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Nonsingular Finite-Time Event-Triggered Fuzzy Control for Large-Scale Nonlinear Systems

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***Abstract*—This paper investigates the problem of event-based decentralized adaptive fuzzy output-feedback finite-time control for the large-scale nonlinear systems. The full-state tracking error constraints, unmeasured states and external disturbances are simultaneously considered in the controlled systems. The unknown auxiliary functions are modelled by using fuzzy logic systems (FLSs), and a state observer is established to estimate unmeasured states. By taking a new error transformation method based on prescribed performance functions (PPFs) and constructing corresponding barrier Lyapunov functions (BLFs), the predefined system error dynamic performance is ensured. Then, on the basis of the event-triggered control technique and the backstepping recursive design technique, a new event-based adaptive fuzzy nonsingular finite-time control strategy is proposed, and the "singularity" problem existing in backstepping design procedure is avoided. Moreover, by using the finite-time stability criterion, it is proven that the proposed control strategy can ensure the boundedness of the whole system variables and achieve all the state tracking errors evolve within the predesigned performance regions in finite time. Finally, the effectiveness of the proposed control strategy is verified by using some simulation results.**

***Index Terms*—Decentralized adaptive fuzzy control, finite-time control, event-triggered control, large-scale nonlinear systems, full-state tracking error constraints.**

I. Introduction

OWING to its practical significance and theoretical challenge, decentralized control of large-scale nonlinear systems has received extensive application in the control community. Previously, decentralized control was prevailingly focused on a class of nonlinear systems [1] with some matched conditions. To cope with the control issues of the large-scale nonlinear interconnected systems without possessing the matched conditions, many adaptive decentralized control schemes have been presented in [2]–[7] by fusing some novel design techniques with neural network (NN) control [8]–[13] or fuzzy control [14]–[27]. Among them, the state-feedback decentralized stabilization problem was solved in [2] for the interconnected nonlinear system with unmodeled dynamics. The authors in [3] proposed an adaptive fuzzy decentralized control method for the nonlinear large-scale system with actuator faults and unknown dead zones. It is notable that the controlled system states in [2] and [3] are required to be measurable. To decrease the conservatism of state-feedback control, the observer-based decentralized adaptive NN and fuzzy control schemes in [4] and [5] were presented, respectively. In [6], an observer-based adaptive decentralized fault-tolerant controller was constructed to stabilize the nonlinear large-scale system preceded by sensor and actuator faults.

To drive the system trajectories reach steady response from transient response quickly, massive efforts have been made to study the finite-time control design of nonlinear systems. Originally, the authors in [28] presented the Lyapunov theory of finite-time stability for a class of nonlinear systems. Following the idea of the seminal work, many finite-time stabilization results [29]–[36] on diverse nonlinear systems have been consecutively proposed. For example, an adaptive NN finite-time output feedback control strategy was devised in [29] for the quantized nonlinear system. Sui *et al.* [32] raised a finite-time filter decentralized control approach for the uncertain nonlinear large-scale systems in nonstrict-feedback structure. However, the established finite-time controllers in [29]–[33] cannot avoid the "singularity" problem in the backstepping control design framework. In view of this problem, the authors in [34] presented an adaptive finite-time fault-tolerant control algorithm for the multi-input and multi-output nonlinear systems. In [35], a finite-time command filter controller was designed for a class of single input single output (SISO) nonlinear systems. Although the "singularity" problem caused by the repeated derivative of virtual controllers have been discussed in [34] and [35], some matching conditions are demanded such as the observability of system states and no effect of external disturbances. Additionally, the restrictive condition of exponential power term ℓ in controller are always required, and the full-state tracking errors cannot be guaranteed to remain within the prescribed performance ranges.

