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Book Reviews

Richard L. Epstein, **Reasoning in Science and Mathematics: Essays on Logic as The Art of Reasoning Well**, Advanced Reasoning Forum, 2011, 134 pp., ISBN-13: 978-0983452126, ISBN-10: 0983452121.

This is one of a series of small handbooks: good as practical guides which will no doubt help students and early career scholars navigate their way around notions they may encounter in applied courses. Notions such as theory, model, explanation, prediction, falsification and assumption are explained within the context of science and mathematics. Taken as such practical guides, the books are informative, useful and often times stimulating — offering incentive for further study where the complexity of the issues or concepts covered goes beyond their representation in the books (the endnotes serve this particular feature well, suggesting further reading and going into some of the missing detail and subtleties that the main text sometimes glosses).

This particular work is a highly practical approach to the consideration of mathematics and science. Although it does describe a philosophy of these areas, it is perhaps best not viewed primarily as a philosophical piece but rather as a work seeking ways we can utilize theories, experiment and mathematics in scientific and everyday knowledge. Indeed, we are directly cautioned not to take the work as seeking to provide 'ultimate explanations' for some of the key questions in the philosophies of mathematics and science, but to take the work as "grounding mathematics" with "minimal metaphysics" in order to stimulate better work in the field: i.e. "to help students grasp new concepts and make use of them, not only as a subject to be learned but as a tool for modelling further" (p. 93).

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By and large, the book's perspective comes from a particular stance on human reason, set out in more detail elsewhere, but briefly summarized in the opening chapters. The basic concepts laid out there are then applied to reason about the particular domains of science and mathematics. It is probably worth noting that the view of human 'reason' outlined in the introductory background is set out along classical 'critical thinking' lines: it comes from consideration of what underpins practical argument skills, and is based on classical rules of inference and well known rules and principles establishing the strength or validity of arguments.

In line with the series of which it is part, this particular work is a useful introduction to the discourse and presuppositions of science and mathematics and so could be particularly helpful for the student unacquainted with what lies beneath many of the core notions in these fields and unsure of how to approach and interpret these notions in a way that works — i.e. the work could help to open the door to understanding the larger themes and theories in these fields.

Not only does it go a long way toward explaining and demystifying the scientific approach, it also provides various examples translating and untangling the cross-currents; outright disagreements; and developments within the fields, as well as within some of their different sub-fields.

The use of cartoons help to facilitate the practical aims of the book nicely — highlighting assumptions we might make on visual or prima facie analyses of data. The examples, though, are also interesting in their own right: e.g. examples of 'duplicable and replicable experiments' (pp. 55–62) range over a recipe for vegetarian Chile; an ironic experiment on grass growth; the feeding behaviour of primates; the refraction of light rays; and the growth of living nerve cells in vitro.

The 'outside in' view of scientific reasoning offered leads to the provision of 'rules of thumb' again particularly applicable to the student or novice finding their way through the field. Notable among these are those presented at the close of the chapter on experiments: "Never trust the first or even the first few experiments that claim to establish a significant correlation" and "to reason well, imagine the possibilities" (p. 65). Wonderful advice!

Indeed, the emphasis throughout the work is continually on usefulness, both to the reader and of the theories within the fields examined. This emphasis is particularly apparent in the essay on mathematics, which begins with the premise that a "good story of mathematics" should not only answer the host of issues dubbed the 'usual suspects' in the philosophy of mathematics (e.g. 'what is mathematical truth?' and 'what is mathematical intuition?') but also "be consistent with how we actually do mathematics" and "be useful to mathematicians, leading to new and interesting work in mathematics" (p. 68).

The story then offered could thus perhaps best be understood from a largely instrumental perspective: prioritising the utility of the account over a more traditional philosophical defence of its more speculative aspects.

The story given is primarily that of "mathematics as the art of abstraction". That mathematics can be understood as abstracted from experience is an idea that those in the field will have encountered before, but the instrumental flavour of the account here offers a fresh look at the notion. At times the story given is reminiscent of Shapiro's structuralism, at times of Maddy's 'Second philosophy', though the naturalism here is more Millian than Quinean.

One of the interesting ideas put forward can be seen as arising naturally from the introductory chapters on reason and from a key emphasis that the work places on the similarities between science and reason. This idea is that mathematical abstractions are akin to scientific abstractions and so should be seen as schematic and not the sort of thing able to be true or false until their application is made clear. The idea, flipping Hilbert's Formalism almost on its head, deserves more attention than I can give it here, but a small discussion follows nonetheless.

The central claim is that mathematical proofs should be seen as arguments, specifically arguments that a given mathematical inference is valid. The sense, shared by most mathematicians and philosophers thereof, that mathematical claims are the sorts of thing which can be true or false (if any claim can be) is accounted for by embedding mathematical claims in their theories: the suggestion is that we imagine that each mathematical claim has an implicit caveat; '$2 + 2 = 4$' then really means " '$2+2 = 4$' follows from the Peano's axioms". This, Epstein says, accounts for the conviction held by many that the truths of mathematics are 'necessary': the necessity comes not from there being "no possible way such [...] claim[s] could be false" but from "the necessity that the claim must follow from the assumptions of the theory" (p. 77).

But the sense that the truths of mathematics are 'really true' is also accounted for in the book by the suggestion that mathematics holds only in those select domains in which it applies and that it is only applied to domains in which it holds: the idea here is that the final claim of a proof is 'preserved to be true' by "applying it only in cases in which it is true" (p. 70).

