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# GARETH EVANS'S ARGUMENT AGAINST VAGUE IDENTITY* 


#### Abstract

In the paper Evans's argument concerning indeterminate identity statements is presented and discussed. Evans's paper in which he formulated his argument is one of the most frequently discussed papers concerning identity. There are serious doubts concerning what Evans wanted to prove by his argument. Theorists have proposed two competing and incompatible interpretations. According to some, Evans purposefully constructed an invalid argument in order to demonstrate that the vague objects view cannot diagnose the fallacy and is therefore untenable. According to others, Evans wanted to formulate a (valid) argument to the effect that there cannot be vague identity statements whose vagueness is due solely to the existence of vague objects. As it has been argued, if it is the former interpretation which is correct, than the argument really is invalid, but it is doubtful whether it achieves its aim. It might be claimed that "the vague objects view" it refutes is not the view that most vague objects theorists hold. The main part of the paper is devoted to the second interpretation and the discussions concerning the validity of the argument on this interpretation. It appears that the vague objects theorist is in a position to object to the validity of every single step of the proof.


## 1. Introduction

Anyone who wishes to investigate the question of vague identity and vague objects has to face G. Evans's argument. His one-page article entitled "Can

[^0]There Be Vague Objects?" ([3]) is usually regarded as a reductio ad absurdum of the claim that identity between objects may be a vague matter. If what the argument proves is indeed that vague identity is an inconsistent notion, then there is not much point in undertaking the study of it. Therefore, any investigation devoted to vague identity has to start with the exploration of the meaning and consequences of Evans's argument. Before exposing oneself to the dangers of 'the quicksand' (as M. Tye calls the intricacies of the issue of vagueness (Cf. [17]) one ought to make sure that one is not embarking on a venture that is bound to end in a contradiction.

The above mentioned article is no doubt one of the most discussed papers concerning identity to have been published within the last 30 years. It has been criticised both for leaving too many things unspoken and for saying too much (Cf. [9], p. 129). In the first place there is no common agreement as to whether Evans's argument is valid. Moreover, those critics who accept its validity cannot agree as to whether or not it proves what Evans intended it to prove. According to some theorists, the argument does not prove anything interesting, according to others it proves too much (Cf. [15], p. 82). ${ }^{1}$

The aim of this paper is to present the main problems surrounding Evans's argument and its interpretations.

## 2. The interpretations of Evans's argument

Evans's argument goes as follows:
"Let " $a$ " and " $b$ " be singular terms such that the sentence " $a=b$ " is of indeterminate truth value, and let us allow for the expression of the idea of indeterminacy by the sentential operator " $\nabla$ ".

Then we have:
(1) $\nabla(a=b)$
(1) reports a fact about $b$ which we may express by ascribing it the property " $\lambda x[\nabla(x=a)] ":$
(2) $\lambda x[\nabla(x=a)] b$, But we have:
(3) $\neg \nabla(a=a)$ and hence:
(4) $\neg \lambda x[\nabla(x=a)] a$.

But by Leibniz's Law, we may derive from (2) and (4):
(5) $\neg(a=b)$

[^1]contradicting the assumption, with which we began, that the identity statement " $a=b$ " is of indeterminate truth value.

If "Indefinitely" and its dual, "Definitely" (" $\Delta$ ") generate a modal logic as strong as S5, (1)-(4) and, presumably, Leibniz's Law, may each be strengthened with a "Definitely" prefix, enabling us to derive:
(5) $\Delta \neg(a=b)$
which is straightforwardly inconsistent with (1)" ([3], p. 208).
Two different interpretations of what Evans took this argument to prove have been offered. According to the first one, and this is apparently the one that Evans really intended (See [9], p. 129), the argument shows that the vague-objects view is untenable. ${ }^{2}$ It allegedly demonstrates that the assumption that there are vague identity statements leads to contradiction. It is obvious, however, that there are such statements. Take, for instance, the identity statement "Princeton $=$ Princeton Borough" ([9], p. 128). Most theorists will agree that it is indeterminate in truth value, for no determinate answer to the question of whether Princeton is identical to Princeton Borough is acceptable. Therefore, the only conclusion one can draw is that Evans's argument, which is a reductio of the existence of such statements, must be fallacious. At least one step of the proof must be illegitimate. The idea is that Evans intended his argument to fail in order to demonstrate something. Namely, he wanted to show that only those theorists who take vagueness to be a semantic phenomenon are able to spot the fallacy. The view according to which vagueness is linguistic can diagnose the fallacy in the proof: the step from (1) to (2) is invalid, because it commits a scope fallacy. One cannot infer:
(2) $\lambda x[\nabla(x=a)] b$
from
(1) $\nabla(a=b)$,
unless " $b$ " is a precise designator. ${ }^{3}$ If " $b$ " is vague, it does not single out a unique object determinately. In this case, contrary to what Evans has argued (or rather has pretended to argue), (1) - in which such a vague designator " $b$ " features - does not report any facts about $b$. Moreover the statement (1) can

[^2]be true, even if (2) is false. Since " $\nabla(a=b)$ " does not entail " $\lambda x[\nabla(x=a)] b$ ", the argument is invalid.

In contrast, the view according to which vagueness is in the world, and not in language, cannot explain why the proof is fallacious. According to this theory " $a$ " and " $b$ " are precise designators denoting vague objects. So, on this view one cannot block the step from (1) to (2). If " $b$ " is precise, the inference (1) - (2) is valid. Moreover, it has been argued that the vague-objects theorist has no reason to object to the other steps of Evans's argument, either. From his point of view all the other steps seem valid. In consequence, vague-objects theorists are forced to accept the evidently false conclusion that there cannot be vague identity statements. As D. Lewis puts it:
"In fact, the vague-objects view does not afford any diagnosis of the fallacy, so it is stuck with the unwelcome proof of an absurd conclusion" ([9], p. 129).

Thus, the vague-objects view, within the framework of which one is not able to refute a clearly absurd claim to the effect that vague identity statements do not exist, should better be rejected. The claim is that the vagueness-in-language view is the only view which can deal with Evans's argument and therefore it should be accepted as the correct theory of vagueness.

