MECHANISTIC RESPONSE OF CONVENTIONAL VS PERPETUAL FLEXIBLE PAVEMENTS UNDER SIMILAR LOADING CONDITION: A 3-D FINITE ELEMENT ANALYSIS

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ABSTRACT

A 3D finite element analysis was conducted on several models of conventional and perpetual pavements to assess the similarities and differences in their structural and fatigue response when both pavement types are subjected to varied axle load and wheel configurations. The models were designed as a five-layer pavement system made up of an asphalt surface laid on an asphalt binder course. The base used was a graded crushed stone of G1 quality. Asphalt materials were assumed to be viscoelastic, while the granular base, subbase and subgrade were assumed to be linearly elastic. All material properties conformed to the South African Mechanistic Design Method (SAMDM) Guideline. Tandem axle dual wheel loading produces the least safety factor and fatigue life, and the highest damage among other configurations. Perpetual pavement models from the same materials were found to be more structurally rigid when comparing strains and damage in the long term. They show more resistance to distress by traffic loading when compared to conventional pavements. It was concluded that a 3-Dimensional Finite Element Analysis (FEA) is a suitable tool for supporting Mechanistic-Empirical design methods and provides a platform for further investigation of the behaviour of perpetual pavements.

1. INTRODUCTION

A significant bulk of all freight is transported by road networks together with everyday human transportation needs. This produces demand rates that inherently increase the burden on roads, while simultaneously decreasing the road lifespan (Department of Transport, 2014). The demand for innovative designs that require less maintenance and structural rehabilitation increases rapidly, especially in economies that aim for sustainability and greener infrastructure (Timm & Newcomb, 2006). Relatively low life cycle costs due to reduced deep structural repairs are the desirable properties in pavement design. The goal is to find a pavement design solution that best fits environmental conditions, for a higher design life with minimal rehabilitation and maintenance activities. Conventional road pavements subjected to high traffic loading are highly susceptible to deformation and structural failure as a result of dynamic stress and strain increase over time. Such structural failure of the pavement can manifest itself through excessive strains at interfaces or within layers that result in fatigue cracking and rutting. Conventional pavements which generally last for up to 30 years with considerable rehabilitation activities are designed using either the traditional empirical design method based on the AASHTO guidelines (AASHTO, 1993) or the more recent and robust mechanistic-empirical method (AASHTO, 2008). The Mechanistic-Empirical design method (AASHTO, 2008) uses an iterative linear elastic analysis method to come up with mechanistic pavement responses that are later used to compute the pavement thicknesses.

The concept of perpetual pavements, which is relatively new, has been developed to meet the need for more durable pavement that requires less maintenance and structural rehabilitation (Timm & Newcomb, 2006; Asphalt Pavement Alliance (APA), 2002). Perpetual pavements are intended as long-life pavements as they are designed to last for 35 years and above, with minimum maintenance and rehabilitation (Mazumder, Kim & Lee, 2015). Perpetual pavements originated from the realization that forever increasing the thickness of the pavement to continue to provide structurally sound roads with ever-increasing traffic volumes is too costly (Asphalt Pavement Alliance (APA), 2002). The main idea behind perpetual pavement is to construct an asphalt pavement that is resistant to the main distress types. The premise of this approach is that pavement distress with deep structural origins should be kept below thresholds where the distresses begin to occur. As such, the mechanistic response of flexible pavements to loading needs to be better understood so that pavement distress with such deep structural origins can be prevented (Timm & Newcomb, 2006). In conventional pavements, a lack of understanding of the mechanical response of pavements could lead to costly overdesign, shorter design life and several rehabilitation activities. Perpetual pavements avoid structural failure in the presence of heavy traffic and require only periodic resurfacing, making them advantageous in terms of minimizing life cycle costs and user delays. The only current perpetual pavement design method is through the mechanistic-empirical design methods embodied in design tools such as PerRoad (Timm & Newcomb, 2006). This method relies on the knowledge and availability of methods that can accurately measure pavement mechanistic responses during the design process.

The 3-Dimensional Finite Element Method (FEM) is one of the major supporting tools that have been employed by researchers (Wang & Al-Qadi, 2009; Walubita & Scullion, 2010; Shen *et al.*, 2022) for the mechanistic analysis of flexible pavements. Its relevance is mainly based on its ability to model dynamic loading and complex material behaviour such as viscoelasticity, anisotropy, non-linear, and fatigue life (Minkwan, 2007). This study intends to contribute to the body of knowledge on the mechanistic response of pavements using the finite element analysis method. The objective of the study is to investigate the stress, strain and elastic deflection responses of typical structural models of two kinds of pavement (conventional and perpetual) that have been designed using two methods:

1) The traditional empirical approach based on the AASHTO methodology; and
2) Mechanistic-empirical design method for perpetual pavements. Both pavement types are designed and analysed using same loading condition and similar layer materials.

2. METHODOLOGY

2.1 Design of Conventional and Perpetual Pavement Layers

The research methodology comprises two steps. The first step involves the design of flexible pavement structures using two approaches. The first type of pavement considered as the ‘conventional pavement’ is designed using the PaveXpress tool, a web-based open-source pavement design tool that utilises the AASHTO empirical design approach (AASHTO, 1993). The second pavement type is considered the ‘perpetual pavement’ designed using PerRoad 4.4, which is also an open-source tool for perpetual pavement design based on the mechanistic-empirical design method (AASHTO, 2008). Some design parameters such as the traffic load in terms of the Equivalent Single Axle Loads (ESALs) are kept the same for both pavement types while the layer material types are also similar. Both pavement types are designed as five-layer systems comprising the surface layer, binder course, granular base, granular subbase and subgrade. The subgrade properties are also kept the same for both pavement types. The objective is to understand the mechanistic responses of the two pavement types using a static-structural 3D finite element analysis when subjected to repeated wheel loads. The entire approach is described in the flow chart below.