On the other hand, the issues of constraints including state constraint, output constraint and error constraint have also attracted widespread concern, any transgression of constraints may result in performance degradations, hazards or system damages. To guarantee the constraints are never violated, different types of barrier Lyapunov functions (BLFs) have been presented in [37]–[40], and further applied to some practical nonlinear systems such as the robot system [41], active suspension system [42] and so on. Noting that the

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tracking errors in above references were only constrained indirectly via the restricted state variables. In [43], the constraint was straightway imposed on the output tracking error, and an error transformation control method based on prescribed performance function (PPFs) was provided for the feedback linearization nonlinear system. By taking new error transformation method, the PPFs-based BLFs were established in [44] and [45] to further meet the requirement of error performance constraint, and the "singularity" problem arising from the constrained PPF in [43] was eliminated. However, when the error transformation mechanisms in [44] and [45] are combined with finite-time control, there will be some obstacles, which drives us to adopt a new error transformation strategy in the backstepping-based finite-time control design.

Nowadays, the savings on computing and communication resources become particularly significant for the controlled systems both in theory and practice. Compared with the conventional time-sampling control, event-triggered control [46]–[52] can effectively reduce the heavy computational burden and the waste of communication resources. For a category of uncertain nonlinear systems, Xing *et al.* in [46] proposed three event-triggered controller update strategies with the fixed threshold, the relative threshold and the switching threshold. In [48], a novel adaptive event-triggered control method was given for nonstrict-feedback multi-agent systems subject to unknown disturbances. The adaptive fuzzy event-triggered control problem in [51] was addressed for the state-constrained stochastic nonlinear system with actuator faults. It should be mentioned that the aforesaid control schemes cannot be directly used in the large-scale nonlinear systems with full-state tracking error constraints.

Motivated by the aforementioned discussions, a novel event-based fuzzy adaptive nonsingular finite-time control strategy is proposed for the error-constrained nonlinear large-scale systems in this paper. Compared with the existing results, the main contributions of this paper are summarized as follows.

i) A new PPFs-based error transformation method is given and first applied to the large-scale nonlinear systems, and the full system state tracking errors after transformation are restricted to a positive interval. An observer-based adaptive event-triggered finite-time controller is constructed such that the "singularity" problem arising form the repeated differentiation of the finite-time controllers in [29]–[32] cannot be caused, and the restrictive condition of exponential power terms in the finite-time controllers can be removed. To decrease the conservatism of control strategy, the unmeasurable state variables and unknown external disturbances are considered in the researched systems.

ii) In the previous works [43]–[45], the constraints are only applied to the output tracking errors, the constrained problem of other state tracking errors is overlooked, and the selected control systems are some uncomplex SISO nonlinear systems. This paper is concerned with the problem that full-state tracking errors are constrained, which is more comprehensive than the works mentioned above. Additionally, different from some existing results in [43]–[45], the symmetric BLFs are constructed to further meet the requirement of state tracking error constraints, which is also suitable for no constrained systems, without changing the control structure of the BLFs.

iii) For every subsystem, the number of adaptive parameters needed to be estimated online is reduced to two, and by incorporating BLF technique with event-triggered control theory, the designed control strategy can guarantee that system trajectories possess good transient performance in finite time, the full-state tracking error constraints are satisfied, and the computational burden and communication resources are effectively saved.

II. Problem Formulation and Preliminaries

A. System Description and Basic Assumptions

The following large-scale nonlinear systems comprising $\mathcal{N}$ subsystems are considered:

$$\begin{cases} \dot{x}_{i,j} = \phi_{i,j}(\breve{x}_{i,j}) + x_{i,j+1} + \mathcal{H}_{i,j}(\breve{y}) + d_{i,j}(t), \\ \dot{x}_{i,n_i} = \phi_{i,n_i}(\breve{x}_{i,n_i}) + u_i + \mathcal{H}_{i,n_i}(\breve{y}) + d_{i,n_i}(t), \\ y_i = x_{i,1},\ i = 1,2,...,\mathcal{N},\ j = 1,2,...,n_i - 1 \end{cases} \tag{1}$$

where $x_i = [x_{i,1}, x_{i,2}, \ldots, x_{i,n_i}]^T \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}$ and $y_i \in \mathbb{R}$ are the system state vectors, input vector and output vector for ith subsystem, respectively. $\breve{x}_{i,j} = [x_{i,1}, x_{i,2}, \ldots, x_{i,j}]^T \in \mathbb{R}^j$, $j = 1,2,...,n_i$, $\breve{y} = [y_1, y_2, ..., y_{\mathcal{N}}]^T \in \mathbb{R}^{\mathcal{N}}$. $\phi_{i,j}(\cdot)$ denotes unknown smooth nonlinear function, $\mathcal{H}_{i,j}(\cdot)$ represents the unknown smooth interconnected term which exists in every subsystem. $d_{i,j}(t)$ is the unknown bounded external disturbance varying with time t. The whole state variables are unmeasurable except $x_{i,1}$.

