DOI:
10.1109/TFUZZ.2022.3156701

Citation for published version (APA):
Ran, G., Li, C., Lam, H-K., Li, D., & Chen, H. (2022). Fuzzy-Model-Based Asynchronous Fault Detection for Markov Jump Systems with Partially Unknown Transition Probabilities: An Adaptive Event-Triggered Approach. *IEEE Transactions on Fuzzy Systems*, *30*(11), 4679-4689. https://doi.org/10.1109/TFUZZ.2022.3156701

Fuzzy-Model-Based Asynchronous Fault Detection for Markov Jump Systems with Partially Unknown Transition Probabilities: An Adaptive Event-Triggered Approach

Guangtao Ran, Jian Liu, *Member, IEEE*, Chuanjiang Li, Hak-Keung Lam, *Fellow, IEEE*, Dongyu Li, *Member, IEEE*, and Hongtian Chen, *Member, IEEE*

***Abstract*—This paper addresses the event-triggered asynchronous fault detection (FD) problem of fuzzy-model-based nonlinear Markov jump systems (MJSs) with partially unknown transition probabilities. For this objective, the nonlinear plant is modeled as an interval type-2 (IT2) fuzzy MJSs with the aid of the IT2 fuzzy sets capturing the uncertainties of the membership functions. An adaptive event-triggered scheme (ETS) is introduced to bring down the costs of the communication network from the system to the fuzzy fault detection filter (FDF), in which the triggering parameter can be adaptive tuned with the system dynamics. A hidden Markov model (HMM) is employed to characterize the asynchronous phenomenon between the system and FDF. Unlike the existing results, the transition probabilities of the plant and the FDF are allowed to be partially known. By using the Lyapunov and the membership-function-dependent (MFD) methods, the existence conditions of the FDF are derived. Finally, the proposed FD methods are verified by a numerical simulation.**

***Index Terms*—Markov jump systems, fault detection, adaptive event-triggered scheme, partially unknown transition probabilities.**

I. Introduction

OVER the past decades, Takagi-Sugeno (T-S) fuzzy systems have obtained remarkable attention due to their widespread applications [1], and a type-1 T-S model is has been proved that be a powerful tool to deal with the problems in nonlinear systems. Some works can be found in [2], [3]. Nevertheless, the type-1 fuzzy sets do not work effectively while the uncertainties are considered in nonlinear systems. Fortunately, Lam *et al.* [4] have proposed an interval type-2 (IT2) fuzzy modeling method to address the uncertainties problem. Since then, some outspread works on filtering, model reduction, and control algorithms have been reported in [5]–[9]. In [5], the imperfect premise matching problem was investigated for IT2 fuzzy systems, where the controller and the plant do not require the identical premise variable information. The model reduction problem was considered in [7]. In [8], an adaptive filter was developed to address the signal processing problems. An output feedback controller was designed for IT2 fuzzy systems, in which time-varying delays and external disturbance are considered [9].

Moreover, in many physical systems, some stochastic phenomena are inevitable, such as abrupt environmental disturbance, irregular component breakdowns, etc. Markov jump model is developed to catch these physical phenomena to facilitate the theoretical research for physical systems. Some results on the MJSs can be seen in [10]–[13]. In [10], an asynchronous dissipative controller was designed for MJSs. In [11], a new quantized feedback control algorithm was proposed for MJSs. In [12], a dissipative controller was designed for the IT2 nonhomogeneous MJSs. Additionally, an anti-disturbance control method for singular MJSs was proposed in [13].

In practical systems, faults are common obstacles that hinder the achievement of better performance. Hence, fault detection (FD) problems in MJSs have been considered widely [14]–[19]. In [14], the reliable dissipative control problem was studied for IT2 fuzzy MJSs subject to actuator faults. In [15], an asynchronous fault detection filter (FDF) was designed for two-dimensional MJSs, and the corresponding results of continuous-time fuzzy semi-MJSs were given in [16]. In [17], a sliding-mode control approach was presented for MJSs with actuator faults. In [18], a comprehensive control algorithm was proposed by combining a fault compensation method and a sliding-mode control approach. In [19], a type-1 fuzzy FD method was proposed for MJSs. In the studies mentioned above, all signals needed periodical transmitting through a communication network. Since the communication network resources are limited, an event-triggered scheme (ETS) was designed to eliminate the unnecessary waste of resources and save energy in [20]–[25]. Moreover, some works on event-triggered control for MJSs were reported in [26]–[31]. The event-triggered H_∞ and asynchronous extended passive control problems were addressed in [26]. In [27], an event-triggered sliding mode controller was designed for discrete-

This work was supported by the China Scholarship Council under Grant 202006120100 and the National Natural Science Foundation of China under Grant 61876050. (*Corresponding author: Chuanjiang Li.*)

Guangtao Ran and Chuanjiang Li are with the Department of Control Science and Engineering, Harbin Institute of Technology, Harbin 150001, China. Email: ranguangtao@hit.edu.cn (G. Ran); lichuan@hit.edu.cn (C. Li).

Jian Liu is with the School of Automation, Southeast University, Nanjing 210096, China. Email: bkliujian@163.com (J. Liu).

Hak-Keung Lam is with Department of Engineering, King's College London, Strand, London, WC2R 2LS, U.K. Email: hak-keung.lam@kcl.ac.uk (H. K. Lam).

Dongyu Li is with the School of Cyber Science and Technology, Beihang University, Beijing 100191, China. Email: dongyuli@buaa.edu.cn (D. Li).

Hongtian Chen is with the Department of Chemical and Materials Engineering, University of Alberta, Edmonton, AB T6G 1H9, Canada. Email: chtbaylor@163.com (H. Chen).

time MJSs. Lu *et al.* [28] have designed a mixed-triggered scheme, which is applied to IT2 fuzzy nonlinear MJSs. Ran *et al.* [29] have focused on the dissipative control problem of IT2 fuzzy MJSs with sensor saturation. Su *et al.* [30] have proposed a sliding mode control algorithm for MJSs based on ETS. Although the mentioned event-triggered mechanism can reduce the waste of network resources to a certain extent, its triggering parameter is a fixed value. It cannot be automatically adjusted as the system status changes, and it lacks flexibility. Fortunately, an adaptive ETS was developed [32]–[35], which increases the design flexibility by introducing an adaptive law to adjust triggering parameters. In [32], the exponential stabilization problem was addressed for a class of continuous-time T-S fuzzy systems based on aperiodic sampling. In [33], a new adaptive ETS was proposed for T-S fuzzy memristive neural networks. The adaptive event-triggered finite-time dissipative filtering problems were addressed for the IT2 fuzzy MJSs in [34].

Considering the FD with the ETS, there are also some results of the networked systems [36]–[40] and the MJSs [41], [42]. In [36], the FD problem was investigated for discrete-time networked systems based on ETS. In [37], an FD filter was designed for IT2 fuzzy networked systems via ETS. The repeated scalar nonlinearities and stochastic disturbance were considered for nonlinear switched systems in [38]. An FDF was designed for fuzzy stochastic systems via adaptive ETS, and the missing measurements were considered in [39]. The FD problem was studied for IT2 fuzzy systems with delays, external disturbances, and asynchronous premise variables in [40], but they did not consider jumping mode. In [41], the FD problem was investigated for semi-Markovian jump systems via an adaptive ETS, but they require that the modes of FDF are synchronous and the transition probabilities are completely known. An asynchronous FDF was developed for MJSs via an ETS in [42], where a hidden Markov model (HMM) is applied to handle asynchronous, but the information of the transition probability matrix (TPM) are assumed to be fully known. However, transition probabilities are hard to obtain in practical situations.

Therefore, it is important to consider network resource waste, uncertainty, and unknown transition probabilities in practical scenarios. Overall, few results have dealt with the event-triggered asynchronous FD problem of fuzzy-model-based nonlinear MJSs with partially unknown transition probabilities to the authors' best knowledge. Motivated by these existing studies, we investigate the asynchronous FD problem for IT2 fuzzy MJSs through an adaptive event-triggered mechanism. The main contributions of this paper are classified as follows.

1) Different from the existing studies [19], [42], we extend the type-1 fuzzy FD to IT2 fuzzy FD, which can handle the parameter uncertainties of the nonlinear MJSs.

2) Considering the difficulty of obtaining transition probabilities, the information of TPM for MJSs is considered to be incompletely known. Meanwhile, an HMM is constructed to represent the asynchronous phenomenon between the FDF and the original system, where the conditional probability matrix (CPM) is allowed to be partially unknown.

3) An adaptive ETS is designed to improve transmission efficiency and save communication resources while guaranteeing the FD performance for the IT2 fuzzy MJSs. The parameter of the adaptive ETS can dynamically tune instead of a fixed one in [29], [30], [42].

The rest of this paper is organized as follows: Sec. II introduces the IT2 fuzzy MJSs model, the adaptive ETS, FDF model, and the fault reference model for formulating the FD problem tackled in this paper. Sec. III presents the main results. Sec. IV furnishes an example that verifies the effectiveness of the proposed algorithms. Sec.V concludes the main work.

Table I shows the acronyms and abbreviations used in this paper. The notations are provided in Table II, which will use in this work.

