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## The Influence of Jerzy Łoś on Early Developments in Temporal Logic

### Introduction and Aim of the Paper

Attempts by Jarmużek and Tkaczyk<sup>1</sup> to acknowledge Jerzy Łoś as the inventor of the first modern system of temporal logic led to a response in the form of Øhrstrøm and Hasle's paper.<sup>2</sup> In our opinion, most of their arguments do not succeed in refuting Jarmużek and Tkaczyk's claims and are therefore not valid in the overarching discussion.

In this paper, we will analyse the arguments on both sides and attempt to respond to Øhrstrøm and Hasle's statements. Further, we would like to

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<sup>1</sup> Tomasz Jarmużek, Marcin Tkaczyk, "Jerzy Łoś Positional Calculus and the Origin of Temporal Logic", *Logic and Logical Philosophy* 28 (2009): 259–276.

<sup>2</sup> Peter Øhrstrøm, Per Hasle, "The Significance of the Contributions of A.N. Prior and Jerzy Łoś in the Early History of Modern Temporal Logic", in: *Logic and Philosophy of Time: Further Themes from Prior*, Vol. 2, eds. Patrick Rowan Blackburn, Per Hasle, Peter Øhrstrøm, (Aalborg: Aalborg University Press, 2019), 31–40.

support the claim made by Jarmużek and Tkaczyk, albeit with a slight modification. We claim that Łoś was the inventor of temporal logic in the sense that he constructed, described and examined the first mature calculus of temporal logic and what is more, Hiż's review of his work influenced the early development of temporal logic (systems created by Prior in 1957 and Rescher in 1966). We will undertake this by providing a detailed analysis of the argument presented by Øhrstrøm and Hasle.

In the first section, we will analyse the core thesis of the article. After demonstrating the supporting statements by Øhrstrøm and Hasle, we will highlight their faults and present our counterargument. We will then focus on each individual supporting claim in dedicated sections, the second and third sections, where our responses will aim at deriving answers from further historical sources.

The conclusions in this paper will be based on the work of the logician who contributed to the development of positional logic during the beginning of the modern history of temporal logic, Nicholas Rescher. We will outline his view on the early development of temporal logic, the influence that Łoś had on Prior, and his research in the field of *R*-calculus axiomatisations. This will be done in the fourth, and last, section of the paper.

## 1. Analysis of the Argument

Øhrstrøm and Hasle's paper<sup>3</sup> can be viewed as a response to the ideas of Jarmużek and Tkaczyk.<sup>4</sup> Therefore, it is crucial for our work to accurately present the original statement given by the Polish authors, the main purpose of which was to introduce the work of Jerzy Łoś to the field of temporal logic and present it in the context of the early developments in this branch of Logic. The original claim by Jarmużek and Tkaczyk was that Łoś was the inventor of temporal logic:<sup>5</sup>

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<sup>3</sup> Ibidem.

<sup>4</sup> Jarmużek, Tkaczyk, "Jerzy Łoś Positional Calculus and the Origin of Temporal Logic", 259–276.

<sup>5</sup> Ibidem, 259.

**Claim 1:** *Łoś constructed, described and examined the first mature calculus of temporal logic.*

**Claim 2:** *Prior was aware of, and inspired by, Łoś's ideas when beginning his own work in the field.*

This statement is especially bold in the context of the history of modern temporal logic, which usually omits this Polish mathematician and introduces Prior as the first to formalise a system of temporal logic. For now, we will not delve into whether these claims are justified; instead, we will analyse the counterarguments given by Øhrstrøm and Hasle.

The Danes agree on Łoś's contribution to the development of temporal logic and his role as a founder of the  $U_t p$  notation.<sup>6</sup> However, they do not accept Łoś as an inventor in the sense defined by Jarmużek and Tkaczyk. After articulating their counter-thesis, they justify their point of view with two main arguments:<sup>7</sup>

**Claim 1:** *According to a straightforward and commonly accepted understanding of temporal logic, Prior should be regarded as the inventor of modern temporal logic.*

**Claim 2:** *In Prior's very first presentation of the system of temporal logic in August 1954, Łoś's work was not mentioned.*

Below, we will analyse these two claims and highlight the main ideas facilitating the justification of the claims.

### 1.1. Definition of Temporal Logic

To support claim 1 by Øhrstrøm and Hasle, the authors recall the popular definition of temporal logic provided by Goranko and Dalton.<sup>8</sup> It contains a widely used categorisation that applies to a logical society. According to this definition, temporal logic can be understood in two ways:

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<sup>6</sup> Øhrstrøm, Hasle, "The Significance of the Contributions of A.N. Prior and Jerzy Łoś in the Early History of Modern Temporal Logic", 33.

<sup>7</sup> Ibidem.