This is the interpretation proposed by D. Lewis, which has been endorsed by Evans as a correct explanation of what he was trying to do in his paper ([9], p. 130). ${ }^{4}$ B. Garrett points out, however, that the conclusion that the vague-objects view cannot offer any explanation of the fallacy is unwarranted. The vague-objects theorists do not claim that all vagueness is in the world and none in the language. And they do not support an implausible claim that all vague identity statements are vague as a result of the vagueness in the world. On the contrary, they argue that both the world and language are vague, and identity sentences can be indeterminate in truth value because of the vagueness of the designators as well as because of the vagueness of the objects. In particular, they do not hold that all designators have to be precise. Hence, the scope fallacy diagnosis is as available for them as it is for the vagueness-in-language theorists (Cf. [5], p. 131).

So, if the above interpretation of Evans's argument is correct, then the argument is of little significance. It is intended to prove that the vague-objects

[^3]view is untenable, but it wrongly characterises the view it is supposed to refute. Although it does indeed show that the view that all vagueness is in the world is committed to the absurd claim that there are no indeterminateidentity statements, such a view is in general (i.e. independently of Evans's argument) quite implausible. The vague-objects theorists are people who think that vagueness afflicts not only language but the world as well. By no means do they want to attribute all vagueness to the world. That our language is vague is a fact accepted probably by any philosopher whatsoever, whether he is an ontic-vagueness or a linguistic-vagueness theorist. In particular, saddling the vague-objects theorists with the claim that singular terms such as "Princeton" and "Princeton Borough" are precise is simply unfair.

According to the other interpretation Evans's argument is not concerned with all vague-identity statements but only with those vague-identity statements whose vagueness is a result of vagueness in the world. It is obvious that ontic vagueness is not the only possible source of indeterminacy. The statement " $a=b$ " may not have any determinate truth value as a result of one or both singular terms, " $a$ " and " $b$ ", being imprecise designators. The fact that the statement "Princeton $=$ Princeton Borough" is indeterminate does not indicate that Princeton is a vague object, for its indeterminacy is caused by the name "Princeton" not having a precise designation. Thus, the assumption that a given identity statement lacks determinate truth value does not entitle one to the claim that there are vague objects in the world responsible for the indeterminacy of that statement. In order to interpret the argument as an argument which purports to say something about ontic vagueness, the assumption to the effect that singular terms flanking the identity sign are precise designators must be added. Only by claiming that the statement " $a=b$ " is indeterminate and that " $a$ " and " $b$ " are precise designators can one hope to capture the idea of ontic vagueness. ${ }^{5}$ On this interpretation, Evans's argument seeks to prove that there cannot be vague identity statements whose vagueness is due solely to the existence of vague objects; i.e. it is supposed to establish that there cannot be indeterminateidentity statements " $a=b$ " that contain only precise designators " $a$ " and " $b$ " and the identity sign (Cf. [5], p. 130). ${ }^{6}$ If such a statement composed entirely of precise terms existed and nevertheless were vague, its vagueness would have to be a result of the vagueness in the world.

[^4]Thus, the argument has it that if there are indeterminate-identity statements " $a=b$ ", where " $a$ " and " $b$ " are precise, then there is ontic vagueness. The argument assumes for reductio that there is such a sentence and arrives at a contradiction. Hence, the conclusion is that the statement consisting of two precise designators and the identity sign cannot be indeterminate in truth value. By proving that there cannot be such a statement, the argument allegedly proves that there is no vagueness in the world - i.e. that all vagueness is linguistic. Whether or not the argument with a tacit assumption that " $a$ " and " $b$ " are precise designators is valid is a contentious matter, the main reason being that it is not clear which logic Evans assumes for his argument.

The differences between the two interpretations can be summarised in the following way. On both interpretations the argument attempts to prove that there cannot be true vague identity-statements " $\nabla(a=b)$ ". Each interpretation assumes that Evans has left an important assumption unspoken an assumption without which the proof cannot properly be understood. On the first interpretation the tacit assumption is that it is obvious that indeterminate identity-statements statements do exist, so the proof as a whole is in fact a reductio of an obvious truth - i.e. a reductio that is obviously fallacious. People who believe in the existence of vague objects and think in addition that every such object is designated by a precise term, cannot diagnose the fallacy and are forced to endorse the proof as a bona fide reductio. They have to argue - contrary to facts - that there are no indeterminate identity-statements. On this interpretation Evans succeeds in showing that a vague- objects view combined with the claim that all singular terms naming vague objects (and all non-vague objects too) are precise is untenable.

On the second interpretation the tacit assumption is that the singular terms " $a$ " and " $b$ " are precise designators. This interpretation does not saddle the vague-objects theorists with an implausible claim that all vague objects are precisely designated. The proof is a bona fide reductio of the claim that " $\nabla(a=b)$ ", where terms " $a$ " and " $b$ " are precise, can be true. Whether or not it succeeds is an open matter; in any case it is not intended as a clearly fallacious proof. If it works, then also on this interpretation Evans succeeds in showing that a vague- objects view combined with the claim that singular terms naming vague objects are precise is untenable. Thus, if the proof is valid on the second interpretation, on each interpretation one of the aims of the argument is to show that vague objects cannot be precisely designated, but the means of achieving that aim are interpreted differently. While on the first interpretation Evans succeeds by means of an obviously fallacious proof, on the second interpretation, if he succeeds, it is in virtue of using a valid proof. The additional difference is that the first interpretation requires
that we take the vague-objects theorists to be the people who put all the vagueness in the world and none in language. Such a characterisation seems unfaithful to the facts.

Evans precedes his argument by the following introduction.
"It is sometimes said that the world itself can be vague. Rather than vagueness being a deficiency in our mode of describing the world, it would then be a necessary feature of any true description of it. It is also said that amongst the statements which may not have a determinate truth value as a result of their vagueness are identity statements. Combining these two views we would arrive at the idea that the world might contain certain objects about which it is a fact that they have fuzzy boundaries. But is this idea coherent?"
This introduction is followed by the proof quoted above, in which Evans from the assumption that it is indeterminate whether $a$ is identical to $b$ derives a contradiction. It is then hard to resist the temptation to treat that proof as a negative answer to the question he asked in the last sentence of the introduction - i.e. as a proof that the idea that there are vague objects is in fact incoherent. Although we know from Lewis that this is not the reading Evans had in mind, ${ }^{7}$ it certainly is the interpretation that suggest itself most vividly to the reader of Evans's paper. In other words, it is at least a justified interpretation. Moreover, it is not without significance, that it is the interpretation that does justice to the vague-objects theorists.

