concerning the structure of the world and the relationship between the world and human understanding.

Thinking of Anselm's argument as ontological might have been caused by the way the argument is formulated. Aquinas's arguments could be easily formulated that way as well. For instance, one could begin the argument *ex motu* with the definition:

God is the first mover, put in motion by no other

and continue with the same premises, as in [2, pp. I, 1, 3], to derive the conclusion that God exists. Aquinas's argument such formulated could be accused of being ontological, exactly like Anselm's one.

Anselm's argument is certainly disputable, even highly disputable. However, one should discuss the credibility of Anselm's premises rather than so called derivation the existence from the concept of God. Let it be said once again: *Ratio Anselmi* is a valid argument from consistent premises, concerning the structure of the world and the relationship between the world and human understanding. Consequently *Ratio Anselmi* is not an otological argument in strict meaning. Nor is Anselm's argument an ontological argument in wide meaning, for the premises (at least some of them) are not *a priori* statements. In the next section we recommend to consider the premises (F) and (M) to be empirical statements. So, although premises (H) and (I) may be a kind of ontology, the premises (F) and (M) are based on observation of the world.

8. Anselm's World

Let us now initiate the discussion of Anselm's premises, according to the formalization presented as the theory AP2.

8.1. What is there supposed to exist?

In our formalization *Ratio Anselmi* presupposes, there are concepts and persons (subjects, rational agents, intellects). The objects listed may exemplify some properties and relations. A person can *understand* a concept. A concept's reference may either *exist in reality* or not. And understanding a concept, a person can *think* of the concept's reference *that it exists in reality* or not. These relations are in a sense partially independent, for according to Anselm:

> Aliud enim est rem esse in intellectu, aliud intelligere rem esse. Nam cum pictor praecogitat quae facturus est, habet quidem in intellectu, sed nondum intelligit esse quod nondum fecit. Cum vero iam pinxit, et habet in intellectu et intelligit esse quod iam fecit. [1, p. 101]

According to the words quoted, it is possible for a person to understand a concept regardless whether the concept's reference exists in reality and the person thinks it exists in reality. And it is possible for a person, who understands a concept, to think of the concept's reference that it exist in reality regardless whether the reference actually exists in reality. However, it is not possible to think, that a concept's reference exists in reality without understanding the concept.

One could accept special axioms for the suppositions listed, but we have asurred them, as far as necessary for the formalization, in the construction of the language AP2 itself.

Furthermore, the references of concepts being understood by persons—both existing and non existing in reality, and both thought to exist and not thought to exist in reality references—may be compared, as regards their *greatness* (perfection?). One reference of a concept understood by a person may be considered greater than another. The concepts and persons may here be either the same or different. When a concept of a thing without thinking of the thing to exist, is compared to the same concept but with thinking of its reference to exist, there is a difference between the references in question, not between the concepts. So it seems to be a supposition concerning a real hierarchy of greatnes (perfection?) in being.

We want to emphasize, there is no prohibited supposition of God's existence in our concepts. Many formalizations of *Ratio Anselmi* are claimed to presuppose God's existence by accepting an individual variable or name of God. It is not the case in the AP2 theory. All we need to suppose to exist, are understandable concepts, persons who can understand the concepts and some properties and relations among them: the person's beliefs concerning references of the concepts, the existence and relationships of the refeences.

8.2. Is the Fool a Fool?

After David Hume and Logical Empiricism one should certainly discuss the axiom (F). The axiom claims, one can understand the concept of God. However, there are scholars who find it highly uncertain, if there is any consistent concept of God at all. If there is no such a concept, obviously God is not conceivable. The problem has been observed—but not solved—by Leibniz.

Of course the problem of the axiom (M) is similar. One could claim, although there is a concept of God, one cannot coherently ascribe God existence.

So, Anselm's Argument certainly involves the old discussion of *divinis nominibus*.

On the other hand, one could ask, if the concept of that than which no greater can be conceivable, is in fact understandable. What are the conditions for a concept to be defined and to be understandable? The respective concepts are in Anselm's philosophy deeply underworked. The premises (F) and (M) seem so to be *empirical statements*. It may be observed, that some persons seem to understand concepts in question or ascribe existence to their references. So Anselm's argument cannot be considered as ontological one.

8.3. Is the God God?

Many theologians, when studying theistic arguments, say: in the argument one has certainly proved something, however, it has hardly anything to do with God.

Both Anselm and Aquinas, and many other scholars, who provided theistic arguments, having elaborated a brilliant piece of reasoning, so easy finish: "and this everyone understands to be God" [2, pp. I, 1, 3] (cf. [1, p. 101–103]).

So is *Ratio Anselmi* obviously involved in the problem of the relationship between philosophical absolute beings and God of the Revelation. Perhaps Anselm's involvement is especially deep, because the concept of the being than which no greater can be conceived is so underworked. One should necessarily ask, what the word 'greater' means here actually.

8.4. Is the being hierarchical?

We have already been complaining on the word 'greater'. Actually, we find that word absolutely essential for *Ratio Anselmi*. According to the axiom (H), references—objects of human understanding—can be ascribed with a qualification of *greatness*, and provided the ascription of existence is the only difference between two objects of human understanding, the one thought to exist is always greater than the other. Such hierarchy is vital for Anselm's argument.

