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Development and Evaluation of a Rib Statistical Shape Model for Thoracic Surgery*

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Abstract— **Patients with advanced cancer undergoing chest wall resection may require reconstruction. Currently, rib prostheses are created by segmenting computed tomography images, which is time-consuming and labour intensive. The aim was to optimise the production of digital rib models based on a patient's age, weight, height and gender. A statistical shape model of human ribs was created and used to synthetise rib models, which were compared to the ones produced by segmentation and mirroring. The segmentation took 11.56 ± 1.60 min compared to 0.027 ± 0.009 min using the new technique. The average mesh error between the mirroring technique and segmentation was 0.58 ± 0.25 mm (right ribs), and 0.87 ± 0.18 mm (left ribs), compared to 1.37 ± 0.66 mm ($p < 0.0001$) and 1.68 ± 0.77 mm ($p < 0.05$), respectively, for the new technique. The new technique is promising for the efficiency and ease-of-use in the clinical environment.**

Clinical Relevance— **This is an optimised 3D modelling method providing clinicians with a time-efficient technique to create patient-specific rib prostheses, without any expertise or software knowledge required.**

I. Introduction

The chest wall provides protection to vital internal organs, stability for movement and plays a significant role in ventilation. The thoracic cage consists of 12 pairs of ribs, the sternum, costal cartilages and 12 thoracic vertebrae [1]. A healthy rib cage with no deformities is assumed to be symmetric [2].

Surgical removal of tumours may require en-bloc resection of the chest wall, including rib cage components. Lung cancer is the most common primary diagnosis leading to chest wall resection [3]. Reconstruction of the chest wall following resection decreases post-operative complications and rates of respiratory failure caused by flail chest [4]. Current methods of reconstruction can be categorised into rigid and non-rigid reconstruction [5]. Rigid prostheses provide increased stability, reduced incidence of flail chest and improved cosmetic outcomes [6]. A novel technique combined three-dimensional (3D) printing and moulding of methyl-methacrylate to create patient-specific rib prostheses [7]. The patient anatomy was reconstructed from preoperative computed tomography (CT) scans using image segmentation. CT has become the best practice for imaging patients preoperatively [8]. By segmenting a patient's CT to create 3D digital models, 'neo-ribs' can be manufactured to accurately recreate the patient's own anatomy. This technique requires a level of expertise and has a degree of inter-operator variability. The user needs to be able to correctly interpret the CT and efficiently segment the desired structures. For a more efficient segmentation, Staal et al [9] followed a 5-step framework whereby voxels with the relevant image structure were detected and then used to construct primitives.
The primitives which formed part of the ribs were classified and grouped to allow the full segmentation of the ribs. In contrast, Belal et al [10] presented a deep learning-based method. Anatomical landmarks were detected using convolutional neural networks (CNN) and identified using shape models. The identified landmarks were used as input to a CNN to perform rib segmentation.

Principal Component Analysis (PCA) and multivariate regression analysis have been used in recent studies to develop a statistical rib cage model based on age, sex, stature and body mass index [11]. Statistical shape models (SSMs) are geometric models that describe the general shape of a deformable object as well as its variation in shape. The shape of the object is found by applying PCA to a set of images, referred to as training set [12]. Firstly, the mean shape is computed for the training set. The covariance matrix is then calculated to study how landmarks vary in the different shapes. The normalised eigenvectors correspond to modes of variation while the eigenvalues represent the data variance [13]. Multivariate regression analysis is used to assess how changes in more than one independent variable affect the behaviour of a dependent variable [14].

This study aims to develop an optimised, less labour-intensive method to produce a patient-specific, 3D digital rib model of cancer patients undergoing chest wall reconstruction. Custom software was developed using MATLAB in order to generate a single rib model based on a patient's characteristics: rib number (rib 2 – 10), age, height, weight and gender. The technique was then evaluated using clinical data and comparison to the manual segmentation method.

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II. Methods

A. Manual Segmentation Method

The manual segmentation method uses the *active contour "snake"* tool in ITK-SNAP 3.6.0 to extract a patient's thoracic anatomical structures from CT images (see Fig. 1). Ribs from both the right and left side were segmented and exported as surface meshes in STL format. The ribs were then mirrored using the mesh processing software Meshmixer (Autodesk, California, USA) to generate ribs of the contralateral side of the chest.