If the first interpretation is the one that is correct, then it is obvious that the proof fails as intended (because of the obvious fallacy in the step (1)-(2)) and the aim of the paper is achieved. There is not much point in investigating other steps of the proof. If it is the second interpretation which is adequate, however, then all the steps are important. Recall that on this interpretation someone who wants to show that the vague-objects view is untenable has to show that the proof is correct. The step (1)- (2) is no longer obviously fallacious on that interpretation, because of the assumption that " $a$ " and " $b$ " are precise. Other steps of the proof are interesting in their own right and are certainly worth investigating. In what follows we will adopt the second interpretation with the aim of examining the possible fallacies of the argument.

[^5]In general, it is far from clear how the argument is supposed to work. First of all, although Evans introduces " $\nabla$ " as an operator which expresses the indeterminacy of truth value, he also assumes that it generates a modal logic. So, the question arises what the correct interpretation of delta operators, " $\nabla$ " and " $\Delta$ ", is. Are they truth value indicating operators or modal operators? Moreover, doubt has been cast upon the "Definitely" prefix. It is not clear whether it can be treated as a dual of " $\nabla$ " and it seems that even if it can, together they cannot generate so strong a logic as Evans wanted. Secondly, since the steps from (1) to (2) and from (3) to (4) involve property abstraction in contexts governed by delta operators their validity can be questioned. Thirdly, step (3), which is assumed as trivial, in the presence of vague objects becomes controversial. Fourthly, it is doubtful whether Leibniz's Law (LL) can be used in order to derive (5) from (4). Fifthly, it is by no means obvious that (5) contradicts (1). And finally the proposed strengthening of (1) - (4) and LL with " $\Delta$ " is a very contentious operation.

There is no place here to discuss all these difficulties. Therefore in what follows we will take a closer look at the problem of the correct interpretation of delta operators and the problem of definite self-identity, while other difficulties will be merely outlined.

## 3. The interpretations of "Indefinitely" and "Definitely" operators

It is not entirely clear how Evans meant his delta operators to work. On the one hand, as we have seen, he begins his proof in the following way: "Let " $a$ " and " $b$ " be singular terms such that the sentence " $a=b$ " is of indeterminate truth value, and let us allow for the expression of the idea of indeterminacy by the sentential operator " $\nabla$ "" (my emphasis). This quote suggests that "Indefinitely" and "Definitely" should be regarded as operators that indicate the truth value of the statement to which they are attached (Cf. [14], p. 482). On the other hand, Evans's later tentative claim that ""Indefinitely" and its dual, "Definitely" (" $\Delta$ ") generate a modal logic as strong as S 5 " suggests a reading according to which "Indefinitely" and "Definitely" are modal operators.

According to the "truth-value indicators" reading " $\Delta p$ " says that " $p$ " has one of the determinate truth values, whereas " $\nabla p$ " says that the truth value of " $p$ " is indeterminate. On this reading " $\nabla p$ " is true iff $p$ is neither true nor false. Hence, on this interpretation "the sentence " $a=b$ " is of indeterminate truth value" means that " $a=b$ " is neither true nor false, but has some third truth value; the operator " $\nabla$ " expresses that fact. The sentence " $\nabla(a=b)$ "
is taken to be a description of the actual world, which expresses the fact that the statement " $a=b$ " has no determinate truth value in this world.

Two contrary opinions can be found in the literature. On the one hand, it is often argued (Cf. e.g. [17]) that in order to express indeterminacy at least three truth values are needed: one needs some kind of 'status' which would reflect the fact that there is no fact of the matter as to whether a given statement is (definitely) true, or (definitely) false (no matter whether it will be another value, truth-value glut or truth- value gap). This third "status" is marked by " $\nabla$ ". On the other hand, it has also been claimed that indeterminacy should not be expressed by a sentential operator like " $\nabla$ ". E. J. Lowe argues that if 'indeterminacy' is regarded as 'there being no fact of the matter' then it is wrong to treat the lack of any objective fact of the matter determining the truth value of a sentence " $a=b$ " as itself being an objective fact of the matter which can be reported by a true sentence, " $\nabla(a=b)$ " ([10], p. 112).

It seems, however, that if indeterminacy is to be a 'genuine' ontological possibility, then one needs some means of expressing it. If there is no fact of the matter as to whether $a$ and $b$ are identical, then it is clearly neither (determinately) true nor (determinately) false that they are identical. It is then tempting to ascribe some third 'status' to the statement " $a=b$ ". A person who agrees with Lowe that " $\nabla$ " should not be used as a means of expressing the lack of any objective fact of the matter, deprives himself of the possibility of talking about worldly indeterminacy.

The "truth value indicators" reading takes only the first of the abovequoted Evans's claims into account. If one takes the operator " $\nabla$ " to be the operator that indicates the third truth value that sentences can have and the logic in which Evans's argument operates to be a 'common' (i.e. nonmodal) three-valued logic, then one is forced to ignore the claim concerning the generation of modal logic and regard it as a slip on Evans's part.

According to the alternative reading " $\nabla$ " and " $\Delta$ " are modal operators. J. F. Pelletier interprets them as operators attached to statements that already possess determinate truth value (Cf. [14], p. 482). The statement " $p$ " is either true or false at any given world and the statement " $\Delta p$ " says that it possesses the same truth value in all possible worlds. The statement " $\nabla p$ " orders one to look at some related worlds to check whether " $p$ " is true or false there, before ascribing " $\nabla p$ " a truth value at our world. On such an interpretation " $p$ " does not follow from " $\Delta p$ ", for " $\Delta p$ " may be true in virtue of " $p$ " being false in all worlds.