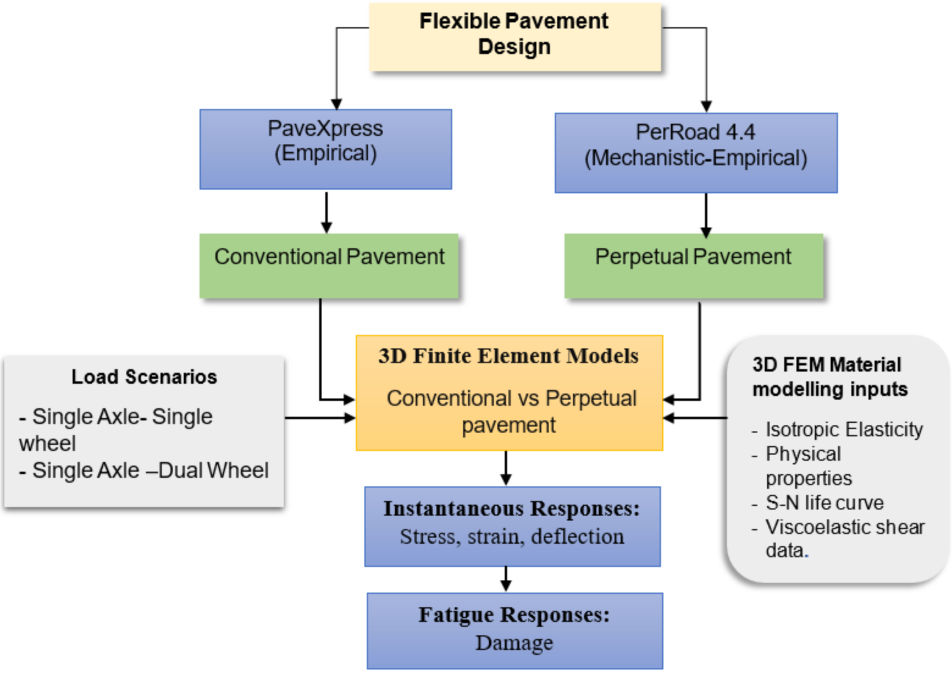


Figure 1: Study design

The output of the first step is layer thicknesses of the conventional and perpetual pavement types. Design parameters and resulting layer thicknesses are presented in Tables 1 - 4 below. Both pavement types are designed for a typical South African category A road with traffic class E4 as defined by the South Africa Technical Recommendations for Highway (TRH16) document (Department of Transport, 1991). Traffic class E4 is considered high traffic volume with a high proportion of heavy vehicles and with a cumulative equivalent single axle load (ESAL) of 12-50 million ESALS per lane (Department of Transport, 1991). For the design scenario considered in this study, an ESAL value of 30 million was adopted. Serviceability indices and reliability levels were also specified according to the class of road. Selected values for the parameters were informed by the prescribed range of values in the technical recommendation for highway (TRH4) document (Department of Transport, 1996). Table 1 below summarises these parameter values for conventional pavement.

Table 1: Design parameters for Conventional pavement: PaveXpress

Design period (years)	25
Reliability level	95%
Initial serviceability Index	4.5
Terminal serviceability Index	2
Total design ESALs	30 Million

Additional design parameters required to establish layer thickness are the layer material specification and properties. In the PaveXpress tool, layer materials are characterised by their structural and drainage coefficients while the subgrade is characterised by its resilient modulus or the California Bearing Ratio (CBR). The structural coefficient, according to the AASHTO empirical design guide (AASHTO, 1993), is a measure of stiffness and bearing capacity, which describes the relationship between the structural number of the pavement structure and the layer thickness. There is a direct relationship between structural coefficient and resilient modulus. The AASHTO 1993 guide recommends that the structural coefficients be calibrated based on the resilient modulus of asphalt mixtures. The typical range of values for asphalt surface courses is 0.35 – 0.5 (AASHTO, 1993). Values utilised in this study were informed by literature (Dave, Sias and Nemati, 2019) on investigations of pavement with similar structures. The drainage coefficient is also assumed in this study and is based on a good quality drainage capacity of the pavement layers. Minimum layer thicknesses are also part of the design inputs in the PaveXpress tool. The values are inputted as initial estimates of the needed thickness and are subject to change when the PaveXpress tool simulates the empirical design procedure. The thicknesses are then modified until a balanced pavement that satisfies all design input variables is obtained. Table 2 shows the conventional pavement layer structure with material coefficients as well as the final layer thicknesses computed in PaveXpress.

Table 2: Conventional pavement material properties and layer thickness

			Input		*Output*
Layer No.	**Layer**	**Material**	**Structural Coeff.**	**Drainage Coeff.**	**Thickness (mm)**
1	Surfacing	Asphalt (AC)	0.35	1.25	50
2	Binder	Asphalt (AC)	0.25	1.05	140
3	Base	Granular (G1)	0.2	0.85	210
4	Subbase	Gravel (G6)	0.12	0.5	180
5	Subgrade	Natural Soil (G7)	0.09	0.4	-

For perpetual pavement, the design parameters are inputs in the PerRoad design tool. The information required in the design process includes the traffic loading configuration, layer structure definition, layer material properties and a specification of performance criteria. In terms of traffic loading, a similar road category (Category A) and traffic class (E4) were assumed for conventional pavement, with an estimated load of 30 million ESALS. The load value was kept the same to provide a basis for comparison between the two pavement types in terms of mechanistic response to load. A five-layer structure was adopted for the pavement. Material properties are defined by the elastic modulus and Poisson ratio. The values selected for the design are assumed from the suggested range of values in the literature such as Theyse, De Beer & Rust (1996). A summary of the material parameters and the resulting layer thickness is presented in Table 3 below.