<sup>8</sup> Valentin Goranko, Antje Galton, "Temporal Logic", in: E. N. Zalta (ed.) *The Stanford Encyclopedia of Philosophy (Winter 2015 Edition)* 2015, <https://plato.stanford.edu/archives/win2015/entries/logictemporal/>, accessed Nov 28, 2018.

**Definition 1 (broad):** *Temporal logic can be understood as any reasoning about time and temporal information represented within a logical framework.*

**Definition 2 (narrow):** *Temporal logic is identified as the modal logic approach to the representation of time in formal systems and, therefore, is identified with the tense logic proposed by Prior in 1960.*

It is unclear which version the authors chose in their argument as they do not state it explicitly. Yet, we can assume that they did not choose the narrow definition as, in this case, their argument would become vain, considering that Łoś's system does not use modal connectives to represent tenses. Therefore, we must assume that the authors used the broader sense of temporal logic.

Still, assuming that they accept the broader definition of temporal logic, they use Prior's analysis of medieval and ancient investigations regarding time in logic to narrow it down. According to the analysis of its roots, temporal logic should have two main features:<sup>9</sup>

**Feature 1:** *Tense distinctions are a proper subject of logical reflection.*

**Feature 2:** *What is true at one time is, in many cases, false at another time, and vice versa.*

Taking Prior's analysis into account, it can be admitted that Łoś's system of logic possesses Feature 2 of a re-invented antique and medieval view on logic but not Feature 1. This is derived from Henryk Hiż's review of Łoś's paper, wherein he states:<sup>10</sup>

The main purpose of this paper is to analyse Mill's canons as rules of operation for a part of the language of physics. To do it the author builds up an axiomatization of a fragment of the physical language.

Therefore, in the author's opinion, Łoś's system cannot be viewed as a system of temporal logic as its creation was not preceded by an analysis of tenses. At the core, the system was constructed for a special purpose – the axiomatisation of a fragment of the physical language. Even if time is present in the

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<sup>9</sup> Øhrstrøm, Hasle, "The Significance of the Contributions of A.N. Prior and Jerzy Łoś in the Early History of Modern Temporal Logic", 34.

<sup>10</sup> Henryk Hiż, "Review: Jerzy Łoś, Foundations of the Methodological Analysis of Mill's Canons", *Journal of Symbolic Logic* 16 (1951): 58.

system, it is described by physical theories. This leads to another argument that Prior's logic was, in fact, more general, as it goes beyond the notion of time presented in physics and can be used to formalise more general reasoning.

The aim of creating Łoś's system and the notion of time that it attempts to axiomatise is one of the pillars of the authors' argument. The second pillar is the fact that this logic does not possess any symbolic representation of tenses. Øhrstrøm and Hasle support it by recognising that the absence of a genuine analysis of tenses and their representation by modal connectives is Prior's main contribution.<sup>11</sup>

## 1.2. Łoś's Influence on Prior's Early Work

The second claim in Øhrstrøm and Hasle's argument is focused on historical justifications that there is no evidence of Prior being acquainted with Łoś's paper in 1954 or that it had any impact on Prior's first work on tense logic. At the conference in Wellington held in August 1954, as the Danes state, Prior presented his early views on temporal logic.<sup>12</sup> In his presentation, Prior introduced a logic similar to Łoś's; however, it was more similar to first order logic with indexing. According to Prior, those indexes should be read as moments of time. His calculus was named l-calculus, where "l" referred to "later". Expressions of the form  $p_x$  were meant to denote " $p$  at  $x$ ."<sup>13</sup>

To support the statement that Prior did not know of Łoś's work during his 1954 presentation, the authors point to the list of Prior's papers in which Łoś's name appears. Prior mentions him for the first time in *Formal Logic* under the headline "Logic of Assertion and Formalized Physics", which was published in 1955. The second time he mentions Łoś is in *Time and Modality* from 1957 as a reference to Hiž's review, and then in *Past, Present and Future* from 1967.<sup>14</sup>

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<sup>11</sup> Øhrstrøm, Hasle, "The Significance of the Contributions of A.N. Prior and Jerzy Łoś in the Early History of Modern Temporal Logic", 35.

<sup>12</sup> Ibidem, 36.

<sup>13</sup> Ibidem.

<sup>14</sup> Ibidem.

This list is of utmost importance for the argument as the authors use psychological argumentation based on it, stating that Prior had no cause not to mention his inspiration, especially when he confessed to the influence of Łoś on his work in his later papers. Other arguments in this context also indicate psychological assumptions regarding Prior's character. It was said by Øhrstrøm and Hasle that by not referring to Łoś's work, if Prior had already read the review by Hiż, three objections based on Prior's assumed character could be raised. The first is that Prior had great respect for Polish logic and was a propagator of the ideas of many logicians from this country. Second, as Prior has been recalled as being very meticulous in crediting his influences,<sup>15</sup> it seems unlikely for him to omit crediting Łoś at this stage of the development of his ideas. The third argument is ethical in nature, stating, as the author put it, that it would be advantageous for Prior as he would not need to create new notation and could, instead, use Łoś's work as a basis, at least in 1954.