TABLE I
ACRONYMS AND ABBREVIATIONS

Acronym	Long Title
CPM	Conditional probability matrix
ETS	Event-triggered scheme
FD	Fault detection
FDF	Fault detection filter
FDS	Fault detection system
HMM	Hidden Markov model
IT2	Interval type-2
LGM	Lower grade of membership
LKF	Lyapunov-Krasovskii function
LMF	Lower membership function
LMIs	Linear matrix inequalities
MFD	Membership-Function-Dependent
MJP	Markov jump process
MJSs	Markov jump systems
T-S	Takagi-Sugeno
TPM	Transition probability matrix
UGM	Upper grade of membership
UMF	Upper membership function
ZOH	Zero-order hold

TABLE II
NOTATIONS

Notation	Description
$\mathbb{R}^n$	n-dimensions Euclidean space
Q^T	Transpose of a matrix Q
Q^{-1}	Inverse of a matrix Q
I	Identity matrix
0	Zero matrix
$\mathrm{acot}(\cdot)$	Arccot function
$\mathrm{diag}\{Q, A\}$	Diagonal matrix of Q and A
$*$	Symmetric term in a matrix
$\Xi\{\cdot\}$	Mathematical expectation operator
$\lVert\cdot\rVert$	Euclidean norm
$\mathcal{L}_2\,[0,\infty)$	The space of square summable infinite vector sequences
$B_{\max}$	Maximum value of B
$B_{\min}$	Minimum value of B
$Q > 0 (\geq 0)$	Positive (semi-positive)-definite matrix
$Q < 0 (\leq 0)$	Negative (semi-negative)-definite matrix

II. Problem Statement

A. System Description

Consider the following discrete-time IT2 fuzzy MJSs:

Original System Rule i: IF $z_1(x(k))$ is T_{i1}, $z_2(x(k))$ is

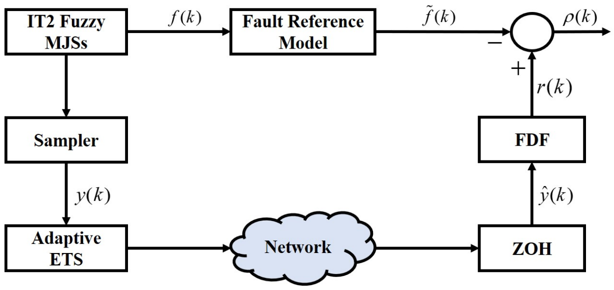

Fig. 1. The diagram of the IT2 fuzzy MJSs under the adaptive ETS.

T_{i2},. . ., and $z_p(x(k))$ is T_{ip}, THEN

$$\begin{cases} x(k+1) = A_{\sigma(k)i}(k)x(k) + B_{\sigma(k)i}(k)w(k) \\ \qquad\qquad + D_{\sigma(k)i}(k)f(k), \\ y(k) = E_{\sigma(k)i}(k)x(k) + H_{\sigma(k)i}(k)f(k), \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^{n_x}$, $w(k) \in \mathbb{R}^{n_w}$ and $f(k) \in \mathbb{R}^{n_f}$ stand for the state, the external disturbance that belongs to $\mathcal{L}_2[0,\infty)$, and the fault signal, respectively; $y(k) \in \mathbb{R}^{n_y}$ represents the measured output; $A_{\sigma(k)i}$, $B_{\sigma(k)i}$, $D_{\sigma(k)i}$, $E_{\sigma(k)i}$, and $H_{\sigma(k)i}$ are system matrices with appropriate dimensions; $T_{i\tilde{p}}$ denotes an IT2 fuzzy set of rule i corresponding to the function $z_{\tilde{p}}(x(k))$, $\tilde{p} = 1, \ldots, p$, $i \in \mathcal{U}_1 = \{1, 2, \ldots, l\}$; Both p and l are positive integers; The variable $\{\sigma(k)\ (k > 0)\}$ is used to represent a discrete-time Markov jump process (MJP) taking value in an integer set $O_1 = \{1, 2, \ldots, o_1\}$. Moreover, the corresponding TPM is described as $\Psi_1 = \{\lambda_{ab}\}$ with

$$\text{Prob}\{\sigma(k+1) = b\,|\sigma(k) = a\} = \lambda_{ab},\ a, b \in O_1, \tag{2}$$

where $\lambda_{ab} \geq 0$ and $\sum_{b=1}^{o_1} \lambda_{ab} = 1$. Assume that the transition probabilities of MJP are not entirely available. That means the elements of the TPM Ψ_1 are partially known. The following example is given with three operation modes

$$\Phi_1 = \begin{bmatrix} ? & \lambda_{12} & \lambda_{13} \\ ? & \lambda_{22} & ? \\ \lambda_{31} & ? & ? \end{bmatrix}, \tag{3}$$

where "?" is applied to represent the unknown items in the TPM Ψ_1. To facilitate subsequent analysis, define that $O_1 = O_1^k + O_1^{uk}$ with

$$\begin{cases} O_1^k \triangleq \{b : \lambda_{ab} \text{ is known}\}, \\ O_1^{uk} \triangleq \{b : \lambda_{ab} \text{ is unknown}\}, \end{cases} \tag{4}$$

where $a, b \in O_1$. Furthermore, the firing strength corresponding to the ith rule can be described as the following interval sets:

$$W(x(k)) = \left[\chi_i^L(x(k)), \chi_i^U(x(k))\right],\ i \in \mathcal{U}_1, \tag{5}$$

where

$$\chi_i^L(x(k)) = \prod\nolimits_{\tilde{p}=1}^{p} \eta_{T_{i\tilde{p}}}^L(z_{\tilde{p}}(x(k))) \geq 0,$$
$$\chi_i^U(x(k)) = \prod\nolimits_{\tilde{p}=1}^{p} \eta_{T_{i\tilde{p}}}^U(z_{\tilde{p}}(x(k))) \geq 0,$$
$$\eta_{T_{i\tilde{p}}}^U(z_{\tilde{p}}(x(k))) \geq \eta_{T_{i\tilde{p}}}^L(z_{\tilde{p}}(x(k))) \geq 0,\ i \in \mathcal{U}_1,$$

in which $\chi_i^L(x(k))$, $\chi_i^U(x(k))$, $\eta_{T_{i\tilde{p}}}^L(z_{\tilde{p}}(x(k)))$, and $\eta_{T_{i\tilde{p}}}^U(z_{\tilde{p}}(x(k)))$ stand for the lower grade of membership (LGM), the upper grade of membership (UGM), the lower membership function (LMF), and the upper membership function (UMF), respectively. Let $\sigma(t) = a$. Afterwards, by using approach in [4], [5], it can be inferred that

$$\begin{cases} x(k+1) = A_{am}(k)x(k) + B_{am}(k)w(k) + D_{am}(k)f(k), \\ y(k) = E_{am}(k)x(k) + H_{am}(k)f(k), \end{cases} \tag{6}$$

where

$$A_{am}(k) = \sum_{i=1}^{l} \chi_i(x(k))A_{ai},\ B_{am}(k) = \sum_{i=1}^{l} \chi_i(x(k))B_{ai},$$
$$D_{am}(k) = \sum_{i=1}^{l} \chi_i(x(k))D_{ai},\ E_{am}(k) = \sum_{i=1}^{l} \chi_i(x(k))E_{ai},$$
$$H_{am}(k) = \sum_{i=1}^{l} \chi_i(x(k))H_{ai},\ \sum_{i=1}^{l} \chi_i(x(k)) = 1,$$
$$\chi_i(x(k)) = \psi_i^L(x(k))\chi_i^L(x(k)) + \psi_i^U(x(k))\chi_i^U(x(k)) \geq 0,$$
$$\psi_i^L(x(k)) \geq 0,\ \psi_i^U(x(k)) \geq 0,\ \psi_i^L(x(k)) + \psi_i^U(x(k)) = 1,$$

in which $\psi_i^L(x(k))$ and $\psi_i^L(x(k))$ are nonlinear functions.

Remark 1: Unlike the existing results, the constructed system model (6) contains parameter uncertainties by using the IT2 fuzzy sets. In addition, the information of the TPM requires completely known in [19], [42], but this paper considers the case where the information of the TPM is partially unknown, which is more practical.

B. Adaptive ETS

As shown in Fig. 1, an adaptive ETS is employed to judge whether the sampling data need to be sent, which can reduce the consumption of network resources. Therefore, the current sampling data will be sent through the communication network if the following adaptive ETS condition is satisfied

$$e^T(k)G_a e(k) > \varepsilon(k)y^T(k)G_a y(k), \tag{7}$$

where $e(k) = y(k) - y(k_s)$ denotes the difference between the current sampling data $y(k)$ and the latest transmitted data $y(k_s)$; G_a $(a \in O_1)$ are positive matrices to be designed. In addition, $\varepsilon(k)$ is tuned by the following adaptive law

$$\varepsilon(k) = \varepsilon_1 + (\varepsilon_2 - \varepsilon_1)\frac{2}{\pi}\text{acot}\left(\kappa \|e(k)\|^2\right), \tag{8}$$

where ε_1 and ε_2 are prescribed parameters with $0 < \varepsilon_1 \leq \varepsilon_2 < 1$; $\text{acot}(\cdot)$ denotes arccot function; κ is a positive scalar to adjust the sensitivity of the function $\|e(k)\|^2$.

Assumption 1: The sampler is clock-driven.

Remark 2: It is worth noting that if $\varepsilon(k)$ becomes constant that belongs to $(0, 1)$, the adaptive ETS (7) will degrade to static ETS (i.e., the case in [29] and [42]). If $\varepsilon(k)$ is always equal to zero, the adaptive ETS (7) will change to traditional time-triggered scheme. Therefore, the static ETS and time-triggered scheme are special cases of the proposed adaptive ETS.

Remark 3: It should be noticed that there is at least one sampling time between adjacent triggers, so that the designed adaptive ETS (7) can exclude Zeno behavior.

