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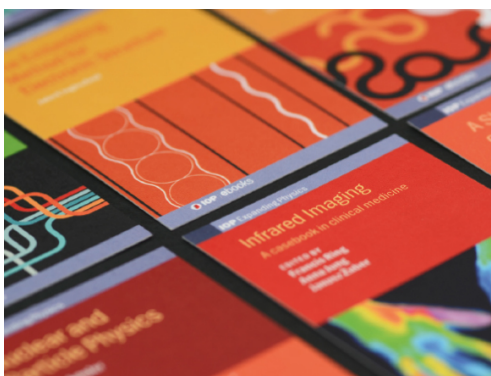
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# Dual solutions of magnetohydrodynamic stagnation point flow and heat transfer of viscoelastic nanofluid over a permeable stretching/shrinking sheet with thermal radiation

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**Abstract.** The present study is intended to encompass the stagnation point flow and heat transfer of viscoelastic nanofluid with the presence of thermal radiation. The viscous incompressible electrically conducting and Jeffrey fluid model is taken into account. The governing partial differential equations are reduced to ordinary differential equations by using the appropriate similarity variables. The resulting differential equations are solved numerically using the built in `bvp4c` function in Matlab. Dual solutions are discovered for a certain range of the governing parameters. Numerical results for the velocity and temperature profiles as well as the skin friction coefficients and the local Nusselt number are elucidated through tables and graphs.

## 1. Introduction

Nowadays, a vast range of industrial and engineering processes require the higher rate of heat transfer especially involving the cooling process of the small volume materials like solid-state lighting, manufacturing, transportations and microelectronics. Since the conventional heat transfer methods are incapable to fulfill this requirement, thus the employment of nanofluids is necessary due to their higher thermal conductivity. Choi and Eastman [1] pioneered the discovery of nanofluids by disseminating extremely small ( $10^{-9}$ ) sized particles in the common base fluid. Owing to their advanced thermophysical properties, nanofluid is immensely useful in industrial cooling applications, microelectromechanical systems, extraction of geothermal power, cooling of microchips, automotive coolants, optical devices, nanodrug delivery, nuclear reactors and cancer therapeutics [2-3].

In the present study, Jeffrey fluid model is considered. It can be categorized as a non-Newtonian fluid which exhibits the viscoelasticity characteristic. In addition, Jeffrey model is known as a relatively incomplex linear model due to the utilization of time derivatives instead of convective derivatives. There are numerous references on Jeffrey nanofluid such as Sandeep et al. [4], Narla et al. [5], Zin et al. [6], Chakraborty et al. [7], etc. The existence of magnetic field contributes to the prominent and faster effect. Hayat et al. [8] explored the magnetohydrodynamic (MHD) stagnation point flow of Jeffrey nanofluid with Newtonian heating. Interaction of magnetic field and electrically conducting liquid has significantly influencing the boundary layer control, MHD generators, pumps and bearings as elaborated by Shehzad et al. [9]. Reddy and Makinde [10] identified that velocity and nanoparticle concentration declined while the temperature raised with the enhancement of magnetic field.



Another consideration in this study is thermal radiation since it has a remarkable effect in surface heat transfer. Sheikholeslami et al. [11] discovered that Nusselt number was proportionally related to the radiation parameter in their investigation of MHD nanofluid flow with the presence of thermal radiation. Farooq et al. [12] considered nonlinear thermal radiation in their study and discovered that the surface heat flux increased with the influence of thermal radiation. Motivated by the above mentioned studies, this paper is devoted to investigate the stagnation point flow and heat transfer of MHD viscoelastic nanofluid over a stretching/shrinking sheet with the new formulations and suction effect which is the extension of the research done by Madhu and Kishan [13]. The numerical result has been generated by using the built-in function `bvp4c` in Matlab. Numerical analysis of the effect of some governing parameters has been conducted. Furthermore, dual solutions also have been discovered.