Is there such a hierarchy understandable? Is it justified? Can it provide good tools for the concept of God be constructed? Why should one (say a believer) suppose, the top point of a hierarchy of that kind is the author of Revelation?

We find the axiom (H) the very Anselm's premise, that is most refutable. Thomistic philosophers could probably refute the idea to compare existing and non existing objects. They could hardly accept such objects as members of one, objective hierarchy of being. Perhaps Kant could claim the same. A contemporary philosopher could question the idea of a hierarchy of greatness (perfection?) at all.

Furthermore, we claim the premise (H) is false. According to (H), if one concept is understood by two persons u and v, and the person u thinks, the reference of their concept exist in reality, while the person v does not think that, then the person u thinks of something greater than that, the person v thinks of.

First, we want to emphasize, the concept of *greatness* here involved is most high obscure and underworked, although it is quite common for metaphysics of some sort. We do not actually know, what the greatness in question is. This is enough to refute the premise (H) already.

However, supposing we were provided with a good concept of greatness (perfection), understood as a kind of inner feature of a being, we should refute the premise in question as well. For—we recommend to claim—if one concept is understood by two persons u and v, the persons in question necessarily think of *the same thing*, regardless of their beliefs concerning the existence of the reference of their concept. Consequently, neither of the two objects (as we have said, identical objects) can be greater (or in any sense different) than the other. If such objects could differ (e.g. if one of them was greater than the other), then the questioning and arguing on any object's existence would not be possible. For, suppose, two persons—a theistic person u and an antitheistic person v—discuss whether there exists God. If the concept of God understood by u (so, with a belief the God to exist) was a concept of something greater (or in any sense different) than the reference of the same concept of God understood by v (without the belief mentioned), those persons u and v would discuss about two different objects. For none object could be greater (nor in any sense different) from itself ($\neg\exists_x \; x \neq x$). Analogically, when two logicians discuss about, say, the law of excluded middle, and one of them thinks, the law of excluded middle is true and holds in reality, and the other does not, they necessarily think of the same law of excluded middle. If the law of excluded middle that holds in reality had been something different (e.g. greater) than the law of excluded middle that exists only in human understanding, the discussion would have not been possible.

So, we have found in *Ratio Anselmi* a premise, that is not credible. We have so to claim, that the Anselm's argument is not sound—although we have found it valid—and it fails to prove its conclusion. Our analysis could end here, however, we want nevertheless to sketch the debate on the last axiom of the AP2 theory yet.

8.5. A case of rationalism

The hidden premise (I) should be discussed as well. Anselm must suppose, there is a relationship between human understanding and the real world, such as the existence of an object, than cannot be consistently thought not to exist, is guaranteed. That, of course, is a kind of traditional *intelligibilitas entis*.

The axiom (I) could be refuted by an antirationalist or by a kantian philosopher, who doubt in intelligibility of the *noumena*. In fact, the axiom (I) could be also refuted on the ground of intuitionistic logic, for it seems to imply a possibility of non constructive proofs of existence.

The axiom (I) seems to be connected to the here involved concept of person (intellect). If any person should be considered, the axiom (I) is an instance of the *Lincoln Principle*: one could not deceive everyone. So, if everyone thinks that ϕ, than it is the case that ϕ. The supposition mentioned is accepted in many epistemic logics, nonetheless it is highly disputable. One could always imagine a non rational person, that having understood the proof presented, refutes to ascribe God existence—as a real instance of the biblical Fool. Perhaps the axiom (I) is to be interpreted such that the rational persons would be considered exclusively. But what kind of rationality should be considered? Logical? Monotonic or non monotonic? It should be definitely discussed.

9. Conclusion

We have been arguing *Ratio Anselmi* is a valid, logically correct, ethymematic argument from consistent set of premises. There is no logical mistake in Anselm's reasoning. The argument involves some sort of ontology, expressed in formulas (H) and (I), and some *a posteriori* premises, i.e. (F) and (M). Those premises are highly disputable, but far from absurd. However, Anselm's argument is not sound. We find at least the premise (H) false. So, *Ratio Anselmi* fails to prove its conclusion, the conclusion derived correctly from the premises.

Having submitted all that criticism, we want to make it clear, that we nevertheless consider Anselm's argument to be a stroke of genius rather than a hilarious mistake. The profound and accurate debate of fundamental questions should be definitely developed regardless of the hope (or its lack) to gain definite solutions.

References

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[2] Thomas Aquinas, *Summa theologica*, Textum Leonianum, Romae 1888 (we quote Aquinas traditionally, providing the number of the *liber*, the *questio* and the *articulus* instead of the page).

[3] Tkaczyk, M., "Próba formalizacji wnioskowania zawartego w II rozdziale *Proslogionu* Anzelma z Canterbury" ("A Formalization of the argument presented in *Proslogion* Ch. II by Anselm of Canterbury"), *Ruch Filozoficzny* 58 (2006), vol. 2, 237–247.

[4] Tkaczyk, M., Rev.: K. Świrydowicz, *Podstawy Logiki modalnej*. In: *Roczniki Filozoficzne* 54 (2006), vol. 1, 283–287.

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