C. Fault Detection Filter and Fault Weighting Model

In order to obtain the residual signal, an FDF is designed as follows

FDF Rule j: IF $g_1(x(k_s))$ is X_{j1}, $g_2(x(k_s))$ is X_{j2},..., and $g_q(x(k_s))$ is X_{jq}, THEN

$$\begin{cases} \hat{x}(k+1) = \hat{A}_{\delta(k)j}(k)\hat{x}(k) + \hat{B}_{\delta(k)j}(k)\hat{y}(k), \\ r(k) = \hat{E}_{\delta(k)j}(k)\hat{x}(k) + \hat{F}_{\delta(k)j}(k)\hat{y}(k), \end{cases} \tag{9}$$

where $\hat{x}(k) \in \mathbb{R}^{n_{\hat{x}}}$ denotes the state of the FDF (9); $r(k) \in \mathbb{R}^{n_r}$ presents the residual signal; $\hat{A}_{\delta(k)j}(k)$, $\hat{B}_{\delta(k)j}(k)$, $\hat{E}_{\delta(k)j}(k)$, and $\hat{F}_{\delta(k)j}(k)$ are the parameter matrices to be determined. The variable $\delta(k)$ is applied to observe the original system model, which takes the value in an integer set $\{O_2 = 1, 2, \ldots, o_2\}$ and satisfies a CPM $\Psi_2 = \{\varpi_{ac}\}$ with

$$\text{Prob}\{\delta(k) = c\,|\sigma(k) = a\} = \varpi_{ac}, \tag{10}$$

where $\varpi_{ac} \geq 0$ and $\sum_{c=1}^{O_2} \varpi_{ac} = 1$. Due to the information of the CPM Ψ_2 may not be fully available, thus, partially unknown probabilities are considered. For example, when $o_1 = o_2 = 3$, Ψ_2 may be as

$$\Phi_2 = \begin{bmatrix} ? & \varpi_{12} & \varpi_{13} \\ ? & ? & \varpi_{23} \\ ? & \varpi_{32} & ? \end{bmatrix}. \tag{11}$$

Then, denote that $O_2 = O_2^k + O_2^{uk}$ with

$$\begin{cases} O_2^k \triangleq \{c : \varpi_{av} \text{ is known}\}, \\ O_2^{uk} \triangleq \{c : \varpi_{av} \text{ is unknown}\}. \end{cases} \tag{12}$$

Analogous to (6), the IT2 FDF (9) with $\delta(k) = c$ can be inferred as [5]

$$\begin{cases} \hat{x}(k+1) = \hat{A}_{cn}(k)\hat{x}(k) + \hat{B}_{cn}(k)\hat{y}(k), \\ r(k) = \hat{E}_{cn}(k)\hat{x}(k) + \hat{F}_{cn}(k)\hat{y}(k), \end{cases} \tag{13}$$

where

$$\hat{A}_{cn}(k) = \sum_{j=1}^{l} \vartheta_j(x(k_s))\hat{A}_{cj}(k),$$
$$\hat{B}_{cn}(k) = \sum_{j=1}^{l} \vartheta_j(x(k_s))\hat{B}_{cj}(k),$$
$$\hat{E}_{cn}(k) = \sum_{j=1}^{l} \vartheta_j(x(k_s))\hat{E}_{cj}(k)\ ,$$
$$\hat{F}_{cn}(k) = \sum_{j=1}^{l} \vartheta_j(x(k_s))\hat{F}_{cj}(k).$$

in which

$$\vartheta_j(x(k_s)) = \frac{\phi_j^L(x(k_s))\vartheta_j^L(x(k_s)) + \phi_j^U(x(k_s))\vartheta_j^U(x(k_s))}{\sum_{j=1}^{l}\left(\phi_j^L(x(k_s))\vartheta_j^L(x(k_s)) + \phi_j^U(x(k_s))\vartheta_j^U(x(k_s))\right)},$$
$$0 \leq \phi_j^L(x(k_s)) \leq 1,\ 0 \leq \phi_j^U(x(k_s)) \leq 1,$$
$$\phi_j^L(x(k_s)) + \phi_j^L(x(k_s)) = 1,\ \sum_{j=1}^{l} \vartheta_j(x(k_s)) = 1.$$

Remark 4: Note that the modes of the FDF (13) do not need to be consistent with the original system (6). The HMM is employed to handle the asynchronous phenomenon. Moreover, the information of CPM requires completely known in [19], but this paper does not need to be fully known.

To obtain a better performance, inspired by [40], a reference model is introduced as $\tilde{f}(k) = R(k)f(k)$, where $R(k)$ is a weighting matrix. Thereby, the state-space can be represented as

$$\begin{cases} \tilde{x}(k+1) = A_R(k)\tilde{x}(k) + B_R(k)f(k), \\ \tilde{f}(k) = E_R(k)\tilde{x}(k) + F_R(k)f(k), \end{cases} \tag{14}$$

where $\tilde{x}(k) \in \mathbb{R}^{n_{\tilde{x}}}$ denotes the state vector of weighted fault; $\tilde{f}(k) \in \mathbb{R}^{n_{\tilde{f}}}$ indicates the weighted fault; In addition, $A_R(k)$ $B_R(k)$, $E_R(k)$, and $F_R(k)$ are known matrices.

D. Fault Detection System (FDS)

Considering the effect of the ZOH as shown in Fig. 1 and assuming that there is no delay and packet loss in network transmission, the real input of FDF (13) can be represented as

$$\hat{y}(k) = y(k_s),\ k \in [k_s, k_{s+1}),\ s \in N = \{0, 1, \ldots, \infty\}. \tag{15}$$

For brevity, $\chi_i(x(k))$ and $\vartheta_j(x(k_s))$ are respectively represented as χ_i and ϑ_j. Defining residual error as $\rho(k) = r(k) - \tilde{f}(k)$, and combining (6), (13), (14), and (15), the FDS can be obtained as follows:

$$\begin{cases} \varsigma(k+1) = A_1^f(k)\varsigma(k) + B_1^f(k)\mu(k), \\ \rho(k) = E_1^f(k)\varsigma(k) + F_1^f(k)\mu(k), \end{cases} \tag{16}$$

where

$$\varsigma(k) = \begin{bmatrix} x^T(k) & \hat{x}^T(k) & \tilde{x}^T(k) \end{bmatrix}^T,$$
$$\mu(k) = \begin{bmatrix} e^T(k) & w^T(k) & f^T(k) \end{bmatrix}^T,$$
$$A_1^f(k) = \sum_{i=1}^{l}\sum_{j=1}^{l} \chi_i\vartheta_j A_{ij}^f,\ B_1^f(k) = \sum_{i=1}^{l}\sum_{j=1}^{l} \chi_i\vartheta_j B_{ij}^f,$$
$$E_1^f(k) = \sum_{i=1}^{l}\sum_{j=1}^{l} \chi_i\vartheta_j E_{ij}^f,\ F_1^f(k) = \sum_{i=1}^{l}\sum_{j=1}^{l} \chi_i\vartheta_j F_{ij}^f,$$
$$A_{ij}^f = \begin{bmatrix} A_{ai} & 0 & 0 \\ \hat{B}_{cj}E_{ai} & \hat{A}_{cj} & 0 \\ 0 & 0 & A_R \end{bmatrix},$$
$$B_{ij}^f = \begin{bmatrix} 0 & B_{ai} & D_{ai} \\ -\hat{B}_{cj} & 0 & \hat{B}_{cj}H_{ai} \\ 0 & 0 & B_R \end{bmatrix},$$
$$E_{ij}^f = \begin{bmatrix} \hat{F}_{cj}E_{ai} & \hat{E}_{cj} & -E_R \end{bmatrix},$$
$$F_{ij}^f = \begin{bmatrix} -\hat{F}_{cj} & 0 & \hat{F}_{cj}H_{ai} - F_R \end{bmatrix}.$$

Based on the above description, the FD problem can be represented as:

1) The designed FDF (13) can be guaranteed that the FDS (16) is asymptotically stable for $w(k) = 0$; Moreover, under the zero initial condition, the FDS (16) satisfies the H_∞ performance shown below

$$\Xi\left\{\sum_{k=0}^{\infty} \rho^T(k)\rho(k)\right\} < \Xi\left\{\gamma^2 \sum_{k=0}^{\infty} w^T(k)w(k)\right\}. \tag{17}$$

2) To detect fault $f(k)$, we define the evaluation function J_e and the threshold J_{th} as

$$J_e = \frac{1}{\mathfrak{F}}\sqrt{\sum_{k=k_0}^{k_1} r^T(k)r(k)},\ J_{th} = \sup_{w(k)\neq 0, f(k)=0} J_e, \tag{18}$$

where $\mathfrak{F} = k_1 - k_0 + 1$. According to (16), an FD task can be described as

$$\begin{aligned} &J_e - J_{th} \leq 0 \Rightarrow \text{no faults}, \\ &J_e - J_{th} > 0 \Rightarrow \text{faults} \Rightarrow \text{alarm}. \end{aligned} \tag{19}$$

III. Performance Analysis and FDF Design

This section contains two parts. The first part is to analyze the stability and H_∞ performance for the FDS (16) with partially unknown probabilities, the adaptive ETS (7), and asynchronous jump modes. The second part is to design the parameters of the FDF (13) such that the derived conditions are in the form of linear matrix inequalities (LMIs).

A. Performance Analysis

We will now present Theorem 1 which involves the asymptotically stability conditions and satisfies the H_∞ performance for FDS (16).