Another argument supporting the claim that Łoś did not inspire Prior's first presentation of temporal logic in 1954, posited by Øhrstrøm and Hasle, is based on the notion of the temporal logic that Prior allegedly had. Here, we should clarify that the notion the authors impose on Prior is that of tense logic and, thus, of temporal logic in a narrow sense.

According to the authors, Prior did not recognise his own early work on temporal logic, which he began in 1950–1951. This claim refers to his work *The Craft of Formal Logic*. The Danes note that he had started working on the development of temporal logic at least three years before its presentation in 1954 and that he himself dated the beginning of his work on tense logic to 1953, when he came across Findlay's note.<sup>16</sup> Merging these two facts, the authors imply that Prior's representations of the notion of temporal logic did not overlap with his early work on variables indexed by time. This argument is highly unclear as to the intended result. We do not know if the authors were implying that in Prior's view, Łoś's system did not overlap with the notion of temporal logic he had at the time or that even if Prior was acquainted with Łoś's work, he did not refer to it as it did not have any impact on his tense logic.

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<sup>15</sup> Ibidem, 36.

<sup>16</sup> Ibidem.

## 2. Revision of Øhrstrøm and Hasle's First Claim

The argument for not including Łoś's system in temporal logic is, in our opinion, weak, both in its form and content. Therefore, we would like to not only refute it but also present this system as the first formal framework dealing with reasoning about time. To achieve this aim, we will begin by commenting on Claim 1 in Øhrstrøm's and Hasle's argument, supporting it and then presenting Łoś's logic and examining his philosophical presumptions.

### 2.1. Critique of Øhrstrøm and Hasle's Argument

First, we need to ensure the consistency of the accepted definitions. The authors indicate the definition given by Goranko and Dalton in two aspects – broad and narrow – yet do not compare either of them explicitly to Łoś's system. This is striking, as the broader sense of the quoted definition is vast enough (as we will further prove) to cover this case and the second is too narrow to fit any logical systems not related to Prior's main ideas about tense logic.

This is noteworthy, as after presenting the aforementioned definitions, the authors do not use them in any way and abandon the thread of thought, instead moving forward to Prior's historical analysis of the roots of temporal logic. The analysis is incredibly valuable and worth exploring. However, it is invalid in a case where it is used to argue that a logic system, which does not possess all the features of a tense logic, cannot be considered temporal logic. On the basis of his historical analysis, Prior detailed the features of antique and medieval logic. He stated that a proper logic has to possess both these features in order to be considered a re-invention of those traditions using formal tools.<sup>17</sup> Øhrstrøm and Hasle ignore this definition and focus on the two features as necessary conditions for the logic to be considered temporal.

Therefore, Øhrstrøm and Hasle's first claim is not supported by the argument that Łoś's system allegedly does not fit the definition of temporal logic, which is at least partly agreed upon among logicians. Instead, the authors

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<sup>17</sup> Ibidem, 3.

use properties of a tense logic taken from Prior's historical analysis and compare them to Łoś's system. Let us consider the situation wherein both features form a proper definition of temporal logic. As Feature 2 sounds reasonable, being related to the feature of the logic of transporting temporal information, it leads to confusion.

An important question to ask at this juncture would be the following: Why should all systems of temporal logic be focused on tenses? Applying this feature in its most radical meaning leads to absurdity. The restrictions of this definition would eliminate a great part of the works within the field of temporal logic as they were not created as *tense logics*. The construction of the system had as its core an observation that Mill's canons are, in fact, temporal in their nature. Therefore, the attempt to formalise them has to deal with time and provide a possibility of expressing temporal relation between two events. Time is the main subject of Łoś's work even though it is not present in the same form as in Prior's work.

Another claim the authors present is that the  $U_t p$  notation presented by Łoś was created to formalise the notion of time in a physical sense. Therefore, this perspective is less general than that of tense logic presented by Prior. We can agree with the statement that Łoś's work was not focused on the subject of tenses. It is also true that it was created with the purpose of formalising a fragment of physical language that contains temporal factors.