B. New Method: Rib Synthesis from a Statistical Shape Model

A collection of 3D extended cardiac-torso (XCAT) images [15] which are computational phantoms based on CT imaging data, containing the anatomical data, served as the training set to create the SSM. Nine subjects were selected from the XCAT library, matching the typical characteristics (see Table 1) of patients who undergo chest wall reconstruction at Guy's hospital, London. Two SSMs were built, one for the left and one for the right rib, using a sample of 81 ribs (ribs 2 – 10 of the 9 subjects) from the XCAT data. The shapes encoded by the SSM were then inputted into multivariate regression analysis accounting for age, weight, height, gender and rib number, since those are the characteristics that affect the human rib shape the most [16]. The 3D meshing of each rib and creation of the SSM was performed using a custom MATLAB software adapting the tools developed for cardiovascular analysis [17,18]. In short, a binary mask file was generated for each rib from the already labelled XCAT data, and a common idealised 3D mesh template of a rib was fitted to each anatomy by a combination of image registration and mesh warping techniques (see Fig. 2). The 3D meshes used smooth cubic interpolation functions [17,18]. Satisfactory fitting results, reaching up to the ends of the rib, were obtained after 3 registration and warping iterations, because each one the average rib generated at the end of each batch was used as the new template for the next iteration, until the rib was fully captured. This method can be easily reproduced by accessing the code [19] since any patient case can be re-segmented and the fittings can be re-run.

The two SSMs were built independently using the 81 meshes, right or left. The average rib was found after alignment by the centre of mass and orientation of the meshes, and a principal component analysis (PCA) over the differences from the 81 meshes to the average was performed.

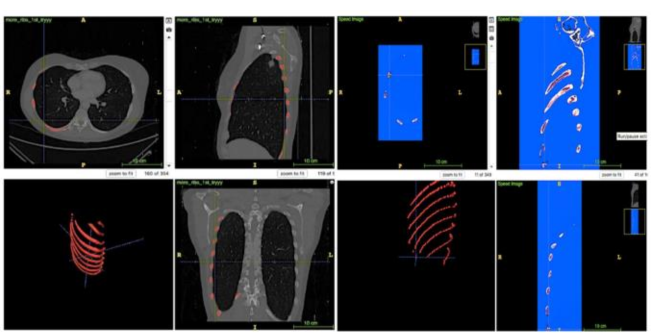

Figure 1. ITK-SNAP semi-automatic segmentation.

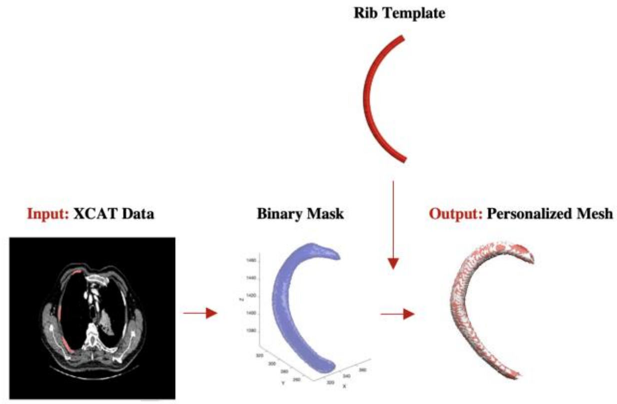

Figure 2. Diagram of the steps followed to generate the rib meshes.

TABLE I. XCAT Data Patient Characteristics Used to Build the SSM.

PID	Age	Weight (kg)	Height (cm)	Gender
I	63	81	153	F
II	52	72	179	F
III	66	66	162	F
IV	62	79	160	F
V	56	70	167	F
VI	54	71	176	M
VII	63	76	168	M
VIII	60	88	190	M
IX	78	69	173	M

The resulting SSM consisted of a compact basis, the eigenvectors or modes describing the shape variation and a set of scores, and the coefficients describing the 81 shapes. The 10 first modes of the SSM were considered for the rest of the analysis, since those 10 accounted for 99% of the variance. Linear regression models that best predicted the shape of the rib were obtained.

