DOI:
10.1016/j.eswa.2022.116627

LMIs-based stability analysis and fuzzy-logic controller design for networked systems with sector nonlinearities: application in tunnel diode circuit

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Abstract: In this article, the problem of exponential mean-square stability analysis is discussed for uncertain networked control systems expressed by a stochastic T-S fuzzy model. In general, the characteristics of random occurrence for multipath packet dropouts often exist in the signal transmission network. For dealing with this difficult point, a dynamic output feedback strategy combining stochastic Bernoulli theory is employed. Then, delay-dependent stability conditions are derived and closed-loop system is exponentially mean-square stable by designing fuzzy-basis-dependent Lyapunov functional. Furthermore, in terms of linear matrix inequalities (LMIs) technology, sufficient conditions are gained to guarantee the prescribed H-infinity performance. Different from previous literatures, the congruence transformation method is employed to determine controller gain matrices for reducing the computation complexity of solving LMIs. Finally, the proposed method is applied in tunnel diode circuit model to verify the applicability.

Keywords: Lyapunov functional, stability analysis, disturbance, linear matrix inequalities.

1. Introduction

The development of network communication technology expands the application range of the network control systems (Ramirez, Minami, & Sugimoto, 2018; Khan, Khan, Iqbal, Mustafa, Abbasi et al., 2021; Chiang, & Liu, 2021). Networked control systems are very important control system, the system information and control signals are transmitted via the shared digital networks (Khan, Khan, Iqbal, Mustafa, Abbasi et al., 2021). Generally, the networked control systems include many devices, such as the sensor units, controller units, actuator units and control objects (Chiang, & Liu, 2021). In recent years, with the rapid development of network technology, the networked control systems have great advantages than the conventional control systems (Haghighi, Tavassoli, & Farhadi, 2020). The networked control systems have some advantages, such as the signal transmission flexibility, low installation cost, easy diagnosis maintenance and so on (Du, Kao, & Park, 2021). Thus, the networked control systems have attracted much attention (Haghighi, Tavassoli, & Farhadi, 2020; Zhang, Wang, Jiang, & Zhang, 2015; Dhanalakshmi, Senpagam, & Mohanapriya, 2021; Zheng, Zhang, Sun, & Wen, 2022). However, the wide application of networked control systems will also bring some unexpected disadvantages (Du, Kao, & Park, 2021; Chandrasekaran, Durairaj, & Padmavathi, 2021; Zheng, Zhang, Sun, Wen, Li et al., 2021; Yu, Dong, Li, & Li, 2017). Particularly, in the data transmitting process from remote sensors to local controllers, multipath packet dropouts will arise in the communication channels (Du, Kao, & Park, 2021; Du, Kao, & Zhao, 2021; Du, Kao, Karimi, & Zhao, 2020; Zhang, Zheng, Lam, Wen, Sun et al., 2020).

In practical applications, the nonlinearities always exist because of the influence of external or internal factors, thus many achievements have been obtained in the research of nonlinear system (Cheng, Wang, Stojanovic, He, Shi et al., 2021; Zheng, Wang, Zhang, & Yin, 2019; Liu, Lam, Ban, & Zhao, 2016). In order to deal with the nonlinearities, most of the methods are available for investigation of qualitative behaviors of both nonlinear and linear dynamical systems, such as Jacobian method, T-S fuzzy technique, and other techniques (Cheng, Wang, Stojanovic, He, Shi et al., 2021; Li, Sun, & Tong, 2019). In addition, there are many nonlinearities in the physical systems, and there are serious difficulties in the stability analysis and controller design of control systems (Fang, Zhu, Stojanovic, Nie, He et al., 2021; Ruangsang, & Assawinchaichote, 2019). For example, an online adaptive optimal control was proposed for a class of nonlinear systems, and the system model was transferred to N coupled linear subsystems by using subsystem transformation scheme (Fang, Zhu, Stojanovic, Nie, He et al., 2021). Furthermore, many methods can be used to investigate the qualitative behavior of nonlinear and linear dynamical systems, such as sliding mode control, neural network control, state feedback control, T-S fuzzy technique and so on (Lam, 2011; Wei, Qiu, & Karimi, 2017; Zhang, Wang, Stojanovic, Cheng, He et al., 2021). Especially, the T-S fuzzy model is an efficient technique in describing the nonlinear systems (Wei, Qiu, & Karimi, 2017; Li, Ma, & Tong, 2019). Compared with conventional linear submodel control methods, the main advantage of T-S fuzzy technique is the high compatibility (Cheng, He, Stojanovic, Luan, & Liu, 2021; Wei, Qiu, Shi, & Lam, 2017; Cheng, He, Stojanovic, Luan, & Liu, 2021; Wei, Qiu, Shi, & Chadli, 2017). For example, the input state stabilizing problem was investigated for a class of T-S fuzzy systems with multiple transmission channels under denial-of-service attacks (Wu, Yang, & Wang, 2021). The integral sliding mode control was studied for a class of generalized T-S fuzzy singular stochastic systems by involving the Markovian jump type of system parameters, and the matched/mismatched uncertainties can be approximated effectively (Mani, Rajan, & Joo, 2021).

The system output is often measurable, thus the output feedback control strategy provides a feasible way to construct the controller for the control system (Yu, Li, & Du, 2017). On the other hand, it is difficult to measure all the state variables information of the system (Wang, Tong, & Li, 2017). For example, the adaptive output feedback controller and a fuzzy observer were employed to estimate unmeasured states (Li, & Tong, 2017). The robust output feedback control and fuzzy model were employed to approximate unstructured uncertainties (Li, Tong, Liu, & Li, 2014). The results in (Li, & Tong, 2017; Li, Tong, Liu, & Li, 2014) mean that the state variables information are unavailable in the measurement process. With above analysis, it can be seen that the output feedback control is more effective for the control system (Tong, Sui, & Li, 2018; Wang, Qiu, Gao, & Wang, 2017). In fact, the conventional output feedback control is easy to implement in practical applications, but it contains a small amount of system state variables information (Hua, & Guan, 2016; Wang, Qiu, Fu, & Ji, 2017; Kwon, Park, Park, Lee, & Cha, 2017.). In addition, the conventional output feedback can not satisfy the actual design requirements (Zheng, Wang, Wang, & Wen 2019; Wei, Qiu, & Fu, 2015). Thus, the dynamic output feedback is proposed (Wei, Qiu, Karimi, & Wang, 2015; Zheng, Wang, Wang, & Wen, 2019).

Although there are some researchs about dynamic output feedback control have been studied on the networked control systems, the problems of obtaining H-infinity controller by using cone complementarity linearization are not fully solved. Moreover, with the development networked control systems, the packet dropouts problem often exist. Thus, multipath packet dropouts problem is challenging to be solved. On the other hand, the robust adaptive fuzzy control was proposed for the nonlinear systems with induced delay and data packet dropouts (Hamdy, Elhaleem, & Fkirin, 2017), without considering dynamic output feedback control. The L-infinity stability analysis was proposed for the networked control systems subject to stochastic deception attacks (Wu, Xiong, & Xie, 2021), without considering H-infinity stability analysis. Compared with (Hamdy, Elhaleem, & Fkirin, 2017; Wu, Xiong, & Xie, 2021), both the dynamic output feedback control and H-infinity stability analysis are proposed for the uncertain networked control systems with sector nonlinearities, time-varying delay and unmatched disturbance in this paper in this paper. The contributions are presented below. (1) The system plant is approximated via the premise variables and fuzzy set. (2) The stochastic Bernoulli theory is employed, and the characteristics of random occurrence for packet dropouts are described clearly. (3) By designing the fuzzy-basis-dependent Lyapunov functional, the closed-loop system is exponentially mean-square stable.

Notations $\mathbb{R}^n$ denotes *n*-dimensional Euclidean space, $A>0$ ($A\geq 0$) denotes positive (semi positive) definite matrix, $A<0$ ($A\leq 0$) denotes negative (semi negative) definite matrix. " * " denotes elements below main diagonal of symmetric matrix, $\|\bullet\|$ denotes Euclidean norm of " • ". $sup\{\bullet\}$ denotes minimum upper bound of " • ", $diag\{r_1 \quad r_2 \quad \dots \quad r_n\}$ denotes block diagonal matrix with elements r_1, r_2, … and r_n.

2. System formulation

Consider the uncertain networked control systems

$$\begin{cases} x(k+1)=\left(A+\Delta A(k)\right)x(k)+\left(A_d+\Delta A_d(k)\right)x\left(k-d(k)\right)+\left(E+\Delta E(k)\right)f\left(x(k)\right)+\left(E_d+\Delta E_d(k)\right)f_d\left(x\left(k-d(k)\right)\right)+B_1u(k)+D_1\omega(k) \\ y(k)=Cx(k)+C_dx\left(k-d(k)\right)+\phi\left(Sx(k)\right)+D_2\omega(k) \\ z(k)=Lx(k)+B_2u(k) \\ x(k)=\psi(k), \qquad k=-d_M,-d_M+1,\dots,0 \end{cases} \tag{1}$$

Applying T-S fuzzy model, one can obtain

Plant rule i: if $\theta_1(k)$ *is* M_{i1}, $\theta_2(k)$ *is* M_{i2}, *... and* $\theta_p(k)$ *is* M_{ip}, *then*

$$\begin{cases} x(k+1)=\left(A_i+\Delta A_i(k)\right)x(k)+\left(A_{di}+\Delta A_{di}(k)\right)x\left(k-d(k)\right)+\left(E_i+\Delta E_i(k)\right)f\left(x(k)\right)+\left(E_{di}+\Delta E_{di}(k)\right)f_d\left(x\left(k-d(k)\right)\right)+B_{1i}u(k)+D_{1i}\omega(k) \\ y(k)=C_ix(k)+C_{di}x\left(k-d(k)\right)+\phi\left(S_ix(k)\right)+D_{2i}\omega(k) \\ z(k)=L_ix(k)+B_{2i}u(k) \\ x(k)=\psi(k), \qquad k=-d_M,-d_M+1,\dots,0 \end{cases} \tag{2}$$

$$A_i(k)=A_i+\Delta A_i(k), \quad A_{di}(k)=A_{di}+\Delta A_{di}(k), \quad E_i(k)=E_i+\Delta E_i(k), \quad E_{di}(k)=E_{di}+\Delta E_{di}(k) \tag{3}$$

where $\theta_1(k)$, $\theta_2(k)$, … and $\theta_p(k)$ are the premise variables, M_{ij} ($i=1,2,\dots,r$ and $j=1,2,\dots,p$) is the fuzzy set, *r* is the number of fuzzy rules, and *p* is the number of premise variables. A_i, A_{di}, E_i, E_{di}, B_{1i}, D_{1i}, C_i, C_{di}, S_i, D_{2i}, L_i and B_{2i} are the system gain matrices with appropriate dimensions. $x(k)\in\mathbb{R}^x$ is the state variable, $y(k)\in\mathbb{R}^y$ is the measured output, $z(k)\in\mathbb{R}^z$ is the control output, $u(k)\in\mathbb{R}^u$ is the control input, $\psi(k)$ is the initial condition with $k=-d_M,-d_M+1,\dots,0$.

$\Delta A_i(k)$, $\Delta A_{di}(k)$, $\Delta E_i(k)$ and $\Delta E_{di}(k)$ are the uncertainties satisfying (Guelton, Bouarar, & Manamanni, 2009)

$$\begin{bmatrix} \Delta A_i(k) \\ \Delta A_{di}(k) \\ \Delta E_i(k) \\ \Delta E_{di}(k) \end{bmatrix} = M_iF_i(k)N_i \tag{4}$$

$$M_i=\begin{bmatrix}M_{i1}\\M_{i2}\\M_{i3}\\M_{i4}\end{bmatrix},\quad N_i=\begin{bmatrix}N_{i1}\\N_{i2}\\N_{i3}\\N_{i4}\end{bmatrix},\quad F_i(k)=\begin{bmatrix}F_{i1}(k) & F_{i2}(k) & F_{i3}(k) & F_{i4}(k)\end{bmatrix} \tag{5}$$

where $F_{i1}(k)$, $F_{i2}(k)$, $F_{i3}(k)$ and $F_{i4}(k)$ satisfying

$$\begin{cases}F_{i1}^T(k)F_{i1}(k)\le I, & F_{i2}^T(k)F_{i2}(k)\le I\\ F_{i3}^T(k)F_{i3}(k)\le I, & F_{i4}^T(k)F_{i4}(k)\le I\end{cases} \tag{6}$$

$f(x(k))$, $f_d(x(k-d(k)))$ and $\phi(Sx(k))$ satisfying (Benzaouia, 2012)

$$\begin{cases}f(0)=0,\quad f_d(0)=0,\quad \phi(0)=0\\ \left(f(x_1(k))-f(x_2(k))-U_1(x_1(k)-x_2(k))\right)^T\left(f(x_1(k))-f(x_2(k))-U_2(x_1(k)-x_2(k))\right)\le 0\\ \left(f_d(x_1(k))-f_d(x_2(k))-V_1(x_1(k)-x_2(k))\right)^T\left(f_d(x_1(k))-f_d(x_2(k))-V_2(x_1(k)-x_2(k))\right)\le 0\\ \left(\phi(x_1(k))-\phi(x_2(k))-W_1(x_1(k)-x_2(k))\right)^T\left(\phi(x_1(k))-\phi(x_2(k))-W_2(x_1(k)-x_2(k))\right)\le 0\end{cases} \tag{7}$$

$$U_1-U_2>0,\quad V_1-V_2>0,\quad W_1-W_2>0 \tag{8}$$

where U_1, U_2, V_1, V_2, W_1 and W_2 are the known constant matrices.
$d(k)$ is the time-varying delay and

$$d_m\le d(k)\le d_M,\quad \Delta d(k)\le\bar{d} \tag{9}$$

where d_m is the lower bound of $d(k)$, d_M is the upper bound of $d(k)$, and $\bar{d}$ is upper bound of $\Delta d(k)$.
$\omega(k)$ is the unmatched disturbance and

$$\sum_{k=0}^{\infty}\omega^T(k)\omega(k)\le\bar{\omega} \tag{10}$$

The packet dropouts from sensor to controller are considered and $y(k)$ can be rewritten

$$y(k)=\alpha(k)\left(C_ix(k)+C_{di}x(k-d(k))+\phi(S_ix(k))+D_{2i}\omega(k)\right) \tag{11}$$

According to Bernoulli probability distribution, one has

$$\begin{cases}\alpha(k)=1, & \text{if signal transmission success}\\ \alpha(k)=0, & \text{if packet dropout}\end{cases} \tag{12}$$

$$Prob\{\alpha(k)=1\}=\bar{\alpha},\quad Prob\{\alpha(k)=0\}=1-\bar{\alpha},\quad 0\le\bar{\alpha}\le 1 \tag{13}$$

$$\sigma^2=\bar{\alpha}(1-\bar{\alpha}) \tag{14}$$

where $\alpha(k)=1$ denotes signal transmission success, and $\alpha(k)=0$ denotes packet dropouts. $Prob\{\alpha(k)=1\}$ is the Bernoulli probability distribution of $\alpha(k)=1$, and $Prob\{\alpha(k)=0\}$ is the Bernoulli probability distribution of $\alpha(k)=0$. $\bar{\alpha}$ is the value of $Prob\{\alpha(k)=1\}$, $1-\bar{\alpha}$ is the value of $Prob\{\alpha(k)=0\}$, and σ^2 is the variance of $\alpha(k)$.
Substituting (11) into (2) yields

$$\begin{cases}x(k+1)=(A_i+\Delta A_i(k))x(k)+(A_{di}+\Delta A_{di}(k))x(k-d(k))+(E_i+\Delta E_i(k))f(x(k))\\ \qquad +(E_{di}+\Delta E_{di}(k))f_d(x(k-d(k)))+B_{1i}u(k)+D_{1i}\omega(k)\\ y(k)=\alpha(k)\left(C_ix(k)+C_{di}x(k-d(k))+\phi(S_ix(k))+D_{2i}\omega(k)\right)\\ z(k)=L_ix(k)+B_{2i}u(k)\\ x(k)=\psi(k),\quad k=-d_M,-d_M+1,\ldots,0\end{cases} \tag{15}$$