## 2. Problem formulations

A steady two-dimensional stagnation point flow of an electrically conducting Jeffrey fluid model of a viscoelastic nanofluid over a permeable stretching/shrinking sheet is considered. The stretching/shrinking surface is in the  $x$  direction while  $y$ -axis is set perpendicular to the surface. There are assumptions that  $u_e = ax$  is the velocity of an external flow and  $u_w = bx$  is the velocity of the stretching/shrinking sheet where  $a$  and  $b$  are constants. The temperature and nanoparticle concentration at the stretching/shrinking surface are  $T_w$  and  $C_w$ , respectively, while  $T_\infty$  and  $C_\infty$  are the uniform temperature and concentration of the nanofluid far from the sheet.

The governing equations of the nanofluid flow, the heat and the concentration fields are as follows (see Madhu and Kishan [13]):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\nu}{1 + \chi} \left[ \frac{\partial^2 u}{\partial y^2} + \chi_1 \left( u \frac{\partial^3 u}{\partial x \partial y^2} - \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} + v \frac{\partial^3 u}{\partial y^3} \right) \right] + u_e \frac{du_e}{dx} + \frac{\sigma B_0^2}{\rho_f} (u_e - u) \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho C)_f} \frac{\partial q_r}{\partial y} + \frac{(\rho C)_p}{(\rho C)_f} \left( D_B \frac{\partial T}{\partial y} \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 \right) \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} \quad (4)$$

The corresponding boundary conditions are

$$u = u_w(x) = bx, \quad v = v_w, \quad T = T_w, \quad D_B \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \frac{\partial T}{\partial y} = 0 \quad \text{at } y = 0, \quad (5)$$

$$u = u_e(x) = ax, \quad v = 0, \quad T = T_\infty, \quad C = C_\infty \quad \text{as } y \rightarrow \infty.$$

where  $\nu = \mu/\rho_f$  denotes the kinematic viscosity,  $\mu$  is the dynamic viscosity,  $\rho_f$  is density of the base fluid,  $\chi$  is the ratio of relaxation time to retardation times,  $\chi_1$  is the retardation time,  $\sigma$  is the electrical conductivity,  $B_0$  is the intensity of the magnetic field,  $\alpha = k/(\rho C)_f$  is the thermal diffusivity and  $k$  is the thermal conductivity. Other parameters in the above equations are  $D_B$ ,  $D_T$ ,  $(\rho C)_p$  and  $(\rho C)_f$  which are representing the Brownian diffusion coefficient, thermophoretic diffusion coefficient, effective heat capacity of the nanoparticle and heat capacity of the fluid, respectively. Moreover, we also consider the mass transfer velocity  $v = v_w$ , where  $v_w < 0$  represents the suction flow and  $v_w > 0$  corresponds to the injection flow.

By implying the Rosseland approximation for radiation [14], the following simplification of radiative heat flux is obtained:

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y}. \quad (6)$$

Here, the Stefan Boltzmann constant is symbolized as  $\sigma^*$  and the mean absorption coefficient is represented by  $k^*$ . Then, by exclusion of the higher-order terms and expansion of  $T^4$  about  $T_\infty$  using Taylor series, the following expression can be obtained:

$$T^4 = 4T_\infty^3 T - 3T_\infty^4 \quad (7)$$

Using (6) and (7), we get

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_\infty^3}{3k^*} \frac{\partial^2 T}{\partial y^2}. \quad (8)$$

The governing equations (1)-(4) are solved by implementing the following similarity transformations:

$$\eta = y\sqrt{\frac{a}{\alpha}}, \quad u = axf'(\eta), \quad v = -\sqrt{a\alpha}f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_\infty}. \quad (9)$$