The regression model was built between a feature matrix and a shape coefficient matrix. The feature matrix, F, with dimensions 81x6, contained the patient characteristics (age, weight, height, gender and rib number). Once the regression was built, the synthesis of a new rib consisted of 10 linear inferences, one per each of the 10 SSM coefficients, from the patient features.

$$b_i = y_{i0}*1 + y_{i1}*f_1 + y_{i2}*f_2 + y_{i3}*f_3 + y_{i4}*f_4 + y_{i5}*f_5 \quad (1)$$

where b_i is the i_{th} SSM coefficient of one of the desired synthetic rib, and $f_1 \dots f_5$ are the characteristic values for that rib. Equation (1) was then rewritten in matrix form (2) for the i_{th} coefficient, leading to a solution matrix y (3) for all 10 coefficients, with dimensions 6x10.

$$B_i = F * y_i \quad (2)$$

$$y = F^{-1}B \quad (3)$$

C. Comparison & Quantitative Evaluation of the New Method

Pre-operative CT data were available from 16 cancer patients previously operated at Guy's hospital. Ribs 2-10 (both left & right) were segmented using ITK-SNAP to generate a set of ground truth rib models (referred to as the *original models*). The statistical model method was evaluated by comparing the rib meshes generated to the original models using CloudCompare, a 3D point cloud processing software. As a comparison, another method to generate a synthetic rib is used, this time by semi-automatically segmenting one side of the chest and mirroring the anatomy to generate the opposite rib, referred to as the *mirrored models* [20] (note that cancer will many times affect the structure of rib to be replaced). In this academic case, the original models were available for reference.

D. Statistical analysis

Normality of the data was tested using the Shapiro-Wilk test, showing that the data were not normally distributed. The distances of CloudCompare were reported as mean ± standard deviation. The two groups under comparison were independent, so a two-sided non-parametric, Mann Whitney U test was used, with a significance level of 0.05. Since the aim was to show that the two measured distances, original-mirrored and original-statistical model, were *not different* from one another, the method was evaluated by identifying all the p-values *above 0.05*, indicating that the two methods were not significantly different.

E. Qualitative evaluation

The CT scans of two patients who previously underwent chest wall reconstruction were segmented to get the original right model and the mirrored one. The patients' characteristics were then used to synthetise the rib model from the SSM. The STL files were imported into the software Ultimaker Cura 4.5 (Ultimaker, Netherlands), which converts a mesh file into G-code, a series of instructions recognised by the 3D printer. Three rib samples of two patients were printed: right original rib, right rib mirrored from left, and right rib generated using the statical model. The 6 ribs were printed using a Fused Deposition Modelling printer (Anycubic i3 Mega), in polylactic acid filament.

The new method was also evaluated by using a survey undertaken by 5 thoracic surgeons. The survey comprised of 8 questions regarding the anatomical accuracy of the rib models, time efficiency of the method and the feasibility of its use in clinical settings. The response was rated from 1 to 5, where 1 corresponded to strongly disagree and 5 to strongly agree.

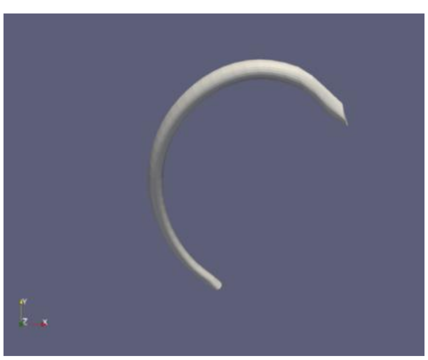

Figure 3. Mesh of right rib 5 generated with the statistical model. Input characteristics were: 5, 64, 87, 175, M.

III. Results

The preoperative CT scans of 16 patients who previously underwent chest wall reconstruction were used. The median age of the population was 64 years old (range 27 – 77), with 9 (56%) male patients. The median height was 174.5cm (range 151 – 190cm) and the median weight 75kg (range 40 – 108kg). Eighteen ribs were generated for each patient using segmentation; 9 (2 – 10) left and 9 right. These experimental results were produced using semi-automatic segmentation. The statistical models were used to generate the personalised rib meshes (see Fig. 3 for an example) based on each patient's characteristics: rib number, age, weight, height and gender. This is repeated twice, to generate left and right sided ribs for each patient.