In terms of performance criteria definition, the procedure suggested by Timm and Newcomb (2006) is adopted. Performance criteria are defined for each layer based on the nature of distress and mechanical responses associated with the layer when subjected to loading. For the asphalt layers, performance is defined based on fatigue cracking that propagates from the bottom of the layer as a result of tensile strain that develops at the bottom (Timm & Newcomb, 2006). Granular base and subbase layers are assumed to fail by deformation and shear while subgrade failure is through permanent deformation

(Theyse, De Beer & Rust, 1996). The transfer function is also defined to describe the relationship between critical values of response parameters such as strain and the maximum number of load repetitions before failure. Table 4 shows the performance criteria and transfer functions defined for the pavement.

Table 3: Perpetual pavement design parameters

Layer No.	Layer	Material	Modulus(MPa)	Poisson's ratio.	Thickness (mm)
1	Surfacing	Asphalt (AC)	6000	0.35	70
2	Binder	Asphalt (AC)	8000	0.35	100
3	Base	Granular (G1)	500	0.40	150
4	Subbase	Gravel (G6)	300	0.45	180
5	Subgrade	Natural Soil (G8)	170	0.45	-

Values of modulus are adopted from Theyse, De Beer and Rust (1996)

Table 4: Perpetual pavement performance criteria and endurance limits

Layer	Position	Criteria	Endurance Limit ($\mu\varepsilon$)	Transfer Functions (Ref?)	
				k1	k2
3	Bottom of layer 3	Horizontal strain	-70	2.83e-06	3.148
4	Top of layer 4	Vertical strain	200	6.02e-08	3.87

The other aspect of the methodology involves the finite element model development, loading and analysis of mechanical responses of the pavement model. Details of this step are discussed in the next section.

2.2 Finite Element Model Development and Analysis

The FEA tool used for the second stage of the study is the ANSYS Workbench (Altabey, Noori & Wang, 2018). This is a finite-element modelling and analysis package for numerically solving a wide variety of complex structural and mechanical problems. The first step in the modelling and analysis process involved developing 3D models of the two pavement structures. The static structural analysis system was employed in the analysis. In the modelling step, all layers are assigned their specific material properties as previously described in Section 2.1. The properties are stored as engineering data in the software database. The analysis attributes such as interlayer bonding, boundary conditions, tyre imprint pressure, meshing and other analysis settings are all set up within the modelling step. Meshing algorithms and pressure applications are also prepared in that step. The results from the analysis are presented in the form of contour-coloured gradients that indicate pavement responses. Two loading scenarios described in Figure 1 were employed in the analysis, resulting in four different 3D models. Wheel loads were assumed to have circular tyre imprints on the pavement surface. The pavement material is modelled using Isotropic elasticity. Other physical properties, S-N life curves and viscoelastic shear data (Altabey, Noori & Wang, 2018) were also defined.

2.2.1 Model Dimensions, Tire Imprint and Loading Condition

1140 mmx1140 mm models were used for the analysis of both axle wheel configurations. The thicknesses vary according to the specific pavement design, but both accommodate the same traffic loading. The layers are separated as individual parts and assembled to specifically characterise each layer material. All layer shapes and horizontal dimensions are the same to preserve downward continuity. Models are wide enough for all analysis constraints and generate a computable number of nodes and elements. To idealise the loading condition and reduce the model size, a half-width pavement and half-axle are used in the loading step and symmetry of loading geometry is assumed. The pavement is loaded using two major axle configurations. A single axle with single wheel, as well as a single axle with dual wheels. The value of the pressure exerted on the pavement surface is taken from literature such as Wang and Al-Qadi (2009) and is assumed to be uniform across a circular load imprint area. The radius of the circularly loaded area is obtained using the following equation from the literature.

$$r = \sqrt{\frac{P}{p\pi}} \tag{1}$$

Where r is the radius of the circular imprint area, P is the Standard axle load on one tire, and p is tire inflation pressure assumed as 40psi (275kPa) for passenger cars and 60psi (414kPa) for trucks. The load variables are presented in Table 5 below

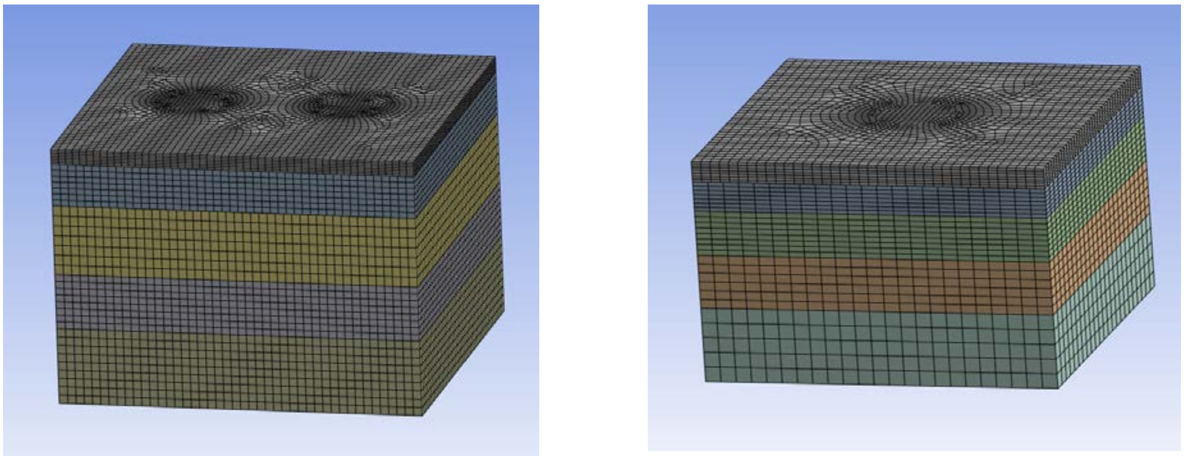
Table 5: Loading variables for tyre imprint area and contact pressures

Axle Configuration	**Typical half-axle static load (kN)**	**Static axle group load (kN)**	**Imprint diameter (mm)**	**Tyre contact pressure (kPa)**
Single axle-single wheel	40	80	190	650
Single axle-dual wheel	45	90	140	700

2.2.2 Element Type, Meshing and Boundary Conditions

The pavement structure is modelled using an 8-node linear brick, reduced integration, and linear 3D stress element type. Each node has 3 degrees of freedom in normal static directions. All three degrees of freedom are restricted from movement to preserve fixity and model basic material bonding. The mesh is generated using five different multi-zoning mesh inputs. This multi-zoning mesh method uses a global element order setting and a thin sweep decomposition. Figure 2 below shows the resulting finite element models of both pavements.