Applying T-S fuzzy inference, one has

$$\begin{cases} x(k+1)=\sum_{i=1}^{r}h_i(\theta(k))\big((A_i+\Delta A_i(k))x(k)+(A_{di}+\Delta A_{di}(k))x(k-d(k))+(E_i+\Delta E_i(k))f(x(k)) \\ \qquad +(E_{di}+\Delta E_{di}(k))f_d(x(k-d(k)))+B_{1i}u(k)+D_{1i}\omega(k)\big) \\ y(k)=\sum_{i=1}^{r}h_i(\theta(k))\big(\alpha(k)(C_ix(k)+C_{di}x(k-d(k))+\phi(S_ix(k))+D_{2i}\omega(k))\big) \\ z(k)=\sum_{i=1}^{r}h_i(\theta(k))(L_ix(k)+B_{2i}u(k)) \\ x(k)=\psi(k), \quad k=-d_M,-d_M+1,\ldots,0 \end{cases} \tag{16}$$

where $\theta(k)=\begin{bmatrix}\theta_1(k) & \theta_2(k) & \ldots & \theta_p(k)\end{bmatrix}^T$, and

$$h_i(\theta(k))=\prod_{j=1}^{p}M_{ij}(\theta_j(k))\Big/\sum_{i=1}^{r}\prod_{j=1}^{p}M_{ij}(\theta_j(k)) \tag{17}$$

$$h_i(\theta(k))\geq 0, \quad \sum_{i=1}^{r}h_i(\theta(k))=1 \tag{18}$$

Remark 1. More precise approximation of the sector can be achieved by considering nonlinear bounds of the sector, which can describe the specific nonlinearities better than using the sector with linear bounds (Lam, Liu, Wu, & Zhao, 2015). Furthermore, the bounds of sector nonlinearities are allowed to change with the state variables, which can describe the wider range of nonlinearities than the constant bounds (Lam, Liu, Wu, & Zhao, 2015). Thus, the sector nonlinearities are closer to the actual nonlinearities, and the less conservative stability results can be obtained in the controller design. The T-S fuzzy model offers nice theory framework to denote the system plant as average weighted sum of semi-linear subsystems (Sakr, Elnagar, Elbardini, & Sharaf, 2019; He, Liu, Wu, & Li, 2020). Thus, the T-S fuzzy model is employed in this paper.

3. Controller design

The delay-dependent dynamic output feedback controller is designed as follows

$$\begin{cases} \hat{x}(k+1)=A_k\hat{x}(k-d(k))+B_ky(k) \\ u(k)=C_k\hat{x}(k) \end{cases} \tag{19}$$

Applying T-S fuzzy model, one has

Controller rule i: if $\theta_1(k)$ *is* $\hat{M}_{i1}$, $\theta_2(k)$ *is* $\hat{M}_{i2}$, … *and* $\theta_p(k)$ *is* $\hat{M}_{ip}$, *then*

$$\begin{cases} \hat{x}(k+1)=A_{ki}\hat{x}(k-d(k))+B_{ki}y(k) \\ u(k)=C_{ki}\hat{x}(k) \end{cases} \tag{20}$$

where $\theta_1(k)$, $\theta_2(k)$, … and $\theta_p(k)$ are the premise variables, $\hat{M}_{ij}$ ($i=1,\ 2,\ \ldots,\ r$ and $j=1,2,\ldots,p$) is the fuzzy set, *r* is the number of fuzzy rules, and *p* is the number of premise variables. A_{ki}, B_{ki} and C_{ki} are the controller gain matrices, and $\hat{x}(k)\in\mathbb{R}^x$ is the controller state variable. The packet dropouts from controller to actuator are considered and $u(k)$ is rewritten as follows

$$u(k)=\beta(k)C_{ki}\hat{x}(k) \tag{21}$$

According to Bernoulli probability distribution, one has

$$\begin{cases} \beta(k)=1, & \textit{if signal transmission success} \\ \beta(k)=0, & \textit{if packet dropout} \end{cases} \tag{22}$$

$$Prob\{\beta(k)=1\}=\bar{\beta}, \qquad Prob\{\beta(k)=0\}=1-\bar{\beta}, \quad 0\leq\bar{\beta}\leq 1 \tag{23}$$

$$\delta^2=\bar{\beta}(1-\bar{\beta}) \tag{24}$$

where $\beta(k)=1$ denotes signal transmission success, and $\beta(k)=0$ denotes packet dropouts. $Prob\{\beta(k)=1\}$ is the Bernoulli probability distribution of $\beta(k)=1$, and $Prob\{\beta(k)=0\}$ is the Bernoulli probability distribution of $\beta(k)=0$. $\bar{\beta}$ is the value of $Prob\{\beta(k)=1\}$, $1-\bar{\beta}$ is the value of $Prob\{\beta(k)=0\}$, and δ^2 is the variance of $\beta(k)$. Substituting (21) into (20) yields

$$\begin{cases} \hat{x}(k+1)=A_{ki}\hat{x}(k-d(k))+B_{ki}y(k) \\ u(k)=\beta(k)C_{ki}\hat{x}(k) \end{cases} \tag{25}$$

Applying T-S fuzzy inference, one can obtain

$$\begin{cases} \hat{x}(k+1)=\sum_{i=1}^{r}\hat{h}(\theta(k))\left(A_{ki}\hat{x}(k-d(k))+B_{ki}y(k)\right) \\ u(k)=\sum_{i=1}^{r}\hat{h}(\theta(k))\left(\beta(k)C_{ki}\hat{x}(k)\right) \end{cases} \tag{26}$$

where $\theta(k)=\begin{bmatrix}\theta_1(k) & \theta_2(k) & \dots & \theta_p(k)\end{bmatrix}^T$, and

$$\hat{h}_i(\theta(k))=\prod_{j=1}^{p}\hat{M}_{ij}(\theta_j(k))\Big/\sum_{i=1}^{r}\prod_{j=1}^{p}\hat{M}_{ij}(\theta_j(k)) \tag{27}$$

$$\hat{h}_i(\theta(k))\ge 0,\quad \sum_{i=1}^{r}\hat{h}_i(\theta(k))=1 \tag{28}$$

Applying (26) to (16), the closed-loop system is obtained

$$\begin{cases} \eta(k+1)=\tilde{A}_{ij}(k)\eta(k)+\tilde{A}_{dij}(k)H\eta(k-d(k))+\bar{E}_i(k)f(x(k))+\bar{E}_{di}(k)f_d(x(k-d(k)))+\alpha(k)\bar{B}_{ki}\phi(S_ix(k))+D_{ij}\omega(k) \\ z(k)=\bar{L}_{ij}\eta(k) \end{cases} \tag{29}$$

where

$$\begin{cases} \eta(k)=\begin{bmatrix}x(k)\\ \hat{x}(k)\end{bmatrix}, & \tilde{A}_{ij}(k)=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta(k))\hat{h}_j(\theta(k))\begin{bmatrix}A_i(k) & \beta(k)B_{1i}C_{kj}\\ \alpha(k)B_{kj}C_i & A_{kj}\end{bmatrix}, & \tilde{A}_{dij}(k)=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta(k))\hat{h}_j(\theta(k))\begin{bmatrix}A_{di}(k)\\ \alpha(k)B_{kj}C_{di}\end{bmatrix} \\ H=\begin{bmatrix}I\\ 0\end{bmatrix}^T, & \bar{E}_i(k)=\sum_{i=1}^{r}h_i(\theta(k))\begin{bmatrix}E_i(k)\\ 0\end{bmatrix}, & \bar{E}_{di}(k)=\sum_{i=1}^{r}h_i(\theta(k))\begin{bmatrix}E_{di}(k)\\ 0\end{bmatrix} \\ \bar{B}_{ki}=\sum_{i=1}^{r}\hat{h}_i(\theta(k))\begin{bmatrix}0\\ B_{ki}\end{bmatrix}, & D_{ij}=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta(k))\hat{h}_j(\theta(k))\begin{bmatrix}D_{1i}\\ B_{kj}D_{2i}\end{bmatrix}, & \bar{L}_{ij}=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta(k))\hat{h}_j(\theta(k))\begin{bmatrix}L_i^T\\ \beta(k)C_{kj}^TB_{2i}^T\end{bmatrix}^T \end{cases} \tag{30}$$

Let us define

$$\begin{cases} \bar{A}_{ij}(k)=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta(k))\hat{h}_j(\theta(k))\begin{bmatrix}A_i(k) & \beta(k)B_{1i}C_{kj}\\ \bar{\alpha}B_{kj}C_i & A_{kj}\end{bmatrix}, & \bar{A}_{dij}(k)=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta(k))\hat{h}_j(\theta(k))\begin{bmatrix}A_{di}(k)\\ \bar{\alpha}B_{kj}C_{di}\end{bmatrix} \\ \bar{C}_{ij}=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta(k))\hat{h}_j(\theta(k))\begin{bmatrix}0 & 0\\ B_{kj}C_i & 0\end{bmatrix}, & \bar{C}_{dij}=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta(k))\hat{h}_j(\theta(k))\begin{bmatrix}0\\ B_{kj}C_{di}\end{bmatrix} \\ \bar{A}_{ij}=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta(k))\hat{h}_j(\theta(k))\begin{bmatrix}A_i & \beta(k)B_{1i}C_{kj}\\ \bar{\alpha}B_{kj}C_i & A_{kj}\end{bmatrix}, & \bar{A}_{dij}=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta(k))\hat{h}_j(\theta(k))\begin{bmatrix}A_{di}\\ \bar{\alpha}B_{kj}C_{di}\end{bmatrix} \\ \bar{E}_i=\sum_{i=1}^{r}h_i(\theta(k))\begin{bmatrix}E_i\\ 0\end{bmatrix}, & \bar{E}_{di}=\sum_{i=1}^{r}h_i(\theta(k))\begin{bmatrix}E_{di}\\ 0\end{bmatrix} \\ \Delta\bar{A}_i(k)=\sum_{i=1}^{r}h_i(\theta(k))\begin{bmatrix}\Delta A_i(k) & 0\\ 0 & 0\end{bmatrix}, & \Delta\bar{A}_{di}(k)=\sum_{i=1}^{r}h_i(\theta(k))\begin{bmatrix}\Delta A_{di}(k)\\ 0\end{bmatrix} \\ \Delta\bar{E}_i(k)=\sum_{i=1}^{r}h_i(\theta(k))\begin{bmatrix}\Delta E_i(k)\\ 0\end{bmatrix}, & \Delta\bar{E}_{di}(k)=\sum_{i=1}^{r}h_i(\theta(k))\begin{bmatrix}\Delta E_{di}(k)\\ 0\end{bmatrix} \\ \bar{M}_i=\begin{bmatrix}M_i^T & 0\end{bmatrix}^T, & \bar{N}_{i1}=\begin{bmatrix}N_{i1} & 0\end{bmatrix} \end{cases} \tag{31}$$

Definition 1 (Exponential mean-square stability) (Dong, Wang, Ho, & Gao, 2010)**.** Under any initial condition and $\omega(k)=0$, if there exist $\mu>0$ and $0<\chi<1$ such that

$$\mathbb{E}\left\{\|\eta(k)\|^2\right\}\le\mu\chi^k\sup_{-d_M\le k\le 0}\mathbb{E}\left\{\|\psi(k)\|^2\right\},\qquad \omega(k)=0 \tag{32}$$

then the system is said to be exponentially mean-square stable, where $\eta(k)$ is the state variable, $\psi(k)$ is the initial condition.

Definition 2 (H-infinite performance) (Burl, 1999)**.** Under zero initial condition and $\omega(k)\neq 0$, if $z(k)$ satisfies

$$\mathbb{E}\left\{\sum_{k=0}^{\infty}z^T(k)z(k)\right\}-\gamma^2\mathbb{E}\left\{\sum_{k=0}^{\infty}\omega^T(k)\omega(k)\right\}\le 0,\qquad \omega(k)\neq 0 \tag{33}$$

then the prescribed H-infinite performance is guaranteed, where $\gamma>0$ is H-infinity performance index.

Lemma 1 (Schur complement) (Marouf, Esfanjani, Akbari, & Barforooshan, 2016)**.** For given matrices $\mathcal{S}_{11}=\mathcal{S}_{11}^T$ and $\mathcal{S}_{22}=\mathcal{S}_{22}^T$, the following inequality

$$\mathcal{S}=\begin{bmatrix}\mathcal{S}_{11} & \mathcal{S}_{12}\\ \mathcal{S}_{21}^T & \mathcal{S}_{22}\end{bmatrix}<0 \tag{34}$$

is equivalent to

$$\mathcal{S}_{22} < 0, \quad \mathcal{S}_{11} - \mathcal{S}_{12}\mathcal{S}_{22}^{-1}\mathcal{S}_{21}^T < 0 \tag{35}$$

Lemma 2 (Guelton, Bouarar, & Manamanni, 2009)**.** For given scalar $\varepsilon > 0$ and matrices $\mathcal{D}$, $\mathcal{F}$ and $\mathcal{G}$, the following inequality holds

$$\mathcal{DFG} + \left(\mathcal{DFG}\right)^T \le \varepsilon^{-1}\mathcal{D}\mathcal{D}^T + \varepsilon\mathcal{G}^T\mathcal{G} \tag{36}$$

$$\mathcal{F}^T\mathcal{F} \le I \tag{37}$$

Lemma 3 (Song, Niu, Lam, & Lam, 2018)**.** For given $X_i \in \mathbb{R}^{\upsilon\times\upsilon}$, if there exist $Y_i \in \mathbb{R}^{\upsilon\times\upsilon}$ satisfying

$$\begin{cases} X_i > 0, \quad Y_i > 0 \\ \begin{bmatrix} X_i & I \\ I & Y_i \end{bmatrix} \ge 0, \quad i = 1, 2, \dots, r \end{cases} \tag{38}$$

then the following inequalities hold

$$\begin{cases} tr\left(X_iY_i\right) > \upsilon \\ tr\left(X_iY_i\right) = \upsilon, \quad X_i = Y_i = I \end{cases} \tag{39}$$

Remark 2. The objectives in this paper can be summarized as follows
(i) closed-loop system (29) is exponentially mean-square stable under any initial condition and $\omega(k) = 0$;
(ii) prescribed H-infinity performance is guaranteed under zero initial condition and $\omega(k) \ne 0$;
(iii) A_{ki}, B_{ki} and C_{ki} are determined by employing the proposed methods.
Remark 3. The dynamic output feedback control is easy to implement and required conditions are less conservative (Zhao, & Dian, 2018). The T-S fuzzy model has nice ability to facilitate controller design, thus it is more effective to design the controller in practice (Choi, Ahn, Shi, Wu, & Lim, 2018; Wei, Qiu, Shi, & Wu, 2016; Wang, Wu, Wang, & Ma, 2020). Thus, the stochastic T-S fuzzy delay-dependent dynamic output feedback controller is designed in this section.