This interpretation is not an adequate interpretation for the aim that Evans wanted to achieve, however. Evans's argument is supposed to deal
with statements which may not have a determinate truth value as a result of their vagueness. The starting point of his reasoning is the assumption that the sentence " $a=b$ " is indeterminate in truth value and " $\nabla$ " is introduced as a device expressing this indeterminacy. In contrast, on the modal interpretation presented above each statement already has a determinate truth value before a modal operator is attached to it. Although one may not know what the truth values of the statements " $\Delta p$ " and " $\nabla p$ " are, one does know that " $p$ " has a determinate truth value at each world. And of course modal operators, interpreted in this way, do not express any indeterminacy. Moreover, it seems that that modal interpretation does not capture the intuitions connected with ontic vagueness. As we have seen, Evans's argument can be taken to prove that "the idea that the world might contain certain objects about which it is a fact that they have fuzzy boundaries" is incoherent. If we take the "Indefinitely" operator to be a modal operator in the above sense, then it may say nothing in particular about the real world. The actual world being vague or not being vague makes no difference to the truth value of the sentence " $\nabla p$ ". Hence, in particular the truth or falsity of the sentence " $\nabla(a=b)$ " has nothing to do with vague objects at our world and the argument to the effect that that sentence cannot be true, has no bearing whatsoever on the answer to the question whether there can be vague objects in our world. ${ }^{8}$

Thus, the modal reading presented above is a non-starter for the indeterminate identity theorist. There is however another modal interpretation of " $\Delta$ " and " $\nabla$ ": according to it they are modal operators, but instead of ranging over possible worlds they range over admissible precisifications. Thus, " $\Delta p$ " will be true if " $p$ " is either true on all admissible precisifications or false on all admissible precisifications, whereas " $\nabla p$ " will be true if " $p$ " is true on some, and false on some admissible precisifications. ${ }^{9}$ Such a reading is reminiscent of the method used by the supervaluationists (see e.g. [4] and [19]) and is much more compatible with intuitions concerning indeterminacy. Evans's assumption that the sentence " $a=b$ " is of indeterminate truth value should, on this reading, be interpreted as saying that " $a=b$ " has opposite truth values on different admissible precisifications, and that fact we express by the " $\nabla$ " operator. Since indeterminacy is to be a worldly phenomenon, the relevant precisifications must be precisifications of the (vague) state of affairs

[^6]corresponding to " $a=b$ ", and not of the statement " $a=b$ ". ${ }^{10}$ The objects $a$ and $b$ are vague objects, which can be precisified, and the truth value of the precise statement " $a=b$ " changes depending on the objects-precisification in which it is currently evaluated. ${ }^{11}$

It is worth noticing that the supervaluationists' reading of " $\Delta$ " and " $\nabla$ " seems to be compatible with both remarks concerning delta operators made by Evans. As we have just seen on this reading " $\Delta$ " and " $\nabla$ " are modal operators ranging over admissible precisifications. However, it is also true that they indicate a truth value of the sentence to which they are attached. Moreover, K. Fine, the 'father' of supervaluationism, claimed that the set of valid formulas in a language with a definitely operator was given by the modal system S5. A model for a language with " $\Delta$ " and " $\nabla$ " might be thus considered analogous to that for S5 (Cf. [4], p. 290; [19], p. 149), which seems to correspond to Evans's intentions.

However, as T. Williamson has pointed out, a model for S 5 is inconsistent with the possibility of higher-order vagueness. In such a model T schema $[\Delta p \rightarrow p]$, S4 schema $[\Delta p \rightarrow \Delta \Delta p]$ and S5 schema $[\nabla p \rightarrow \Delta \nabla p]$ are all valid. In order to take higher- order vagueness into account one should introduce the relation of the admissibility between precisifications, allowing for the possibility that it might be indeterminate whether something is determinate. Supervaluationists have recognized this and introduced the relevant relation into their framework. However, now the analogy with S 5 system is inadequate, for neither schema S 4 nor schema S 5 are valid. It seems that S5 and even S4 are both too strong to hold in a logic which is to take into account higher-order indeterminacy.

So far we have ignored Evans's claim that delta operators are duals. Interpreting " $\nabla$ " and " $\Delta$ " as duals seems to commit one to the following definition (see [13], p. 416nn.):
(Def $\nabla$ )

$$
\nabla A \leftrightarrow \neg \Delta \neg A .
$$

But this definition is equivalent to " $\neg \nabla A \leftrightarrow \Delta \neg A$ " and this cannot be a valid law of indeterminacy (Cf. [6], p. 346). It works for the right-to-left

[^7]reading of biconditional, but the left-to-right reading amounts to the claim that if it is not the case that $\nabla(a=b)$, then it is determinate that $\neg(a=b)$. This cannot be true, since in the case in which it is determinately the case that $a=b$, the antecedent is true and the consequent is false. From the fact that a given statement is not indeterminate it does not follow that its negation is determinate. Thus, it seems that delta operators cannot be duals after all.

There is also another argument against the duality of delta operators ([11], p. 97). If delta operators are duals and $\nabla A={ }_{\mathrm{df}} \neg \Delta \neg A$, then Evans's step

$$
\begin{equation*}
\neg \nabla(a=a) \tag{3}
\end{equation*}
$$

becomes

$$
\begin{equation*}
\neg \neg \Delta \neg(a=a) \tag{3’}
\end{equation*}
$$

which is equivalent to the absurd

$$
\begin{equation*}
\Delta \neg(a=a) .{ }^{12} \tag{3"}
\end{equation*}
$$

Clearly, on the supervaluationist reading " $\nabla$ " and " $\Delta$ " are not duals. " $\nabla A$ " cannot be defined as " $\neg \Delta \neg A$ ", for although in the supervaluationistic logic, if it is indeterminate whether $p$ (i.e. " $p$ " has different truth values on different precisifications), then it is not the case that determinately $\neg p$, but not vice versa. Determinately $\neg p$ may be false, because determinately $p$ is true. We seem to have the following relations between " $\nabla$ " and " $\Delta$ " in this logic: $\nabla A \leftrightarrow \nabla \neg A ; \nabla A \rightarrow \neg \Delta A ; \nabla A \rightarrow \neg \Delta \neg A$.

Therefore, it seems that although the supervaluationist logic seems to conform to some of the Evans's claims (namely the claim that delta operators indicate truth value of sentences to which they are attached and the claim that they generate a modal logic), it does not satisfy all Evans's requirements. The logic that " $\nabla$ " and " $\Delta$ " generate is not as strong as S5 (and moreover it appears that it should not be so strong, because if it were, it would not be an adequate logic for the phenomenon of vagueness) and the operators are not duals.

[^8]
## 4. Step (3), i.e. definite self identity

Probably no reasonable theorist would claim that there are objects that are not identical to themselves. The reflexivity of identity, the claim that everything is identical with itself, is usually considered as a part of the definition of what identity is. One could argue, however, that there is a difference between, for instance, $a$ being identical with itself and $a$ being identical to $a$.