Figure 2 shows the models of the 5-layer pavement structure before loading. The pavement model boundary conditions are assumed to be fully constrained on all five faces. This fairly represents the actual pavements as it allows no movement in the downward and sideways direction. The top face is allowed to deform under loading in the downward direction only. The deformation is a function of their material mechanical properties and loading condition.



(a) (b)

Figure 2: Typical pavement mesh models (a) Conventional single-axle dual-wheel, (b) Perpetual single-axle single-wheel

2.2.3 Material Properties

Materials are characterised using physical properties such as density. Linear Isotropic behaviour was assumed for all the materials and was defined using Young's modulus, Poisson's ratio, bulk and shear modulus of each material. The S-N curve, which is used to characterise damage and life parameters relating to a specific failure mode on a material is also input into the software. These parameters are obtained from the South African Mechanistic-Empirical Design Method guideline as prescribed in Theyse, De Beer & Rust (1996) and Theyse and Muthen (2000). Asphalt Concrete materials in the models are characterised by their viscoelastic behaviour as described in Little, Allen & Bhasin (2018).

The surface course and binder course used in the models are classified under asphalt materials. The Surface and binder course are laid using a continuously graded surfacing asphalt mix. The mix is assumed to have a void content of 11% at 20º (Bai *et al.*, 2021). They are suitable for use in a pavement that has an expected performance reliability of road category A. The elastic modulus for these is obtained from the SAMDM as suggested by Theyse & Muthen (2000). The Asphalt concrete layers were assumed to be in good condition and within 0-200mm of depth from the pavement surface. Asphalt materials fail due to fatigue cracking under repeated loading as a result of excessive tensile strains at the bottom or in the layer (Theyse & Muthen, 2000). The transfer functions, and hence the S-N curve are adopted from the SAMDM for a continuously graded surfacing layer. In this study, the transfer functions were not directly used, but the S-N curves generated from them were used to characterise the life parameters for the specific failure modes. The transfer functions cannot be readily input into the ANSYS programme, but rather the graphs are converted back into numerical data and then input into the programme as data representing the relationship between stress/strain and the loading cycles of the material. In the software, this data is utilised as a transfer function.

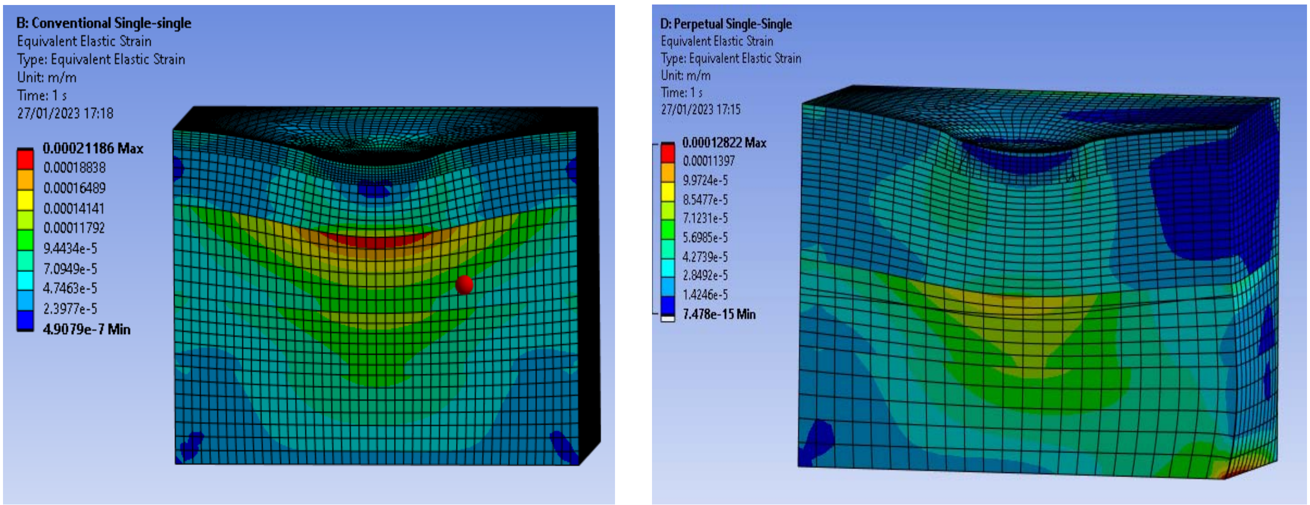
For the base and subbase, granular materials are employed. A dense graded crushed stone (G1 class) of maximum size 37.5 mm is considered for the base while natural gravel (G6 class) with a maximum size of 63 mm is used for the sub-base. The Subgrade material (G7) is assumed to be gravel soil with a CBR of at least 15%, at 100% modified AASHTO density. Granular material exhibits deformation due to densification and gradual shear under repeated loading. Maree developed the concept of safety factor against shear failure for granular materials from the Mohr-Coulomb theory (Theyse, De Beer & Rust, 1996).