4. Main results

4.1. Stability conditions

Theorem 1. For given scalars $\varepsilon > 0$, $\lambda > 0$, $d_m > 0$, $\sigma > 0$, $\delta > 0$, $0 \le \bar{\alpha} \le 1$, $0 \le \bar{\beta} \le 1$ and matrices N_{i1}, N_{i2}, N_{i3}, N_{i4} ($i = 1, 2, \dots, r$), U_1, U_2, V_1, V_2, W_1, W_2 satisfying $U_1 - U_2 > 0$, $V_1 - V_2 > 0$, $W_1 - W_2 > 0$, there exist the matrices $\widehat{U}_1$, $\widehat{U}_2$, $\widehat{V}_1$, $\widehat{V}_2$, $\widehat{W}_1$, $\widehat{W}_2$ and fuzzy-basis-dependent matrices $P(h) = P^T(h) > 0$, $Q(h) = Q^T(h) > 0$, $G_1(h) = G_1^T(h) > 0$, $G_2(h) = G_2^T(h) > 0$ satisfying

$$\begin{cases} \widehat{U}_1 = \dfrac{\left(U_1^TU_2 + U_2^TU_1\right)}{2}, \quad \widehat{U}_2 = -\dfrac{\left(U_1^T + U_2^T\right)}{2}, \quad \widehat{W}_1 = \dfrac{\left(S_i^TW_1^TW_2S_i + S_i^TW_2^TW_1S_i\right)}{2} \\ \widehat{V}_1 = \dfrac{\left(V_1^TV_2 + V_2^TV_1\right)}{2}, \quad \widehat{V}_2 = -\dfrac{\left(V_1^T + V_2^T\right)}{2}, \quad \widehat{W}_2 = -\dfrac{\left(S_i^TW_1^T + S_i^TW_2^T\right)}{2} \end{cases} \tag{40}$$

$$\psi = \begin{bmatrix} \Pi & * & * & * & * & * & * & * \\ 0 & \Pi_a & * & * & * & * & * & * \\ -\widehat{U}_2^TH & 0 & -I & * & * & * & * & * \\ 0 & -\lambda\widehat{V}_2^T & 0 & -\lambda I & * & * & * & * \\ -\widehat{W}_2^TH & 0 & 0 & 0 & -I & * & * & * \\ \bar{A}_{ij} & \bar{A}_{dij} & \bar{E}_i & \bar{E}_{di} & 0 & \Pi_b & * & * \\ \sigma\bar{C}_{ij} & \sigma\bar{C}_{dij} & 0 & 0 & \delta\bar{B}_{ki} & 0 & -P^{-1}(h)G_1(h) & * \\ \bar{N}_{i1} & N_{i2} & N_{i3} & N_{i4} & 0 & 0 & 0 & -\varepsilon I \end{bmatrix} < 0 \tag{41}$$

where

$$\begin{cases} \Pi = -P(h) + \left(d_M - d_m + 1\right)H^TQ(h)G_2^{-1}(h)H - H^T\widehat{U}_1H, \quad \Pi_a = -Q(h)G_2^{-1}(h) - \lambda\widehat{V}_1, \quad \Pi_b = -P^{-1}(h)G_1(h) + \varepsilon\bar{M}_i\bar{M}_i^T \\ \sigma = \sqrt{\bar{\alpha}\left(1-\bar{\alpha}\right)}, \quad \delta = \sqrt{\bar{\beta}\left(1-\bar{\beta}\right)}, \quad \bar{N}_{i1} = [N_{i1} \quad 0] \end{cases} \tag{42}$$

then the closed-loop system (29) is exponentially mean-square stable.
Proof. Consider $V(k)$ as follows

$$V(k) = V_1(k) + V_2(k) + V_3(k) \tag{43}$$

$$\begin{cases} V_1(k)=\eta^T(k)P(h)G_1^{-1}(h)\eta(k) \\ V_2(k)=\sum_{i=k-d(k)}^{k-1}\eta^T(i)H^TQ(h)G_2^{-1}(h)H\eta(i) \\ V_3(k)=\sum_{j=k-d_M+1}^{k-d_m}\sum_{i=j}^{k-1}\eta^T(i)H^TQ(h)G_2^{-1}(h)H\eta(i) \end{cases} \tag{44}$$

Taking the forward difference of (43) along (29), one has

$$\Delta V(k)=\Delta V_1(k)+\Delta V_2(k)+\Delta V_3(k) \tag{45}$$

where

$$\Delta V_1(k)=V_1(k+1)-V_1(k) \tag{46}$$

$$\Delta V_2(k)=V_2(k+1)-V_2(k) \tag{47}$$

$$\Delta V_3(k)=V_3(k+1)-V_3(k) \tag{48}$$

Taking the mathematical expectation of (45) along (2), one has

$$\mathbb{E}\{\Delta V(k)\}=\mathbb{E}\{\Delta V_1(k)\}+\mathbb{E}\{\Delta V_2(k)\}+\mathbb{E}\{\Delta V_3(k)\} \tag{49}$$

Taking the mathematical expectation of (46) along (29), one has

$$\begin{aligned} \mathbb{E}\{\Delta V_1(k)\}&=\mathbb{E}\{V_1(k+1)-V_1(k)\} \\ &=\mathbb{E}\left\{\hat{A}_0^T(k)P(h)G_1^{-1}(h)\hat{A}_0(k)+\sigma^2\hat{B}_0^T(k)P(h)G_1^{-1}(h)\hat{B}_0(k)-\eta^T(k)P(h)G_1^{-1}(h)\eta(k)\right\} \\ &=\mathbb{E}\left\{\hat{A}_0^T(k)P(h)G_1^{-1}(h)\hat{A}_0(k)\right\}+\sigma^2\mathbb{E}\left\{\hat{B}_0^T(k)P(h)G_1^{-1}(h)\hat{B}_0(k)\right\}-\mathbb{E}\left\{\eta^T(k)P(h)G_1^{-1}(h)\eta(k)\right\} \end{aligned} \tag{50}$$

where

$$\begin{cases} \hat{A}_0(k)=\bar{A}_{ij}(k)\eta(k)+\bar{A}_{dij}(k)H\eta(k-d(k))+\bar{E}_i(k)f(x(k))+\bar{E}_{di}(k)f_d(x(k-d(k)))+\bar{\alpha}\bar{B}_{ki}\phi(S_ix(k)) \\ \hat{B}_0(k)=\bar{C}_{ij}\eta(k)+\bar{C}_{dij}H\eta(k-d(k))+\bar{B}_{ki}\phi(S_ix(k)) \end{cases} \tag{51}$$

Taking the mathematical expectation of (47) along (29), one has

$$\begin{aligned} \mathbb{E}\{\Delta V_2(k)\}&=\mathbb{E}\{V_2(k+1)-V_2(k)\} \\ &=\mathbb{E}\left\{\eta^T(k)H^TQ(h)G_2^{-1}(h)H\eta(k)+\sum_{i=k-d_M+1}^{k-d_m}\eta^T(i)H^TQ(h)G_2^{-1}(h)H\eta(i)\right\} \end{aligned}$$

From **Theorem 1**, one knows that $Q(h)>0$ and $G_2(h)>0$. Since $G_2(h)>0$, one can obtain $G_2^{-1}(h)>0$. Both considering $Q(h)>0$ and $G_2^{-1}(h)>0$, one can obtain $H^TQ(h)G_2^{-1}(h)H>0$, which implies the following inequality holds

$$\eta^T(k-d(k))H^TQ(h)G_2^{-1}(h)H\eta(k-d(k))\geq 0$$

then it can be verified that

$$\begin{aligned} \mathbb{E}\{\Delta V_2(k)\}&=\mathbb{E}\{V_2(k+1)-V_2(k)\} \\ &=\mathbb{E}\left\{\eta^T(k)H^TQ(h)G_2^{-1}(h)H\eta(k)+\sum_{i=k-d_M+1}^{k-d_m}\eta^T(i)H^TQ(h)G_2^{-1}(h)H\eta(i)\right\} \\ &\leq\mathbb{E}\left\{\eta^T(k)H^TQ(h)G_2^{-1}(h)H\eta(k)-\eta^T(k-d(k))H^TQ(h)G_2^{-1}(h)H\eta(k-d(k))+\sum_{i=k-d_M+1}^{k-d_m}\eta^T(i)H^TQ(h)G_2^{-1}(h)H\eta(i)\right\} \\ &=\mathbb{E}\left\{\eta^T(k)H^TQ(h)G_2^{-1}(h)H\eta(k)\right\}-\mathbb{E}\left\{\eta^T(k-d(k))H^TQ(h)G_2^{-1}(h)H\eta(k-d(k))\right\} \\ &\quad+\mathbb{E}\left\{\sum_{i=k-d_M+1}^{k-d_m}\eta^T(i)H^TQ(h)G_2^{-1}(h)H\eta(i)\right\} \end{aligned} \tag{52}$$

Taking the mathematical expectation of (48) along (29), one has

$$\begin{aligned} \mathbb{E}\{\Delta V_3(k)\}&=\mathbb{E}\{V_3(k+1)-V_3(k)\} \\ &=\mathbb{E}\left\{(d_M-d_m)\eta^T(k)H^TQ(h)G_2^{-1}(h)H\eta(k)-\sum_{i=k-d_M+1}^{k-d_m}\eta^T(i)H^TQ(h)G_2^{-1}(h)H\eta(i)\right\} \\ &=\mathbb{E}\left\{(d_M-d_m)\eta^T(k)H^TQ(h)G_2^{-1}(h)H\eta(k)\right\}-\mathbb{E}\left\{\sum_{i=k-d_M+1}^{k-d_m}\eta^T(i)H^TQ(h)G_2^{-1}(h)H\eta(i)\right\} \end{aligned} \tag{53}$$

Substituting (50), (52) and (53) into (49), one has

$$\begin{aligned}\mathbb{E}\{\Delta V(k)\} &\le \mathbb{E}\left\{\xi_0^T(k)\psi_1(k)\xi_0(k)+\xi_0^T(k)A_0^T(k)P(h)G_1^{-1}(h)A_0(k)\xi_0(k)+\sigma^2\xi_0^T(k)B_0^TP(h)G_1^{-1}(h)B_0\xi_0(k)\right\}\\ &\le \mathbb{E}\left\{\xi_0^T(k)\psi_1(k)\xi_0(k)+\xi_0^T(k)A_0^T(k)P(h)G_1^{-1}(h)A_0(k)\xi_0(k)+\xi_0^T(k)B_0^TP(h)G_1^{-1}(h)B_0\xi_0(k)\right\}\\ &\le \mathbb{E}\left\{\|\eta(k)\|^2\right\}+\mathbb{E}\left\{\sum_{i=k-d_M}^{k-1}\|\eta(i)\|^2\right\}-\mathbb{E}\left\{\eta^T(k)P(h)\eta(k)\right\}-\mathbb{E}\left\{\sum_{i=k-d_M+1}^{k-d_m}\eta^T(i)H^TQ(h)H\eta(i)\right\}\\ &\le \mathbb{E}\left\{\|\eta(k)\|^2\right\}+\mathbb{E}\left\{\sum_{i=k-d_M}^{k-1}\|\eta(i)\|^2\right\}\end{aligned} \tag{54}$$

where

$$\begin{cases}\xi_0(k)=\begin{bmatrix}\eta^T(k) & x^T(k-d(k)) & f^T(x(k)) & f_d^T(x(k-d(k))) & \phi^T(S_ix(k))\end{bmatrix}^T\\ A_0(k)=\begin{bmatrix}\bar{A}_{ij}(k) & \bar{A}_{dij}(k) & \bar{E}_i(k) & \bar{E}_{di}(k) & 0\end{bmatrix}\\ \psi_1(k)=diag\{\Pi_1 \quad -Q(h) \quad 0 \quad 0 \quad 0\}, \qquad B_0=\begin{bmatrix}\bar{C}_{ij} & \bar{C}_{dij} & 0 & 0 & \bar{B}_{ki}\end{bmatrix}\end{cases} \tag{55}$$

$$\Pi_1=-P(h)G_1^{-1}(h)+(d_M-d_m+1)H^TQ(h)G_2^{-1}(h)H \tag{56}$$

From (7), one can obtain

$$\begin{cases}\begin{bmatrix}\eta(k)\\ f(x(k))\end{bmatrix}^T\begin{bmatrix}H^T\hat{U}_1H & H^T\hat{U}_2\\ \hat{U}_2^TH & I\end{bmatrix}\begin{bmatrix}\eta(k)\\ f(x(k))\end{bmatrix}\le 0\\ \begin{bmatrix}x(k-d(k))\\ f_d(x(k-d(k)))\end{bmatrix}^T\begin{bmatrix}\hat{V}_1 & \hat{V}_2\\ \hat{V}_2^T & I\end{bmatrix}\begin{bmatrix}x(k-d(k))\\ f_d(x(k-d(k)))\end{bmatrix}\le 0\\ \begin{bmatrix}\eta(k)\\ \phi(S_ix(k))\end{bmatrix}^T\begin{bmatrix}H^T\hat{W}_1H & H^T\hat{W}_2\\ \hat{W}_2^TH & I\end{bmatrix}\begin{bmatrix}\eta(k)\\ \phi(S_ix(k))\end{bmatrix}\le 0\end{cases} \tag{57}$$

From (54) and (57), one has

$$\begin{aligned}\mathbb{E}\{\Delta V(k)\} &\le \mathbb{E}\left\{\xi_0^T(k)\psi_1(k)\xi_0(k)+\xi_0^T(k)A_0^T(k)P(h)G_1^{-1}(h)A_0(k)\xi_0(k)+\sigma^2\xi_0^T(k)B_0^TP(h)G_1^{-1}(h)B_0\xi_0(k)\right\}\\ &\quad -\mathbb{E}\left\{\begin{bmatrix}\eta(k)\\ f(x(k))\end{bmatrix}^T\begin{bmatrix}H^T\hat{U}_1H & H^T\hat{U}_2\\ \hat{U}_2^TH & I\end{bmatrix}\begin{bmatrix}\eta(k)\\ f(x(k))\end{bmatrix}+\lambda\begin{bmatrix}x(k-d(k))\\ f_d(x(k-d(k)))\end{bmatrix}^T\begin{bmatrix}\hat{V}_1 & \hat{V}_2\\ \hat{V}_2^T & I\end{bmatrix}\begin{bmatrix}x(k-d(k))\\ f_d(x(k-d(k)))\end{bmatrix}\right.\\ &\quad \left.+\begin{bmatrix}\eta(k)\\ \phi(S_ix(k))\end{bmatrix}^T\begin{bmatrix}H^T\hat{W}_1H & H^T\hat{W}_2\\ \hat{W}_2^TH & I\end{bmatrix}\begin{bmatrix}\eta(k)\\ \phi(S_ix(k))\end{bmatrix}\right\}\\ &= \mathbb{E}\left\{\xi_0^T(k)\left(\psi_2(k)+A_0^T(k)P(h)G_1^{-1}(h)A_0(k)+\sigma^2B_0^TP(h)G_1^{-1}(h)B_0\right)\xi_0(k)\right\}\end{aligned} \tag{58}$$

where

$$\psi_2(k)=\begin{bmatrix}\Pi & 0 & -H^T\hat{U}_2 & 0 & -H^T\hat{W}_2\\ 0 & -Q(h)G_2^{-1}(h)-\lambda\hat{V}_1 & 0 & -\lambda\hat{V}_2 & 0\\ -\hat{U}_2^TH & 0 & -I & 0 & 0\\ 0 & -\lambda\hat{V}_2^T & 0 & -\lambda I & 0\\ -\hat{W}_2^TH & 0 & 0 & 0 & -I\end{bmatrix}<0 \tag{59}$$