To this end, the following assumptions will be imposed on the controlled system (1).

Assumption 1: The unknown smooth nonlinear interconnected term $\mathcal{H}_{i,j}(\breve{y})$ meets

$$|\mathcal{H}_{i,j}(\breve{y})|^2 \leq \sum_{k=1}^{p_{i,j}} \sum_{l=1}^{\mathcal{N}} \lambda_{l,j,i}^k |y_l|^k \tag{2}$$

where $\lambda_{l,j,i}^k$ is an unknown constant, and $p = \max\{p_{i,j} |\ i = 1,2,...,\mathcal{N},\ j = 1,2,...,n_i\}$, where $p_{i,j}$ is a known positive constant.

Assumption 2: External disturbances $d_{i,j}(t)$ $(j = 1,...,n_i)$ satisfy $|d_{i,j}(t)| \leq \bar{d}_{i,j}$ with $\bar{d}_{i,j}$ being a constant. The reference trajectory $y_{i,r}(t)$ and its time derivatives $\dot{y}_{i,r}(t)$, ..., $y_{i,r}^{(n_i)}(t)$ are continuous and bounded.

Lemma 1: [2] Let $\tilde{F}(\Lambda)$ be any continuous function specified on a bounded closed set Ω_Λ. There is a fuzzy logic system (FLS) $W^T\Xi(\Lambda)$ for an expected precision ε such that

$$\sup_{\Lambda \in \Omega_\Lambda} |\tilde{F}(\Lambda) - W^T\Xi(\Lambda)| \leq \varepsilon \tag{3}$$

where $W = [w_1, w_2, ..., w_{\mathcal{N}}]^T \in \mathbb{R}^{\mathcal{N}}$ is the ideal constant weight vector with $\mathcal{N} > 1$ being the number of the rules, and $\Xi(\Lambda) = [s_1(\Lambda), s_2(\Lambda), ..., s_{\mathcal{N}}(\Lambda)]^T / \sum_{i=1}^{\mathcal{N}} s_i(\Lambda)$ is the basic function vector with $s_i(\Lambda)$ being Gaussian functions, that is

$$s_i(\Lambda) = \exp\left[\frac{-(\Lambda - \varsigma_i)^T(\Lambda - \varsigma_i)}{{\tau_i}^2}\right],\ i = 1,...,\mathcal{N} \tag{4}$$

with $\varsigma_i = [\varsigma_{i1}, ..., \varsigma_{in_i}]^T$ being the center vector and τ_i being the Gaussian function width.

B. Finite-Time Stability

Definition 1: [29] The equilibrium $\varsigma = 0$ of the nonlinear system $\dot{\varsigma} = f(\varsigma)$ is semi-global practical finite-time stable (SGPFS), if for all initial values $\varsigma(t_0) = \varsigma_0$ hold, then there exist $\epsilon > 0$ and settling time $T(\epsilon, \varsigma_0) < \infty$ to make $||\varsigma(t)|| < \epsilon$, for all $t \geq t_0 + T$.

Lemma 2: [30] Assume there exist a positive definite function $\mathcal{V}(\varsigma)$ with scalars $\mathcal{C}_1 > 0$, $1 > \ell > 0$, and $\mathcal{C}_2 > 0$, if the nonlinear system $\dot{\varsigma} = f(\varsigma)$ meets

$$\dot{\mathcal{V}}(\varsigma) \leq -\mathcal{C}_1 \mathcal{V}^{\ell}(\varsigma) + \mathcal{C}_2,\ t \geq 0, \tag{5}$$

and define the settling time T^* as

$$T^* = \frac{1}{(1-\ell)\kappa \mathcal{C}_1} \left[\mathcal{V}^{1-\ell}(\varsigma(0)) - \left(\frac{\mathcal{C}_2}{(1-\kappa)\mathcal{C}_1} \right)^{1-\ell/\ell} \right] \tag{6}$$

where $\mathcal{V}(\varsigma(0)$ is the initial value of $\mathcal{V}(\varsigma)$, and $0 < \kappa < 1$ is any constant. Then, the system $\dot{\varsigma} = f(\varsigma)$ is SGPFS for $\forall t \geq T^*$.