Theorem 1: For given positive scalars κ, $0 < \varepsilon_2 < 1$, and $0 < \delta_j \leq 1$, the FDS (16) is asymptotically stable with the H_∞ performance index $\gamma > 0$, if $\vartheta_j - \delta_j \chi_j \geq 0$ and there exist matrices $\hat{A}_{cj}$, $\hat{B}_{cj}$, $\hat{E}_{cj}$, $\hat{F}_{cj}$, $M_{aci} > 0$, $G_a > 0$, $P_{ai} > 0$, Γ_i for $i \in \mathcal{U}_1$, $j \in \mathcal{U}_1$, $a \in O_1$, $c \in O_2$, such that

$$\sum\nolimits_{c \in O_2^k} \varpi_{ac} M_{aci} + \sum\nolimits_{c \in O_2^{uk}} M_{aci} - P_{ai} < 0, \tag{20}$$

$$\Phi_{ij} - \Gamma_i < 0, \tag{21}$$

$$\delta_i \Phi_{ii} + (1 - \delta_i) \Gamma_i < 0, \tag{22}$$

$$\delta_j \Phi_{ij} + \delta_i \Phi_{ji} + (1 - \delta_j) \Gamma_i + (1 - \delta_i) \Gamma_j < 0, \ i < j, \tag{23}$$

where

$$\Phi_{ij} = \begin{bmatrix} \Upsilon_{ij}^{11} & \Upsilon_{ij}^{12} & \Upsilon_{ij}^{13} \\ * & \Upsilon_{ij}^{22} & 0 \\ * & * & -I \end{bmatrix}, \Upsilon_{ij}^{11} = \begin{bmatrix} \Theta_{11} & \Theta_{12} \\ * & \Theta_{22} \end{bmatrix},$$

$$\Pi_1 = \varepsilon_2 \begin{bmatrix} E_{ai} & 0 & 0 \end{bmatrix}^T G_a \begin{bmatrix} E_{ai} & 0 & 0 \end{bmatrix},$$

$$\Theta_{11} = -M_{aci} + \Pi_1, \Theta_{22} = \Pi_2 + \Pi_3 + \Pi_4,$$

$$\Theta_{12} = \varepsilon_2 \begin{bmatrix} E_{ai} & 0 & 0 \end{bmatrix}^T G_a \begin{bmatrix} 0 & 0 & H_{ai} \end{bmatrix},$$

$$\Pi_2 = \varepsilon_2 \begin{bmatrix} 0 & 0 & H_{ai} \end{bmatrix}^T G_a \begin{bmatrix} 0 & 0 & H_{ai} \end{bmatrix},$$

$$\Pi_3 = \begin{bmatrix} I & 0 & 0 \end{bmatrix}^T G_a \begin{bmatrix} I & 0 & 0 \end{bmatrix},$$

$$\Pi_4 = -\begin{bmatrix} 0 & \gamma & 0 \end{bmatrix}^T \begin{bmatrix} 0 & \gamma & 0 \end{bmatrix},$$

$$\Upsilon_{ij}^{12} = \begin{bmatrix} A_{ij}^f & B_{ij}^f \end{bmatrix}^T, \Upsilon_{ij}^{13} = \begin{bmatrix} E_{ij}^f & F_{ij}^f \end{bmatrix}^T,$$

$$\Upsilon_{ij}^{22} = -\tilde{P}_{bg}^{-1}, \tilde{P}_{bg} = \sum\nolimits_{b \in O_1^{uk}} P_{bg} + \sum\nolimits_{b \in O_1^k} \lambda_{ab} P_{bg}.$$

Proof: Choose the following Lyapunov-Krasovskii functional (LKF) for the FDS (16)

$$V(k) = \varsigma^T(k) P_{am}(k) \varsigma(k), \tag{24}$$

where $P_{am}(k) = \sum_{i=1}^{l} \chi_i P_{ai}$. Along with the trajectory of the FDS (16), it yields from (24) that

$$\begin{aligned} &\Xi \left\{ \Delta \left[V(k) \right] \right\} \\ &= \Xi \left\{ \varsigma^T(k+1) P_{am}^+(k+1) \varsigma(k+1) - \varsigma^T(k) P_{am}(k) \varsigma(k) \right\}, \end{aligned} \tag{25}$$

where

$$P_{am}^+(k+1) = \sum_{b \in O_1} \lambda_{ab} P_{bm}(k+1),$$

$$P_{bm}(k+1) = \sum\nolimits_{g=1}^{l} \chi_g P_{bg} = \sum\nolimits_{g=1}^{l} \chi_g(k+1) P_{bg}.$$

Considering the effect of the partially unknown element in the TPM, we can calculate that

$$\sum_{b \in O_1} \lambda_{ab} P_{bg} = \left(\sum_{b \in O_1^k} \lambda_{ab} + \sum_{b \in O_1^{uk}} \lambda_{ab} \right) P_{bg} < \tilde{P}_{bg}. \tag{26}$$

According to (20), one can get

$$\begin{aligned} P_{am} &> \sum\nolimits_{c \in O_2^k} \varpi_{ac} M_{acm} + \sum\nolimits_{c \in O_2^{uk}} M_{acm} \\ &\geq \sum\nolimits_{c \in O_2} \varpi_{ac} M_{acm}, \end{aligned} \tag{27}$$

where $M_{acm} = \sum\limits_{i=1}^{l} \chi_i M_{aci}$. In addition, recalling the adaptive ETS (7) and the adaptive law (8), one has

$$\begin{aligned} 0 &< \varepsilon(k) y^T(k) G_a y(k) - e^T(k) G_a e(k) \\ &< \varepsilon_2 y^T(k) G_a y(k) - e(k)^T G_a e(k) \\ &= \varepsilon_2 \varsigma^T(k) \textstyle\prod_1 \varsigma(k) + 2\varepsilon_2 \varsigma^T(k) \prod_2 \mu(k) \\ &\quad + \varepsilon_2 \mu^T(k) \textstyle\prod_3 \mu(k) - e^T(k) G_a e(k). \end{aligned} \tag{28}$$

Similar to [4], we introduce slack matrices $\Gamma_i = \Gamma_i^T$ ($i \in \mathcal{U}_1$). By using the property of the fuzzy rule and membership-function-dependent (MFD) method [43], we have

$$\sum_{i}^{l} \sum_{j}^{l} \chi_i \left(\chi_j - \vartheta_j \right) \Gamma_i = 0. \tag{29}$$

According to the above equation, combining (21)–(23) with $\vartheta_j - \delta_j \chi_j \geq 0$, one can get

$$\begin{aligned} &\sum_{i}^{l} \sum_{j}^{l} \sum_{g}^{l} \chi_g \chi_i \vartheta_j \Phi_{ij} \\ &= \sum_{g=1}^{l} \chi_g \left[\sum_{i=1}^{l} \chi_i^2 \left(\delta_i \Phi_{ii} + (1 - \delta_i) \Gamma_i \right) \right] \\ &+ \sum_{g=1}^{l} \chi_g \left[\sum_{i=1}^{l} \sum_{j=1}^{l} \chi_i \left(\vartheta_j - \delta_j \chi_j \right) \left(\Phi_{ij} - \Gamma_i \right) \right] \\ &+ \sum_{g=1}^{l} \chi_g \left[\sum_{i=1}^{l-1} \sum_{j=i+1}^{l} \chi_i \chi_j \left(\delta_j \Phi_{ij} + \delta_i \Phi_{ji} \right) \right] \\ &+ \sum_{g=1}^{l} \chi_g \left[\sum_{i=1}^{l-1} \sum_{j=i+1}^{l} \chi_i \chi_j \left((1 - \delta_j) \Gamma_i + (1 - \delta_i) \Gamma_j \right) \right] \\ &< 0. \end{aligned} \tag{30}$$

Define $\xi(k) = \begin{bmatrix} \varsigma^T(k) & w^T(k) \end{bmatrix}^T$, and it can be derived from (23)–(27) that

$$\begin{aligned} &\Xi \left\{ \Delta \left[V_1(k) \right] + \rho^T(k) \rho(k) - \gamma^2 w^T(k) w(k) \right\} \\ &< \xi^T(k) \sum_{i}^{l} \sum_{j}^{l} \sum_{g}^{l} \chi_g \chi_i \vartheta_j \\ &\times \left\{ \Upsilon_{ij}^{11} + \left(\Upsilon_{ij}^{12} \right)^T \left(\Upsilon_{ij}^{22} \right)^{-1} \left(\Upsilon_{ij}^{12} \right) + \left(\Upsilon_{ij}^{13} \right)^T \left(\Upsilon_{ij}^{13} \right) \right\} \xi(k). \end{aligned} \tag{31}$$

Since $\Upsilon_{ij}^{11} + \left(\Upsilon_{ij}^{12} \right)^T \left(\Upsilon_{ij}^{22} \right)^{-1} \left(\Upsilon_{ij}^{12} \right) + \left(\Upsilon_{ij}^{13} \right)^T \left(\Upsilon_{ij}^{13} \right) < 0$ is equivalent to $\Phi_{ij} < 0$ by using Schur Complement, (31) can be rewritten as

$$\Xi \left\{ \Delta \left[V_1(k) \right] + \rho^T(k) \rho(k) - \gamma^2 w^T(k) w(k) \right\} < 0. \tag{32}$$

Then, under zero the initial condition, we have

$$\Xi \left\{ \sum_{k=0}^{\infty} \rho^T(k) \rho(k) \right\} < \Xi \left\{ \gamma^2 \sum_{k=0}^{\infty} w^T(k) w(k) \right\}. \tag{33}$$

The proof is completed. ■

B. FDF Design

Due to the conditions in Theorem 1 are not in the form of LMIs, it is difficult to solve them. Thus, we will focus on designing the FDF in the form of LMIs.