## 2.2. Łoś's System and its Philosophical Background

Łoś presented his logic as a part of his master thesis "Podstawy analizy metodologicznej kanonów Milla" ("Foundations of Analysis of Mill's Canons") in 1947 (printed in 1948). The main purpose of this system was to formalise John Stuart Mill's canons of induction, as Hiż briefly described in his review.<sup>18</sup> This was done owing to the philosophical analysis of Mill's work, with Łoś highlighting that the crux of Mill's canons is their temporal aspect and

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<sup>18</sup> Hiż, "Review: Jerzy Łoś, Foundations of the Methodological Analysis of Mill's Canons": 58–59.



causality. Indeed, Łoś attempts to formalise the language of physics, but physics in a philosophical sense, as a science, built on observation sentences.<sup>19</sup>

Observation sentences are sentences about the objects, belong to the language of physics and contain a spacio-temporal coordinate related to the fact described by the sentence<sup>20</sup> [Translated by the author].

Using this definition, Łoś analysed the form of example observation sentences, which were partially added after Logical Positivists. He discovered that sentences that are determined neither temporally nor spatially cannot be evaluated as true or false. Łoś called this set of expressions “occasional functions”. In this set of expressions, Łoś distinguished a subset that needs only a time determinant to be true or false, and these are called “temporal functions”. He presents his logical system, which, in fact, attempts to symbolise the form of the temporal function.

Below, we will present Łoś’s system in its modern version, given by Jarmużek and Tkaczyk,<sup>21</sup> which Øhrstrøm and Hasle originally replied to. The main addition from Jarmużek and Tkaczyk incorporates a modern symbolical language instead of Łukasiewicz’s notation, which Łoś used himself. The other modifications to the system included reconstructing the meaning of the logical connectives and using the mathematical symbol  $R$  for the realisation operator instead of  $U$ .

The language of Łoś’s system is a superset of the language of classical propositional logic. Therefore, among standard symbols of CPL and propositional variables, it contains instant variables:  $x, y, x_1, x_2, \dots$ ; interval variables:  $e, e_1, e_2, \dots$ ; and consequently, constants of those types: realisation operator  $R$  and functional symbol  $\delta$ . In this language, a set of instant terms is defined

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<sup>19</sup> Jerzy Łoś, “Podstawy analizy metodologicznej kanonów Milla” (“Foundations of methodological analysis of Mill’s canons”), *Annales Universitatis Mariae Curie-Skłodowska* 2.5 F (1947): 269–301.

<sup>20</sup> The original sentence in Polish is as follows: “Zdania spostrzeżeniowe są zdaniami o rzeczach, należą do języka fizykalnego i występują w nich współrzędniczasowo-przestrzenna dotycząca faktu, który zdanie to opisuje” (ibidem, 277).

<sup>21</sup> Jarmużek Tomasz, Marcin Tkaczyk, “Jerzy Łoś Positional Calculus and the Origin of Temporal Logic”: 263–264.

as the smallest set  $S$ , containing all instant variables and expressions of the form  $\delta(\tau, \varepsilon)$ , where  $\tau \in S$  and  $\varepsilon$  are interval variables.

The set of formulas in Łoś's system is the smallest that comprises all formulas of CPL and all expressions of the form  $R_\tau(\phi)$  (where  $\tau \in S$  and  $\phi$  is a formula of CPL). This set is closed under the standard use of first order logic, with rules of quantification across all types of variables appearing. Specific axioms of the system are presented below:

**Axiom 2.1**  $R_x \neg \phi \leftrightarrow \neg R_x \phi$

**Axiom 2.2**  $R_x(\phi \rightarrow \psi) \rightarrow (R_x \phi \rightarrow R_x \psi)$

**Axiom 2.3**  $R_x((\phi \rightarrow \psi) \rightarrow ((\psi \rightarrow \chi) \rightarrow (\phi \rightarrow \chi)))$

**Axiom 2.4**  $R_x(\phi \rightarrow (\neg \phi \rightarrow \psi))$

**Axiom 2.5**  $R_x((\neg \phi \rightarrow \phi) \rightarrow \phi)$

**Axiom 2.6**  $\forall_x R_x \phi \rightarrow \phi$

**Axiom 2.7**  $\forall_x \forall_e \exists_y \forall_p (R_{\delta(x, e)} \phi \leftrightarrow R_y \phi)$

**Axiom 2.8**  $\forall_x \forall_e \exists_y \forall_\phi (R_{\delta(y, e)} \phi \leftrightarrow R_x \phi)$

**Axiom 2.9**  $\forall_x \exists_\phi \forall_y (R_y \phi \leftrightarrow \forall_\psi (R_x \psi \leftrightarrow R_y \psi))$ .

Besides the given axioms, Łoś proved some metalogical properties of his system, the analysis of which, as well as more results, can be found in related works. From our perspective, it is worth pointing out that this logic was created according to good logical standards. While analysing Łoś's work as a temporal logic, it is crucial to point out how he intended to use his symbols.  $R_x \phi$  is read as " $\phi$  occurs at the time  $x$ ", while  $R_{\delta(x, e)} \phi$  is read as " $\phi$  occurs at the moment later by  $e$  than  $x$ ".