3. RESULTS & DISCUSSION

Presented in this section are the obtained mechanistic responses (strain, stress and fatigue life) from the finite element analysis of the conventional and perpetual pavement types under the two axle-wheel load configurations.

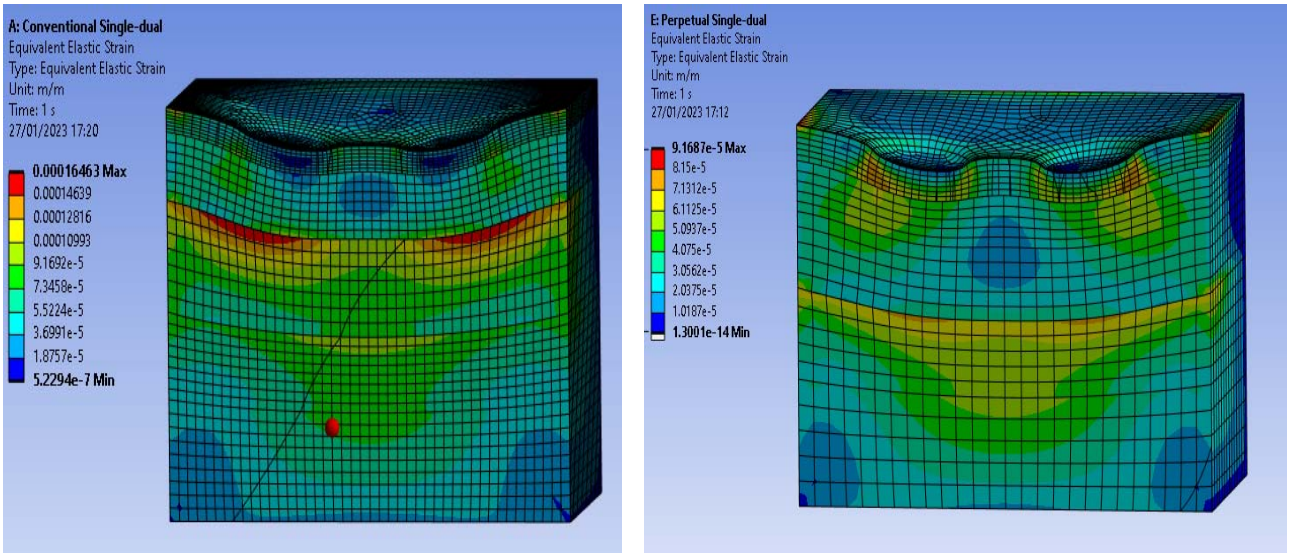
3.1 Equivalent Strain

Figures 3 and 4 show the resulting equivalent strain contours for single-axle single-wheel and single-axle dual-wheel load configurations respectively. Equivalent strain is a scalar variable that can be used to report strain results over a body. It does not represent the complete information of the strain state on the model but rather a useful summary. It is a type of resultant strain that represents the overall strain level in a material, taking into account all of the different types of strains acting on that material.



(a) (b)

Figure 3: Equivalent strain due to Single-axle single-wheel load in (a) Conventional pavement (b) Perpetual pavement



(a) (b)

Figure 4: Equivalent strain due to single-axle dual-wheel load in (a) Conventional pavement (b) Perpetual pavement

Also shown in the figures are the maximum and minimum horizontal elastic strain values for each pavement model. Furthermore, the resulting strain values at the top, midpoint and bottom of each pavement layer were extracted. The distribution is shown in Figure 5 below.