Theorem 2: For given positive scalars κ, $0<\varepsilon_2<1$, and $0<\delta_j\leq 1$, the FDS (16) is asymptotically stable with the H_∞ performance index $\gamma>0$, if $\vartheta_j-\delta_j\chi_j\geq 0$ and there exist matrices $\tilde{A}_{cj}$, $\tilde{B}_{cj}$, $\tilde{E}_{cj}$, $\tilde{F}_{cj}$, $M_{aci}>0$, $G_a>0$, $P_{ai}>0$, $U_1>0$, $U_2>0$, $V>0$, Γ_i for $i\in\mathcal{U}_1$, $j\in\mathcal{U}_1$, $a\in O_1$, $c\in O_2$, such that

$$\sum\nolimits_{c\in O_2^k}\varpi_{ac}M_{aci}+\sum\nolimits_{c\in O_2^{uk}}M_{aci}-P_{ai}<0, \tag{34}$$

$$\tilde{\Phi}_{ij}-\tilde{\Gamma}_i<0, \tag{35}$$

$$\delta_i\tilde{\Phi}_{ii}+(1-\delta_i)\tilde{\Gamma}_i<0, \tag{36}$$

$$\delta_j\tilde{\Phi}_{ij}+\delta_i\tilde{\Phi}_{ji}+(1-\delta_j)\tilde{\Gamma}_i+(1-\delta_i)\tilde{\Gamma}_j<0,\ i<j, \tag{37}$$

where

$$\tilde{\Phi}_{ij}=\begin{bmatrix}\Theta_{11}&\Theta_{12}&\Theta_{13}&\Theta_{14}\\ *&\Theta_{22}&\Theta_{23}&\Theta_{24}\\ *&*&\Theta_{33}&0\\ *&*&*&-I\end{bmatrix},$$

$$\Theta_{13}=\begin{bmatrix}A_{ai}^TU_1+E_{ai}^T\tilde{B}_{cj}^T & A_{ai}^TU_2+E_{ai}^T\tilde{B}_{cj}^T & 0\\ \tilde{A}_{cj}^T & \tilde{A}_{cj}^T & 0\\ 0 & 0 & A_R^TV\end{bmatrix},$$

$$\Theta_{23}=\begin{bmatrix}-\tilde{B}_{cj}^T & -\tilde{B}_{cj}^T & 0\\ B_{ai}^TU_1 & B_{ai}^TU_2 & 0\\ D_{ai}^TU_1+H_{ai}^T\tilde{B}_{cj}^T & D_{ai}^TU_2+H_{ai}^T\tilde{B}_{cj}^T & B_R^TV\end{bmatrix},$$

$$\Theta_{14}=\begin{bmatrix}\tilde{F}_{cj}E_{ai} & \tilde{E}_{cj} & -E_R\end{bmatrix}^T,$$

$$\Theta_{24}=\begin{bmatrix}-\tilde{F}_{cj} & 0 & \tilde{F}_{cj}H_{ai}-F_R\end{bmatrix}^T,$$

$$\Theta_{33}=\tilde{P}_{bg}-2Q,\tilde{\Gamma}_i=\mathfrak{G}\Gamma_i\mathfrak{G}^T.$$

Furthermore, the FDF gains in (13) can be solved by

$$\hat{A}_{cj}=U_2^{-1}\tilde{A}_{cj},\ \hat{B}_{cj}=U_2^{-1}\tilde{B}_{cj},\ \hat{E}_{cj}=\tilde{E}_{cj},\ \hat{F}_{cj}=\tilde{F}_{cj}. \tag{38}$$

Proof: First of all, we define

$$Q=\begin{bmatrix}U_1&U_2&0\\ *&U_2&0\\ *&*&V\end{bmatrix},\ \mathfrak{G}=\operatorname{diag}\{I,I,Q,I,\}, \tag{39}$$

and

$$\tilde{A}_{cj}=U_2\hat{A}_{cj},\ \tilde{B}_{cj}=U_2\hat{B}_{cj},\ \tilde{E}_{cj}=\hat{E}_{cj},\ \tilde{E}_{cj}=\hat{F}_{cj}. \tag{40}$$

Then, multiplying the left and right hand sides of (21) with $\mathfrak{G}$ and $\mathfrak{G}^T$, respectively, it yields

$$\breve{\Phi}_{ij}-\tilde{\Gamma}_i<0, \tag{41}$$

where

$$\breve{\Phi}_{ij}=\begin{bmatrix}\Theta_{11}&\Theta_{12}&\Theta_{13}&\Theta_{14}\\ *&\Theta_{22}&\Theta_{23}&\Theta_{24}\\ *&*&\breve{\Theta}_{33}&0\\ *&*&*&-I\end{bmatrix},\breve{\Theta}_{33}=-Q\tilde{P}_{bg}^{-1}Q.$$

Moreover, since the inequality $-Q\tilde{P}_{bg}^{-1}Q<\tilde{P}_{bg}-2Q$ is hold, it can be derived (35). Analogously, we can obtain (36) and (37). We complete the proof. ■

Remark 5: Note that the proposed algorithms do not need to know the complete information of the TPM Φ_1 and the CPM Φ_2. Moreover, the minimal H_∞ performance γ can be obtained by solving the following optimal problem:

min γ *s.t. the LMIs (34)–(37) in Theorem 2.*

Based on the above analysis, the solution method of FDF gains ($\hat{A}_{cj}$, $\hat{B}_{cj}$, $\hat{E}_{cj}$, $\hat{F}_{cj}$) and the optimal H_∞ performance γ are given in Algorithm 1.

Algorithm 1 Solve for the FDF gain matrices $\hat{A}_{cj}$, $\hat{B}_{cj}$, $\hat{E}_{cj}$, $\hat{F}_{cj}$, and optimal H_∞ performance γ

1: Given $\varepsilon_2\in(0,1)$, $\delta_i>0$, and $\kappa\in(0,1)$, $i\in\mathcal{U}_1$
2: Define variables $P_{ai}>0$, $\tilde{A}_{cj}$, $\tilde{B}_{cj}$, $\tilde{E}_{cj}$, $\tilde{F}_{cj}$, $M_{aci}>0$, $G_a>0$, $P_{ai}>0$ $U_1>0$, $U_2>0$, $V>0$, $\tilde{\Gamma}_i$, $i\in\mathcal{U}_1$, $j\in\mathcal{U}_1$, $a\in O_1$, and $c\in O_2$
3: Obtain $\tilde{A}_{cj}$, $\tilde{B}_{cj}$, $\tilde{E}_{cj}$, $\tilde{F}_{cj}$, and U_2 by solving the LMIs (34)–(37) in Theorem 2
4: Set $\hat{A}_{cj}=U_2^{-1}\tilde{A}_{cj}$, $\hat{B}_{cj}=U_2^{-1}\tilde{B}_{cj}$, $\hat{E}_{cj}=\tilde{E}_{cj}$, $\hat{F}_{cj}=\tilde{F}_{cj}$
5: Find $\min\bar{\gamma}=\gamma^2$ by solving the conditions in Theorem 2 iteratively
6: Set $\gamma=\sqrt{\bar{\gamma}}$

IV. Simulation results

In this section, an example is given to show the effectiveness of the proposed adaptive event-triggered FD method.

Example: Consider the tunnel diode circuit system with the following IT2 fuzzy model [19]:

Original System Rule 1: IF $x_1(k)$ is T_{11}, THEN

$$\begin{cases}x(k+1)=A_{\sigma(k)1}(k)x(k)+B_{\sigma(k)1}(k)w(k)\\ \qquad\qquad +D_{\sigma(k)1}(k)f(k),\\ y(k)=E_{\sigma(k)1}(k)x(k)+H_{\sigma(k)1}(k)f(k),\end{cases}$$

Original System Rule 2: IF $x_2(k)$ is T_{21}, THEN

$$\begin{cases}x(k+1)=A_{\sigma(k)2}(k)x(k)+B_{\sigma(k)1}(k)w(k)\\ \qquad\qquad +D_{\sigma(k)2}(k)f(k),\\ y(k)=E_{\sigma(k)2}(k)x(k)+H_{\sigma(k)2}(k)f(k),\end{cases}$$

where T_{11} and T_{21} are IT2 fuzzy sets. The LMF and UMF of the original system are chosen as

$$\chi_1^L(x_1(k))=1-\exp(-\frac{0.1x_1^2}{\Delta_{\max}}),\ \chi_2^L(x_1(k))=1-\chi_1^U(x_1(k)),$$

$$\chi_1^U(x_1(k))=1-\exp(-\frac{0.1x_1^2}{\Delta_{\min}}),\ \chi_2^U(x_1(k))=1-\chi_1^L(x_1(k)).$$

Notice that the LMF and the UMF can capture the uncertainties parameter $\Delta\in[0.5,0.9]$, which is more general than the existing type-1 fuzzy method in [19]. Moreover, assume that $\sigma(k)=1,2$, and the system matrices are defined as follows.