Now, equation (1) is satisfied identically and we get

$$\text{Pr}(f''' + \beta(f''^2 - ff^{iv})) + (1 + \chi)(ff'' - f'^2 + 1 + M^2(1 - f')) = 0 \quad (10)$$

$$\left(1 + \frac{4}{3}Rd\right)\theta'' + f\theta' + Nb\theta'\phi' + Nt\theta'^2 = 0 \quad (11)$$

$$\phi'' + Le f\phi' + \frac{Nt}{Nb}\theta'' = 0 \quad (12)$$

correspond to the boundary conditions

$$\begin{aligned} f(0) = s, \quad f'(0) = \lambda, \quad \theta(0) = 1, \quad Nb\phi'(0) + Nt\theta'(0) = 0, \\ f'(\infty) \rightarrow 1, \quad f''(\infty) \rightarrow 0, \quad \theta(\infty) \rightarrow 0, \quad \phi(\infty) \rightarrow 0. \end{aligned} \quad (13)$$

where  $s = -\frac{v_w}{\sqrt{a\alpha}}$  denotes the suction parameter. Further,  $\lambda$  represents the constant stretching ( $\lambda > 0$ )

or shrinking ( $\lambda < 0$ ) parameter, Pr is the Prandtl number,  $\beta$  is the Deborah number,  $M^2$  is the magnetic parameter, Rd is the radiation parameter, Nb is the Brownian motion parameter, Nt is the thermophoresis parameter and Le is the Lewis number which are defined as

$$\begin{aligned} \lambda = \frac{b}{a}, \quad \text{Pr} = \frac{\nu}{\alpha}, \quad \beta = \lambda_1 a, \quad M^2 = \frac{\sigma B_0^2}{\rho_f a}, \quad Rd = \frac{4\sigma^* T_\infty^3}{kk^*} \\ Nt = \frac{(\rho c)_p D_T (T_w - T_\infty)}{(\rho c)_f \alpha T_\infty}, \quad Nb = \frac{(\rho c)_p D_B (C_w - C_\infty)}{(\rho c)_f \alpha}, \quad Le = \frac{\alpha}{D_B}. \end{aligned} \quad (14)$$

The investigated physical quantities in this study are the local skin friction coefficients  $C_{fx}$ , and the local Nusselt number  $Nu_x$  which can be expressed as

$$C_{fx} = \frac{\mu}{\rho u_e^2} \left( \frac{\partial u}{\partial y} \right)_{y=0}, \quad Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, \quad (15)$$

where the surface heat flux is

$$q_w = -k \left( \left( 1 + \frac{16\sigma^* T_\infty^3}{3kk^*} \right) \frac{\partial T}{\partial y} \right)_{y=0} \quad (16)$$

Substituting the similarity transformations (6) into equations (15)-(16), we get

$$\left( \frac{\text{Re}_x}{\text{Pr}} \right)^{1/2} C_{fx} = f''(0), \quad (\text{Re}_x \text{Pr})^{-1/2} Nu_x = -\left( 1 + \frac{4Rd}{3} \right) \theta'(0). \quad (17)$$

where  $\text{Re}_x = u_e x / \nu$  is the local Reynolds number.

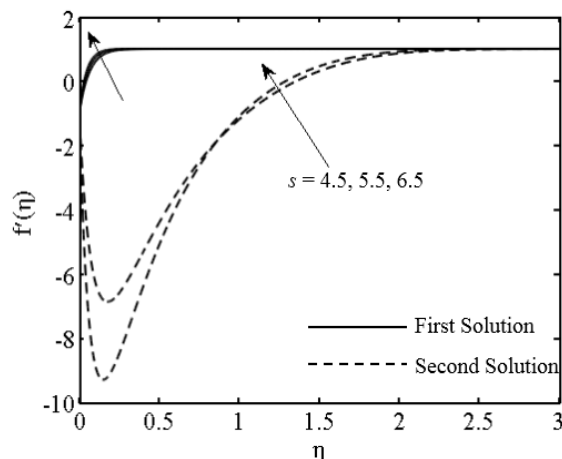
### 3. Results and discussion

Table 1 clearly reveals that the present solution method namely the bvp4c function shows a good agreement with the existing solutions as discussed by Khan and Pop [15] and Madhu and Kishan [13] by neglecting the effect of the Deborah number, magnetic parameter, radiation parameter and the ratio of relaxation time to retardation time. From this result, we also notice that  $-\theta'(0)$  is a decreasing function, while  $-\phi'(0)$  is an increasing function for  $Nb$ ,  $Nt$ ,  $Le$  and  $Pr$ .