C. State Observer Design

To estimate unmeasured system state vectors $x_{i,2}, \ldots, x_{i,n_i}$, the following observer is established [4]:

$$\dot{\hat{x}}_{i,j} = \hat{x}_{i,j+1} + l_{i,j}(x_{i,1} - \hat{x}_{i,1}),\ j = 1, 2, ..., n_i \tag{7}$$

where $\hat{x}_{i,n_i+1} = u_i$. $\hat{x}_{i,j}$ is the estimate of $x_{i,j}$, $l_{i,j}$ denotes the positive design parameter such that

$$A_i = \begin{bmatrix} -l_{i,1} & & \\ \vdots & I_{n_i-1} & \\ -l_{i,n_i} & 0\ldots & 0 \end{bmatrix} \tag{8}$$

is a strict Hurwitz matrix. Therefore, given matrix $Q_i = Q_i^T > 0$, there exists the other matrix $\Phi_i = \Phi_i^T > 0$ satisfying

$$A_i^T \Phi_i + \Phi_i A_i = -Q_i. \tag{9}$$

Define an observer error vector as

$$\varrho_i = x_i - \hat{x}_i \tag{10}$$

where $\hat{x}_i = [\hat{x}_{i,1}, \ldots, \hat{x}_{i,n_i}]^T$ and $\varrho_i = [\varrho_{i,1}, ..., \varrho_{i,n_i}]^T$ with $\varrho_{i,j} = \hat{x}_{i,j} - x_{i,j}$, $j = 1, 2, ..., n_i$.

Based on (1) and (7), the observer error meets

$$\dot{\varrho}_i = A_i \varrho_i + \phi_i(x_i) + \mathcal{H}_i(\breve{y}) + d_i(t) \tag{11}$$

where $\phi_i(x_i) = [\phi_{i,1}(x_{i,1}), ..., \phi_{i,n_i}(x_{i,n_i})]^T$, $\mathcal{H}_i(\breve{y}) = [\mathcal{H}_{i,j}(\breve{y}), ..., \mathcal{H}_{i,n_i}(\breve{y})]^T$, and $d_i(t) = [d_{i,1}(t), ..., d_{i,n_i}(t)]^T$.

For observer (7), we first define the Lyapunov function candidate as follows $\mathcal{V}_0 = \sum_{i=1}^{\mathcal{N}} \varrho_i^T \Phi_i \varrho_i$. Then, we can obtain

$$\begin{aligned} \dot{\mathcal{V}}_0 \quad = \quad & \sum_{i=1}^{\mathcal{N}} \{ \varrho_i^T (A_i^T \Phi_i + \Phi_i A_i) \varrho_i + 2\varrho_i^T \Phi_i [\phi_i(x_i) \\ & + \mathcal{H}_i(\breve{y}) + d_i(t)] \}. \end{aligned} \tag{12}$$

As function $\phi_i(x_i)$ is unknown, for $\forall \varepsilon_{i,0} > 0$, there is a FLS $W_{i,0}^T \Xi_{i,0}(\Lambda_{i,0})$ such that

$$\phi_i(x_i) = W_{i,0}^T \Xi_{i,0}(\Lambda_{i,0}) + \delta_{i,0}(\Lambda_{i,0}),\ ||\delta_{i,0}(\Lambda_{i,0})|| \leq \varepsilon_{i,0} \tag{13}$$