$$\left[\begin{array}{c|c}A_{11}&B_{11}\\ \hline E_{11}&H_{11}\end{array}\right]=\left[\begin{array}{cc|c}0.9887&0.9024&0.0093\\ -0.018&0.8100&0.0181\\ \hline 1&0&0.2\end{array}\right],$$

$$\left[\begin{array}{c|c} A_{12} & B_{12} \\ \hline E_{12} & H_{12} \end{array}\right] = \left[\begin{array}{cc|c} 0.90337 & 0.8617 & 0.0091 \\ -0.0172 & 0.8103 & 0.0181 \\ \hline 1 & 0 & 0.4 \end{array}\right],$$

$$\left[\begin{array}{c|c} A_{21} & B_{21} \\ \hline E_{21} & H_{21} \end{array}\right] = \left[\begin{array}{cc|c} 0.998 & 1 & 0 \\ -0.02 & 0.98 & 0.02 \\ \hline 1 & 0 & 0.5 \end{array}\right],$$

$$\left[\begin{array}{c|c} A_{22} & B_{22} \\ \hline E_{22} & H_{22} \end{array}\right] = \left[\begin{array}{cc|c} 0.908 & 1 & 0 \\ -0.02 & 0.98 & 0.02 \\ \hline 1 & 0 & 0.6 \end{array}\right],$$

$$D_{11} = \begin{bmatrix} 0.9 \\ -0.01 \end{bmatrix},\ D_{12} = \begin{bmatrix} 0.98 \\ 0 \end{bmatrix},$$

$$D_{21} = \begin{bmatrix} 0.86 \\ -0.02 \end{bmatrix},\ D_{22} = \begin{bmatrix} 0.9 \\ -0.01 \end{bmatrix}.$$

The TPM and CPM are given as follows

$$\Phi_1 = \begin{bmatrix} ? & ? \\ 0.6 & 0.4 \end{bmatrix}, \Phi_2 = \begin{bmatrix} 0.5 & 0.5 \\ ? & ? \end{bmatrix}.$$

The LGM and UGM of the FDF are given as

$$\vartheta_1^L(x_1(k)) = 1 - \exp(\frac{-x_1^2}{7}),\ \vartheta_2^L(x_1(k)) = 1 - \vartheta_1^U(x_1(k)),$$

$$\vartheta_1^U(x_1(k)) = 1 - \exp(\frac{-x_1^2}{6}),\ \vartheta_2^U(x_1(k)) = 1 - \vartheta_1^L(x_1(k)),$$

and the nonlinear functions with $i = 1, 2$ are set as

$$\psi_i^L(x_1(k)) = \sin^2(x_1(k)),\ \psi_i^U(x_1(k)) = 1 - \psi_i^L(x_1(k)),$$

$$\phi_i^L(x_1(k)) = \cos^2(x_1(k)), \phi_i^U(x_1(k)) = 1 - \phi_i^L(x_1(k)).$$

The parameters of (14) are chosen as

$$A_R = 0.2,\ B_R = 0.3,\ E_R = 0.4,\ F_R = 0.$$

Let $\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.7$, $\delta_1 = 0.3$, and $\delta_2 = 0.2$. Based on Theorem 2, the parameters of the IT2 fuzzy FDF and the adaptive ETS are obtained as:

$$\hat{A}_{\delta(k)1} = \begin{bmatrix} -0.4518 & -0.8298 \\ -0.2709 & -0.1835 \end{bmatrix}, \hat{B}_{\delta(k)1} = \begin{bmatrix} -0.1122 \\ 0.1925 \end{bmatrix},$$

$$\hat{A}_{\delta(k)2} = \begin{bmatrix} -0.3992 & -0.6919 \\ -0.2213 & -0.0938 \end{bmatrix}, \hat{B}_{\delta(k)2} = \begin{bmatrix} -0.1957 \\ 0.2036 \end{bmatrix},$$

$$\hat{E}_{11} = \begin{bmatrix} -0.8911 & -0.8911 \end{bmatrix},$$

$$\hat{E}_{12} = \begin{bmatrix} -1.0055 & -1.0115 \end{bmatrix},$$

$$\hat{E}_{21} = \begin{bmatrix} -0.8853 & -0.8911 \end{bmatrix},$$

$$\hat{E}_{22} = \begin{bmatrix} -1.0055 & -1.0115 \end{bmatrix},$$

$$\hat{F}_{\delta(k)1} = \hat{F}_{\delta(k)2} = 0, G_1 = -1.1568 \times 10^{-15},$$

$$G_2 = -9.4328 \times 10^{-16}, \delta(k) = 1, 2.$$

The fault signal $f(k)$ and the disturbance input $w(k)$ are selected as

$$w(k) = \begin{cases} \sin(k), 20 \leq k \leq 30, \\ 0, \text{otherwise}, \end{cases}$$

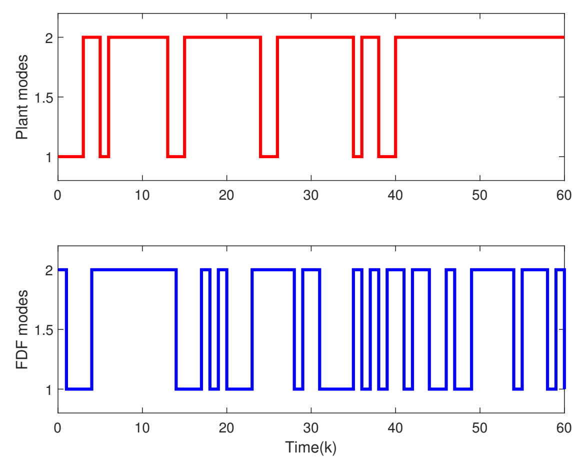

Fig. 2. Plant modes and FDF modes.

and

$$f(k) = \begin{cases} 0.3, & 30 < k < 50, \\ 0, \text{otherwise}. \end{cases}$$

By Algorithm 1, the optimal H_∞ performance can be obtained, i.e., $\gamma = 1.0092$. The initial state vectors of the plant, the FDF, and the weighted fault are respectively set as

$$x_0 = \begin{bmatrix} 0 & 0 \end{bmatrix}^T, \hat{x}_0 = \begin{bmatrix} 0 & 0 \end{bmatrix}^T, \tilde{x}_0 = \begin{bmatrix} 0 & 0 \end{bmatrix}^T.$$

The modes for the plant and the FDF are depicted in Fig. 2. As we can observe from Fig. 2, the modes of the FDF do not require the same as the plant. Fig. 3 shows the trajectories of the adaptive parameter $\varepsilon(k)$. We can observe that $\varepsilon(k)$ tunes with time rather than a constant. The real transmitted instants and intervals are shown in Fig. 4. It is obvious that there are only 11 triggering points, which indicates that the sampled data transmission is reduced by 81.7%. Under $\varepsilon_1 = 0.1$, $\varepsilon_2 = 0.9$, and $e(k) = k$, Fig. 5 depicts the trajectories of the adaptive parameter $\varepsilon(k)$ for different κ. It can be found that as the value of κ increases, the value of $\varepsilon(k)$ changes faster, which implies that the larger the value of κ, the more sensitive the designed adaptive ETS is to $e(k)$. Under $\varepsilon_1 = 0.2$, Table III depicts the triggering times and transmission rates for different ε_2. From Table III, we can see that the transmission rates and the triggering times increase as ε_2 decreases, which means that the bigger ε_2, the lower consumption of network resources. As shown in Table IV, we can observe that the adaptive ETS proposed in this paper can reduce the number of transmissions more than the existing methods in [29].

TABLE III
TRIGGERING TIMES AND TRANSMISSION RATES FOR DIFFERENT ε_2

ε_2	0.9	0.7	0.5	0.3
Triggering times	9	11	13	14
Transmission rates	15%	18.3%	21.6%	23.3%

Furthermore, there are two steps required to validate the designed fault detection method. The first step is to calculate the threshold J_{th}. We can find a reliable J_{th}, i.e., $J_{th} =$

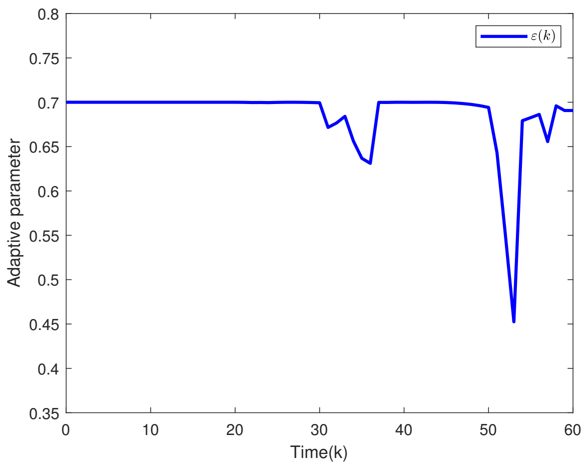

Fig. 3. The trajectories of the adaptive parameter $\varepsilon(k)$.

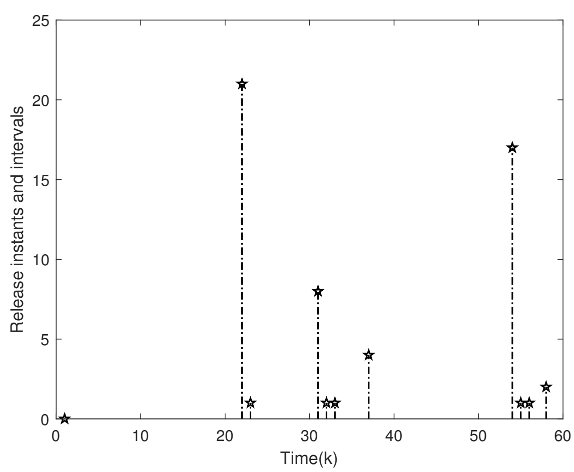

Fig. 4. The real transmitted instants and intervals under the adaptive ETS.

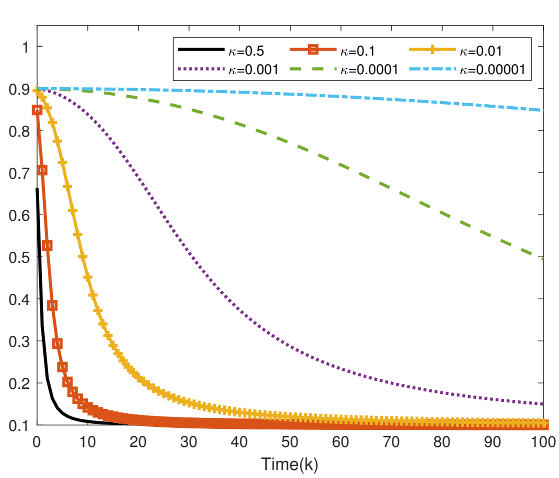

Fig. 5. The trajectories of the adaptive parameter $\varepsilon(k)$ for different κ.