A. Comparison & Quantitative Evaluation of Methods

The time required to generate 3D rib models using the 2 different methods was recorded. The process was timed from starting the software to obtaining the ready-to-print STL file. The ITK-SNAP semi-automatic segmentation and mirroring method took 11.56 ± 1.60 min for an experienced user to segment rib 2, and 22 ± 1.50min for an inexperienced user. The statistical method took significantly less time, 0.027 ± 0.009 min ($p < 0.0001$), for any user to generate a rib model. Comparing this to more automatic methods described in literature, it still appears to be more time efficient. Belal et al used a CNN-based method, which took 2 min per CT to generate automated segmentations [10]. Staal et al also used automatic rib segmentation with which the computation time for a rib application was about 6.5 min [9]. The right and left rib models were generated using 3 different methods: semi-automatic segmentation (original ribs), statistical model and mirroring from left (mirrored models). The segmented original right ribs were then compared to those generated using both the statistical model and the mirroring technique. The mean distance of all left and right ribs (2 – 10) for 16 patients were computed (Fig. 4). Mean values were then calculated based on the rib number, 2 – 10, for the 16 patients, to investigate a potential correlation between the accuracy of the method and the rib compared (Fig. 5).

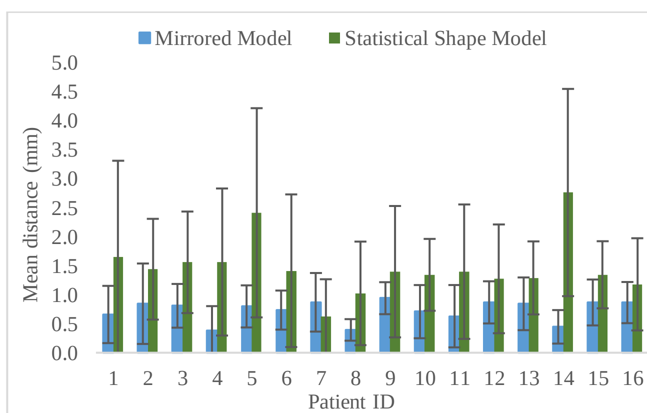

Figure 4. Accuracy comparison for 16 patients. Mean distances between the original ribs and the ribs generated with two methods, mirroring and the statistical model respectively. The two measured distances, original-mirrored and original-statistical model of both left and right side, were compared, calculating an average of the 9 ribs of each patient.

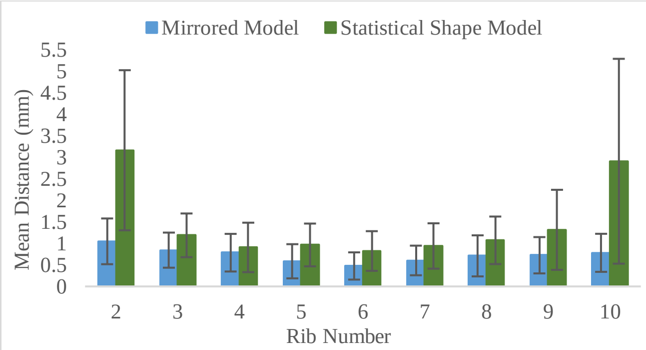

Figure 5. Accuracy comparison for 9 ribs of left and right side. Mean distances between the original ribs and the ribs generated with two methods, mirroring and the statistical model respectively. The two measured distances, original-mirrored and original-statistical model were compared for 9 ribs, computing the average of 16 patients for each rib.

The statistical model generated rib closest to the original one was rib 6 in both left and right side, with a mean distance of 0.73 ± 0.35 mm and 0.91 ± 0.55 mm, respectively, and an average of left and right equal to 0.82 ± 0.46 mm (Fig. 5). Rib 2, on both left and right side, had the largest distance between the statistical model and original, equal to 3.46 ± 2.11 mm and 2.93 ± 1.60 mm for left and right side, respectively, and average of 3.17 ± 1.86mm. The most and least accurate values from all patients were identified for further analysis. When comparing rib 2 of patient 5 generated using the statistical model to the original one, a mean distance of 6.5 mm was found. Rib 6 of patient 7 resulted to be the most accurate with a mean distance of 0.02 mm. CloudCompare results were displayed using the scalar field's colour scale feature (Fig. 6). A blue > green > yellow > red colour scale was chosen where the smallest distances are labelled in green.