The two notions are in some tension with one another — to the extent that (apparent) mathematical truth is linked both to internal mathematical inference and to external applicability. But the interest here, for me anyway, is more in the central idea: that mathematical claims can be thought of as true of particular domains or of specific inferences, but not in themselves.

I'm not sure the picture Epstein paints can account for interest in mathematics for its own sake and the way in which mathematics proceeds often without regard for any of its potential applications, even apparently without regard for the possibility that whole swathes of it may never have any application. Equally, mathematical truth is commonly thought of as (at least) sturdier somehow than empirical truth, not to mention as objective. The idea that mathematics is true only in so far as its inferences are internally valid and only insofar as the domains to which it is applied are sufficiently analogous to its theories seems to undercut some of the most persistent of our convictions about the nature of mathematics: e.g. on the one hand that the assumptions from which we draw our inferences are themselves true and, on the other, that experience cant falsify mathematics, not because we selectively ignore those elements of experience to which mathematics does not seem to apply, but because mathematics is not dependent on any empirical state of affairs or contingent truth.

I personally find other accounts more appealing: e.g. Mark Steiner's argument in his [1998] that mathematics is an anthropomorphic endeavour whose application to science suggests a 'user friendly' world. Steiner's account allows mathematical claims to have genuine truth values and at the same time, provides a convincing account of the applicability of mathematics.

Steiner also provides some novel and compelling reasons as to why we should proceed with caution when propounding most versions of naturalism — specifically those versions that can fairly be described as 'anti-anthropocentric'. That the assumptions underpinning the tale offered in this book are naturalistic is clear throughout the text. The philosophy that emerges could perhaps, in the final analysis, be described as a hybrid between empiricism and naturalism. I think it meets the essential elements of Steiner's definition of naturalism: "naturalism demands that philosophy be part of, or continuous with, natural science" (Steiner [1998, p. 55] and so can fairly be described as belonging in the 'anti-anthropocentric' camp, although I leave it to readers to judge this for themselves.

Leaving aside these weightier philosophical issues for a moment (as, in fairness we should, when reading the book), it is worth noting again that a key explanatory tool employed throughout is that of mathematical analogy. The explanatory work to which Epstein puts this particular phenomenon or practice relies heavily on the understanding of physical or scientific analogy outlined earlier in the text. That is, earlier in the text, reasoning by analogy is described in tandem with abstraction from experience: models and theories are described as constructed in service to various practical or physical applications of analogical reasoning (models and theories are seen as good to the extent that they 'work' in an application of analogical reasoning to some domain or phenomena to which they apply).

The earlier work on analogies and abstraction in science is recapped in the section on mathematics: "A comparison becomes *reasoning by analogy* when a claim is being argued for: on one side of the comparison we draw a conclusion, so on the other side we can draw a similar one [. . .] Science sometimes proceeds by analogy. But scientists always proceed by abstracting: choosing some aspect(s) of experience to pay attention to and claiming, perhaps implicitly, that all other aspects of experience in these kinds of situations don't matter. What we pay attention to gives us the constraints for saying whether a claim is true or false" (pp. 68–69).

The understanding of mathematics presented in the book could, I think, be viewed as informed directly by these ideas. The story of mathematics as schematic, for example, draws from the above analysis of the way in which science can proceed, beginning the particular analogy here with: "Mathematics abstracts from experience, too, only much more than any science" (p. 69).

It is, Epstein says, when we can see "a clear path of abstraction' to mathematical theories and entities that we can make sense of the application of these theories and entities to experience or the 'real world.

Epstein also says that abstractions of abstractions, or analogies between mathematical theories are "merely aesthetic until applications are found" (p. 75)

Returning for a minute to the weightier philosophical issues, this seems to me not to meet some of the requirements laid out earlier:

including those of explaining the practice of mathematics and aiding mathematical discovery. To press the earlier point: it is hard to see how mathematicians proceed and mathematics progresses as if the theories therein will be true or false or able to be tested and thus established as true or false upon discovery of an application of those theories — rather, the truth of the theory seems to rely on mathematical standards of proof alone. Applicability and truth do not seem, to me at least, to be quite so linked in mathematics as they appear in Epstein's description of the field, at least not from the perspective of mathematicians themselves.

Steiner's [1998] can help clarify some the issues here too: he points out that often when analogies are (successfully) drawn between mathematical theories or between a mathematical and physical theory there may simply be no path of abstraction from any material or natural property to the mathematical theory. Indeed, in many cases, there's another (at times quite mysterious, even magical) path leading us in quite the opposite direction: from the theory to the natural world (for just one example, see the discussion of Hilbert spaces in quantum mechanics pages 36–44): indeed Steiner argues that two central types of mathematical analogies: 'Pythagorean' and 'formalist', are both absolutely indispensible to contemporary physics and "deeply antinaturalistic" (p. 54).

These observations are not meant to suggest Epstein's story is any the less engaging or useful to enquiring minds. It is, rather, meant as a perspective on the story he offers from metaphysics and the realm of 'ultimate explanations'.

That is, from that perspective, I'm not sure Epstein's account of mathematics as best understood as abstractions from experience is convincing. There are competing theories (e.g. realism: that the structures mathematics describes exist in their own right, as well as Steiner's anthropomorphism) that, to me at least, more thoroughly address some of the 'usual suspects' Epstein lines up at the beginning of the book.

Nonetheless, I enjoyed reading the work, and I would recommend it to anyone with an interest in the philosophy of science or mathematics, as well as to teachers and researchers interested in the question of what constitutes good reasoning and critical thinking skills across these particular domains.

References

[Steiner 1998] Steiner, M., *The Applicability of Mathematics as a Philosophical Problem*, Harvard University Press, London, 1998.

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