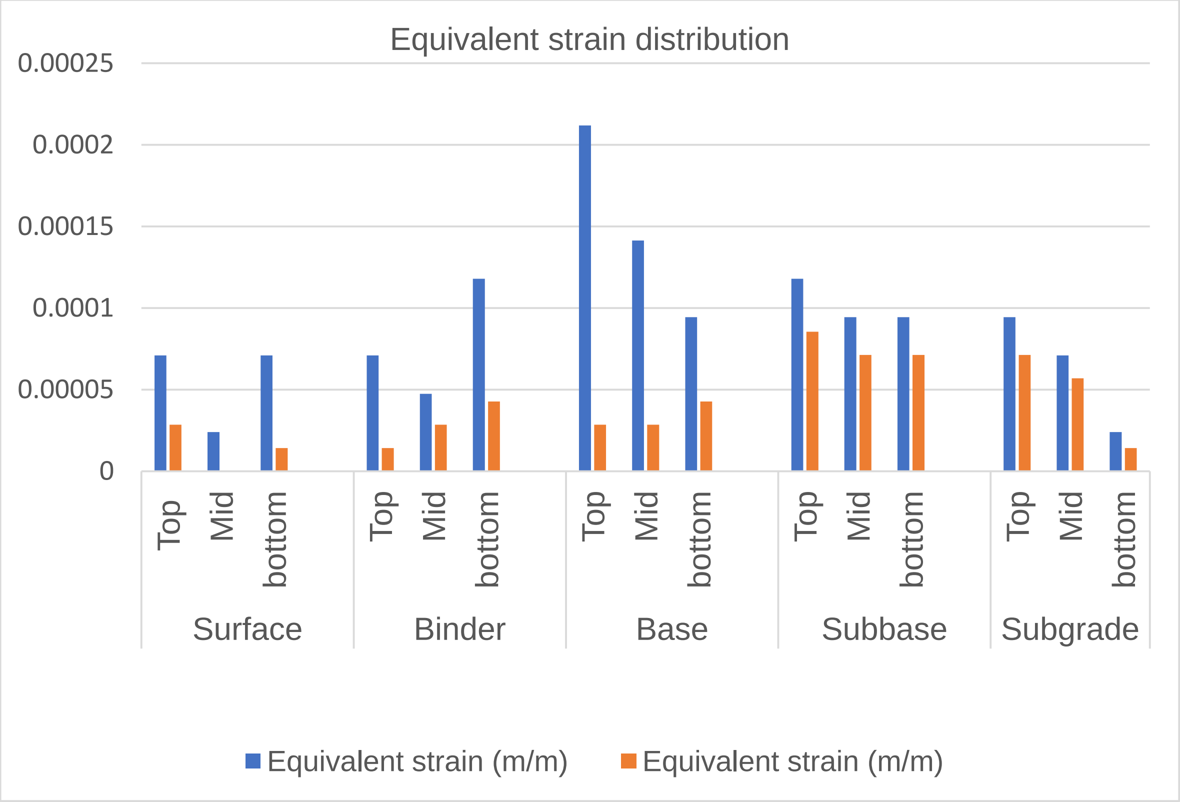
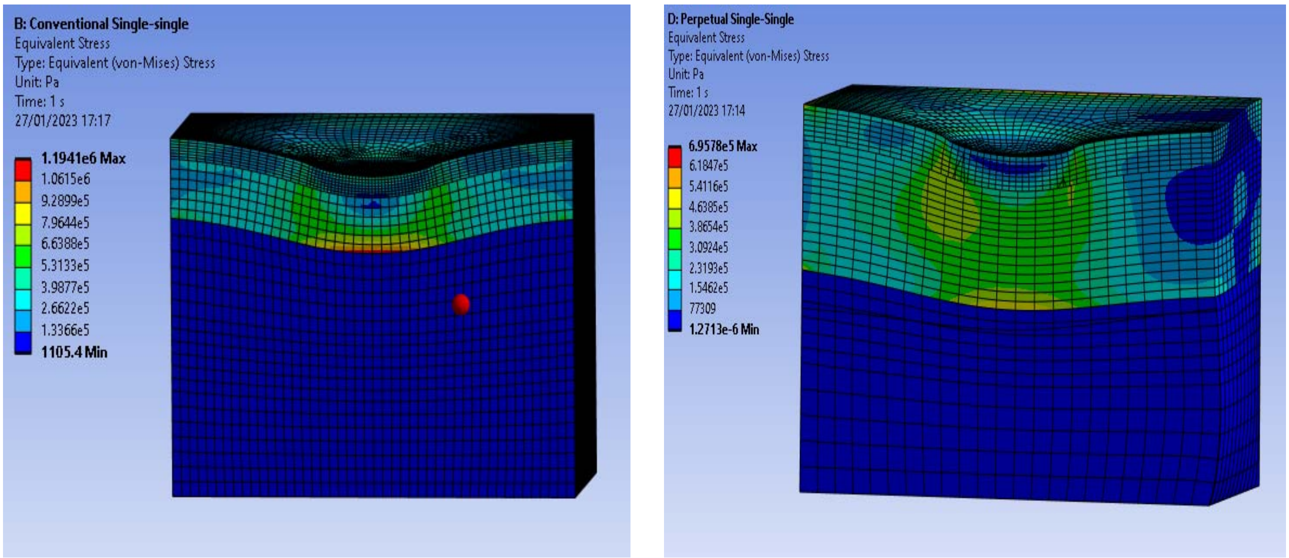


Figure 5: Equivalent strain distribution across pavement layers

Figure 5 shows the distribution of strains at specific positions within each layer. It must be emphasized that the strain at the bottom of the top layer and the strain at the top of the bottom layer, within any two adjacent layers, are not compatible. This is mainly due to the major differences in their mechanical properties and response pattern. From the Figure, the equivalent strain in conventional pavements is seen to be higher than that in the perpetual pavement. At the upper layers, the strains for the conventional pavement models seem to be relatively low and have a peak value at the top of the base course, marking a change in material response since a granular base is used. Even though perpetual pavements have the same material makeup, the strains throughout the layers remain relatively low and show no peaks where other layers carry much higher stress. The strains in the perpetual pavement are lower than those in conventional pavements at all selected/modelled positions for both axle-wheel configurations.

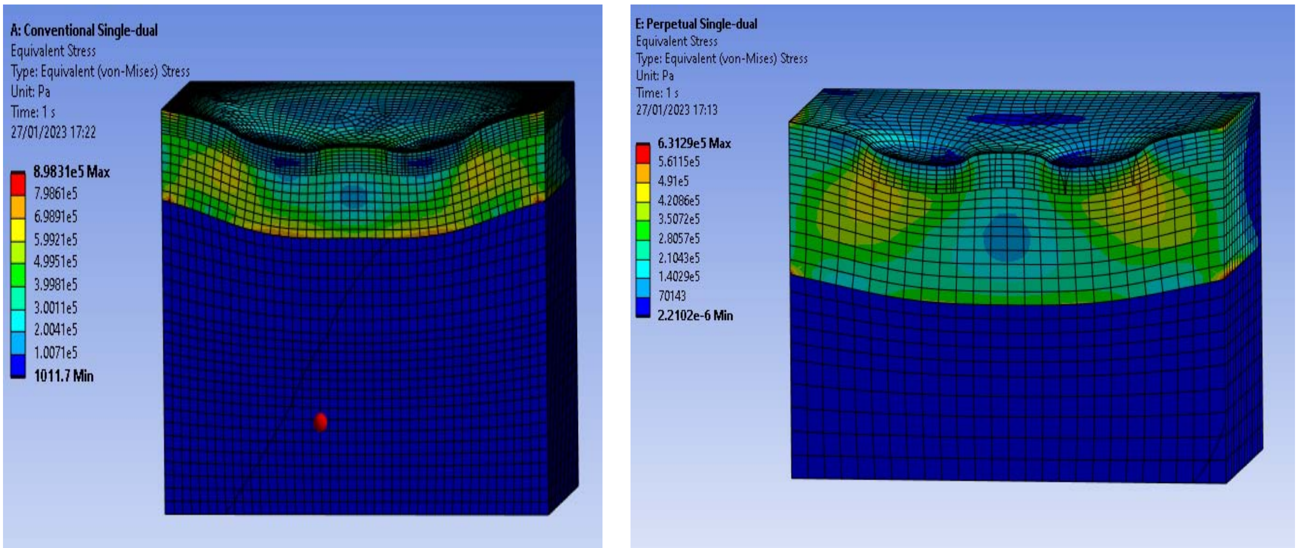
3.2 Equivalent Stress

Equivalent stress allows one to view the stress state of the body by one plot instead of nine contour plots. It summarises the results of a 3x3 tensor into one. It is also a type of resultant stress that represents the stress level in a particular region of material, taking into account all of the different types of stresses acting on that region. The stress contours for both pavement types are shown in Figure 6.