where $\Lambda_{i,0} = x_i \in \Omega_{\Lambda_{i,0}}$. $W_{i,0} = [W_{i,10}, ..., W_{i,n_i 0}]$, $\Xi_{i,0}(\Lambda_{i,0}) = \Xi_{i,0} = [\Xi_{i,10}, ..., \Xi_{i,n_i 0}]^T$, $\delta_{i,0}(\Lambda_{i,0}) = \delta_{i,0} = [\delta_{i,10}, ..., \delta_{i,n_i 0}]^T$ and $\varepsilon_{i,0} = \sqrt{\varepsilon_{i,10}^2 + ... + \varepsilon_{i,n_i 0}^2}$. According to Young's inequality, Assumption 1 and the property of $\Xi_{i,0}^T \Xi_{i,0} \leq 1$, one has

$$\begin{aligned} 2\varrho_i^T \Phi_i \phi_i(x_i) \quad & = \quad 2\varrho_i^T \Phi_i [W_{i,0}^T \Xi_{i,0}(\Lambda_{i,0}) + \delta_{i,0}(\Lambda_{i,0})] \\ & \leq \quad 2||\varrho_i||^2 + ||\Phi_i||^2 \left(\Theta_i + \varepsilon_{i,0}^2 \right), \end{aligned} \tag{14}$$

$$2\varrho_i^T \Phi_i d_i(t) \quad \leq \quad ||\varrho_i||^2 + ||\Phi_i||^2 ||\bar{d}_i(t)||^2, \tag{15}$$

$$\begin{aligned} 2\varrho_i^T \Phi_i \mathcal{H}_i(\breve{y}) \quad \leq \quad & \sum_{k=1}^{p} pN 2^{2k} \sum_{l=1}^{\mathcal{N}} \sum_{j=1}^{n_i} \left(\lambda_{l,j,i}^k \right)^2 (|y_{i,r}|^{2k} \\ & + |z_{i,1}|^{2k}) + ||\varrho_i||^2 ||\Phi_i||^2 \end{aligned} \tag{16}$$

where $\Theta_i = \max\{||W_{i,0}||^2, i = 1, ..., \mathcal{N}\}$. Combining with (14)-(16), (12) is rewritten as

$$\begin{aligned} \dot{\mathcal{V}}_0 \quad \leq \quad & \sum_{i=1}^{\mathcal{N}} [-\pi_{i,0} ||\varrho_i||^2 + \sigma_{i,0}] + \sum_{i=1}^{\mathcal{N}} \sum_{k=1}^{p} \lambda_{i,k} \\ & \times (|y_{i,r}|^{2k} + |z_{i,1}|^{2k}) \end{aligned} \tag{17}$$

where $\lambda_{i,k} = pN 2^{2k} \sum_{l=1}^{\mathcal{N}} \sum_{j=1}^{n_i} \left(\lambda_{l,j,i}^k \right)^2$, $\pi_{i,0} = -||\Phi_i||^2 - 3 + \lambda_{\min}(Q_i)$, and $\sigma_{i,0} = ||\Phi_i||^2 \left(\Theta_i + \varepsilon_{i,0}^2 \right) + ||\Phi_i||^2 ||\bar{d}_i(t)||^2$.

D. Performance Function and Error Transformation

In this subsection, we will introduce the preliminary knowledge of the error constraint. First, we define the transformation of coordinates as $z_{i,1} = x_{i,1} - y_{i,r}$, and $z_{i,j} = \hat{x}_{i,j} - \alpha_{i,j-1}$, $j = 2, ..., n_i$, where $\alpha_{i,j-1}$ $(j = 2, 3, ..., n_i)$ are the virtual control signals to be designed later.

Definition 2: [43] The prescribed performance function $\mu_{i,j}(t)$ is a smooth strictly decreasing function which meets $\lim_{t \to \infty} \mu_{i,j}(t) = \mu_{i,j\infty} > 0$, and can be expressed as $\mu_{i,j}(t) = (\mu_{i,j0} - \mu_{i,j\infty}) e^{-\varkappa_i t} + \mu_{i,j\infty}$, where $\varkappa_i > 0$, $\mu_{i,j0} > 0$ and $\mu_{i,j\infty} > 0$ are the appropriate constants.