Applying **Lemma 1** to (59), one has

$$\psi_3(k)=\begin{bmatrix}\Pi+\varepsilon_0I & * & * & * & * & * & *\\ 0 & -Q(h)G_2^{-1}(h)-\lambda\hat{V}_1 & * & * & * & * & *\\ -\hat{U}_2^TH & 0 & -I & * & * & * & *\\ 0 & -\lambda\hat{V}_2^T & 0 & -\lambda I & * & * & *\\ -\hat{W}_2^TH & 0 & 0 & 0 & -I & * & *\\ \bar{A}_{ij}(k) & \bar{A}_{dij}(k) & \bar{E}_i(k) & \bar{E}_{di}(k) & 0 & -P^{-1}(h)G_1(h) & *\\ \sigma\bar{C}_{ij} & \sigma\bar{C}_{dij} & 0 & 0 & \delta\bar{B}_{ki} & 0 & -P^{-1}(h)G_1(h)\end{bmatrix}<0 \tag{60}$$

and

$$\psi_3(k)=\psi_3+\Delta\psi_3(k) \tag{61}$$

where

$$\psi_3 = \begin{bmatrix} \Pi + \varepsilon_0 I & * & * & * & * & * & * \\ 0 & -Q(h)G_2^{-1}(h) - \lambda\hat{V}_1 & * & * & * & * & * \\ -\hat{U}_2^T H & 0 & -I & * & * & * & * \\ 0 & -\lambda\hat{V}_2^T & 0 & -\lambda I & * & * & * \\ -\hat{W}_2^T H & 0 & 0 & 0 & -I & * & * \\ \bar{A}_{ij} & \bar{A}_{dij} & \bar{E}_i & \bar{E}_{di} & 0 & -P^{-1}(h)G_1(h) & * \\ \sigma\bar{C}_{ij} & \sigma\bar{C}_{dij} & 0 & 0 & \delta\bar{B}_{ki} & 0 & -P^{-1}(h)G_1(h) \end{bmatrix} \tag{62}$$

$$\Delta\psi_3(k) = \begin{bmatrix} 0 & * & * & * & * & * & * \\ 0 & 0 & * & * & * & * & * \\ 0 & 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * & * \\ \Delta\bar{A}_i(k) & \Delta\bar{A}_{di}(k) & \Delta\bar{E}_i(k) & \Delta\bar{E}_d(k) & 0 & 0 & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{63}$$

From (31) and (63), one can obtain

$$\Delta\psi_3(k) = \tilde{M}_i F_i(k)\tilde{N}_i + \left(\tilde{M}_i F_i(k)\tilde{N}_i\right)^T \tag{64}$$

where

$$\tilde{M}_i = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \bar{M}_i^T & 0 \end{bmatrix}^T, \quad \tilde{N}_i = \begin{bmatrix} \bar{N}_{i1} & N_{i2} & N_{i3} & N_{i4} & 0 & 0 & 0 \end{bmatrix} \tag{65}$$

Applying **Lemma** 2 to (64), one has

$$\tilde{M}_i F_i(k)\tilde{N}_i + \left(\tilde{M}_i F_i(k)\tilde{N}_i\right)^T \le \varepsilon\tilde{M}_i\tilde{M}_i^T + \varepsilon^{-1}\tilde{N}_i^T\tilde{N}_i \tag{66}$$

From (60) and (66), one has

$$\psi_3(k) \le \psi_4 + \varepsilon^{-1}\tilde{N}_i^T\tilde{N}_i \tag{67}$$

where

$$\psi_4 = \begin{bmatrix} \Pi + \varepsilon_0 I & * & * & * & * & * & * \\ 0 & \Pi_a & * & * & * & * & * \\ -\hat{U}_2^T H & 0 & -I & * & * & * & * \\ 0 & -\lambda\hat{V}_2^T & 0 & -\lambda I & * & * & * \\ -\hat{W}_2 H & 0 & 0 & 0 & -I & * & * \\ \bar{A}_{ij} & \bar{A}_{dij} & \bar{E}_i & \bar{E}_{di} & 0 & \Pi_b & * \\ \sigma\bar{C}_{ij} & \sigma\bar{C}_{dij} & 0 & 0 & \delta\bar{B}_{ki} & 0 & -P^{-1}(h)G_1(h) \end{bmatrix} \tag{68}$$

It can be verified that there exists $\varepsilon_0 > 0$ satisfying

$$\psi + \varepsilon_0 diag\{I \quad 0\} < 0 \tag{69}$$

where $\psi < 0$ is a matrix with appropriate dimension.

In order to prove the exponential mean-square stability, one should prove that the inequality (69) holds.

According to **Lemma 1**, (60) is equivalent to (70)

$$\psi_2(k) + \varepsilon_0 diag\{I \quad 0\} + A_0^T(k)P(h)G_1^{-1}(h)A_0(k) + \sigma^2 B_0^T P(h)G_1^{-1}(h)B_0 < 0 \tag{70}$$

The inequality (69) holds if $\psi_4 + \varepsilon^{-1}\tilde{N}_i^T\tilde{N}_i$ satisfying (71)

$$\psi_4 + \varepsilon^{-1}\tilde{N}_i^T\tilde{N}_i < 0 \tag{71}$$

Substituting (71) into (67), one can obtain $\psi_3(k) < 0$, and the inequality (70) holds

From (58) and (70), one has

$$\mathbb{E}\{\Delta V(k)\} \le -\varepsilon_0\mathbb{E}\left\{\|\eta(k)\|^2\right\} \tag{72}$$

From (43) and (44), one has

$$\begin{aligned}\mathbb{E}\{V(k)\} &= \mathbb{E}\left\{\eta^T(k)P(h)G_1^{-1}(h)\eta(k) + \sum_{i=k-d(k)}^{k-1}\eta^T(i)H^TQ(h)G_2^{-1}(h)H\eta(i) + \sum_{j=k-d_M+1}^{k-d_m}\sum_{i=j}^{k-1}\eta^T(i)H^TQ(h)G_2^{-1}(h)H\eta(i)\right\} \\ &= \mathbb{E}\left\{\eta^T(k)P(h)G_1^{-1}(h)\eta(k)\right\} + \mathbb{E}\left\{\sum_{i=k-d(k)}^{k-1}\eta^T(i)H^TQ(h)G_2^{-1}(h)H\eta(i) + \sum_{j=k-d_M+1}^{k-d_m}\sum_{i=j}^{k-1}\eta^T(i)H^TQ(h)G_2^{-1}(h)H\eta(i)\right\} \\ &\le \mathbb{E}\left\{\|\eta(k)\|^2\right\} + \sum_{i=k-d_M}^{k-1}\mathbb{E}\left\{\|\eta(i)\|^2\right\}\end{aligned} \tag{73}$$

From (44) and (72), one has

$$\mathbb{E}\{V(k)\} \le \rho_1\mathbb{E}\left\{\|\eta(k)\|^2\right\} + \rho_2\sum_{i=k-d_M}^{k-1}\mathbb{E}\left\{\|\eta(i)\|^2\right\} \tag{74}$$

where $\rho_1 > 1$ and $\rho_2 > 1$ are the scalars.

From (72) and (74), one can obtain

$$\mu^{k+1}\mathbb{E}\{V(k+1)\} - \mu^k\mathbb{E}\{V(k)\} = \mu^{k+1}\mathbb{E}\{\Delta V(k)\} + \mu^k(\mu-1)\mathbb{E}\{V(k)\} \le \omega_1(\mu)\mu^k\mathbb{E}\left\{\|\eta(k)\|^2\right\} + \omega_2(\mu)\sum_{i=k-d_M}^{k-1}\mu^k\mathbb{E}\left\{\|\eta(i)\|^2\right\} \tag{75}$$

with

$$\omega_1(\mu) = -\mu\varepsilon_0 + (\mu+1)\rho_1, \quad \omega_2(\mu) = (\mu-1)\rho_2 \tag{76}$$

where $\mu > 1$ is a scalar.

Taking the sum on both sides of (75) from $k = 0$ to $k = N-1$, one has

$$\mu^N\mathbb{E}\{V(N)\} - \mathbb{E}\{V(0)\} \le \omega_1(\mu)\sum_{k=0}^{N-1}\mu^k\mathbb{E}\left\{\|\eta(k)\|^2\right\} + \omega_2(\mu)\sum_{k=0}^{N-1}\sum_{i=k-d_M}^{k-1}\mu^k\mathbb{E}\left\{\|\eta(i)\|^2\right\} \tag{77}$$

where $N \ge d_M + 1$.

For $d_M \ge 1$, it can be verified that the following inequality holds

$$\begin{aligned}\sum_{k=0}^{N-1}\sum_{i=k-d_M}^{k-1}\mu^k\mathbb{E}\left\{\|\eta(i)\|^2\right\} &\le \sum_{i=-d_M}^{-1}\sum_{k=0}^{i+d_M}\mu^k\mathbb{E}\left\{\|\eta(i)\|^2\right\} + \sum_{i=0}^{N-1-d_M}\sum_{k=i+1}^{i+d_M}\mu^k\mathbb{E}\left\{\|\eta(i)\|^2\right\} + \sum_{i=N-1-d_M}^{N-1}\sum_{k=i+1}^{N-1}\mu^k\mathbb{E}\left\{\|\eta(i)\|^2\right\} \\ &\le d_M\sum_{i=-d_M}^{-1}\mu^{i+d_M}\mathbb{E}\left\{\|\eta(i)\|^2\right\} + d_M\sum_{i=0}^{N-1-d_M}\mu^{i+d_M}\mathbb{E}\left\{\|\eta(i)\|^2\right\} + d_M\sum_{i=N-1-d_M}^{N-1}\mu^{i+d_M}\mathbb{E}\left\{\|\eta(i)\|^2\right\} \\ &\le d_M\mu^{d_M}\max_{-d_M\le i\le 0}\mathbb{E}\left\{\|\psi(i)\|^2\right\} + d_M\mu^{d_M}\sum_{i=0}^{N-1}\mu^i\mathbb{E}\left\{\|\eta(i)\|^2\right\}\end{aligned} \tag{78}$$

Next, from (77) and (78), one has

$$\mu^N\mathbb{E}\{V(N)\} \le \mathbb{E}\{V(0)\} + \left(\omega_1(\mu) + d_M\mu^{d_M}\omega_2(\mu)\right)\sum_{k=0}^{N-1}\mu^k\mathbb{E}\left\{\|\eta(k)\|^2\right\} + d_M\mu^{d_M}\omega_2(\mu)\max_{-d_M\le i\le 0}\mathbb{E}\left\{\|\psi(i)\|^2\right\} \tag{79}$$

Let us define

$$\rho_0 = \lambda_{min}\left(P(h)G_1^{-1}(h)\right), \quad \rho = max\{\rho_1 \quad \rho_2\} \tag{80}$$

where $\lambda_{min}(\bullet)$ is the minimum eigenvalue value of "$\bullet$".

It is obvious that

$$\mathbb{E}\{V(N)\} \ge \rho_0\mathbb{E}\left\{\|\eta(N)\|^2\right\} \tag{81}$$

From (74) and (80), it can be verified that

$$\mathbb{E}\{V(0)\} \le \rho\max_{-d_M\le i\le 0}\mathbb{E}\left\{\|\psi(i)\|^2\right\} \tag{82}$$

For (76), it can be seen that there exists the scalar $\mu_0 > 1$ satisfying

$$\omega_1(\mu_0) + d_M\mu_0^{d_M}\omega_2(\mu_0) = 0 \tag{83}$$

Substituting (81)-(83) into (79)

$$\mathbb{E}\left\{\|\eta(N)\|^2\right\} \le c_0\mu_0^{-N}\max_{-d_M\le i\le 0}\mathbb{E}\left\{\|\psi(i)\|^2\right\} \tag{84}$$

where

$$c_0 = \rho_0^{-1}\left(\rho + d_M\mu_0^{d_M}\omega_2(\mu_0)\right) \tag{85}$$

Then, it can be seen that closed-loop system (29) is exponentially mean-square stable. The objective **(i)** in **Remark 2** is achieved, and the proof of **Theorem 1** is completed.

Remark 4. From (40)-(42), it can be seen that fuzzy-basis-dependent matrices $P(h)$, $Q(h)$, $G_1(h)$, $G_2(h)$, the lower bounds d_m and d_M are employed to derive the fuzzy-basis-dependent and delay-dependent stability conditions, thus the control design conditions are relaxed by adjusting d_m and d_M. Moreover, the more important stability results can be obtained in the exponential mean-square stability analysis, because it is used to investigate the exponential convergence performance of state variables (Guan, & Liu, 2016). Thus, the exponential mean-square stability analysis is discussed in this paper. However, the prescribed H-infinity performance is not guaranteed, and **Theorem 2** is presented.

4.2. Less conservative stability conditions

Theorem 2. For given scalars $\varepsilon>0$, $\lambda>0$, $d_m>0$, $\sigma>0$, $\delta>0$, $\gamma>0$, $0\le\bar{\alpha}\le1$, $0\le\bar{\beta}\le1$ and matrices N_{i1}, N_{i2}, N_{i3}, N_{i4} ($i=1,2,\ldots,r$), U_1, U_2, V_1, V_2, W_1, W_2 satisfying $U_1-U_2>0$, $V_1-V_2>0$, $W_1-W_2>0$, there exist the matrices $\widehat{U}_1$, $\widehat{U}_2$, $\widehat{V}_1$, $\widehat{V}_2$, $\widehat{W}_1$, $\widehat{W}_2$ and fuzzy-basis-dependent matrices $P(h)=P^T(h)>0$, $Q(h)=Q^T(h)>0$ satisfying

$$\begin{cases}\widehat{U}_1=\dfrac{\left(U_1^TU_2+U_2^TU_1\right)}{2}, & \widehat{U}_2=-\dfrac{\left(U_1^T+U_2^T\right)}{2}, & \widehat{W}_1=\dfrac{\left(S_i^TW_1^TW_2S_i+S_i^TW_2^TW_1S_i\right)}{2}\\ \widehat{V}_1=\dfrac{\left(V_1^TV_2+V_2^TV_1\right)}{2}, & \widehat{V}_2=-\dfrac{\left(V_1^T+V_2^T\right)}{2}, & \widehat{W}_2=-\dfrac{\left(S_i^TW_1^T+S_i^TW_2^T\right)}{2}\end{cases} \tag{86}$$

$$\Phi(k)=\begin{bmatrix}\Upsilon & * & * & * & * & * & * & * & *\\ 0 & \Pi_a & * & * & * & * & * & * & *\\ -\widehat{U}_2^TH & 0 & -I & * & * & * & * & * & *\\ 0 & -\lambda\widehat{V}_2^T & 0 & -\lambda I & * & * & * & * & *\\ -\widehat{W}_2^TH & 0 & 0 & 0 & -I & * & * & * & *\\ 0 & 0 & 0 & 0 & 0 & -\gamma^2I & * & * & *\\ \bar{A}_{ij} & \bar{A}_{dij} & \bar{E}_i & \bar{E}_{di} & 0 & 0 & \Pi_b & * & *\\ \sigma\bar{C}_{ij} & \sigma\bar{C}_{dij} & 0 & 0 & \delta\bar{B}_{ki} & 0 & 0 & -P^{-1}(h) & *\\ \bar{N}_{i1} & N_{i2} & N_{i3} & N_{i4} & 0 & 0 & 0 & 0 & -\varepsilon I\end{bmatrix}<0 \tag{87}$$

where

$$\begin{cases}\Upsilon=-P(h)+(d_M-d_m+1)H^TQ(h)H-H^T\widehat{U}_1H-H^T\widehat{W}_1H+\bar{L}_{ij}^T\bar{L}_{ij}\\ \Pi_a=-Q(h)-\lambda\widehat{V}_1, \quad \Pi_b=-P^{-1}(h)+\varepsilon\bar{M}_i\bar{M}_i^T\\ \sigma=\sqrt{\bar{\alpha}(1-\bar{\alpha})}, \quad \delta=\sqrt{\bar{\beta}(1-\bar{\beta})}, \quad \bar{N}_{i1}=\begin{bmatrix}N_{i1} & 0\end{bmatrix}\end{cases} \tag{88}$$

then the prescribed H-infinity performance is guaranteed.