(a) (b)

Figure 6: Equivalent stress due to single-axle single-wheel load in (a) Conventional pavement (b) Perpetual pavement



(a) (b)

Figure 7: Equivalent stress due to single-axle dual-wheel load in (a) Conventional pavement (b) Perpetual pavement

Equivalent stress is recorded from the FEA models in the same manner as the strains. The graphs show that the equivalent stresses in conventional pavement layers are higher than the stresses in the perpetual pavement layers at all positions in the models. It is also shown that the stresses in both pavement models are mainly confined in the top asphaltic layers, and are very low in the granular substructure. This can be attributed to the difference in the modulus of linear elasticity between the top asphaltic layers and the underlying granular sub-structure. This observation highlights the similarity in the mechanistic response of both pavement types, while it also reveals that perpetual pavements perform better in terms of overall stress.

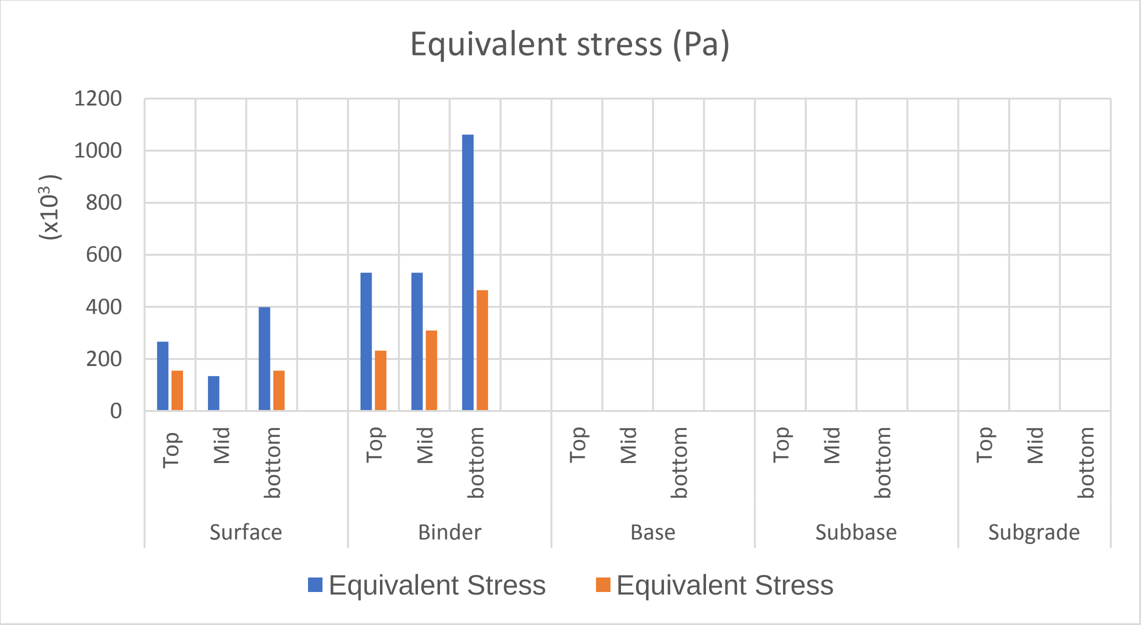


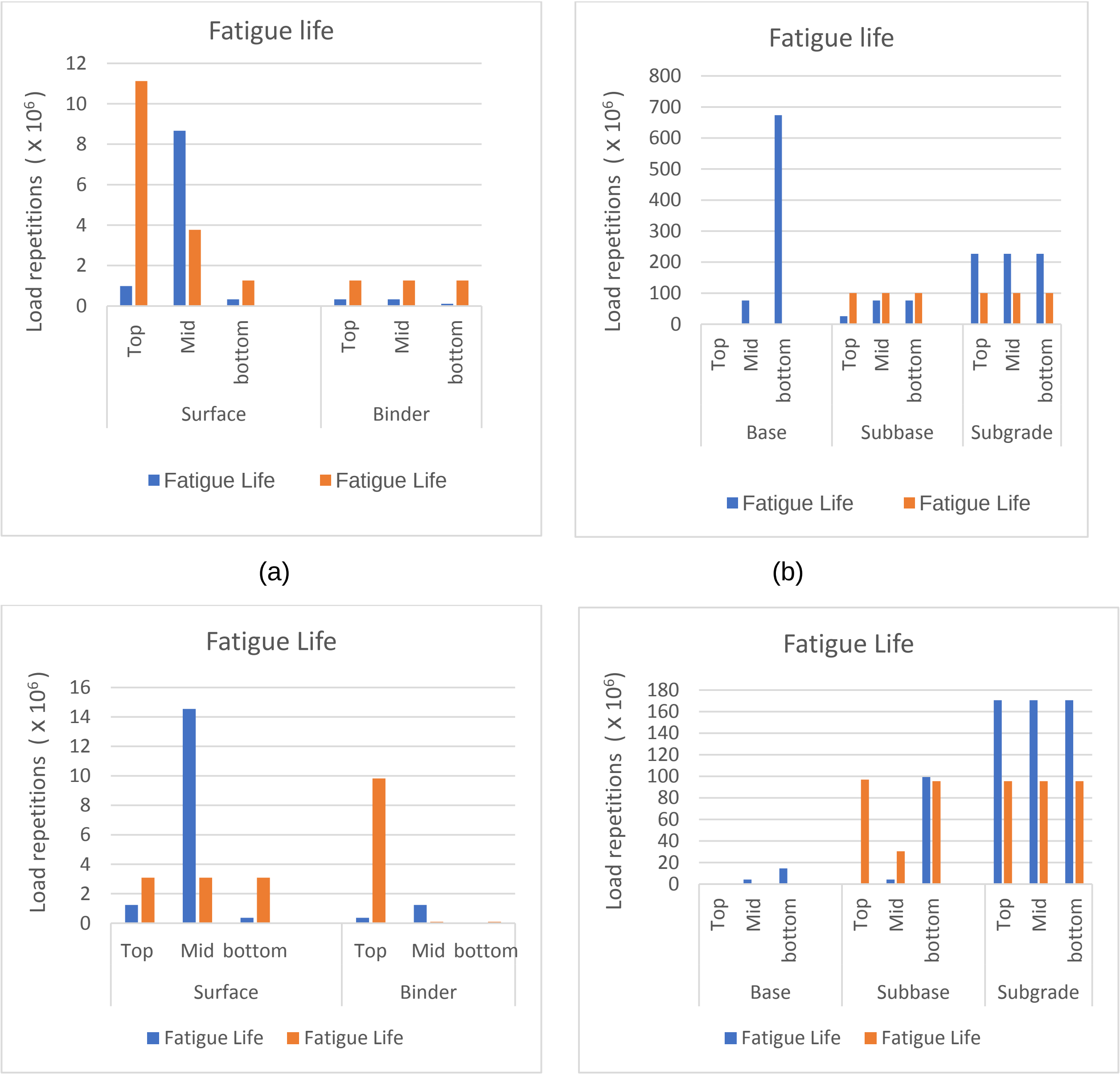
Figure 8: Equivalent stress distribution in pavements

The pavement materials widely vary in mechanical properties. Due to this, their stress response varies. The asphalt material deforms primarily at the bottom due to tensile stresses in the lateral direction. This is the origin of bottom-up cracking. For granular materials, the major stresses are vertical downward and longitudinal, relative to pavement alignment.

3.3 Fatigue Life and Damage Analysis

Fatigue life shows the number of load cycles that the pavement is expected to carry before failure occurs. The following graphs show the fatigue life for both pavement models. It is calculated based on the defined transfer functions of the materials as given in the SAMDM (Theyse & Muthen, 2000). Asphaltic and granular layers differ in their failure mechanism due to the difference in material stiffness. While the asphaltic layers can experience fatigue from repeated loads, granular layers fail through a process of gradual deformation and shear (Theyse, De Beer & Rust, 1996; Timm & Newcomb, 2006). For this reason, the asphalt material fatigue life is plotted separately to show an amplified, and relatively readable graph.

The fatigue life for asphaltic material in perpetual pavement models is relatively higher compared to its conventional model counterparts. Additionally, it can be observed that the lower underlying layers in perpetual pavement models show a relatively low number of cycles to failure which directly translates into low fatigue life. This highlights a perpetual pavement mechanistic response characteristic first observed by Timm & Newcomb (2006) that stresses and strains in perpetual pavements are confined at the top resistant layers such that weaker underlying material does not experience failure. Conventional pavement shows that the fatigue life of the base is higher than other layers. This highlights the fact that conventional pavement balance is heavily dependent on the performance of the base.