TABLE IV
TRIGGERING TIMES AND TRANSMISSION RATES FOR DIFFERENT ε_2

ε_1	0.1	0.3	0.5	0.7
Triggering times in [29]	18	14	12	11
Triggering times in this paper	10	10	9	9

6.2076×10^{-5} without the fault occurs. The second step is to FD. When the fault occurs, the evaluation function can be obtained, i.e., $J_e = \frac{1}{33}\sqrt{\sum_{k=1}^{33} r^T(k)r(k)} = 7.5182 \times 10^{-5}$ at $k = 33$. Based on the two steps, Fig. 6 depicts the fault detection results for the proposed adaptive event-triggered FD method. It can be observed that J_e exceeds the threshold J_{th} at $k = 33$. This indicates that once the fault occurs, the proposed FD method can be effectively detected at $k = 33$. The simulation results can prove that the proposed algorithms can effectively realize FD and improve the transmission efficiency.

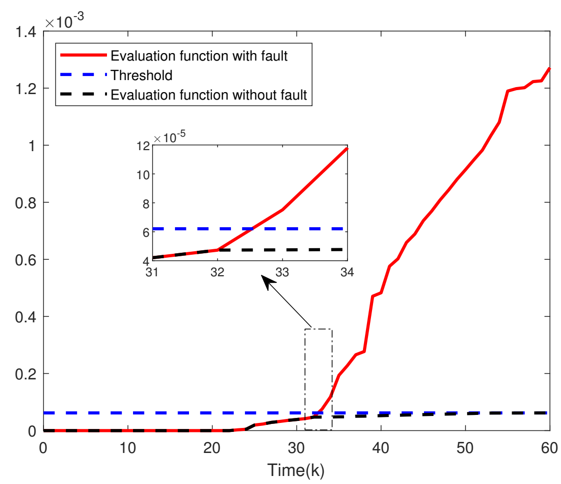

Fig. 6. Detection results of $f(k)$ using the proposed FD design.

V. CONCLUSIONS

The adaptive event-triggered FD problems were considered for the IT2 fuzzy networked MJSs with partially unknown transition probabilities and asynchronous modes. The FDF was designed to generate the residual signal, where the HMM models the asynchronous phenomenon. Considering the limited bandwidth and communication burden, the adaptive ETS was designed for IT2 fuzzy networked MJSs, where the adaptive law can tune the adaptive parameter, and the weighting matrices are mode-dependent. By using Lyapunov function methods and inequality techniques, the parameter existence conditions of the FDF are obtained. The simulation results verified the effectiveness of this approach. It should be noticed that the proposed approach focuses on designing the adaptive event-triggered FDF, which furnishes a new view for handling the FD problem of IT2 fuzzy networked MJSs. This approach can be applied in many fields, such as circuit systems, complex industrial systems, and automobile systems. Although the FD approach for IT2 fuzzy networked MJSs is proposed, a control strategy is not designed to recover the system performance. Therefore, future work will focus on event-based fault-tolerant control for IT2 fuzzy networked MJSs. Besides, we are also devoted to designing the non-periodical sampling ETS for nonlinear MJSs in the future.

REFERENCES

[1] H. Li, J. Yu, C. Hilton, and H. Liu, "Adaptive sliding-mode control for nonlinear active suspension vehicle systems using T–S fuzzy approach," *IEEE Trans. Ind. Electron.*, vol. 60, no. 8, pp. 3328–3338, 2012.

[2] L. Wang and H. K. Lam, "Further study on observer design for continuous-time Takagi–Sugeno fuzzy model with unknown premise variables via average dwell time," *IEEE Trans. Cybern.*, vol. 50, no. 11, pp. 4855–4860, 2019.

[3] L. Wang, J. Liu, and H. K. Lam, "Further study on stabilization for continuous-time Takagi-Sugeno fuzzy systems with time delay," *IEEE Trans. Cybern.*, 2020, doi:10.1109/TCYB.2020.2973276.

[4] H. K. Lam and L. D. Seneviratne, "Stability analysis of interval type-2 fuzzy-model-based control systems," *IEEE Trans. Syst., Man, Cybern., Part B:Cybern.*, vol. 38, no. 3, pp. 617–628, 2008.

[5] H. K. Lam, H. Li, C. Deters, E. L. Secco, H. A. Wurdemann, and K. Althoefer, "Control design for interval type-2 fuzzy systems under imperfect premise matching," *IEEE Trans. Ind. Electron.*, vol. 61, no. 2, pp. 956–968, 2013.

[6] J. M. Mendel and X. Liu, "Simplified interval type-2 fuzzy logic systems," *IEEE Trans. Fuzzy Syst.*, vol. 21, no. 6, pp. 1056–1069, 2013.

[7] H. Li, S. Yin, Y. Pan, and H. K. Lam, "Model reduction for interval type-2 Takagi–Sugeno fuzzy systems," *Automatica*, vol. 61, pp. 308–314, 2015.

[8] C. M. Lin, M. S. Yang, F. Chao, X. M. Hu, and J. Zhang, "Adaptive filter design using type-2 fuzzy cerebellar model articulation controller," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 27, no. 10, pp. 2084–2094, 2016.

[9] W. Zheng, Z. Zhang, H. Wang, and H. Wang, "Robust H_∞ dynamic output feedback control for interval type-2 T-S fuzzy multiple time-varying delays systems with external disturbance," *J. Frankl. Inst.*, vol. 357, no. 6, pp. 3193–3218, 2020.

[10] Z. G. Wu, S. Dong, H. Su, and C. Li, "Asynchronous dissipative control for fuzzy Markov jump systems," *IEEE Trans. Cybern.*, vol. 48, no. 8, pp. 2426–2436, 2018.

[11] M. Zhang, P. Shi, L. Ma, J. Cai, and H. Su, "Quantized feedback control of fuzzy Markov jump systems," *IEEE Trans. Cybern.*, vol. 49, no. 9, pp. 3375–3384, 2019.

[12] T. B. Nguyen and S. H. Kim, "Dissipative control of interval type-2 nonhomogeneous Markovian jump fuzzy systems with incomplete transition descriptions," *Nonlinear Dyn.*, vol. 100, pp. 1289–1308, 2020.

[13] K. Ding and Q. Zhu, "Extended dissipative anti-disturbance control for delayed switched singular semi-Markovian jump systems with multi-disturbance via disturbance observer," *Automatica*, vol. 128, p. 109556, 2021.

[14] J. Tao, R. Lu, P. Shi, H. Su, and Z. G. Wu, "Dissipativity-based reliable control for fuzzy Markov jump systems with actuator faults," *IEEE Trans. Cybern.*, vol. 47, no. 9, pp. 2377–2388, 2017.

[15] Y. Shen, Z. G. Wu, P. Shi, and G. Wen, "Dissipativity based fault detection for 2D Markov jump systems with asynchronous modes," *Automatica*, vol. 106, pp. 8–17, 2019.

[16] L. Zhang, H. K. Lam, Y. Sun, and H. Liang, "Fault detection for fuzzy semi-Markov jump systems based on interval type-2 fuzzy approach," *IEEE Trans. Fuzzy Syst.*, vol. 28, no. 10, pp. 2375–2388, 2020.

[17] C. Du, F. Li, and C. Yang, "An improved homogeneous polynomial approach for adaptive sliding-mode control of Markov jump systems with actuator faults," *IEEE Trans. Autom. Control*, vol. 65, no. 3, pp. 955–969, 2020.

[18] H. Yang, Y. Jiang, and S. Yin, "Adaptive fuzzy fault tolerant control for Markov jump systems with additive and multiplicative actuator faults," *IEEE Trans. Fuzzy Syst.*, vol. 29, no. 4, pp. 772–785, 2021.

[19] S. Dong, Z. G. Wu, P. Shi, H. R. Karimi, and H. Su, "Networked fault detection for Markov jump nonlinear systems," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 6, pp. 3368–3378, 2018.

[20] P. Tabuada, "Event-triggered real-time scheduling of stabilizing control tasks," *IEEE Trans. Autom. Control*, vol. 52, no. 9, pp. 1680–1685, 2007.

[21] J. Liu, Y. Zhang, Y. Yu, H. Liu, and C. Sun, "A Zeno-free self-triggered approach to practical fixed-time consensus tracking with input delay," *IEEE Trans. Syst., Man, Cybern., Syst.*, 2021, doi: 10.1109/TSMC.2021.3063117.

[22] S. Hu, D. Yue, C. Dou, X. Xie, Y. Ma, and L. Ding, "Attack-resilient event-triggered fuzzy interval type-2 filter design for networked nonlinear systems under sporadic denial-of-service jamming attacks," *IEEE Trans. Fuzzy Syst.*, 2020, doi: 10.1109/TFUZZ.2020.3033851.

[23] G. Ran, J. Liu, C. Li, L. Chen, and D. Li, "Event-based finite-time consensus control of second-order delayed multi-agent systems," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 68, no. 1, pp. 276–280, 2021.

[24] H. Yu and T. Chen, "A new zeno-free event-triggered scheme for robust distributed optimal coordination," *Automatica*, vol. 129, p. 109639, 2021.