B. *Qualitative Evaluation of Methods*

The original model of the right rib 4, along with the ones generated using the mirroring technique (from left to right in Fig. 7) and the statistical model were printed and compared. Patient 1 and patient 5 were chosen as the sample patients. Patient 5 represents one of the patients with the highest mean distance of all ribs for the right side (1.87 ± 1.78 mm), but also for the mean of all ribs from both left and right side (2.41 ± 1.80 mm). Patient 1 was one of those with the lowest mean distance for the right side (0.94 ± 0.57 mm) and hence highest similarity between the statistical model and the original ribs. The six printed ribs were given to surgeons to be assessed, and feedback was collected (Fig. 8). When collecting the expert opinions, particular focus was given on both model accuracy and time efficiency. The rib models were rated as anatomically accurate and realistic by the majority.

IV. Discussion

The proposed new method to synthetise ribs is completely reproducible being deterministic for any input of patient characteristics, and reduces the time of data processing, achieving real-time performance, without significant loss of fidelity, with less than two millimetres in average error. The current method of chest wall reconstruction involves the construction of the digital rib models by semi-automatically segmenting the patient's preoperative CT scan.

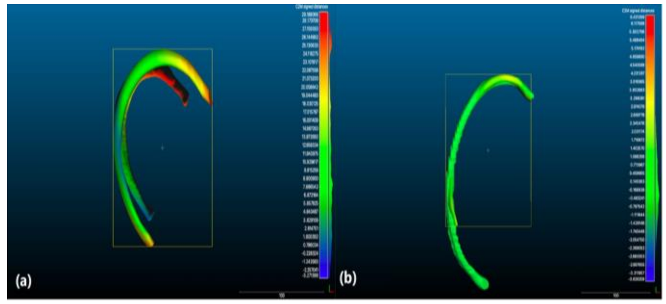

Figure 6. CloudCompare results of "cloud/mesh distance" between the original and statistical model. Colourbar of distances for the (a) worst result (mean = 6.5mm): rib 2 of patient 5, with the smaller rib being the original and red colour corresponding to 29.2mm (b) best result (mean = 0.02mm): rib 6 of patient 7 where original and statistical model are almost perfectly overlapping.

The result obtained using semi-automatic segmentation is a more accurate reproduction of the rib shape compared to the model resulting from the statistical model; however, it is a highly labour-intensive process, significantly more time consuming and a high level of expertise is required, good understanding of the chest anatomy and knowledge of the segmentation software. Therefore, there is a need for an optimised technique, such as the presented SSM. Similar to the studies conducted by Staal and Belal [9][10], this paper presents the automatic development of bespoke 3D rib models. However, in the previously mentioned studies the anatomical models are constructed using the CT images of a patient, whereas this study uses the patient characteristics as the sole input.

In addition, the method developed was validated by performing a quantitative analysis of the developed rib models against those manually segmented using the software CloudCompare. Staal et al [9], however, made no reference to manual segmentation and only followed a qualitative approach to evaluate their method. Although Belal et al [10] applied the Sorensen-Dice index to quantitively compare their developed models to manually segmented ribs, it could be argued that although the volume results between the models are very similar, the shape accuracy may differ. Moreover, these studies did not use the reconstruction methods to address a clinical problem and hence there was no application in a clinical setting. Patient 4 demonstrated the most significant difference between model generation methods on the right side (0.20 ± 0.12 mm vs 2.06 ± 0.13 mm, $p = 0.0004$).

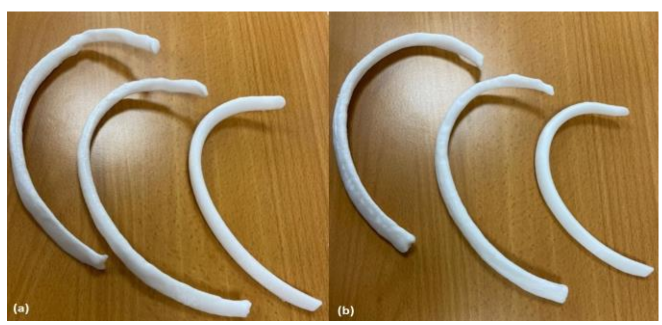

Figure 7. 3D printed rib models of rib 4. The original model is placed in the middle, the statistical model on the right side and the mirrored on the left side. (a) Patient 5 (b) Patient 1.