Thus, one can claim, for instance, that (3) $\neg \nabla(a=a)$ is not true in virtue of $a$ being a vague object. It may seem that if $a$ is a vague object then it may also be vague whether it is identical with $a$. So, although it is determinately true that $a$ is self-identical, it is indeterminate whether $a$ is identical to $a$. To this it could be responded that even if $a$ is vague, then one still can obtain precise identity by matching $a$ with $a$. $a$ corresponds precisely to $a$, for "[a]ll their vagueness matches exactly" ([18], p. 175; see also [5], p. 132n.). Moreover, this reasoning clearly multiplies properties: each object $x$ will possess two (distinct) properties: one of being self-identical and another of being identical to $x$.

This last claim is contested by Copeland. He argues against distinguishing those two properties and claims moreover that the fact that such properties are not distinct makes Evans's argument invalid. Copeland argues that " i$] \mathrm{t}$ is not as though there are two different properties that $a$ has, the property of being determinately self identical and the property of being determinately identical to $a "([2]$, p. 88). He considers two formulas:

$$
\neg \lambda x[\Delta(x=a)] b
$$

and

$$
\begin{equation*}
\lambda x[\Delta(x=a)] a \tag{4’}
\end{equation*}
$$

and argues that because in the case of $a$, the property of being determinately identical to $a$ is the same as the property of being identical to itself, (4') says in fact that $a$ has the property of being self-identical. Since $b$ is also selfidentical, (4') does not ascribe to $a$ any property that $b$ does not have. Thus, one may claim that $\left(2^{\prime}\right)$ and $\left(4^{\prime}\right)$ are both true, without maintaining that $a$ has a property which $b$ lacks. Analogous reasoning can be repeated for Evans's steps:

$$
\begin{equation*}
\lambda x[\nabla(x=a)] b \tag{2}
\end{equation*}
$$

and

$$
\begin{equation*}
\neg \lambda x[\nabla(x=a)] a \tag{4}
\end{equation*}
$$

In this case (4) says that $a$ does not have the property of being indeterminately self-identical. However, (2) does not say that $b$ has that property (i.e. the property of being indeterminately self-identical). It says instead that $b$ has the property of being indeterminately identical to $a$. Again, one can claim that both (2) and (4), which says that $a$ does not have the property of being indeterminately self- identical, are true, without holding that $b$ has $a$ property that $a$ lacks. The appearance that it is otherwise, arises, according to Copeland, because of an illegitimate substitution into lambda abstracts containing unbound singular terms. In his words:
"The substitution of $a$ for $y$ and the substitution of $b$ for $y$ in the open sentence $\neg \lambda x[\Delta(x=a)] y$ are not of the same feather. The first substitution produces $a$ statement equivalent in meaning to

$$
\neg \lambda x[\Delta(x=x)] a
$$

but the second does not" ([2], p. 89).
Copeland concludes that (2) and (4) cannot be used to derive (5). To say that $\lambda x[\nabla(x=a)] b$ and $\neg \lambda x[\nabla(x=a)] a$ does not amount to saying that there is $a$ property $F$ such that $b$ has it and $a$ lacks it. Therefore, (2) and (4) cannot be used in the contraposition of Leibniz's Law to prove that $a$ and $b$ are distinct ([2], p. 90). ${ }^{13}$

Thus, Copeland concentrates on the relation between $a$ 's property of being self- identical and $a$ 's property of being identical to $a$ and argues that there is nothing for " $\neg \lambda x[\nabla(x=a)] a$ " to mean other than " $\neg \lambda x[\nabla(x=x)] a$ ", for there is only one property at issue, namely the property of being indeterminately self-identical (Cf. [2], p. 89). Since $b$ also does not have that property, one cannot conclude that $b$ and $a$ are determinately dissimilar. The fact that " $\neg \lambda x[\nabla(x=a)] a$ " expresses $a$ 's property of not being indeterminately self-identical explains why (4) does not deny that $a$ has the property that (2) attributes to $b$. For although (2) ascribes to $b$ the property of being indeterminately identical to $a$, this latter property is not the property that (4) is about.

In Evans's proof one tries to derive the conclusion that $\neg(a=b)$, from the premises which say that $a$ is not indeterminately self-identical, while $b$ is indeterminately identical to $a$. Copeland's argument has it however, that these two premises together cannot be used as the premise for the contrapositive

[^9]of Leibniz's Law (i.e. $\exists F(F b \& \neg F a)$ ), which would entitle the derivation of " $\neg(a=b)$ ". Since, in the case of $a$, " $\neg \lambda x[\nabla(x=a)]$ " and " $\neg \lambda x[\nabla(x=x)]$ " express one and the same property, there is only one property to be attributed to $a$, namely the property of being determinately self-identical. Although one can express that property in two ways, it should not mislead one into thinking that there are two properties corresponding to these ways.

As we have seen Copeland criticizes the validity of the inference from (2) and (4) to (5). Lowe, on the other hand, claims that it is the inference from (3) to (4) which is invalid. First, he concentrates on two properties: " $\lambda x[\nabla(x=a)]$ " and " $\lambda x[\nabla(x=b)]$ ". Let us agree that " $\lambda x[\nabla(x=a)]$ " expresses $a$ 'genuine' property, which $b$ possesses. Then, since our assumption was that $a$ is indeterminately identical to $b, a$ must possess the symmetrical property $\lambda x[\nabla(x=b)]$. If, however, there is no objective fact of the matter as to whether or not $a$ is identical with $b$, then the property " $\lambda x[\nabla(x=a)]$ " is not determinately different from the property " $\lambda x[\nabla(x=b)]$ ", because these properties differ only by permutation of " $a$ " and " $b$ ". In other words, since $a$ is indeterminately identical to $b$, the property of being indeterminately identical to $a$ is indeterminately identical to the property of being indeterminately identical to $b$. Thus, the possession by $b$ of the property $\lambda x[\nabla(x=a)]$ cannot determinately differentiate $b$ from $a$, since that property itself is not determinately different from the property $\lambda x[\nabla(x=b)]$ possessed by $a$. Since $\lambda x[\nabla(x=b)]$ and $\lambda x[\nabla(x=a)]$ are not determinately different, one cannot (determinately) deny that $a$ possesses the property $\lambda x[\nabla(x=a)]$ ([10], p. 114).

This argument is quite compelling. It seems intuitive that if $b$ is indeterminately identical to $a$, then the property of being indeterminately identical to $a$ possessed by $b$ cannot be determinately different from the property of being indeterminately identical to $b$ possessed by $a$, and therefore if one agrees that $a$ possesses the latter property, one cannot determinately claim that it does not possess the former. Anyway, using any of these properties in an argument to the effect that $a$ and $b$ are determinately different seems unfair.