The definitions Łoś provided in his work aim to clarify that the system is, in fact, one of temporal logic. He presented multiple definitions of predicates. One states that two temporal functions are realised in the same set of moments, another states that two moments are equal in the aspect of realised temporal functions and yet another states that one moment precedes the second.

Łoś realised the consequences of his axioms and stated that his system implies ontological requirements for its models. As he states, axioms 2.7 and

2.8 together imply that the set of temporal functions is infinite.<sup>22</sup> Axiom 2.9, or “the clock axiom”, states that for any moment of time, we can distinguish a temporal function that is realised in that moment. Combined with the infinity of temporal functions, it implies that a set of time moments is also infinite.

### 2.3. Łoś’s System as a Temporal Logic

After analysing and critiquing the first argument supporting the counter-thesis and presenting Łoś’s system, we would like to question Øhrstrøm and Hasle’s argument. Hence, we present the system from 1947 as one of temporal logic. We would like to support the thesis that it was, in fact, the first formal presentation of temporal logic, as Tkaczyk and Jarmużek stated.<sup>23</sup>

To achieve this goal, we will use the same definition of temporal logic as Øhrstrøm and Hasle and compare it with Łoś’s actual system. Further, while analysing the system, we will respond to some arguments regarding the fundamental idea in his work. First, let us analyse the most common definition, given by Goranko and Dalton:<sup>24</sup>

The term “temporal logic” has been broadly used to cover all the approaches to reasoning about time and temporal information, as well as their formal representation, within a logical framework, and also more narrowly to refer specifically to the modal-logic type of approach introduced around 1960 by Arthur Prior under the name “tense logic” and subsequently developed further by many logicians and computer scientists.

As we pointed out in the previous section, this definition comprises two sub-definitions of temporal logic – a broad sense and a narrow sense. Łoś’s system can be included in the broader one as in its centre stands the formalisation of an expression comprising temporal information  $R_\tau\phi$ . The expression itself should be read as “it occurs that  $\phi$  at the time  $\tau$ ”. The meaning of the primitive symbols, their properties given in the axiomatisation and the definitions appearing in this system refer directly to a notion of time.

<sup>22</sup> Łoś, “Podstawy analizy metodologicznej kanonów Milla”: 283.

<sup>23</sup> Jarmużek, Tkaczyk, “Jerzy Łoś Positional Calculus and the Origin of Temporal Logic”: 259.

<sup>24</sup> Goranko, Galton, “Temporal Logic”.

Therefore, this system can be viewed as the axiomatisation of the operator of temporal realisation. Hence, time is in the centre of Łoś's logical inquiries.

Łoś's system is the first developed positional and chronological logic, as Rescher refers to it.<sup>25</sup> It deals with time by simple yet compatible formalism – the temporal realisation operator. It symbolises the relation between the situation and time coordinate wherein we can evaluate its truth value. Therefore, according to the definition, we observe a coherent approach to dealing with reasoning that contains temporal information, which is encompassed by a logical framework. Moreover, this system is consistent and has been built according to modern metalogical standards. Łoś provided example models for it wherein he presented its intended meaning.

On the other hand, referring to the second, narrower version of temporal logic, the definition is inadequate, owing to the fact that it covers only tense logic systems and the systems developed within Prior's approach to deal with time. Therefore, Łoś's system is certainly not an example of tense logic. It must be pointed out that it is possible, with a proper set of axioms, to define tense modalities using *R*-operator (the example can be seen in the works of Rescher and Prior).<sup>26</sup>

However, assuming that we accept the narrower definition, Łoś's system cannot be called a system of temporal logic. It is obvious that other temporal logic systems fail to be covered by this definition. For instance, Von Wright's system *And Next* cannot be seen as a temporal logic system through the lens of Prior's tradition. Nonetheless, we do not refuse consider it as such.<sup>27</sup>

Nowadays, temporal logic can be identified with the systems defined by the narrower versions of Definition 1. This is partially understandable, as systems that use modal logic connectives in a temporal sense are most common as a simple exemplification of temporal logic. However, many specialists

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<sup>25</sup> Nicholas Rescher, James Garson, "Topological Logic", *The Journal of Symbolic Logic* 33 (1968): 542.

<sup>26</sup> *Ibidem*, 545.