Proof. The proof of **Theorem 2** is divided into **Steps 1**-**2**.

Step 1. In **Theorem 2**, $\gamma>0$ is a given scalar. According to **Lemma 1** (Schur complement), one knows that (87) is equivalent to (41). Thus, the proof of **Theorem 2** is converted into the proof of **Theorem 1**. Via similar method in **Theorem 1**, it can be seen that the closed-loop system is exponentially mean-square stable. The proof of the objective **(i)** in **Remark 2** is achieved, and the proof of **Step 1** is completed.

Step 2. Consider $\mathcal{V}(k)$ as follows

$$\mathcal{V}(k)=\mathcal{V}_1(k)+\mathcal{V}_2(k)+\mathcal{V}_3(k) \tag{89}$$

$$\begin{cases}\mathcal{V}_1(k)=\eta^T(k)P(h)\eta(k)\\ \mathcal{V}_2(k)=\displaystyle\sum_{i=k-d(k)}^{k-1}\eta^T(i)H^TQ(h)H\eta(i)\\ \mathcal{V}_3(k)=\displaystyle\sum_{j=k-d_M+1}^{k-d_m}\sum_{i=j}^{k-1}\eta^T(i)H^TQ(h)H\eta(i)\end{cases} \tag{90}$$

Taking the forward difference of (89) along (29)

$$\Delta\mathcal{V}(k)=\Delta\mathcal{V}_1(k)+\Delta\mathcal{V}_2(k)+\Delta\mathcal{V}_3(k) \tag{91}$$

$$\Delta\mathcal{V}_1(k)=\mathcal{V}_1(k+1)-\mathcal{V}_1(k) \tag{92}$$

$$\Delta\mathcal{V}_2(k)=\mathcal{V}_2(k+1)-\mathcal{V}_2(k) \tag{93}$$

$$\Delta\mathcal{V}_3(k)=\mathcal{V}_3(k+1)-\mathcal{V}_3(k) \tag{94}$$

Taking the mathematical expectation of (91) along (29)

$$\mathbb{E}\{\Delta\mathcal{V}(k)\}=\mathbb{E}\{\Delta\mathcal{V}_1(k)\}+\mathbb{E}\{\Delta\mathcal{V}_2(k)\}+\mathbb{E}\{\Delta\mathcal{V}_3(k)\} \tag{95}$$

Taking the mathematical expectation of (92) along (29)

$$\begin{aligned}\mathbb{E}\{\Delta\mathcal{V}_1(k)\}&=\mathbb{E}\{\mathcal{V}_1(k+1)-\mathcal{V}_1(k)\}=\mathbb{E}\left\{\hat{A}_0^T(k)P(h)\hat{A}_0(k)+\sigma^2\hat{B}_0^T(k)P(h)\hat{B}_0(k)-\eta^T(k)P(h)\eta(k)\right\}\\&=\mathbb{E}\left\{\hat{A}_0^T(k)P(h)\hat{A}_0(k)\right\}+\sigma^2\mathbb{E}\left\{\hat{B}_0^T(k)P(h)\hat{B}_0(k)\right\}-\mathbb{E}\{\eta^T(k)P(h)\eta(k)\}\end{aligned} \tag{96}$$

where

$$\begin{cases}\hat{A}_0(k)=\bar{A}_{ij}(k)\eta(k)+\bar{A}_{dij}(k)H\eta(k-d(k))+\bar{E}_i(k)f(x(k))+\bar{E}_{di}(k)f_d(x(k-d(k)))+\bar{\alpha}\bar{B}_{ki}\phi(S_ix(k))\\ \hat{B}_0(k)=\bar{C}_{ij}\eta(k)+\bar{C}_{dij}H\eta(k-d(k))+\bar{B}_{ki}\phi(S_ix(k))\end{cases} \tag{97}$$

Taking the mathematical expectation of (93) along (29)

$$\begin{aligned}\mathbb{E}\{\Delta\mathcal{V}_2(k)\}&=\mathbb{E}\{\mathcal{V}_2(k+1)-\mathcal{V}_2(k)\}\le\mathbb{E}\left\{\eta^T(k)H^TQ(h)H\eta(k)-\eta^T(k-d(k))H^TQ(h)H\eta(k-d(k))+\sum_{i=k-d_M+1}^{k-d_m}\eta^T(i)H^TQ(h)H\eta(i)\right\}\\&=\mathbb{E}\{\eta^T(k)H^TQ(h)H\eta(k)\}-\mathbb{E}\{\eta^T(k-d(k))H^TQ(h)H\eta(k-d(k))\}+\mathbb{E}\left\{\sum_{i=k-d_M+1}^{k-d_m}\eta^T(i)H^TQ(h)H\eta(i)\right\}\end{aligned} \tag{98}$$

Taking the mathematical expectation of (94) along (29)

$$\begin{aligned}\mathbb{E}\{\Delta\mathcal{V}_3(k)\}&=\mathbb{E}\{\mathcal{V}_3(k+1)-\mathcal{V}_3(k)\}=\mathbb{E}\left\{(d_M-d_m)\eta^T(k)H^TQ(h)H\eta(k)-\sum_{i=k-d_M+1}^{k-d_m}\eta^T(i)H^TQ(h)H\eta(i)\right\}\\&=\mathbb{E}\{(d_M-d_m)\eta^T(k)H^TQ(h)H\eta(k)\}\\&\quad-\mathbb{E}\left\{\sum_{i=k-d_M+1}^{k-d_m}\eta^T(i)H^TQ(h)H\eta(i)\right\}\end{aligned} \tag{99}$$

Substituting (96), (98) and (99) into (95)

$$\begin{aligned}\mathbb{E}\{\Delta\mathcal{V}(k)\}&\le\mathbb{E}\{\xi^T(k)\Phi_1(k)\xi(k)+\xi^T(k)A^T(k)P(h)A(k)\xi(k)+\sigma^2\xi^T(k)B^TP(h)B\xi(k)\}\\&=\mathbb{E}\{\xi^T(k)\Phi_1(k)\xi(k)\}+\mathbb{E}\{\xi^T(k)A^T(k)P(h)A(k)\xi(k)\}+\sigma^2\mathbb{E}\{\xi^T(k)B^TP(h)B\xi(k)\}\end{aligned} \tag{100}$$

where

$$\begin{cases}\xi(k)=\begin{bmatrix}\eta^T(k) & x^T(k-d(k)) & f^T(x(k)) & f_d^Tx(k-d(k)) & \phi^T(S_ix(k)) & \omega^T(k)\end{bmatrix}^T\\ \Phi_1(k)=diag\{-P(h)+(d_M-d_m+1)H^TQH \quad -Q \quad 0 \quad 0 \quad 0 \quad 0\}\\ A(k)=\begin{bmatrix}\bar{A}_{ij}(k) & \bar{A}_{dij}(k) & \bar{E}_i(k) & \bar{E}_{di}(k) & 0 & D_{ij}\end{bmatrix},\quad B=\begin{bmatrix}\bar{C}_{ij} & \bar{C}_{dij} & 0 & 0 & \bar{B}_{ki} & 0\end{bmatrix}\end{cases} \tag{101}$$

For (29), the H-infinity performance function $J(n)$ is designed as follows

$$J(n)=\mathbb{E}\left\{\sum_{k=0}^{n}\left(z^T(k)z(k)-\gamma^2\omega^T(k)\omega(k)\right)\right\},\qquad \omega(k)\neq0 \tag{102}$$

Under zero initial condition, consider (57), (100) and (102), one has

$$\begin{aligned}J(n)&=\mathbb{E}\left\{\sum_{k=0}^{n}\left(z^T(k)z(k)-\gamma^2\omega^T(k)\omega(k)+\Delta\mathcal{V}(k)\right)\right\}-\mathbb{E}\{\mathcal{V}(n+1)\}\\&\le\mathbb{E}\left\{\sum_{k=0}^{n}\left(\eta^T(k)\bar{L}_{ij}^T\bar{L}_{ij}\eta(k)-\gamma^2\omega^T(k)\omega(k)+\Delta\mathcal{V}(k)\right)\right\}\\&\le\mathbb{E}\left\{\sum_{k=0}^{n}\left(\xi^T(k)\Phi_2(k)\xi(k)-\xi^T(k)A^T(k)P(h)A(k)\xi(k)-\sigma^2\xi^T(k)B^TP(h)B\xi(k)\right)\right\},\qquad\omega(k)\neq0\end{aligned} \tag{103}$$

where

$$\Phi_2(k)=\begin{bmatrix}\Upsilon & 0 & -H^T\widehat{U}_2 & 0 & -H^T\widehat{W}_2 & 0\\ 0 & -Q(h)-\lambda\widehat{V}_1 & 0 & -\lambda\widehat{V}_2 & 0 & 0\\ -\widehat{U}_2^TH & 0 & -I & 0 & 0 & 0\\ 0 & -\lambda\widehat{V}_2^T & 0 & -\lambda I & 0 & 0\\ -\widehat{W}_2^TH & 0 & 0 & 0 & -I & 0\\ 0 & 0 & 0 & 0 & 0 & -\gamma^2I\end{bmatrix} \tag{104}$$

From **Theorem 2**, one knows that

$$P(h)=P^T(h)>0 \tag{105}$$

$$\xi^T(k)A^T(k)P(h)A(k)\xi(k)\geq 0,\quad \sigma^2\xi^T(k)B^TP(h)B\xi(k)\geq 0 \tag{106}$$

Substituting (106) into (103)

$$J(n)\leq \mathbb{E}\left\{\sum_{k=0}^{n}\xi^T(k)\Phi_2(k)\xi(k)\right\} \tag{107}$$

Applying **Lemma 1** to (87)

$$\Phi_2(k)=\begin{bmatrix} \Upsilon & 0 & -H^T\widehat{U}_2 & 0 & -H^T\widehat{W}_2 & 0 \\ 0 & -Q(h)-\lambda\widehat{V}_1 & 0 & -\lambda\widehat{V}_2 & 0 & 0 \\ -\widehat{U}_2^TH & 0 & -I & 0 & 0 & 0 \\ 0 & -\lambda\widehat{V}_2^T & 0 & -\lambda I & 0 & 0 \\ -\widehat{W}_2^TH & 0 & 0 & 0 & -I & 0 \\ 0 & 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix}<0 \tag{108}$$

Substituting (108) into (107), one has

$$J(n)\leq \mathbb{E}\left\{\sum_{k=0}^{n}\xi^T(k)\Phi_2(k)\xi(k)\right\}<0 \tag{109}$$

and substituting (102) into (105), one has

$$\mathbb{E}\left\{\sum_{k=0}^{n}\left(z^T(k)z(k)-\gamma^2\omega^T(k)\omega(k)\right)\right\}\leq \mathbb{E}\left\{\sum_{k=0}^{n}\xi^T(k)\Phi_2(k)\xi(k)\right\}<0,\qquad \omega(k)\neq 0 \tag{110}$$

which yields

$$\mathbb{E}\left\{\sum_{k=0}^{n}\left(z^T(k)z(k)-\gamma^2\omega^T(k)\omega(k)\right)\right\}<0,\qquad \omega(k)\neq 0 \tag{111}$$

Substituting $n=\infty$ into (111)

$$\mathbb{E}\left\{\sum_{k=0}^{\infty}z^T(k)z(k)\right\}-\gamma^2\mathbb{E}\left\{\sum_{k=0}^{\infty}\omega^T(k)\omega(k)\right\}<0,\qquad \omega(k)\neq 0 \tag{112}$$

With above analysis, the prescribed H-infinite performance is guaranteed.

Remark 5. In this section, the less conservative stability conditions are derived by constructing fuzzy-basis-dependent Lyapunov functional. Compared with (44), the cross product terms $P(h)G_1^{-1}(h)$ and $Q(h)G_2^{-1}(h)$ between $P(h)$, $Q(h)$, $G_1(h)$ and $G_2(h)$ are avoided in (90), thus the design conditions can be relaxed. H-infinity performance index is one of the most important robust control performance indicators (Zhang, Wang, Jiang, & Zhang, 2015). Specifically, γ is the H-infinity performance index of the system and it is often used to investigate the control problem of minimum sensitivity. Moreover, the H-infinity optimization control is more significant in the practical control system (Zhang, Wang, Jiang, & Zhang, 2015; Yu, Dong, Li, & Li, 2017). Thus, the Lyapunov-Razumikhin method will be considered for the stability analysis in the next study.

4.3. Determine controller gain matrices

Theorem 3. For given scalars $\varepsilon>0$, $\lambda>0$, $d_m>0$, $\sigma>0$, $\delta>0$, $\gamma>0$, $0\leq\bar{\alpha}\leq 1$, $0\leq\bar{\beta}\leq 1$ and matrices N_{i1}, N_{i2}, N_{i3}, N_{i4} ($i=1,2,\ldots,r$), U_1, U_2, V_1, V_2, W_1, W_2 satisfying $U_1-U_2>0$, $V_1-V_2>0$, $W_1-W_2>0$, there exist matrices $\widehat{U}_1$, $\widehat{U}_2$, $\widehat{V}_1$, $\widehat{V}_2$, $\widehat{W}_1$, $\widehat{W}_2$, Λ, Ω, Γ, $X_i>0$, $Y_i>0$ and fuzzy-basis-dependent matrices $P(h)=P^T(h)>0$, $Q(h)=Q^T(h)>0$ satisfying

$$\begin{cases} \widehat{U}_1=\dfrac{\left(U_1^TU_2+U_2^TU_1\right)}{2}, & \widehat{U}_2=-\dfrac{\left(U_1^T+U_2^T\right)}{2}, & \widehat{W}_1=\dfrac{\left(S_i^TW_1^TW_2S_i+S_i^TW_2^TW_1S_i\right)}{2} \\ \widehat{V}_1=\dfrac{\left(V_1^TV_2+V_2^TV_1\right)}{2}, & \widehat{V}_2=-\dfrac{\left(V_1^T+V_2^T\right)}{2}, & \widehat{W}_2=-\dfrac{\left(S_i^TW_1^T+S_i^TW_2^T\right)}{2} \end{cases} \tag{113}$$

$$\Phi_{0i}=\begin{bmatrix} \Phi_{11i} & * \\ \Phi_{21i} & \Phi_{22i} \end{bmatrix}<0 \tag{114}$$