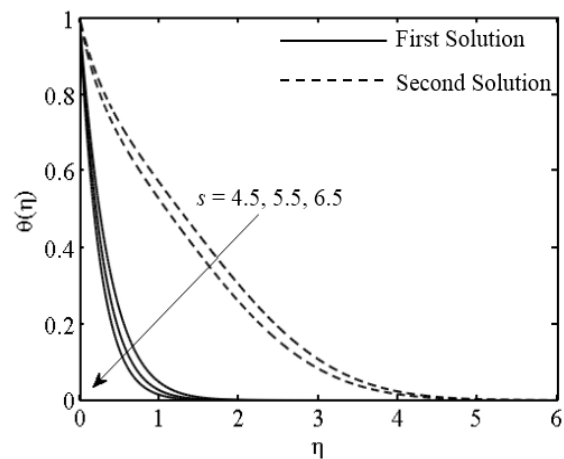
**Table 1.** Comparison of  $-\theta'(0)$  and  $-\phi'(0)$  when neglecting  $M$ ,  $Rd$ ,  $\beta$  and  $\chi$  with  $Le = Pr = 10$ .

$Nb$	$Nt$	$-\theta'(0)$			$-\phi'(0)$		
		Khan and Pop [16]	Madhu and Kishan [13]	Present	Khan and Pop [16]	Madhu and Kishan [13]	Present
0.1	0.1	0.95240	0.95237	0.95226	2.12940	2.12916	2.12854
	0.2	0.69320	0.69311	0.69315	2.27400	2.27384	2.27257
0.2	0.1	0.50560	0.50545	0.50553	2.38190	2.38175	2.38119
	0.2	0.36540	0.36527	0.36536	2.51520	2.51514	2.51425

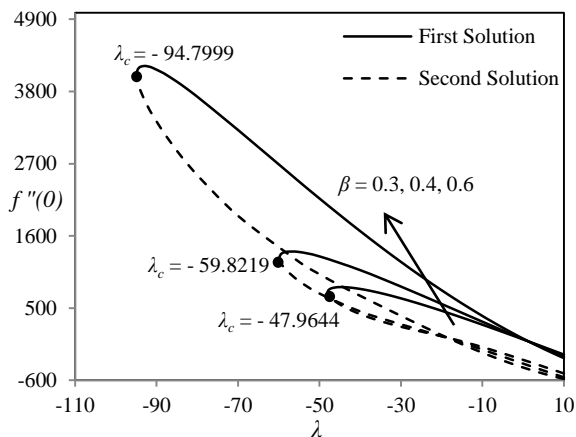
Figures 1 and 2 illustrate the suction effect on the velocity and temperature profiles, respectively. Since suction is a mechanism which expedites the removal of fluid particles into the wall, thus the velocity and temperature boundary layer thicknesses are lessened. Application of suction also leads to the existence of dual solutions as mentioned by Miklavčič and Wang [16] which can be seen obviously in Figures 3-8 for the stretching/shrinking sheet.



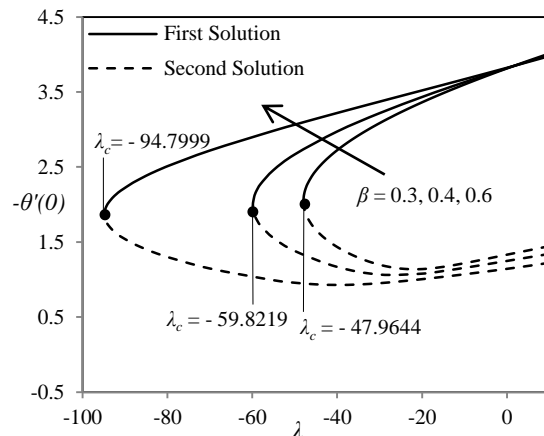
**Figure 1.** Velocity profiles  $f'(\eta)$  for some values of  $s$  when  $\beta = Nb = \chi = 0.3$ ,  $M = 0.5$ ,  $Le = 5$ ,  $Pr = 0.03$ ,  $Nt = 0.2$ ,  $\lambda = -1$  and  $Rd = 0.5$ .