To ensure the transient performance and steady-state bounds, the prescribed constraint conditions are described as

$$\begin{cases} -H_{Li,j} \mu_{i,j}(t) < z_{i,j}(t) < \mu_{i,j}(t) \text{ when } z_{i,j}(0) \geq 0, \\ -\mu_{i,j}(t) < z_{i,j}(t) < H_{hi,j} \mu_{i,j}(t) \text{ when } z_{i,j}(0) < 0 \end{cases}$$

where $H_{Li,j}$, $H_{hi,j} \in (0, 1]$ are design parameters, and the maximum overshoot and undershoot are restricted by $-H_{Li,j} \mu_{i,j}(0), H_{hi,j} \mu_{i,j}(0)$ and $\mu_{i,j}(0)$.

Then, a new transformed error dynamic can be defined as

$$e_{i,j}(t) \quad = \quad \frac{z_{i,j}(t)}{\eta_{i,j}(t)} + \rho_{i,j}, \tag{18}$$

$$\eta_{i,j} \quad = \quad \zeta \eta_{Li,j} + (1-\zeta) \eta_{hi,j} \tag{19}$$

where $\rho_{i,j} > 0$ is the design parameter, $\zeta = 1$ if $z_{i,j}(t) \geq 0$, $\zeta = 0$ if $z_{i,j}(t) < 0$, and

$$\begin{cases} \text{when } z_{i,j}(0) \geq 0, \\ \eta_{Li,j} = \mu_{i,j}(t) \text{ and } \eta_{hi,j} = -H_{Li,j} \mu_{i,j}(t), \\ \text{when } z_{i,j}(0) < 0 \\ \eta_{Li,j} = H_{hi,j} \mu_{i,j}(t) \text{ and } \eta_{hi,j} = -\mu_{i,j}(t). \end{cases}$$

Considering that $\rho_{i,j} \leq e_{i,j}(0) < 1 + \rho_{i,j} = \bar{\rho}_{i,j}$, based on the above formulas, the inequality $0 < \rho_{i,j} \leq e_{i,j} < \bar{\rho}_{i,j}$ is obtained.

In order to ensure the requirement of full-state tracking error constraints, the following positive definite symmetric barrier Lyapunov functions (SBLFs) will be used in the backstepping-based control design procedure, which is constructed as follows

$$\mathcal{V}_{i,j} = \frac{1}{2}\frac{e_{i,j}^2}{\bar{\rho}_{i,j}^2 - e_{i,j}^2},\ i = 1, ..., \mathcal{N},\ j = 1, ..., n_i. \tag{20}$$

Remark 1: Noting that the designed error transformation dynamic (18) and the designed SBLFs (20) are different from the existing results in [43]–[45]. If the available PPFs-based error constrained techniques and SBLFs in [43]–[45] are used in our results, the devised finite-time virtual controllers and actual controller will cause the "singularity" issue when $e_{i,j} = 0$. Therefore, a modified error transformation technique and SBLFs are developed for such issue. By contrast, the definitions of (18) and (20) can be combined with greater control methods, which is more general than the above results. Moreover, it is clear to see that $\mathcal{V}_{i,j}$ is positive definite, and C^1 is continuous in the region $\rho_{i,j} \leq e_{i,j} < \bar{\rho}_{i,j}$. Thus, $\mathcal{V}_{i,j}$ is a valid Lyapunov function.

III. Event-Triggered Finite-Time Control Design and Stability Analysis

In this section, a novel adaptive event-triggered finite-time control strategy will be developed. In the controller design procedure, the unknown functions $\breve{F}_{i,j}(\Lambda_{i,j})$ $(j = 1, ..., n_i)$ at step j will be modeled by FLSs. We first define unknown parameter as $\Theta_i = \max\{||W_{i,0}||^2,\ ||W_{i,j}||^2, i = 1, ..., \mathcal{N}, j = 1, ..., n_i\}$, $\hat{\Theta}_i$ is the estimation of Θ_i, and $\tilde{\Theta}_i = \Theta_i - \hat{\Theta}_i$. Secondly, the other unknown parameter β_i is defined as $\beta_i = \max_{1\leq k\leq p}\{\lambda_{i,k} + n_i\lambda_{1,i,k}\}$, $\hat{\beta}_i$ is the estimation of β_i and satisfies $\beta_i - \hat{\beta}_i = \tilde{\beta}_i$. Then, in the light of the Lemma 1 in [29], for any initial condition $\hat{\Theta}_i(t_0) \geq 0$, $\hat{\beta}_i(t_0) \geq 0$ and $\forall t \geq t_0$, we have $\hat{\Theta}_i(t) \geq 0$ and $\hat{\beta}_i(t) \geq 0$.