where

$$\begin{cases} \Phi_{11i} = \begin{bmatrix} \widehat{\Pi} & * & * & * & * & * \\ 0 & -Q(h)-\lambda\widehat{V}_1 & * & * & * & * \\ \Xi_{1i} & 0 & -I & * & * & * \\ 0 & -P(h)-\lambda\widehat{V}_2^T & 0 & -\lambda I & * & * \\ \Xi_{2i} & 0 & 0 & 0 & -I & * \\ 0 & 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix} \\ \Phi_{21i} = \begin{bmatrix} \Theta_{1i} & \Theta_{2i} & \Theta_{3i} & \Theta_{4i} & 0 & \Theta_{5i} \\ \Theta_{6i} & \Theta_{7i} & 0 & 0 & \Theta_{8i} & 0 \\ \Xi_{3i} & N_{i2} & N_{i3} & N_{i4} & 0 & 0 \\ \Xi_{4i} & \bar{N}_{i1} & 0 & 0 & 0 & 0 \\ \Xi_{5i} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \Phi_{22i} = \begin{bmatrix} \widehat{\Pi} & * & * & * & * & * \\ 0 & \widehat{\Pi} & * & * & * & * \\ 0 & 0 & -\varepsilon I & * & * & * \\ 0 & 0 & 0 & -I & * & * \\ 0 & 0 & 0 & 0 & -\hat{Q}^{-1}(h) & * \\ \Xi_{6i} & 0 & 0 & 0 & 0 & -\varepsilon^{-1} I \end{bmatrix} \end{cases} \tag{115}$$

$$\widehat{\Pi} = \begin{bmatrix} -X_i & -I \\ -I & -Y_i \end{bmatrix}, \quad \bar{N}_{i1} = \begin{bmatrix} N_{i1} & 0 \end{bmatrix}, \quad \hat{Q}(h) = (d_M - d_m + 1)Q(h) - \widehat{U}_1 - \widehat{W}_1 \tag{116}$$

$$\begin{cases} \Xi_{1i} = -\widehat{U}_2^T \begin{bmatrix} X_i & I \end{bmatrix}, & \Xi_{2i} = -\widehat{W}_2^T \begin{bmatrix} X_i & I \end{bmatrix}, & \Xi_{3i} = N_{i1} \begin{bmatrix} X_i & I \end{bmatrix} \\ \Xi_{4i} = \begin{bmatrix} L_i X_i + B_{2i}\Lambda & L_i \end{bmatrix}, & \Xi_{5i} = \begin{bmatrix} X_i & I \end{bmatrix}, & \Xi_{6i} = \varepsilon M_i^T \begin{bmatrix} I & Y_i \end{bmatrix} \end{cases} \tag{117}$$

$$\begin{cases} \Theta_{1i} = \begin{bmatrix} A_i X_i + B_{1i}\Lambda & A_i \\ \Omega & Y_i A_i + \bar{\alpha}\Gamma C_i \end{bmatrix}, & \Theta_{2i} = \begin{bmatrix} A_{di} \\ Y_i A_{di} + \bar{\alpha}\Gamma C_{di} \end{bmatrix}, & \Theta_{3i} = \begin{bmatrix} E_i \\ Y_i E_i \end{bmatrix}, & \Theta_{4i} = \begin{bmatrix} E_{di} \\ Y_i E_{di} \end{bmatrix} \\ \Theta_{5i} = \begin{bmatrix} D_{1i} \\ Y_i D_{1i} + \Gamma D_{2i} \end{bmatrix}, & \Theta_{6i} = \sigma \begin{bmatrix} 0 & 0 \\ \Gamma C_i X_i & \Gamma C_i \end{bmatrix}, & \Theta_{7i} = \sigma \begin{bmatrix} 0 \\ \Gamma C_{di} \end{bmatrix}, & \Theta_{8i} = \delta \begin{bmatrix} 0 \\ \Gamma X_i \end{bmatrix} \end{cases} \tag{118}$$

$$\sigma = \sqrt{\bar{\alpha}(1-\bar{\alpha})}, \quad \delta = \sqrt{\bar{\beta}(1-\bar{\beta})} \tag{119}$$

then A_{ki}, B_{ki} and C_{ki} can be determined

$$A_{ki} = R_i^{-1}(\Omega - Y_i A_i X_i - \bar{\alpha}\Gamma C_i X_i + Y_i B_{1i}\Lambda)G_i^{-T}, \quad B_{ki} = R_i^{-1}\Gamma, \quad C_{ki} = \Lambda G_i^{-T} \tag{120}$$

$$R_i G_i^T = I - Y_i X_i \tag{121}$$

where R_i and G_i are the parameter matrices with appropriate dimensions.

Proof. From (116), one has

$$\widehat{\Pi} = \begin{bmatrix} -X_i & -I \\ -I & -Y_i \end{bmatrix} < 0 \tag{122}$$

Applying **Lemma 1** to (122), one has

$$Y_i - X_i^{-1} > 0 \tag{123}$$

which implies $I - Y_i X_i$ is a nonsingular matrix. Thus, there exist nonsingular matrices G_i and R_i such that the (121) holds. Then, via similar method in (Gahinet, & Apkarian, 1994), let us define

$$P(h) = \widehat{\Pi}_2 \widehat{\Pi}_1^{-1} \tag{124}$$

$$\widehat{\Pi}_1 = \begin{bmatrix} X_i & I \\ G_i^T & 0 \end{bmatrix}, \quad \widehat{\Pi}_2 = \begin{bmatrix} I & Y_i \\ 0 & R_i^T \end{bmatrix} \tag{125}$$

Substituting (125) into (124) yields

$$P(h) = \begin{bmatrix} Y_i & R_i \\ R_i^T & Z_i \end{bmatrix} \tag{126}$$

where

$$Z_i = G_i^{-1} X_i \left(Y_i - X_i^{-1}\right) X_i G_i^{-T}, \quad Z_i - R_i^T Y_i R_i = R_i^T \left(X_i Y_i - I\right)^{-1} \left(Y_i - X_i^{-1}\right)\left(Y_i X_i - I\right)^{-1} R_i \tag{127}$$

$$Z_i > 0, \quad Z_i - R_i^T Y_i R_i > 0 \tag{128}$$

Consider (114) and (125), one has

$$\begin{bmatrix}
-\widehat{\Pi}_1^T P(h)\widehat{\Pi}_1 & * & * & * & * & * & * & * & * & * & * & * \\
0 & -Q(h)-\lambda\widehat{V}_1 & * & * & * & * & * & * & * & * & * & * \\
-\widehat{U}_2^T H\widehat{\Pi}_1 & 0 & -I & * & * & * & * & * & * & * & * & * \\
0 & -\lambda\widehat{V}_2 & 0 & -\lambda I & * & * & * & * & * & * & * & * \\
-\widehat{W}_2^T H\widehat{\Pi}_1 & 0 & 0 & 0 & -I & * & * & * & * & * & * & * \\
0 & 0 & 0 & 0 & 0 & -\gamma^2 I & * & * & * & * & * & * \\
\widehat{\Pi}_2^T \bar{A}_{ij}\widehat{\Pi}_1 & \widehat{\Pi}_2^T \bar{A}_{dij} & \widehat{\Pi}_2^T \bar{E}_i & \widehat{\Pi}_2^T \bar{E}_{di} & 0 & \widehat{\Pi}_2^T D_{ij} & -\widehat{\Pi}_2^T P^{-1}(h)\widehat{\Pi}_2 & * & * & * & * & * \\
\sigma\widehat{\Pi}_2^T \bar{C}_{ij}\widehat{\Pi}_1 & \sigma\widehat{\Pi}_2^T \bar{C}_{dij} & 0 & 0 & \sigma\widehat{\Pi}_2^T \bar{B}_{ki} & 0 & 0 & -\widehat{\Pi}_2^T P^{-1}(h)\widehat{\Pi}_2 & * & * & * & * \\
\bar{N}_{i1}\widehat{\Pi}_1 & N_{i2} & N_{i3} & N_{i4} & 0 & 0 & 0 & 0 & -\varepsilon I & * & * & * \\
\bar{L}_{ij}\widehat{\Pi}_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & * & * \\
H\widehat{\Pi}_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\hat{Q}^{-1}(h) & * \\
0 & 0 & 0 & 0 & 0 & 0 & \varepsilon\bar{M}_i^T\widehat{\Pi}_2 & 0 & 0 & 0 & 0 & -\varepsilon^{-1}I
\end{bmatrix} < 0 \quad (129)$$

Next, the congruence transformation matrices $\mathcal{M}_1$ and $\mathcal{M}_2$ are designed

$$\begin{cases} \mathcal{M}_1 = diag\{\widehat{\Pi}_1^{-T} \quad I \quad I \quad I \quad I \quad I \quad \widehat{\Pi}_2^{-T} \quad \widehat{\Pi}_2^{-T} \quad I \quad I \quad I \quad I\} \\ \mathcal{M}_2 = diag\{\widehat{\Pi}_1^{-1} \quad I \quad I \quad I \quad I \quad I \quad \widehat{\Pi}_2^{-1} \quad \widehat{\Pi}_2^{-1} \quad I \quad I \quad I \quad I\} \end{cases} \quad (130)$$

Taking the congruence transformation of (129) by $\mathcal{M}_1$ and $\mathcal{M}_2$, the inequality (87) holds. With above analysis, A_{ki}, B_{ki} and C_{ki} can be determined

$$A_{ki} = R_i^{-1}(\Omega - Y_i A_i X_i - \bar{\alpha}\Gamma C_i X_i + Y_i B_{1i}\Lambda)G_i^{-T}, \quad B_{ki} = R_i^{-1}\Gamma, \quad C_{ki} = \Lambda G_i^{-T} \quad (131)$$

Remark 6. From Theorem 3, it can be seen that the congruence transformation matrices $\mathcal{M}_1$ and $\mathcal{M}_2$ are employed to determine A_{ki}, B_{ki} and C_{ki}. However, it may be difficult to solve the nonconvex problem caused by fuzzy-basis-dependent LMI. Thus, **Corollary 1** is presented to convert the controller design problem into the nonlinear minimization constraints.

4.4. Cone complementarity linearization

Corollary 1. The nonlinear minimization constraints is described below

$$\begin{cases} min\left(tr\left(\sum_{i=1}^{r} X_i Y_i\right)\right) \\ s.t. \begin{cases} (a) \ \ the\ equalities\ (113) \\ (b) \ \ the\ inequality\ (114) \end{cases} \end{cases} \quad (132)$$

The cone complementarity linearization algorithm is designed.

Table 1. The cone complementarity linearization algorithm.

Start
Step 1: Set the system gain matrices A_i, B_{1i}, C_i and the Bernoulli probability distribution $\bar{\alpha}$. Go to Step 2.
Step 2: Select $\theta_1(k)$, $\theta_2(k)$, ..., $\theta_p(k)$ and design M_{ij} ($i=1, 2, \ldots, r$ and $j=1,2,\ldots,p$) for (2). Go to Step 3.
Step 3: Select $\theta_1(k)$, $\theta_2(k)$, ..., $\theta_p(k)$ and design $\widehat{M}_{ij}$ ($i=1, 2, \ldots, r$ and $j=1,2,\ldots,p$) for (20). Go to Step 4.
Step 4: Set γ for the closed-loop system (29). Go to Step 5.
Step 5: Solve LMIs (38), (113) and (114) to obtain the initial feasible solutions Λ^0, Ω^0, Γ^0, X_i^0 and Y_i^0, then set $\mathcal{N}=0$, where $\mathcal{N}$ is the iteration number. Go to Step 6.
Step 6: Solve LMIs (133) for the $\Lambda^{\mathcal{N}}$, $\Omega^{\mathcal{N}}$, $\Gamma^{\mathcal{N}}$, $X_i^{\mathcal{N}}$ and $Y_i^{\mathcal{N}}$ satisfying (132), set $\Lambda^{\mathcal{N}+1}=\Lambda$, $\Omega^{\mathcal{N}+1}=\Omega$, $\Gamma^{\mathcal{N}+1}=\Gamma$, $X_i^{\mathcal{N}+1}=X_i$ and $Y_i^{\mathcal{N}+1}=Y_i$. Go to Step 7.
Step 7: If (38), (113) and (114) are feasible for the $\Lambda^{\mathcal{N}}$, $\Omega^{\mathcal{N}}$, $\Gamma^{\mathcal{N}}$, $X_i^{\mathcal{N}}$ and $Y_i^{\mathcal{N}}$ that obtained in Step 6, go to Step 8. If (38), (113) and (114) are unfeasible for $\Lambda^{\mathcal{N}}$, $\Omega^{\mathcal{N}}$, $\Gamma^{\mathcal{N}}$, $X_i^{\mathcal{N}}$ and $Y_i^{\mathcal{N}}$ that obtained in Step 6, where $\mathcal{N}<\widehat{\mathcal{N}}$ and $\widehat{\mathcal{N}}$ is the maximum iteration number, set $\mathcal{N}=\mathcal{N}+1$ and return to Step 6.
Step 8: Output $\Lambda^{\mathcal{N}}$, $\Omega^{\mathcal{N}}$, $\Gamma^{\mathcal{N}}$, $X_i^{\mathcal{N}}$ and $Y_i^{\mathcal{N}}$, set $\Lambda^{\mathcal{N}}=\Lambda$, $\Omega^{\mathcal{N}}=\Omega$, $\Gamma^{\mathcal{N}}=\Gamma$, $X_i^{\mathcal{N}}=X_i$ and $Y_i^{\mathcal{N}}=Y_i$. Go to Step 9.
Step 9: Substitute A_i, B_{1i}, C_i, $\bar{\alpha}$, Λ, Ω, Γ, X_i and Y_i into (131), A_{ki}, B_{ki} and C_{ki} can be determined. Exit.

Remark 7. The controller gain matrices can be determined via cone complementarity linearization, and the nonconvex problem can be solved.

5. Simulation examples

5.1. Example 1

Consider a class of uncertain networked control systems

$$\begin{cases} x(k+1)=(A+\Delta A(k))x(k)+(A_d+\Delta A_d(k))x(k-d(k))+(E+\Delta E(k))f(x(k)) \\ \qquad +(E_d+\Delta E_d(k))f_d(x(k-d(k)))+B_1u(k)+D_1\omega(k) \\ y(k)=Cx(k)+C_dx(k-d(k))+\phi(Sx(k))+D_2\omega(k) \\ z(k)=Lx(k)+B_2u(k) \end{cases} \tag{133}$$

Applying T-S fuzzy model and stochastic Bernoulli theory, one has

$$\begin{cases} x(k+1)=(A_i+\Delta A_i(k))x(k)+(A_{di}+\Delta A_{di}(k))x(k-d(k))+(E_i+\Delta E_i(k))f(x(k)) \\ \qquad +(E_{di}+\Delta E_{di}(k))f_d(x(k-d(k)))+B_{1i}u(k)+D_{1i}\omega(k) \\ y(k)=\alpha(k)(C_ix(k)+C_{di}x(k-d(k))+\phi(S_ix(k))+D_{2i}\omega(k)) \\ z(k)=L_ix(k)+B_{2i}u(k) \end{cases} \tag{134}$$

A 2-rules T-S fuzzy model is employed and A_i, A_{di}, E_i, E_{di}, B_{1i}, D_{1i}, C_i, C_{di}, S_i, D_{2i}, L_i and B_{2i} ($i=1,2$) are given as follows

$$A_1=\begin{bmatrix}0.6 & 0\\ 1 & -0.1\end{bmatrix},\ A_2=\begin{bmatrix}0.3 & 0\\ 1 & -0.7\end{bmatrix}6,\ A_{d1}=\begin{bmatrix}0.6 & 0\\ -1 & 1\end{bmatrix},\ A_{d2}=\begin{bmatrix}0.3 & 0\\ -0.2 & 0.3\end{bmatrix},\ E_1=\begin{bmatrix}0.2\\ 0.6\end{bmatrix},\ E_2=\begin{bmatrix}0.7\\ 0.1\end{bmatrix},\ E_{d1}=\begin{bmatrix}0.2\\ -0.1\end{bmatrix},\ E_{d2}=\begin{bmatrix}0.1\\ -0.3\end{bmatrix} \tag{135}$$

$$B_{11}=-0.6,\ B_{12}=-0.2,\ D_{11}=\begin{bmatrix}0.8\\ 0.1\end{bmatrix},\ D_{12}=\begin{bmatrix}0.9\\ 0.7\end{bmatrix},\ C_1=\begin{bmatrix}0.2 & 0\\ 0.1 & -0.3\end{bmatrix},\ C_2=\begin{bmatrix}0 & 0.1\\ 0.1 & -0.2\end{bmatrix},\ C_{d1}=\begin{bmatrix}-0.6 & 0\\ 0 & 1.1\end{bmatrix},\ C_{d2}=\begin{bmatrix}-0.6 & 0\\ 0 & 1.3\end{bmatrix} \tag{136}$$

$$D_{21}=\begin{bmatrix}0.1\\ -0.1\end{bmatrix},\ D_{22}=\begin{bmatrix}0\\ -0.1\end{bmatrix},\ L_1=0.5,\ L_2=0.3,\ B_{21}=B_{22}=0.66,\ S_1=0.3,\ S_2=0.2 \tag{137}$$