<sup>27</sup> Moreover, Rescher has shown how the Von Wright system can be embedded as a part of *R*-calculus in "Note on Chronological Logic". The reverse translation would be impossible, as any system of *R*-calculus is richer than the *T*-system in the aspect of logical constants. Although Rescher proposed a way to develop such a translation, it would require adding indexes to *T* operator. See Nicholas Rescher, James Garson, "Note on Chronological Logic", *Theoria: A Swedish Journal of Philosophy* 2008: 6.

in the field with in-depth knowledge of the history and categorisations of current branches of modern temporal logic systems accept the broader definition. A good example of a publication written by such a specialist, containing different types of temporal logic and not only tense logic, is van Benthem's *The Logic of Time*.<sup>28</sup> On the other hand, there were significant developments within this field from the 1950s that cannot be omitted.

In the work of authors such as Tkaczyk,<sup>29</sup> we can find the typologisation of temporal logic that covers modern results. We can distinguish two types of logic in the aspect of used symbols that represent temporal relations: propositional logic and positional logic. Moreover, we can distinguish at least three types of logics in the aspect of their intended meaning: tense logic, logic of empirical time and the temporal logic of programmes. Other authors have added other types of logic such as interval time logic or hybrid temporal logic<sup>30</sup> as well.

In this kind of typologisation, we can refer to Łoś's logic as a positional logic of empirical time and Prior's most popular systems of logic as propositional tense logic. These distinctions can help on the grounds of the history of logic and modern discussions, which do not operate on the basis of sufficient language for a field so young as temporal logic.

Therefore, while accepting Goranko and Dalton's definition of temporal logic, we cannot allow its narrower sense to dominate the view on what temporal logic should be, as, in our opinion, it simply reflects the common usage of this term in some groups. The broader version of the definition, as we have already shown, covers not only Łoś's system but many other logics that do not use modal logic symbolism to represent time.<sup>31</sup>

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<sup>28</sup> Johan van Benthem, *The Logic of Time* (Dordrecht: Kluwer Academic Publishers, 1991).

<sup>29</sup> For instance, see Marcin Tkaczyk, *Logika czasu empirycznego (The Logic of Physical Time)* (Lublin: Wydawnictwo KUL, 2009).

<sup>30</sup> Anna Kozanecka-Dymek, "On Kinds of Temporal Logic and Some of Its Applications". *Studies in Logic, Grammar and Rhetorics* 41 (2012): 73–81.

<sup>31</sup> Here, by "modal logic symbolism", we are referring to the most common dual operators symbolised by the geometrical figures of the box and diamond. We would also like to point out that *R*-operator is, in fact, a modal operator and can be used to define other modal operators. For further information, please refer to Tomasz Jarmużek, Tkaczyk Marcin, "Expressive Power of the Positional Operator *R*: A Case Study in Modal Logic and Modal Philosophy". *Ruch Filozoficzny* 2 (2019): 93–107.

### 3. Revision of Øhrstrøm and Hasle's Second Claim

Øhrstrøm and Hasle focused on a very specific point in time to defend the claim that Prior was not acquainted with Łoś's work on temporal logic. The fact that Prior had started his own investigation within the field a few years before he presented his first mature formalisation leaves no doubt, although the point at which Prior's view on temporal logic became clarified is vague. Therefore, we will first compare the views of Jarmużek and Tkaczyk with those of their opponents and then analyse the arguments that Øhrstrøm and Hasle managed to prove and the topics that remain unresolved.

Claim 2 by Øhrstrøm and Hasle was a response to Tkaczyk and Jarmużek's statement that one of Prior's inspirations for creating temporal logic was Łoś's work, as expressed below:<sup>32</sup>

Prior's outstanding achievements in the field of temporal logic were inspired by three sources: the problem of future contingents and Łukasiewicz's many-valued logic; the medieval programme to construct the logic of the vernacular with its account of truth-values; and a small footnote on tense and modalities in a work by John Findlay. This is the standard story, based on Prior's texts and repeated by his followers. Our claim here is that there definitely was a vitally important fourth source: the work of Łoś.

This claim by Tkaczyk and Jarmużek is supported by three facts: 1) Prior started his work on the tense logic system after 1953; 2) Prior's note about Łoś's  $U_{tp}$  notation for usage in epistemic logic and for dates and intervals in Prior's *Time and Modality*;<sup>33</sup> 3) Prior's note about Łoś's work on the ground of temporal logic in the work *Past, Present and Future*.<sup>34</sup>

The fact that Prior mentioned Łoś in his works from 1957 and 1967 was acknowledged by Øhrstrøm and Hasle; they also added that Prior mentioned Łoś in his work 1955 onwards. Yet, they insist that there is no proof that Prior knew about the research of the Polish logician in 1954. The whole discussion

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<sup>32</sup> Jarmużek, Tkaczyk, "Jerzy Łoś Positional Calculus and the Origin of Temporal Logic": 274.

<sup>33</sup> Arthur Norman Prior, *Time and Modality* (Oxford: Oxford University Press, 1957).