(a) (b)

(c) (d)

Figure 9: Fatigue life across pavement layers

Table 6 further presents a damage analysis by comparing the measured equivalent strain against the endurance limit strain which was set as an input in the initial layer thickness design in PerRoad software. From the table, it is shown that actual strains for both load configurations were much less than the limiting strains at the bottom of the base layer as well as the top of the sub-base.

Table 6: Endurance limit strain vs Actual strain of perpetual pavement

				Single Wheel	Dual Wheel
Layer	Position	Criteria	Endurance limit strain (10^{-6})	Actual strain (10^{-6})	Actual Strain (10^{-6})
Base layer	Bottom	Horizontal strain	70	42.739	20.375
sub base	Top	Vertical strain	200	85.477	81.692

4. CONCLUSION

This study presents a single mechanistic response analysis of flexible pavements that have been designed using the traditional empirical approach as well as the mechanistic-empirical perpetual pavement design approach. The results of equivalent stress and equivalent strain have shown the relative performance of the conventional (empirical approach) and perpetual pavement models under the varied axle and wheel configurations. In all model instances according to the varied loadings, perpetual pavements showed higher resistance to deformation based on the defined parameters. The nature of their mechanistic response is similar, and this can be attributed to the fact that similar materials were specified in the pavement models. An overall observation is the ability of the empirical design method to arrive at the smallest possible thicknesses that achieve reasonable performance levels in terms of fatigue life of the asphaltic layers when compared with perpetual pavement design.

The use of the finite element method for mechanistic analysis of pavements presents an opportunity for designers to carry out more accurate analyses of expected strains and stresses when pavement materials undergo various loading conditions. While there are still a lot of complexities as regards the understanding of failure mechanisms of various materials, the finite element method can provide numerical solutions to complex problems. With an improved understanding of material behaviour under complex load patterns, finite element analysis could unlock unique opportunities in pavement design and specifically in the aspect of perpetual pavement design.

Finally, it is recommended that numerical analysis of pavement structures be complemented with laboratory or field testing of actual layer materials. For example, outcomes from fatigue analysis in finite element simulations could be validated with laboratory or field experimentation. These aspects are however beyond the scope of the study.

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