The patient's previous breast cancer surgery could have changed the anatomy of their chest wall, which was observed on the rib models generated from the CT. Additionally, the patient's height was 151cm, outside of the height range used to build the SSM (153 – 190cm). Patient 14 showed the largest difference between the mean distance values from the original models for the statistical and the mirroring methods for the right side (3.40 ± 1.10 mm vs 0.26 ± 0.15 mm, $p = 0.001$) but also for the average of left and right side, equal to 2.76 ± 1.78 mm (Fig. 4). That is because the 10th rib of the patient was not fully captured in the CT scan, with 4-5 slices of the CT missing. Hence, the original 10th rib was smaller than normal, which resulted in Patient 14 presenting the largest difference between original and statistical models, but also in rib 10 having the second highest distance between the statistical model and original (2.91 ± 2.38 mm). In this case though, the error in the mean distance could be more clinically significant than a very low p value. Since this is a preliminary study, a larger patient sample and more results are required to identify which is the most clinically significant value. However, when dealing with such large structures such as a rib, where rib 2, the smallest in size, has a base diameter of approximately 82mm, a difference of either 2.06 mm or 3.40 mm is minimal. Rib 2 was the least accurate with largest distance between the statistical model and original, which could be due to the rib's attachment to the serratus anterior muscle [21]. This leads to the tuberosity on the rib's superior surface, making it difficult to segment and hence resulting in a less accurate 3D model.

The surgeons who took part in the survey to assess the optimised statistical technique were satisfied with the quality and time efficiency of the model. Positive factors identified included time efficiency and the low level of expertise required for the SSM model. The neutral responses were mainly associated with a lack of existing surgical outcomes for patients with prosthesis made using the statistical model method. Overall, the method significantly reduced the labour required to produce the rib models and improved time efficiency of the procedure. The surgeons approved this process to be used for the creation of rib prostheses for all patients undergoing chest wall resection and reconstruction in the future. However, to build the left and right side meshes, two separate simplified models were used. To further improve this method, a single model could be created that incorporates the differences between the two sides of the chest, by adding the rib side as the sixth characteristic.

Although the rib models were generated based on the patient's characteristics, the rib shapes were not identical to the original models. The main cause identified is the limited dataset used to build the SSM. This study used the data from 9 patients, whereas Shi et al. used the scans of 89 patients to develop a statistical rib cage model [20]. However, this is the first study comparing rib models generated using a SSM and regression analysis to segmentation generated anatomies. The weight range used in this study was 66 – 88kg and the height range 153 –190cm. Therefore, the developed program may be inaccurate for patients with characteristics outside of these ranges.

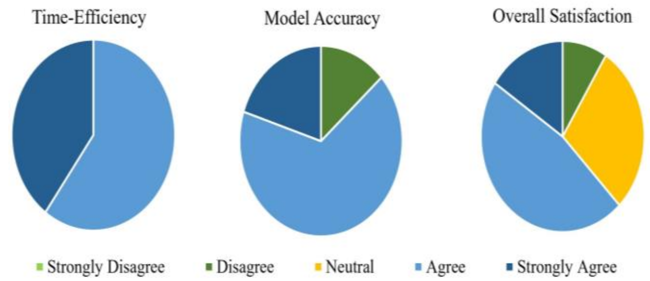

Figure 8. Pie charts presenting the qualitative results based on the feedback collected from the surgeon survey.

To improve the technique, a larger sample size is needed, achieved by the inclusion of scans of patients with a wider range of characteristics, to maximise the robustness and generalisability of the SSM. A larger patient sample would lead to a more inclusive study and a more accurate statistical analysis. Even though this study uses the SSM method which has previously been used, it is the first that proposes the use of this methodology in a clinical setting, to address the lack of patient-specific prostheses unless specially trained staff is involved in a time-consuming and labour-intensive manufacturing process.

V. CONCLUSION

This study aimed to optimise the patient-specific rib prosthesis reconstruction method for patients undergoing en-bloc resection surgery. The optimised, novel technique proved to be more time-efficient and less labour intensive than the semi-automatic segmentation currently used, with no specific expertise required so any member of the clinical care team could generate the 3D rib models in less than a minute. By reducing the level of expertise and time required to build the prosthesis, the optimised technique could increase productivity and make this method more accessible in a clinical setting.

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