Lowe argues further that to claim that (3) entails (4) is to make a formal error. It is true that $a$ is determinately identical to itself, but since it is also indeterminately identical to $b$, which is in turn indeterminately identical to $a$, one cannot conclude that $a$ does not possess the property of being indeterminately identical to $a$. Therefore, a formal restriction must be placed on the property abstraction so that from " $\neg \nabla(a=a)$ " only " $\neg \lambda x[\nabla(x=x)] a$ " could be derived. ${ }^{14}$

[^10]Thus, Lowe argues that even though one can determinately deny that $a$ possesses the property of being indeterminately identical to itself, one cannot determinately deny that $a$ possesses the property of being indeterminately identical to $a$. Since there is no fact of the matter as to whether $a$ is identical to $b$, there is also no fact of the matter as to whether the property of being indeterminately identical to $a$ is different from the property of being indeterminately identical to $b$. Thus, although " $\neg \lambda x[\nabla(x=x)] a$ " is determinately true, " $\neg \lambda x[\nabla(x=a)] a$ " is not.

Copeland and Lowe focus on the properties " $\neg \lambda x[\nabla(x=x)] a$ " and " $\neg \lambda x[\nabla(x=a)] a$ ", and arrive at inconsistent conclusions. Copeland argues that these two formulas express the same property, whereas Lowe claims that the relevant properties are different, and while $a$ definitely possesses the former, it cannot be said that it also definitely possesses the latter.

## 5. Other difficulties

### 5.1. Property Abstraction

The steps (1) - (2) and (3) - (4) involve property abstraction. From the fact that it is indeterminate whether $a=b$ we infer that $b$ is such that it is indeterminate whether it is identical with $a$ (i.e. $b$ has the property of being such that it is indeterminate whether it is identical with $a$ ). And from the fact that it is not the case that it is indeterminate whether $a$ is identical with $a$ we infer that $a$ is such that it is not indeterminate whether it is identical with $a$.

As we have seen one can object to the validity of both steps by appealing to the scope fallacy (see sec.2). Following the analogy between delta operators and modal operators further it can be argued that not every lambda abstract designates a 'genuine' property (Cf. [14], p. 485). It is claimed that not all modalised formulas designate 'genuine' properties. For instance, the property of Paul that he might have been a little bit taller, is usually not considered as his 'genuine' property on a par with his actual property of being $1,90 \mathrm{~m}$ tall. So, if delta operators are regarded as modal operators then they also need not designate 'genuine' properties. It is argued that one cannot assume without further argument, for instance, that there is a 'genuine' property which the predicate "being such that it is indeterminate whether it is identical to $a$ " designates. There are two options available now. One option is to claim that lambda abstracts formed with " $\nabla$ " are ill-formed, in which case both " $\lambda x[\nabla(x=a)] b]$ " and " $\neg \lambda x[\nabla(x=a)] a]$ " will be flawed, and claims (2) and (4) should be rejected. Alternatively one can treat such
lambda abstracts as well-formed but excluded from the range of Leibniz's Law, and therefore regard step (5) as invalid. If $a$ 's property of being such that it is not indeterminately identical to $a$, and $b$ 's property of being such that it is indeterminately identical to $a$, cannot be used in the substitution of Leibniz's Law, then one has no grounds to derive the conclusion that $a$ is not identical to $b$.

The above objections arise from treating delta operators as modal operators. As we have already seen, this is not the only possible interpretation of these operators. One can argue that provided the operators are interpreted as truth value indicators, the above objections lose their force. One could maintain that if the contexts governed by delta operators are not modal, there is no reason why they should not refer to 'genuine' properties. However it might be responded that even if we interpret " $\nabla$ " as a truth value indicator, there is no such thing as the property of being such that it is indeterminate whether it is identical to a (see e.g. [8], p. 188). One can hold that it should not be assumed that the expression of the fact that it is indeterminate whether an object has a certain property constitutes itself a (definite!) ascription of another property. The fact that it is indeterminate whether $a$ is identical to $b$ amounts to there being no fact of the matter as to whether those objects are identical or not. And it itself does not - or so the argument goes - involve another indeterminacy-involving property. In general the vague identity theorist should not allow such indeterminacy-involving 'properties' into his ontology at all. If the phrase "it is indeterminate whether $F a$ " expresses the claim that there is no fact of the matter as to whether $a$ has the property $F$, then we should not treat it as ascribing another property to $a$ (and cannot use it in the contraposition of Leibniz's law). ${ }^{15}$

### 5.2. Leibniz's Law

Evans appeals to Leibniz's Law (hereafter: LL) in order to derive (5) from (2) and (4). He does not indicate which form of this law he uses. Clearly the Principle of Indiscernibility of Identicals in its traditional form (LL) $\forall x \forall y[(x=y) \rightarrow \forall F(F x \leftrightarrow F y)]$, is of no use to him, since we are not assuming that $a$ is identical to $b$. Moreover, although LL is taken to be a valid law in (second-order) classical logic with identity, no matter which

[^11]interpretation of delta operators we choose we will not get classical logic. Regarding " $\nabla$ ", " $\Delta$ " as modal operators leads to some kind of modal logic (which at best could be an extension of classical logic) and regarding them as truth value indicating operators leads to a logic with at least three values. As we remember, one might argue that on the modal interpretation LL will not apply to properties involving one of delta operators, for modal properties are excluded from the range of $F$ (Cf. [14], p. 485; see also above, section 5.1.). For instance, the property $\lambda x[\nabla(x=a)] b$ will be a modal property and as such will not belong to the range of the quantifier featuring in LL. Hence, the argument will have it that on the modal interpretation of " $\nabla$ " and " $\Delta$ " LL cannot be used in deriving (5) from (2) and (4).