For (134), the stochastic controller is designed

$$\begin{cases} \hat{x}(k+1)=A_{ki}\hat{x}(k-d(k))+B_{ki}y(k) \\ u(k)=\beta(k)C_{ki}\hat{x}(k) \end{cases} \tag{138}$$

Solve the LMIs, Λ, Ω, Γ, X_i and Y_i ($i=1,2$) are solved

$$\begin{cases} \Lambda=\begin{bmatrix}0.3171 & 0.0344\\ 0.9502 & 0.4387\end{bmatrix},\quad \Omega=\begin{bmatrix}0.3816 & 0.7952\\ 0.7655 & -0.1869\end{bmatrix},\quad \Gamma=\begin{bmatrix}-0.4898 & 0.6463\\ 0.4456 & 0.7094\end{bmatrix} \\ X_1=\begin{bmatrix}0.0357 & 0.9340\\ 0 & 0.6787\end{bmatrix},\quad X_2=\begin{bmatrix}0.7577 & 0.3922\\ 0.7431 & 0.6555\end{bmatrix},\quad Y_1=\begin{bmatrix}0.2171 & 0.8130\\ 0.0607 & 0.9672\end{bmatrix},\quad Y_2=\begin{bmatrix}0.0640 & 0.6328\\ 0.1790 & 0.8496\end{bmatrix} \end{cases} \tag{139}$$

Using the stability conditions, A_{ki}, B_{ki} and C_{ki} ($i=1,2$) are solved

$$\begin{cases} A_{k1}=\begin{bmatrix}-33.5128 & 32.8327\\ -1.2769 & 1.0836\end{bmatrix},\quad A_{k2}=\begin{bmatrix}-35.1551 & -31.6897\\ 28.8847 & 26.2382\end{bmatrix},\quad B_{k1}=\begin{bmatrix}1.4061 & 1.3051\\ -0.9050 & 1.1942\end{bmatrix} \\ B_{k2}=\begin{bmatrix}2.6355 & -2.1794\\ -2.1779 & 0.8737\end{bmatrix},\quad C_{k1}=\begin{bmatrix}7.8744 & -7.9182\\ 6.1474 & -5.7300\end{bmatrix},\quad C_{k2}=\begin{bmatrix}11.9478 & 10.4816\\ 1.8743 & 1.8885\end{bmatrix} \end{cases} \tag{140}$$

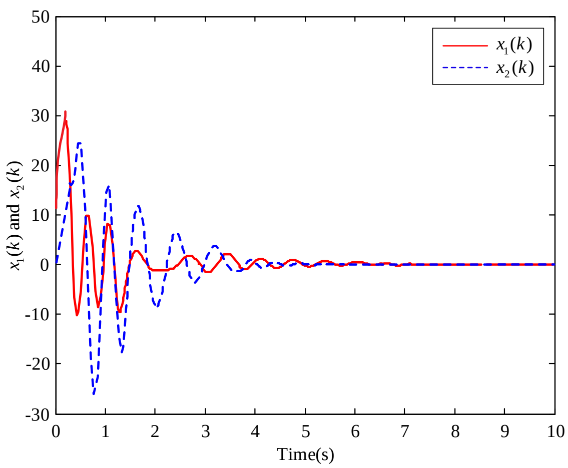

Figure 1. Responses of $x_1(k)$ and $x_2(k)$.

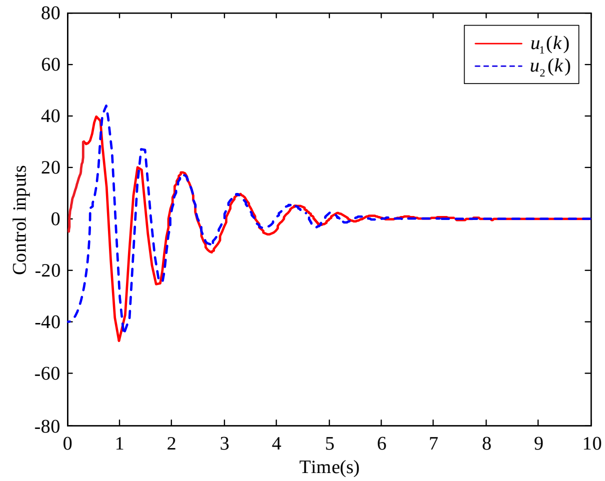

Figure 2. Responses of control inputs.

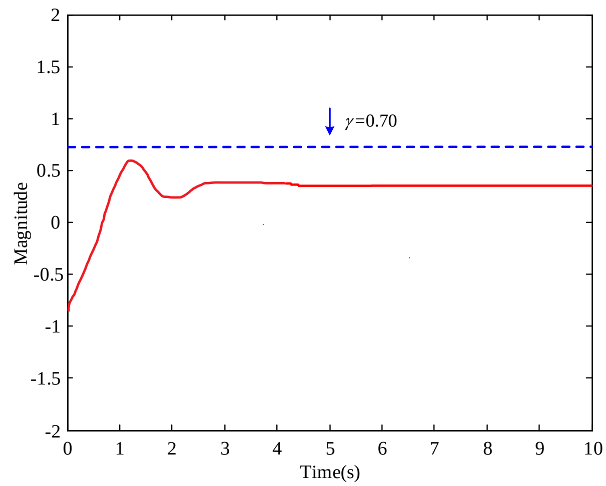

Figure 3. Response of $\sqrt{\mathbb{E}\left\{\sum_{k=0}^{\infty} z^T(k)z(k)\right\} \Big/ \mathbb{E}\left\{\sum_{k=0}^{\infty} \omega^T(k)\omega(k)\right\}}$ with $\omega(k)=0.98k^2$ $(-1 \le k \le 3)$.

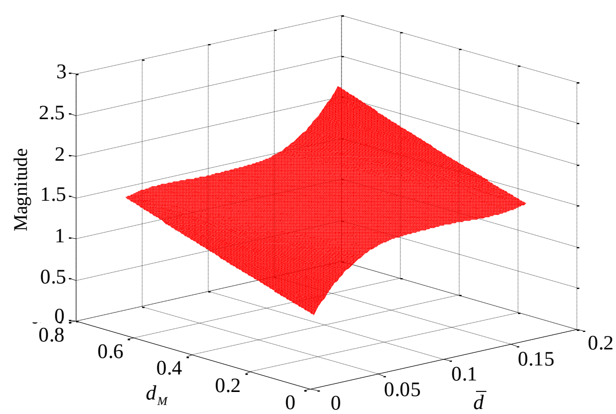

Figure 4. Response of $\sqrt{\mathbb{E}\left\{\sum_{k=0}^{\infty} z^T(k)z(k)\right\} \Big/ \mathbb{E}\left\{\sum_{k=0}^{\infty} \omega^T(k)\omega(k)\right\}}$ for different d_M and $\bar{d}$.

The sector nonlinearities are given as follows

$$\begin{cases} f\left(x(k)\right) = x^2(k)+1 \\ f_d\left(x\left(k-d(k)\right)\right) = x^2\left(k-d(k)\right) \\ \phi\left(S\left(x(k)\right)\right) = tanh\left(x(k)\right) - 0.18 \end{cases} \tag{141}$$

The responses of $x_1(k)$ and $x_2(k)$ are shown in **Figure 1**. The responses of control inputs are shown in **Figure 2**. The response of $\sqrt{\mathbb{E}\left\{\sum_{k=0}^{\infty} z^T(k) z(k)\right\} / \mathbb{E}\left\{\sum_{k=0}^{\infty} \omega^T(k) \omega(k)\right\}}$ is shown in **Figure 3**. The response of $\sqrt{\mathbb{E}\left\{\sum_{k=0}^{\infty} z^T(k) z(k)\right\} / \mathbb{E}\left\{\sum_{k=0}^{\infty} \omega^T(k) \omega(k)\right\}}$ for different d_M and $\bar{d}$ is shown in **Figure 4**. From **Figure 1**, it can be seen that the closed-loop system is exponentially mean-square stable. From **Figure 2**, it can be seen that the control inputs are bounded. From **Figure 3**, it can be seen that the response of $\sqrt{\mathbb{E}\left\{\sum_{k=0}^{\infty} z^T(k) z(k)\right\} / \mathbb{E}\left\{\sum_{k=0}^{\infty} \omega^T(k) \omega(k)\right\}}$ is smaller than $\gamma = 0.70$.

Remark 8. In **Figure 4**, d_M and $\bar{d}$ can affect the response of $\sqrt{\mathbb{E}\left\{\sum_{k=0}^{\infty} z^T(k) z(k)\right\} / \mathbb{E}\left\{\sum_{k=0}^{\infty} \omega^T(k) \omega(k)\right\}}$, which implies d_M and $\bar{d}$ can affect H-infinity performance.

5.2. Example 2

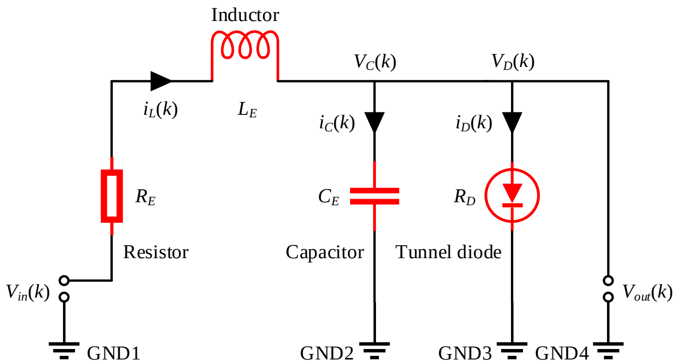

Figure 5. Schematic diagram of tunnel diode circuit system.

Consider a class of tunnel diode circuit systems with networked control (Yu, Sun, & Li, 2018)

$$\begin{cases} \Delta V_C(k) = \frac{1}{C_E} i_L(k) - \frac{1}{C_E} i_D(k) \\ \Delta i_L(k) = -\frac{1}{L_E} V_C(k) - \frac{R_E}{L_E} i_L(k) + \frac{1}{L_E} V_{in}(k) \\ i_D(k) = \frac{V_D(k)}{R_D} \end{cases} \tag{142}$$

where R_E is the resistance, L_E is the inductor, C_E is the capacitor, and R_D is the equivalent resistance of tunnel diode. $i_L(k)$, $i_C(k)$ and $i_D(k)$ are the currents in the inductor, capacitor and tunnel diode, respectively. $V_{out}(k)$ and $V_{in}(k)$ are the measured output and control input of tunnel diode circuit system, respectively. Let us define

$$x_1(k) = V_C(k), \quad x_1(k) \in \left[-\sqrt{m_1}, \sqrt{m_1}\right], \quad x_2(k) = i_L(k), \quad x_3(k) = i_D(k), \quad y(k) = V_{out}(k), \quad z(k) = i_L(k), \quad u(k) = V_{in}(k) \tag{143}$$

Substituting (143) into (142), one has

$$\begin{cases} \Delta x_1(k) = \frac{1}{C_E} x_2(k) - \frac{1}{C_E} x_3(k) \\ \Delta x_2(k) = -\frac{1}{L_E} x_1(k) - \frac{R_E}{L_E} x_2(k) + \frac{1}{L_E} u(k) \\ \Delta x_3(k) = \frac{1}{R_D C_E} x_2(k) - \frac{1}{R_D C_E} x_3(k) \\ y(k) = x_1(k), \qquad z(k) = x_2(k) \end{cases} \tag{144}$$

then (144) is transformed as

$$\begin{cases} \Delta x(k) = Ax(k) + B_1 u(k) \\ y(k) = Cx(k) \\ z(k) = Lx(k) \end{cases} \tag{145}$$

Consider the uncertainties, sector nonlinearities, time-varying delay and unmatched disturbance in (145), one has

$$\begin{cases} x(k+1) = \left(A+\Delta A(k)\right)x(k) + \left(A_d + \Delta A_d(k)\right)x\left(k-d(k)\right) + \left(E+\Delta E(k)\right)f\left(x(k)\right) \\ \qquad\qquad + \left(E_d + \Delta E_d(k)\right)f_d\left(x\left(k-d(k)\right)\right) + B_1 u(k) + D_1\omega(k) \\ y(k) = Cx(k) + C_d x\left(k-d(k)\right) + \phi\left(Sx(k)\right) + D_2\omega(k) \\ z(k) = Lx(k) + B_2 u(k) \end{cases} \tag{146}$$

Applying T-S fuzzy model and stochastic Bernoulli theory, one has

$$\begin{cases} x(k+1) = \left(A_i+\Delta A_i(k)\right)x(k) + \left(A_{di} + \Delta A_{di}(k)\right)x\left(k-d(k)\right) + \left(E_i+\Delta E_i(k)\right)f\left(x(k)\right) \\ \qquad\qquad + \left(E_{di} + \Delta E_{di}(k)\right)f_d\left(x\left(k-d(k)\right)\right) + B_{1i} u(k) + D_{1i}\omega(k) \\ y(k) = \alpha(k)\left(C_i x(k) + C_{di} x\left(k-d(k)\right) + \phi\left(S_i x(k)\right) + D_{2i}\omega(k)\right) \\ z(k) = L_i x(k) + B_{2i} u(k) \end{cases} \tag{147}$$