Step 1. Based on the definition of system error $z_{i,1}$, the time derivative of $z_{i,1}$ is

$$\dot{z}_{i,1} = \phi_{i,1} + z_{i,2} + \alpha_{i,1} + \varrho_{i,2} + \mathcal{H}_{i,1}(\breve{y}) + d_{i,1} - \dot{y}_{i,r}. \tag{21}$$

Take the following Lyapunov function

$$\mathcal{V}_1 = \mathcal{V}_0 + \sum_{i=1}^{\mathcal{N}}\left(\frac{1}{2}\frac{e_{i,1}^2}{\bar{\rho}_{i,1}^2 - e_{i,1}^2} + \frac{1}{2r_{i,a}}\tilde{\beta}_i^2 + \frac{1}{2r_{i,b}}\tilde{\Theta}_i^2\right). \tag{22}$$

Then, the time derivative of $\mathcal{V}_1$ is

$$\begin{aligned}\dot{\mathcal{V}}_1 \quad = \quad & \dot{\mathcal{V}}_0 + \sum_{i=1}^{\mathcal{N}}\{s_{i,1}[\phi_{i,1} + z_{i,2} + \alpha_{i,1} + \varrho_{i,2} + \mathcal{H}_{i,1}(\breve{y}) \\ & + d_{i,1} - \dot{y}_{i,r} - (e_{i,1} - \rho_{i,1})\,\dot{\eta}_{i,1}] \\ & - \frac{1}{r_{i,a}}\tilde{\beta}_i\dot{\hat{\beta}}_i - \frac{1}{r_{i,b}}\tilde{\Theta}_i\dot{\hat{\Theta}}_i\Big\}\end{aligned} \tag{23}$$

where $s_{i,1} = \bar{\rho}_{i,1}^2 e_{i,1}/(\bar{\rho}_{i,1}^2 - e_{i,1}^2)^2\eta_{i,1}$. Based on Assumption 1, one gets

$$s_{i,1}(z_{i,2} + \varrho_{i,2} + d_{i,1}) \leq \frac{5s_{i,1}^2}{4} + \frac{2||\varrho_i||^2 + z_{i,2}^2 + \bar{d}_{i,1}^2}{2}, \tag{24}$$

$$s_{i,1}\mathcal{H}_{i,1}(\breve{y}) \leq \frac{s_{i,1}^2}{4} + \sum_{k=1}^{p}\lambda_{1,i,k}(|y_{i,r}|^{2k} + |z_{i,1}|^{2k}) \tag{25}$$

where $\lambda_{1,i,k} = 2^{2k}pN\sum_{l=1}^{\mathcal{N}}(\lambda_{l,1,i}^k)^2$. Define the unknown auxiliary function as

$$\begin{aligned}\breve{F}_{i,1}(\Lambda_{i,1}) \quad = \quad & \phi_{i,1} - \dot{y}_{i,r} - (e_{i,1} - \rho_{i,1})\,\dot{\eta}_{i,1} + 2s_{i,1} \\ & + \frac{1}{s_{i,1}}\hat{\beta}_i\sum_{k=1}^{p}|z_{i,1}|^{2k}\end{aligned} \tag{26}$$

with $\Lambda_{i,1} = [x_{i,1}, y_{i,r}, \dot{y}_{i,r}, \eta_{i,1}, \dot{\eta}_{i,1}]^T$. Substituting (24)-(26) into (23), it yields