On the truth value-indicators interpretation, LL in the form given above is of no help, either. What is needed to derive (5) is its contrapositive ${ }^{16}$, namely:

$$
\begin{equation*}
\forall x \forall y[\neg \forall F(F x \leftrightarrow F y) \rightarrow \neg(x=y)] \tag{LL1}
\end{equation*}
$$

In our case, the relevant substitution will take the form:

$$
\neg\{\lambda x[\nabla(x=a)] b \leftrightarrow \lambda x[\nabla(x=a)] a\} \rightarrow \neg(a=b)
$$

In classical logic both LL and its contrapositive LL1 are valid in the classical sense of validity, in which an argument is valid iff it is truth-preserving. The notion of validity can be extended so as to apply also to three-valued systems. We say that an inference is valid if it leads from true premises to a true conclusion or from indeterminate premises to a true or indeterminate conclusion (i.e. if it is truth- and indeterminacy-preserving). And it is by no means obvious that in a three-valued logic LL and LL1 will be valid in this sense. ${ }^{17}$

### 5.3. Step (5)

The problem with the step:

$$
\begin{equation*}
\neg(a=b) \tag{5}
\end{equation*}
$$

is that it becomes ambiguous once we assume that the underlying logic is a three- valued logic ([6], p. 345n). In three-valued logics there can be two

[^12]kinds of negation. On the strong interpretation of negation " $\neg A$ " is true iff " $A$ " is false and " $\neg A$ " is indeterminate iff " $A$ " is indeterminate. On the weak interpretation " $\neg A$ " is true iff " $A$ " is either false or indeterminate. So, on the strong interpretation (1) and (5) do in fact contradict each other. If it is true that $\nabla(a=b)$ then " $\neg(a=b)$ " must be false, and if " $\neg(a=b)$ " is true, then " $\nabla(a=b)$ " has to be false. But it remains yet to be proved that (5) with a strong negation in it follows from (2) and (4). Simply assuming that it does, begs the question against indeterminate identity. One can insist that it is a weak negation that should be used throughout the argument. And if one interprets (5) as containing a weak negation, then we do not arrive at a contradiction - contrary to what Evans claimed. On this interpretation, the statement " $\nabla(a=b)$ " is perfectly consistent with the claim " $\neg(a=b)$ ". The latter can be true, even if the former is true (i.e. even if " $a=b$ " is indeterminate).

However, it should be noted that the weak reading is not very plausible for indeterminacy. ${ }^{18}$ On this reading " $\neg A$ " is true if " $A$ " is indeterminate, so all negations of borderline statements, which are themselves borderline statements, are true. Since borderline statements are ex definitione statements with indeterminate truth value, it would follow that negative borderline statements have two true values: indeterminacy and truth.

If we regard " $\nabla$ " and " $\Delta$ " as modal operators, then it seems that (5) does not contradict (1). (1) says that " $a=b$ " is true on some precisifications of $a$ and $b$ and false on some. Clearly, (5) does not say that " $\neg(a=b)$ " is true on all precisifications. Thus, it seems that (1) $\nabla(a=b)$ and (5) $\neg(a=b)$ can be true together.

If we agree that there is no inconsistency between (1) and (5), then the argument - if valid - proves ([14], p. 483n.):

$$
\begin{equation*}
\nabla(a=b) \rightarrow \neg(a=b) \tag{P}
\end{equation*}
$$

The problem is that from that by contraposition we can get

$$
\begin{equation*}
(a=b) \rightarrow \neg \nabla(a=b) \tag{P'}
\end{equation*}
$$

and by duality of delta operators ( $\mathrm{P}^{\prime}$ ) is equivalent to

$$
\begin{equation*}
(a=b) \rightarrow \Delta \neg(a=b) \tag{P"}
\end{equation*}
$$

which is absurd, for it says that if $a$ and $b$ are identical then they are determinately distinct.

[^13]F. J. Pelletier assumes that Evans's argument proves (P). According to him, since ( P ) is equivalent to the absurd ( P "), there must be something wrong with Evans's proof. However, two things are worth noticing.

First of all, the argument in order to derive (P") from (P) essentially uses contraposition. As we have seen, the use of contraposition becomes dubious in a logic with three truth values. Also if " $\nabla$ " is interpreted as a modal operator one would have to object to contraposition. For although $(\mathrm{P})$ seems a valid inference in a modal logic of indeterminacy, ( $\mathrm{P}^{\prime}$ ) cannot hold in this logic.

Moreover, the equivalence between ( $\mathrm{P}^{\prime}$ ) and ( $\mathrm{P}^{\prime \prime}$ ) depends on the assumption that delta operators are duals and - as it has already been mentioned - one might argue that this assumption is flawed.

### 5.4. Step (5')

Evans proposes to strengthen (5) so as to obtain

$$
\begin{equation*}
\Delta \neg(a=b) \tag{5'}
\end{equation*}
$$

which — as he says - "is straightforwardly inconsistent with (1)". Whether it really is inconsistent depends again on the reading of " $\nabla$ " and " $\Delta$ ".

If three-valued-logic reading is presupposed, then ( $5^{\prime}$ ) is either ineffective or unnecessary, depending on the reading of the negation used in (5) ([6], p. $347)$. As has been claimed in the preceding section, if the negation in (5) is a strong negation, then (5) already contradicts (1) and transition to ( $5^{\prime}$ ) is unnecessary. If, on the other hand, the negation in (5) is weak, then moving to (5') does not improve the situation. (5') still does not contradict (1): if $\nabla(a=b)$ is compatible with $\neg(a=b)$, then there is no reason why it should not be compatible with $\Delta \neg(a=b)$.

On the modal reading, (5') evidently contradicts (1), for while (1) says that $a$ and $b$ are indeterminately identical, ( $5^{\prime}$ ) says that they are determinately not identical. However the problem is that we are not entitled to strengthen (5) in this way. Recall that Evans writes as follows: "If "Indefinitely" and its dual, "Definitely" [...] generate a modal logic as strong as S 5 , (1) - (4) and, presumably, Leibniz's Law, ${ }^{19}$ may each be strengthened with a "Definitely" prefix, enabling us to derive $\left[\left(5^{\prime}\right)\right]$ ". This claim indicates a modal interpretation, assumes explicitly that delta operators are duals, and argues

[^14]that ( 5 ') can be inferred, provided that " $\nabla$ " and " $\Delta$ " generate a logic which is as strong as logic S5. As we have already seen however (Cf. section 3.), even if one adopts a modal reading, it is by no means obvious that delta operators are duals and moreover they generate a logic as strong as S5. Surely, if one accepts the supervaluationists logic, such a 'strengthening' of (5) is out of the question.

### 5.5. Conclusion

There is one interpretation according to which Evans's argument is almost unproblematic. It is a sort of 'didactic' argument: it purposely fails in order to check who will notice and diagnose the fallacy. It pretends to prove an absurd claim that there are no indeterminate identity statements, but one of its steps is fallacious. The fallacy lies in the imprecision of the singular terms used in the proof. Thus, only theorists who allow the possibility that such singular terms may be imprecise can diagnose the fallacy. If there were theorists who maintained that indeterminately identical objects have to be denoted by precise terms, they would not be able to refute Evans's absurd conclusion.