A_i, A_{di}, E_i, E_{di}, B_{1i}, D_{1i}, C_i, C_{di}, S_i, D_{2i}, L_i and B_{2i} ($i=1,2$) are given as follows

$$\begin{gathered} A_1 = \begin{bmatrix} 0.8147 & 0.9134 & 0.2785 \\ 0.9058 & 0.6324 & 0.5469 \\ 0.1270 & 0.0975 & 0.9575 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.6948 & 0.0344 & 0.7655 \\ 0.3171 & 0.4387 & 0.7952 \\ 0.9502 & 0.3816 & 0.1869 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} 0.4898 & 0.7094 & 0.6797 \\ 0.4456 & 0.7547 & 0.6551 \\ 0.6463 & 0.2760 & 0.1626 \end{bmatrix} \\ A_{d2} = \begin{bmatrix} 0.1190 & 0.3404 & 0.7513 \\ 0.4984 & 0.5853 & 0.2551 \\ 0.9597 & 0.2238 & 0.5060 \end{bmatrix}, \quad E_1 = \begin{bmatrix} -0.6991 \\ 0.8909 \\ 0.9893 \end{bmatrix}, \quad E_2 = \begin{bmatrix} -0.5472 \\ 0.1386 \\ 0.1493 \end{bmatrix}, \quad E_{d1} = \begin{bmatrix} 0.2575 \\ 0.8407 \\ 0.1493 \end{bmatrix}, \quad E_{d2} = \begin{bmatrix} 0.8143 \\ 0.2436 \\ 0.9293 \end{bmatrix} \end{gathered} \tag{148}$$

$$B_{11} = -0.9308, \quad B_{12} = -1.5856, \quad D_{11} = \begin{bmatrix} 0.3500 \\ 0.1966 \\ -0.2511 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 0.6160 \\ 0.4733 \\ -0.3517 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0.0759 & 0.7792 & 0.5688 \\ 0.0540 & 0.9340 & 0.4694 \\ 0.5308 & 0.1299 & 0.0119 \end{bmatrix} \tag{149}$$

$$C_2 = \begin{bmatrix} 0.3371 & 0.3112 & 0.6020 \\ 0.1622 & 0.5285 & 0.2630 \\ 0.7943 & 0.1656 & 0.6541 \end{bmatrix}, \quad C_{d1} = \begin{bmatrix} 0.6892 & 0.0838 & 0.1524 \\ -0.7482 & 0.2290 & 0.8258 \\ 0.4505 & 0.9133 & 0.5382 \end{bmatrix}, \quad C_{d2} = \begin{bmatrix} 0.9961 & 0.1067 & 0.7749 \\ -0.0782 & 0.9619 & 0.8173 \\ 0.4427 & 0.0046 & 0.8687 \end{bmatrix} \tag{150}$$

$$D_{21} = \begin{bmatrix} 0.1818 \\ 0.1616 \\ 0.9999 \end{bmatrix}, \quad D_{22} = \begin{bmatrix} 0.1361 \\ 0.5693 \\ 0.5797 \end{bmatrix}, \quad L_1 = -0.5499, \quad L_2 = -0.1450, \quad S_1 = 0.0844, \quad S_2 = 0.8001, \quad B_{21} = 0.8530, \quad B_{22} = 0.6222 \tag{151}$$

For (148), the stochastic controller is designed

$$\begin{cases} \hat{x}(k+1) = A_{ki}\hat{x}\left(k-d(k)\right) + B_{ki}y(k) \\ u(k) = \beta(k)C_{ki}\hat{x}(k) \end{cases} \tag{152}$$

Solving the LMIs, Λ, Ω, Γ, X_i and Y_i ($i=1,2$) are solved

$$\begin{cases} \Lambda = \begin{bmatrix} 0.2769 & 0.8235 & -0.9502 \\ 0.0462 & 0.6948 & -0.0344 \\ 0.0971 & 0.3171 & -0.4387 \end{bmatrix}, \quad \Omega = \begin{bmatrix} 0.3816 & -0.1869 & 0.6463 \\ 0.7655 & -0.4898 & 0.7094 \\ 0.7952 & -0.4456 & 0.7547 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} -0.2760 & 0.1626 & 0.9597 \\ 0.6797 & -0.1190 & 0.3404 \\ 0.6551 & 0.4984 & -0.5853 \end{bmatrix} \\ X_1 = \begin{bmatrix} 0.2238 & 0.5060 & 0.9593 \\ 0.7513 & 0.6991 & 0.5472 \\ 0.2551 & 0.8909 & 0.1386 \end{bmatrix}, \quad X_2 = \begin{bmatrix} 0.1493 & 0.2543 & 0.9293 \\ 0.2575 & 0.8143 & 0.3500 \\ 0.8407 & 0.2436 & 0.1966 \end{bmatrix} \\ Y_1 = \begin{bmatrix} 0.0511 & 0.3417 & 0.5497 \\ 0.1600 & 0.8308 & 0.9172 \\ 0.4733 & 0.5853 & 0.2558 \end{bmatrix}, \quad Y_2 = \begin{bmatrix} 0.7572 & 0.5678 & 0.5308 \\ 0.7537 & 0.0759 & 0.7792 \\ 0.3804 & 0.0540 & 0.9340 \end{bmatrix} \end{cases} \tag{153}$$

Using the stability conditions, A_{ki}, B_{ki} and C_{ki} ($i=1,2$) are solved as follows

$$\begin{cases} A_{k1} = \begin{bmatrix} 1.7575 & 2.2124 & -2.8405 \\ 9.2384 & 3.6696 & 4.0739 \\ 2.2841 & 1.1832 & 0.5288 \end{bmatrix}, & A_{k2} = \begin{bmatrix} -1.2843 & -12.4649 & -8.9775 \\ -6.0799 & 8.0942 & 10.4646 \\ -0.4077 & -18.2948 & -14.1395 \end{bmatrix}, & B_{k1} = \begin{bmatrix} 3.3175 & 3.3307 & -8.4658 \\ -2.7984 & -5.7386 & 13.0819 \\ -0.8481 & -0.2540 & 2.3967 \end{bmatrix} \\ B_{k2} = \begin{bmatrix} -1.0849 & -2.1917 & 4.9744 \\ 2.3801 & 1.9429 & -4.7213 \\ -3.6696 & -1.6544 & 7.1568 \end{bmatrix}, & C_{k1} = \begin{bmatrix} 0.6870 & 0.0673 & 0.1185 \\ -0.8144 & -0.4770 & -0.1540 \\ 0.3350 & 0.0521 & 0.0302 \end{bmatrix}, & C_{k2} = \begin{bmatrix} 2.7123 & 0.3729 & -1.2584 \\ 1.2019 & 0.3383 & -0.6030 \\ 1.1672 & 0.0880 & -0.5898 \end{bmatrix} \end{cases} \tag{154}$$

The response of $\alpha(k)$ with $\bar{\alpha} = 0.95$ is shown in **Figure 6**. The response of $\beta(k)$ with $\bar{\beta} = 0.90$ is shown in **Figure 7**. The 3-dimentional response of $x_1(k)$, $x_2(k)$ and $x_3(k)$ is shown in **Figure 8**. In **Figures 6-8**, it can be seen that the closed-loop system is exponentially mean-square stable. The data comparison results of γ with $\bar{d} = 0.3000$ and for $\bar{d} = 0.2000$ different d_M are shown in **Tables 2-3**, respectively. In **Tables 2-3**, it can be seen that the smaller γ can be obtained as $\bar{d}$ gets smaller. The data comparison results corresponding to **Table 3** is shown in **Figure 9**. In **Table 3** and **Figure 9**, it can be seen that the smaller lower bounds γ are obtained by employing **Theorem 3** than (Zheng, Wang, Wang, Wen, & Zhang, 2018) and (Zheng, Zhang, Wang, Wen, & Wang, 2020). The data comparison results of d_M with $\bar{d} = 0.3000$ and $\bar{d} = 0.2000$ for different γ are shown in **Tables 4-5**, respectively. In **Tables 4-5**, it can be seen that the larger d_M can be obtained as $\bar{d}$ gets smaller. The data comparison results corresponding to **Table 5** is shown in **Figure 10**. In **Table 5** and **Figure 10**, it can be seen that the larger d_M are obtained by employing **Theorem 3** than (Zheng, Wang, Wang, Wen, & Zhang, 2018) and (Zheng, Zhang, Wang, Wen, & Wang, 2020).

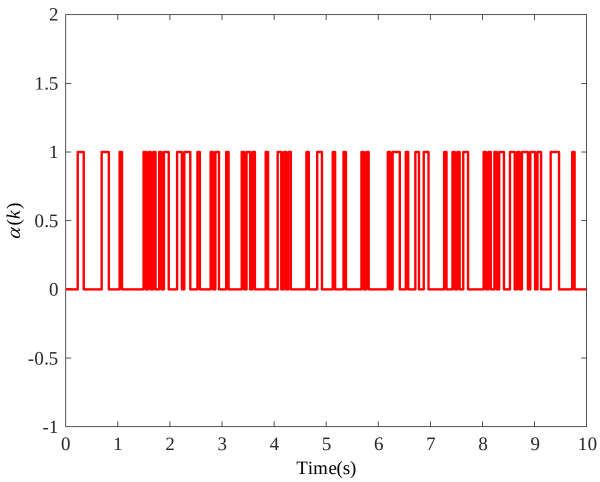

Figure 6. Response of $\alpha(k)$ with $\bar{\alpha} = 0.95$.

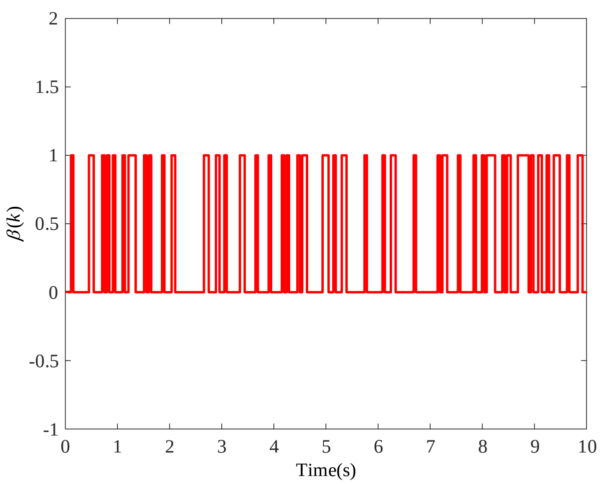

Figure 7. Response of $\beta(k)$ with $\bar{\beta} = 0.90$.

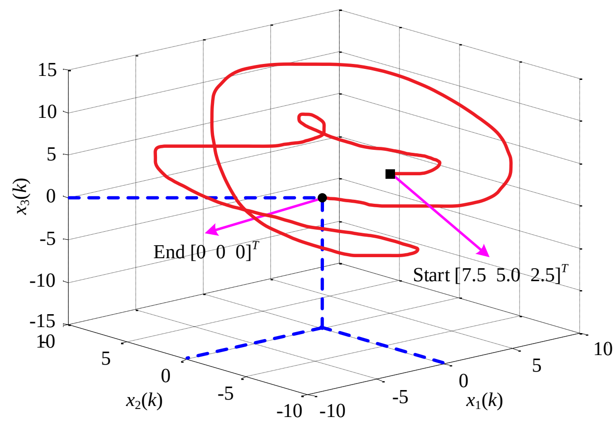

Figure 8. 3-dimentional response of $x_1(k)$, $x_2(k)$ and $x_3(k)$.

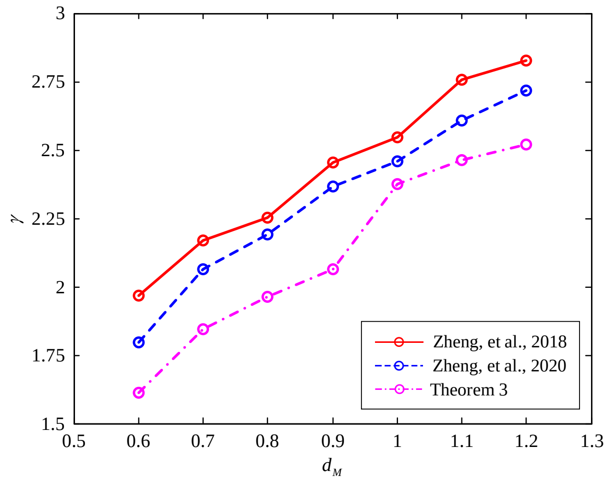

Figure 9. Data comparison results corresponding to Table 3.

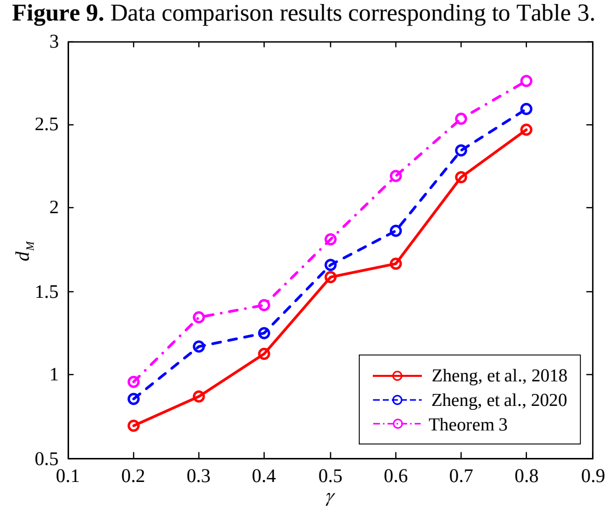

Figure 10. Data comparison results corresponding to Table 5.

Table 2. Data comparison results of lower bounds γ with $\bar{d} = 0.3000$ for different d_M.

Method	d_M						
	0.6000	0.7000	0.8000	0.9000	1.0000	1.1000	1.2000
Zheng, et al., 2018	2.1498	2.3614	2.5462	2.6299	2.7295	2.9688	3.0251
Zheng, et al., 2020	1.9386	2.1646	2.3727	2.4498	2.6286	2.7860	2.9443
Theorem 3	1.7778	1.9453	2.1674	2.2977	2.4738	2.5009	2.7735

Table 3. Data comparison results of lower bounds γ with $\bar{d} = 0.2000$ for different d_M.

Method	d_M						
	0.6000	0.7000	0.8000	0.9000	1.0000	1.1000	1.2000
Zheng, et al., 2018	1.9661	2.1684	2.2520	2.4551	2.5476	2.7552	2.8278
Zheng, et al., 2020	1.7943	2.0646	2.1911	2.3660	2.4605	2.6083	2.7173
Theorem 3	1.6102	1.8446	1.9644	2.0624	2.3744	2.4636	2.5186

Table 4. Data comparison results of upper bounds d_M with $\bar{d} = 0.3000$ for different γ.

Method	γ						
	0.2000	0.3000	0.4000	0.5000	0.6000	0.7000	0.8000
Zheng, et al., 2018	0.4806	0.6421	0.9941	1.3020	1.4268	1.8717	2.2213
Zheng, et al., 2020	0.6348	0.9020	1.1769	1.4331	1.6545	2.1199	2.4406
Theorem 3	0.7322	1.0021	1.2628	1.6376	1.9146	2.3742	2.6177

Table 5. Data comparison results of upper bounds d_M with $\bar{d} = 0.2000$ for different γ.

Method	γ						
	0.2000	0.3000	0.4000	0.5000	0.6000	0.7000	0.8000
Zheng, et al., 2018	0.6942	0.8682	1.1232	1.5822	1.6661	2.1881	2.4711
Zheng, et al., 2020	0.8527	1.1661	1.2524	1.6606	1.8626	2.3430	2.5932
Theorem 3	0.9602	1.3475	1.4182	1.8107	2.1902	2.5388	2.7601

6. Conclusions

In this paper, the stochastic fuzzy delay-dependent dynamic output feedback control is proposed for the uncertain networked control system. The T-S fuzzy model is employed, and system plant can be deacribed effectively. The closed-loop system is exponentially mean-square stable by designing stochastic T-S fuzzy dynamic output controller. Based on the time delay information and fuzzy-basis-dependent Lyapunov functional, the delay-dependent stability conditions can be obtained. The H-infinity performance function is constructed, and the H-infinity performance can be guaranteed. The congruence transformation method is employed and the controller gain matrices can be determined. Usually, the wireless and wire communication networks are used to transmit the data in the networked control system. Hence, the system control performance is easy to suffer from the hacker attacks. Once the attack is successful, it may reduce the system control performance, destabilize the system or even cause the system to crash. Hence, it is necessary to design the active defense algorithm for the hacker attacks in the future. Moreover, the false data injection attacks often exist in the communication channels of networked control system, the necessary and sufficient conditions for the insecurity will be investigate in the future.

Acknowledgments

The authors are grateful to the editor and the anonymous reviewers for their valuable comments and suggestions that have helped improve the presentation of the paper. This paper was financially supported by National Natural Science Foundation of China (91848206); Natural Science Foundation of Hebei Province (F2021203083, F2021203104); and Natural Science Foundation for High Education College Science and Technology Plan (Grant No. QN2021138).

Disclosure statements

The authors declare that there is no conflict of interest regarding the publication of this paper.

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