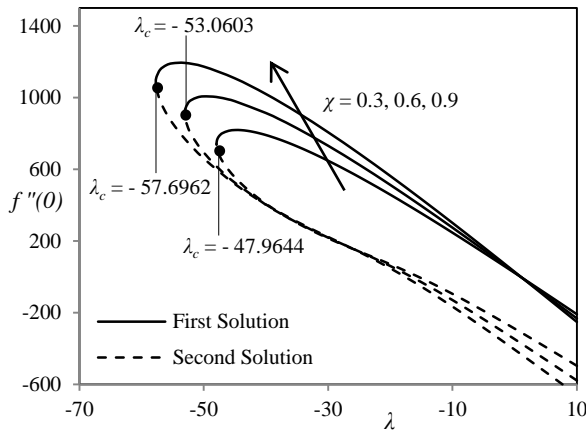
**Figure 2.** Temperature profiles  $\theta(\eta)$  for some values of  $s$  when  $\beta = Nb = \chi = 0.3$ ,  $M = 0.5$ ,  $Le = 5$ ,  $Pr = 0.03$ ,  $Nt = 0.2$ ,  $\lambda = -1$  and  $Rd = 0.5$ .



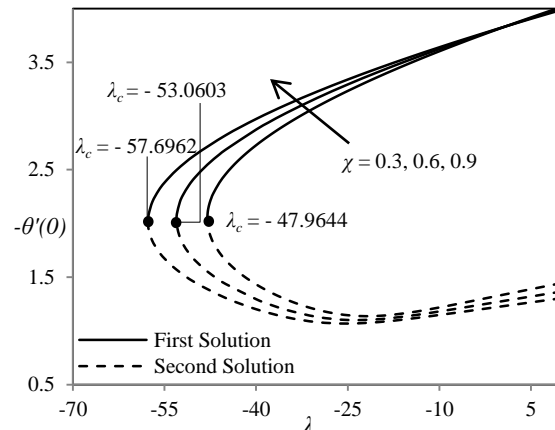
**Figure 3.** Variations of  $f''(0)$  for some values of  $\beta$  when  $s = 6.5, Nb = \chi = 0.3, M = 0.5, Pr = 0.03, Le = 5, Nt = 0.2$  and  $Rd = 0.5$ .



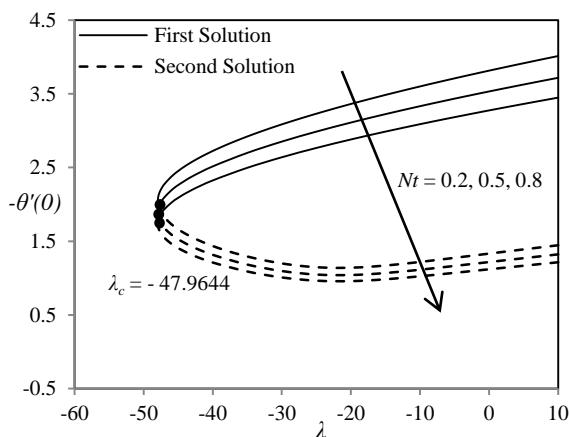
**Figure 4.** Variations of  $-\theta'(0)$  for some values of  $\beta$  when  $s = 6.5, Nb = \chi = 0.3, M = 0.5, Pr = 0.03, Le = 5, Nt = 0.2$  and  $Rd = 0.5$ .