However, there is also another possible interpretation of Evans's argument. According to this interpretations the aim of the argument is twofold: it attempts to prove that a certain sort of indeterminate identity statements is incoherent - namely statements which contain only precise designators and by doing this, it allegedly shows that the world cannot be vague. In this chapter we focused mainly on the former aim. Whether or not the argument achieves this aim is an open question. Every single step of this argument has been challenged in many ways. While some of these attacks are clearly misguided, some pose a genuine threat to the validity of the argument. Anyway, it appears that the vague objects theorists have some room for manoeuvre. The crucial thing is the correct interpretation of delta operators. If one decides to regard them as modal operators, then one may i.a.:
(i) question the 'genuineness' of the properties described in (2) and (4);
(ii) question the applicability of LL to properties involving " $\nabla$ ";
(iii) argue that (5) does not contradict (1) (as a matter of fact, it is hard to understand how it could contradict (1) on this reading of delta operators);
(iv) claim that delta operators cannot be duals and (5) cannot be strengthened to ( $5^{\prime}$ ).

On the other hand, if one chooses the truth-value-indicators interpretation, then one can i.a.:
(i) question the full applicability of abstraction principles to abstracts containing " $\nabla$ ";
(ii) question the validity of the step (3) - (4) on the grounds that from (3) only
" $\neg \lambda x[\nabla(x=x)] a$ " can be derived;
(iii) question the validity of the contrapositive of LL in a three-valued logic;
(iv) argue that (5) does not contradict (1), because it contains a weak negation;
(v) maintain that the strengthening of (5) to (5') is ineffective.

It seems, however, that even if one agrees that Evans's argument is valid (i.e. it does show the incoherence of the statement " $\nabla(a=b)$ "), one may still wonder whether it achieves its second aim, i.e. whether it gives the answer to the title question "Can there be vague objects?". But this is a matter for a separate article.

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[^1]:    ${ }^{1}$ Some commentators have gone so far as to invoke their personal acquaintance with Evans in order to support their ( $N B$ contradictory) interpretations of the argument. See [9], p. 130; [1], p. 116.

[^2]:    ${ }^{2}$ Where by "vague objects" we mean objects which are indeterminately identical to something. If one maintains that vague objects need not be indeterminately identical objects, then Evans's argument has no bearing on vague objects whatsoever.
    ${ }^{3}$ Lewis ([9], p. 128n.) compares the inference from step (1) to step (2) with the following fallacious modal inference: From the true statement (A) It is contingent whether the number of planets is 9 , one infers (B) The number of planets is such that it is contingent whether $i t$ is 9 , which is obviously false. The inference is not valid because the description "the number of planets" is a non-rigid designator in the statement (A).

[^3]:    ${ }^{4}$ Lewis quotes Evans's letter in which Evans replies to Lewis's attempts to clarify the intentions behind his argument: "Exactly! Just so! Yes, Yes, Yes! I am covered with relief that you see so clearly what I was doing"

[^4]:    ${ }^{5}$ Garrett points out that precise designators need not be rigid designators. Although rigid designators are precise, not all precise designators are rigid. "The tallest man in the room" may be both precise and non-rigid. See [6], p. 342.
    ${ }^{6}$ We assume here that identity is precise.

[^5]:    ${ }^{7}$ That it is not Evans's intended reading is also suggested by the already quoted sentence "Let " $a$ " and " $b$ " be singular terms such that the sentence " $a=b$ " is of indeterminate truth value...". It does not say anything about objects; it says let us choose such singular terms that the identity statement will be indeterminate.

[^6]:    ${ }^{8}$ Nevertheless, such modal logic might be useful in the analysis of vague cross-world identity.
    ${ }^{9}$ Compare [9], p. 128. So, also on this interpretation " $\Delta p$ " may be true, while " $p$ " is not.

[^7]:    ${ }^{10}$ Of course, on the first interpretation of Evans's argument - according to which the argument is fallacious - the names " $a$ " and " $b$ " may be imprecise. It is exactly because " $a$ " and " $b$ " have different precisifications, that neither (2) follows from (1) nor (4) follows from (3), and the argument is invalid.
    ${ }^{11}$ Originally precisifications are always sharpenings of meanings of vague expressions. Here the assumption is that one may precisify vague objects (e.g. by precisifying their boundaries) and properties, too.

[^8]:    ${ }^{12}$ Over notices that if one wants to hold on to the duality of delta operators, but does not want to be committed to the absurd claim that $a$ is determinately not identical with $a$, then one is forced to a very strange reading of " $\Delta$ " operator. For, if " $\nabla$ " is read as "it is indefinite whether", then " $\Delta$ " has to be read as "it is not indefinite whether not", i.e. "it is definite whether not". Cf. ibid.

[^9]:    ${ }^{13}$ Copeland (ibid.) writes moreover that " $[\mathrm{t}]$ o hold that $a=b$ does not determinately imply $\lambda x[\nabla(x=a)] a \leftrightarrow \lambda x[\nabla(x=a)] b$ is not to question the determinate truth of the proposition that if $a=b$ then $a$ and $b$ have all their properties and relationships in common".

[^10]:    ${ }^{14}$ Compare also [2], p. 88.

[^11]:    ${ }^{15}$ There is also another line of attack possible. T. Parsons and P. Woodruff claim that even if delta operators are interpreted as truth value indicators one still cannot assume without argument that lambda abstracts in contexts governed by delta operators automatically stand for properties and also fully satisfy property abstraction. See [12], p. $175 n$.

[^12]:    ${ }^{16}$ This contrapositive is sometimes called "the law of the diversity of dissimilar".
    ${ }^{17}$ One logic in which LL is not valid is given by Johnsen, who adopts the Kleene strong tables. See [7].

[^13]:    ${ }^{18}$ Garrett notices this but derives a different conclusion, see [6], p. 346.

[^14]:    ${ }^{19}$ It has already been argued that it cannot be the usual form of LL that Evans's wanted to use. Johnsen argues that Evans had in mind the following law: (LL $\Delta) \Delta \forall y \forall z((y=$ $z) \rightarrow[\lambda x(\phi x)(y) \leftrightarrow \lambda x(\phi x)(z)])$, which is not a law of three-valued logic just as LL is not. See [7], p. 109.