[25] G. Ran, J. Liu, C. Li, H. Chen, and C. Han, "Finite-time filtering for fuzzy nonlinear semi-Markov jump systems with deception attacks and aperiodical transmission," *J. Franklin Inst.*, 2021, doi: 10.1016/j.jfranklin.2021.07.055.

[26] H. Shen, M. Chen, Z. G. Wu, J. Cao, and J. H. Park, "Reliable event-triggered asynchronous extended passive control for semi-Markov jump fuzzy systems and its application," *IEEE Trans. Fuzzy Syst.*, vol. 28, no. 8, pp. 1708–1722, 2020.

[27] D. Yao, B. Zhang, P. Li, and H. Li, "Event-triggered sliding mode control of discrete-time Markov jump systems," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 49, no. 10, pp. 2016–2025, 2019.

[28] Z. Lu, G. Ran, F. Xu, and J. Lu, "Novel mixed-triggered filter design for interval type-2 fuzzy nonlinear Markovian jump systems with randomly occurring packet dropouts," *Nonlinear Dyn.*, vol. 97, no. 2, pp. 1525–1540, 2019.

[29] G. Ran, C. Li, H. K. Lam, D. Li, and C. Han, "Event-based dissipative control of interval type-2 fuzzy Markov jump systems under sensor saturation and actuator nonlinearity," *IEEE Trans. Fuzzy Syst.*, 2020, doi:10.1109/TFUZZ.2020.3046335.

[30] X. Su, C. Wang, H. Chang, Y. Yang, and W. Assawinchaichote, "Event-triggered sliding mode control of networked control systems with Markovian jump parameters," *Automatica*, vol. 125, p. 109405, 2021.

[31] G. Ran, C. Li, S. Rathinasamy, C. Han, B. Wang, and J. Liu, "Adaptive event-triggered asynchronous control for interval type-2 fuzzy Markov jump systems with cyber-attacks," *IEEE Trans. Control Netw. Syst.*, 2022, doi:10.1109/TCNS.2022.314102.

[32] Y. Wang, Y. Xia, C. K. Ahn, and Y. Zhu, "Exponential stabilization of Takagi–Sugeno fuzzy systems with aperiodic sampling: An aperiodic adaptive event-triggered method," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 49, no. 2, pp. 444–454, 2018.

[33] R. Zhang, D. Zeng, J. H. Park, H. K. Lam, and S. Zhong, "Fuzzy adaptive event-triggered sampled-data control for stabilization of T-S fuzzy memristive neural networks with reaction-diffusion terms," *IEEE Trans. Fuzzy Syst.*, 2020, doi: 10.1109/TFUZZ.2020.2985334.

[34] J. Liu, G. Ran, Y. Huang, C. Han, Y. Yu, and C. Sun, "Adaptive event-triggered finite-time dissipative filtering for interval type-2 fuzzy Markov jump systems with asynchronous modes," *IEEE Trans. Cybern.*, 2021, doi: 10.1109/TCYB.2021.3053627.

[35] S. Wang, Z. Shu, and T. Chen, "Cooperative output regulation with mixed time-and event-triggered observers," *arXiv preprint arXiv:2105.02200*, 2021.

[36] H. Li, Z. Chen, L. Wu, H. K. Lam, and H. Du, "Event-triggered fault detection of nonlinear networked systems," *IEEE Trans. Cybern.*, vol. 47, no. 4, pp. 1041–1052, 2017.

[37] Y. Pan and G. H. Yang, "Event-triggered fault detection filter design for nonlinear networked systems," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 48, no. 11, pp. 1851–1862, 2018.

[38] X. Liu, X. Su, P. Shi, S. K. Nguang, and C. Shen, "Fault detection filtering for nonlinear switched systems via event-triggered communication approach," *Automatica*, vol. 101, pp. 365–376, 2019.

[39] Z. Ning, J. Yu, Y. Pan, and H. Li, "Adaptive event-triggered fault detection for fuzzy stochastic systems with missing measurements," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 4, pp. 2201–2212, 2018.

[40] X. G. Guo, X. Fan, and C. K. Ahn, "Adaptive event-triggered fault detection for interval type-2 T-S fuzzy systems with sensor saturation," *IEEE Trans. Fuzzy Syst.*, 2020, doi:10.1109/TFUZZ.2020.2997515.

[41] L. Zhang, H. Liang, Y. Sun, and C. K. Ahn, "Adaptive event-triggered fault detection scheme for semi-Markovian jump systems with output quantization," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 51, no. 4, pp. 2370–2381, 2021.

[42] P. Cheng, S. He, V. Stojanovic, X. Luan, and F. Liu, "Fuzzy fault detection for Markov jump systems with partly accessible hidden information: An event-triggered approach," *IEEE Trans. Cybern.*, 2021, doi: 10.1109/TCYB.2021.3050209.

[43] H. K. Lam, "A review on stability analysis of continuous-time fuzzy-model-based control systems: From membership-function-independent

to membership-function-dependent analysis," *Engineering Applications of Artificial Intelligence*, vol. 67, pp. 390–408, 2018.

Guangtao Ran (Student Member, IEEE) received the B.E. and M.E. degrees from Qiqihar University, Qiqihar, China in 2016 and 2019. He is currently pursuing the Ph.D. degree with Harbin Institute of Technology, Harbin, China. Now he is also a joint training student with the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB T6G 1H9, Canada.

His research interests include fuzzy control, reinforcement learning, networked control systems, multi-agent systems, and robust control.

Jian Liu (Member, IEEE) received his B.S. and Ph.D. degree from the School of Automation and Electrical Engineering, University of Science and Technology Beijing, Beijing, China, in 2015 and 2020, respectively.

From September 2017 to September 2018, he was a joint training student with the Department of Mathematics, Dartmouth College, Hanover, NH, USA. From 2020 to 2021, he was a Postdoctoral Fellow with the School of Automation, Southeast University, Nanjing, China, where he is currently an Associate Professor. His current research interests include multi-agent systems, nonlinear control, event-triggered control, fixed-time control.

Chuanjiang Li is currently a Professor in the Department of Control Science and Engineering, Harbin Institute of Technology, Harbin, China. He received the B.S. degree in automation from HIT. He received the M.S. and Ph.D. degrees in control science and engineering from HIT in 2003 and 2006, respectively. From 2011 to 2012, he was a visiting scholar at Curtin University, Western Australia. In 2014, he was a visiting scholar at Queen's University, Belfast, U.K. His current research interests include spacecraft attitude and orbit control, optimal control, nonlinear control and its applications, formation flying, and multi-agent systems.

Hak-Keung Lam (M'98-SM'12-F'20) received the B.Eng. (Hons.) and Ph.D. degrees from the Department of Electronic and Information Engineering, Hong Kong Polytechnic University, Hong Kong, in 1995 and 2000, respectively. From 2000 and 2005, he was with the Department of Electronic and Information Engineering, Hong Kong Polytechnic University as a Post-Doctoral Fellow and a Research Fellow, respectively. He joined as a Lecturer with Kings College London, London, U.K., in 2005, where he is currently a Reader. He has co-edited two edited volumes: Control of Chaotic Nonlinear Circuits (World Scientific, 2009) and Computational Intelligence and its Applications (World Scientific, 2012), and authored/coauthored three monographs: Stability Analysis of Fuzzy-Model-Based Control Systems (Springer, 2011), Polynomial Fuzzy Model Based Control Systems (Springer, 2016), and Analysis and Synthesis for Interval Type-2 Fuzzy-Model-Based Systems (Springer, 2016).

His current research interests include intelligent control and computational intelligence. Dr. Lam serves as the program committee member, the international advisory board member, the invited session chair, and the publication chair for various international conferences and a reviewer for various books, international journals, and international conferences. He was an Associate Editor for IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS PART II: EXPRESS BRIEFS and currently an Associate Editor of IEEE TRANSACTIONS ON FUZZY SYSTEMS, IET Control Theory and Applications, International Journal of Fuzzy Systems, Neurocomputing, and Nonlinear Dynamics, and a guest editor/editorial board for a number of international journals. He was named as a highly cited researcher by Web of Science.

Dongyu Li (Member, IEEE) received the B.S. and Ph.D. degree from Control Science and Engineering, Harbin Institute of Technology, China, in 2016 and 2020. He was a joint Ph.D. student with the Department of Electrical and Computer Engineering, National University of Singapore from 2017 to 2019, and a research fellow with the Department of Biomedical Engineering, National University of Singapore, from 2019 to 2021. He is currently an Associate Professor with the School of Cyber Science and Technology, Beihang University, China.

His research interests include networked system cooperation, adaptive systems, and robotic control.

Hongtian Chen (Member, IEEE) received the B.S. and M.S. degrees in School of Electrical and Automation Engineering from Nanjing Normal University, China, in 2012 and 2015, respectively; and he received the Ph.D. degree in College of Automation Engineering from Nanjing University of Aeronautics and Astronautics, China, in 2019.

He had ever been a Visiting Scholar at the Institute for Automatic Control and Complex Systems, University of Duisburg-Essen, Germany, in 2018. Now he is a Post-Doctoral Fellow with the Department of Chemical and Materials Engineering, University of Alberta, Canada. His research interests include process monitoring and fault diagnosis, data mining and analytics, machine learning, and quantum computation; and their applications in high-speed trains, new energy systems, and industrial processes.

Dr. Chen was a recipient of the Grand Prize of Innovation Award of Ministry of Industry and Information Technology of the People's Republic of China in 2019, the Excellent Ph.D. Thesis Award of Jiangsu Province in 2020, and the Excellent Doctoral Dissertation Award from Chinese Association of Automation (CAA) in 2020.