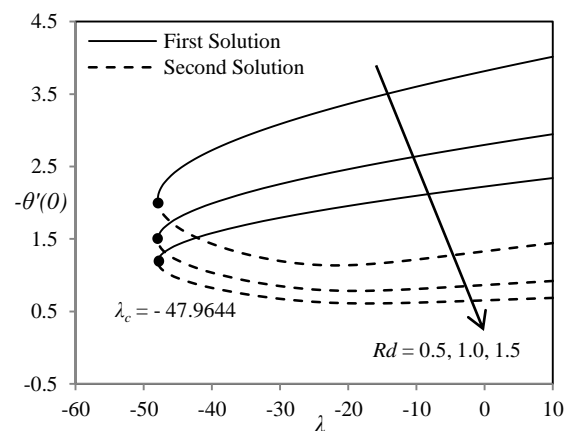
**Figure 5.** Variations of  $f''(0)$  for some values of  $\chi$  when  $Pr = 0.03, \beta = Nb = 0.3, M = 0.5, s = 6.5, Le = 5, Nt = 0.2$  and  $Rd = 0.5$ .



**Figure 6.** Variations of  $-\theta'(0)$  for some values of  $\chi$  when  $Pr = 0.03, \beta = Nb = 0.3, M = 0.5, s = 6.5, Le = 5, Nt = 0.2$  and  $Rd = 0.5$ .



**Figure 7.** Variations of  $-\theta'(0)$  for some values of  $Nt$  when  $\beta = Nb = \chi = 0.3, M = 0.5, Pr = 0.03, Le = 5, s = 6.5$  and  $Rd = 0.5$ .



**Figure 8.** Variations of  $-\theta'(0)$  for some values of  $Rd$  when  $\beta = Nb = 0.3, M = 0.5, Le = 5, Pr = 0.03, Nt = 0.2, \lambda = -1$  and  $s = 6.5$ .



A rise in Deborah number contributes to the increment of the skin friction coefficient as depicted in Figure 3. Deborah number  $\beta$  incorporates the viscosity and elasticity of the fluid and directly proportional to the retardation time where a higher Deborah number correlates to a longer retardation time. Owing to this fact, Abbasi et al. [17] discovered that higher retardation time caused a decline in the temperature and thermal boundary layer thickness. Subsequently, the local Nusselt number will be enhanced which is in accordance with the result shown in Figure 4. Physically, higher rate of heat transfer is generated for the larger Deborah number. Figures 5 and 6 are the graphical representations of the influence of  $\chi$  on  $f''(0)$  and  $-\theta'(0)$ . An upsurge in the ratio of relaxation time to retardation time leads to the increase of the skin friction coefficient and the local Nusselt number. On the other hand, opposite trend can be observed from Figures 7 and 8 where the increments of  $Nt$  and  $Rd$  reduce the local Nusselt number. The influence of thermophoresis will boost higher kinetic energy of nanoparticles and hence increases the temperature. Consequently, the shear stress is elevated and leads to the decline of the heat transfer rate. Figure 8 elucidates the reduction of the rate of heat transfer due to the emission of more heat by the fluid. Since the radiation parameter induces the enhancement of surface heat flux, thus the temperature inside the boundary layer region is getting higher which results in the lower heat transfer.

#### 4. Conclusions

Stagnation point flow and heat transfer of viscoelastic nanofluids was studied numerically in this paper with the consideration of Jeffrey fluid model. The numerical results on the influence of the suction parameter, ratio of relaxation time to retardation time, Deborah number, thermophoresis parameter and radiation parameter were obtained by implementing the `bvp4c` function in Matlab. By imposing sufficient value of suction at the boundary, dual solutions were discovered. The increment of the Deborah number and ratio of relaxation time to retardation time contributed to the upsurge of the skin friction coefficient and the rate of heat transfer. On the contrary, the rise of the thermophoresis parameter and the radiation parameter had resulted in the decrement of the heat transfer rate.

#### Acknowledgement

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