<sup>34</sup> Arthur Norman Prior, *Past, Present and Future* (Oxford: Oxford University Press, 1967).

surrounding Prior reading about Łoś's work is very vague, as we do know that the first presentation of temporal logic ideas was made in 1954, but that in 1955, Prior had mentioned Łoś.

First, we must point out the fact that the authors are referring to the 1954 presentation of Prior's work. The system developed by Prior was, as the authors had already presented in their paper, positional logic. If we were to examine the argumentation for the first claim in Øhrstrøm and Hasle's thesis, this would not hold, as, according to it, this system is not temporal logic in the narrower sense. Thus, we would like to point out that the argumentation here is not sound; however, we will consider this argument separately from the previous one.

The arguments presented by the authors are psychological in nature and based on the conviction of knowing Prior's intentions and feelings as of 1954. However, such assumptions cannot be part of a serious justification in this historical matter. This is a belief, with no supporting facts, rather than a scientific approach. The argument that Prior would never, for some reason, avoid adding a reference to Łoś's work, cannot support the claim that he did not know of Łoś's calculus. As a matter of fact, we cannot tell whether Prior was acquainted with Łoś's work in 1954 as we do not have any proof and can only presume.

Nevertheless, the authors did not respond to the fact that Prior stated<sup>35</sup> that Łoś inspired his calculus of dates:

This calculus influenced my own formulation of a 'calculus of dates' (using the form  $U_t p$ ) in *Time and Modality*, and also has points of resemblance to Rescher's system from 1965.<sup>36</sup>

In fact, Prior confessed that Łoś's calculus influenced his work before 1957, which is when *Time and Modality* was published. Prior's work on positional logic was not only a formulation of a "calculus of dates", as he states, but also a work in which he (and, after him, Rescher) showed that the temporal modal symbols of a given specific set of axioms can be defined using a realisation operator. Hence, the results of Prior's and Rescher's work are a direct

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<sup>35</sup> Ibidem.

<sup>36</sup> Ibidem, 212.

continuation of Łoś's work.<sup>37</sup> Knowing that Prior acknowledged Łoś's work before 1957 and mentioned him even in 1955, we cannot surely reject the possibility of Łoś's idea influencing Prior (which was influenced by other factors as well) before he presented his own system of temporal logic. Despite this, as stated before, we cannot defend this claim using direct evidence.

However, the original claim by Tkaczyk and Jarmużek was not as precise. They stated that Łoś's work was a "vitaly important fourth source" of inspiration that led Prior to his "outstanding achievements in the field of temporal logic". Therefore, we can opt to weaken the claim that "Łoś's work inspired Prior in his first development in temporal logic" to the claim that "Łoś's work inspired Prior in his early developments in temporal logic and in some intermediate way in other developments". Moreover, there is evidence in the form of Prior's and Rescher's systems that Łoś's ideas on positional logic were under further development and that continuity was preserved. The claim that Łoś introduced temporal logic is most justified by all the facts and evidence presented.

#### 4. Rescher's View on Temporal Logic

Regarding the relation between Łoś's and Prior's works, the position of the logician who was the successor of both these thinkers in this matter is worth sharing – Nicholas Rescher. He worked within the fields of temporal and positional logic in the 60s and 70s and had insight into the development of tense and positional logic. He presented a few axiomatic systems of different positional logics using *R*-calculus, generalised it by creating topological logic and propagated it using the *R* symbol.

In the case of the categorisation of temporal logic, Rescher distinguished temporal logic, tense logic and chronological logic. It is hard to explicitly define temporal logic or tense logic in Rescher's views but chronological logic is, in fact, a set of *R*-calculus systems interpreted temporally. Moreover, we can tell that chronological logic, according to Rescher, is a subset of temporal

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<sup>37</sup> Rescher, Garson, "Topological Logic": 543.



logic.<sup>38</sup> In Rescher's works, we get an impression that *R*-calculus is not only a way of symbolising moments of time, as in the case of dates, but also an effective framework to define all temporal connectives within it. The importance of *R*-calculus, its relation to temporal logic and the influence of Łoś's work is evident in the quotation below:<sup>39</sup>

A major revival of interest in temporal logic has sprung up since the later 1940s. The stimulus for this revival can be traced largely to three sources: the study of historical materials (especially the studies of Stoic logic by Martha Hurst Kneale and Benson Mates and the studies of medieval logic by Ernest Moody), the logical analysis of grammatical tenses by Hans Reichenback, and, above all, the endeavour by the Polish logician Jerzy Łoś to devise a system of temporal logic – specifically an *R*-calculus – for the analysis of issues in the philosophy of science, especially Mill's methods of inductive reasoning. Łoś's ideas were considerably refined and extended by Arthur N. Prior and furthered by Nicholas Rescher. Under the stimulus and inspiration of Prior's work, ramifications in various directions have been explored by many logicians, which resulted in making temporal logic a very active field of current research. However, independent of Prior (and apparently of Łoś as well) is the recent development by G.H. von Wright (1963 and 1965) of a (substantially weaker) system of a chronological "logic of change" that has been extended in various directions by several logicians.

Regarding the categorisation of temporal logic, we can see that Rescher distinguished Von Wright's "logic of change" as a system of chronological logic. This is caused by the fact that the symbols of temporal modalities used in the systems of Von Wright and Prior can be presented using *R*-operator when the *R*-calculi contain specific axioms.<sup>40</sup>

Moreover, Rescher provided a set of axioms of chronological logic, which are common for all the *R*-calculus systems created. He used them to compare previous systems – Łoś's system presented in 1948, Prior's systems presented in 1957 and two systems of Rescher from 1966.<sup>41</sup> Whether or not Prior's development of the systems from 1957 was inspired by Łoś's ideas, all the sys-

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<sup>38</sup> Rescher, Urquhart, *Temporal Logic*, 27.

<sup>39</sup> *Ibidem*, 12.

<sup>40</sup> Rescher, Garson, "Topological Logic", 544.

<sup>41</sup> *Ibidem*, 542–543.

tems compared belong to the same class of logic – positional logic – and all of them are classified by Rescher as temporal.

The comparison provided by Rescher not only showed a fluid development of the idea taken from Łoś and Prior of using  $R$ -calculus but also took notice of the similarities between the systems of the two logicians. Four axioms of Prior's first system, called  $P_1$  by Rescher, are also the axioms used by Łoś. The second system by Prior incorporates one additional axiom and a stronger axiom in the place of Łoś's 2.6:

$R_n p \rightarrow p$ , where  $n$  denotes "now" or its numerical index is 0

On the basis of Rescher's research, without going into the historical details, we can say that the systems created by Łoś and Prior – Prior's was published around ten years after Łoś's – have much in common. Even if Łoś did not directly influence Prior's work at the beginning, he certainly had an impact on the development of positional logic. This belief was shared by Rescher, who saw the continuity of the different  $R$ -calculus systems. Besides this, Rescher categorised Łoś's system, and all positional logic systems with temporal interpretation, as systems of temporal logic. Moreover, even in 1971, Rescher shared the claim that Łoś's system had a significant influence on Prior's achievements and that Łoś created the very first system of temporal logic.

## Summary

In this paper, we questioned Øhrstrøm and Hasle's argument. Furthermore, we provided an argument challenging the original thesis of Jarmużek and Tkaczyk and weakened its historical aspect owing to certain credible arguments from Øhrstrøm and Hasle.

An important aspect of the history of the invention of temporal logic is that Łoś presented the first formal system that can be called temporal logic. He achieved this as early as 1947 and his system made it possible to formalise reasoning on time. The main subject of Łoś's work was the temporal aspect of Mill's canons of induction, and he created sufficient formalism to symbolise this. Whether or not this system meets the definition of temporal logic is secondary. If Łoś's system does not meet the

definition, it is an indicator that the definition is too narrow, not only in terms of his system in particular but in terms of many other systems of temporal logic that were developed after the 1950s.

As we have attempted to demonstrate, the broader definition provided by Goranko and Dalton is sufficient to describe Łoś's system. Therefore, we can refer to it a temporal logic system. Consequently, Łoś's system should be included in historical papers devoted to temporal logic history, such as the paper by Øhrstrøm and Hasle, which we hope will be updated in the future.

The second important argument, found within the early development of temporal logic, is regarding the influence of Łoś's work on Prior. We cannot provide direct proof that Prior was influenced by Łoś in 1954 when he presented his first formulation of temporal logic, as Øhrstrøm and Hasle correctly stated. However, as Jarmużek and Tkaczyk showed, there is proof of Łoś's influence on Prior as stated by Prior himself in 1957. Additionally, Prior cites Łoś's work in his papers 1955 onwards, so we cannot refute the possibility that Prior was acquainted with Łoś's work even in 1954. To sum up, after observing Rescher's analysis of positional logic developments as well as Prior's recognition of Łoś's influence within his own work, it can be said that the temporal logic created by Jerzy Łoś was the foundation for further work by Prior, Rescher, Urquhart and other researchers in the field.

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**Abstract:** In this article, we will respond to the paper by Øhrstrøm and Hasle, which questions the position of Jerzy Łoś as the first logician to create a temporal logic system. We will undertake this by analysing the argument supporting their thesis and the preceding claims by Jarmużek and Tkaczyk, who attempted to present Jerzy Łoś as the founder of temporal logic. The paper presents a critique of both arguments presented by the authors and a response based on historical facts and formal distinctions.

**Keywords:** positional logic, realisation operator, temporal logic, history of logic