$$\begin{aligned}\dot{\mathcal{V}}_1 \quad \leq \quad & \sum_{i=1}^{\mathcal{N}}[-(\pi_{i,0} - 1)||\varrho_i||^2] + \sum_{i=1}^{\mathcal{N}}\left[s_{i,1}(\breve{F}_{i,1} + \alpha_{i,1})\right] \\ & + \sum_{i=1}^{\mathcal{N}}\sum_{k=1}^{p}(\lambda_{i,k} + \lambda_{1,i,k})(|y_{i,r}|^{2k} + |z_{i,1}|^{2k}) \\ & + \sum_{i=1}^{\mathcal{N}}\left(\tilde{\beta}_i\sum_{k=1}^{p}|z_{i,1}|^{2k} - \beta_i\sum_{k=1}^{p}|z_{i,1}|^{2k}\right) \\ & + \sum_{i=1}^{\mathcal{N}}\left(\frac{z_{i,2}^2}{2} - \frac{s_{i,1}^2}{2} - \frac{1}{r_{i,a}}\tilde{\beta}_i\dot{\hat{\beta}}_i\right. \\ & \left. - \frac{1}{r_{i,b}}\tilde{\Theta}_i\dot{\hat{\Theta}}_i + \sigma_{i,0} + \frac{\bar{d}_{i,1}^2}{2}\right).\end{aligned} \tag{27}$$

Because $\breve{F}_{i,1}$ includes unknown function $\phi_{i,1}$, it cannot be implemented in practice. Thus, from Lemma 1, there is a FLS $W_{i,1}^T\Xi_{i,1}(\Lambda_{i,1})$ for given parameter $\varepsilon_{i,1} > 0$ such that

$$\breve{F}_{i,1} = W_{i,1}^T\Xi_{i,1}(\Lambda_{i,1}) + \delta_{i,1}(\Lambda_{i,1}),\ |\delta_{i,1}(\Lambda_{i,1})| \leq \varepsilon_{i,1}. \tag{28}$$

Then, the following inequality can be obtained

$$s_{i,1}\breve{F}_{i,1} \leq \frac{1}{2a_{i,1}^2}s_{i,1}^2\Theta_i\Xi_{i,1}^T\Xi_{i,1} + \frac{a_{i,1}^2 + s_{i,1}^2 + \varepsilon_{i,1}^2}{2} \tag{29}$$

where $a_{i,1} > 0$ is the design parameter. Design the virtual control signal $\alpha_{i,1}$ as

$$\alpha_{i,1} = -c_{i,1}\frac{\eta_{i,1}e_{i,1}^{2\ell-1}}{\bar{\rho}_{i,1}^2(\bar{\rho}_{i,1}^2 - e_{i,1}^2)^{\ell-2}} - \frac{1}{2a_{i,1}^2}s_{i,1}\hat{\Theta}_i\Xi_{i,1}^T\Xi_{i,1} \tag{30}$$

where $c_{i,1} > 0$ is the design parameter. Then, combining (29) and (30), (27) is changed into

$$\begin{aligned}\dot{\mathcal{V}}_1 \quad \leq \quad & \sum_{i=1}^{\mathcal{N}}\Bigg[-\pi_{i,1}||\varrho_i||^2 - c_{i,1}\frac{e_{i,1}^{2\ell}}{(\bar{\rho}_{i,1}^2 - e_{i,1}^2)^{\ell}} + \frac{z_{i,2}^2}{2} \\ & + \frac{1}{r_{i,b}}\tilde{\Theta}_i\left(\frac{r_{i,b}}{2a_{i,1}^2}s_{i,1}^2\Xi_{i,1}^T\Xi_{i,1} - \dot{\hat{\Theta}}_i\right) \\ & + \sum_{k=1}^{p}(\lambda_{i,k} + \lambda_{1,i,k})(|y_{i,r}|^{2k} + |z_{i,1}|^{2k}) \\ & + \frac{1}{r_{i,a}}\tilde{\beta}_i\left(r_{i,a}\sum_{k=1}^{p}|z_{i,1}|^{2k} - \dot{\hat{\beta}}_i\right) \\ & - \beta_i\sum_{k=1}^{p}|z_{i,1}|^{2k} + \sigma_{i,0} + \bar{D}_{i,1}\Bigg]\end{aligned} \tag{31}$$

where $\pi_{i,1} = \pi_{i,0} - 1$ and $\bar{D}_{i,1} = \frac{1}{2}(\varepsilon_{i,1}^2 + \bar{d}_{i,1}^2 + a_{i,1}^2)$.

Remark 2: Owing to that the tracking error dynamic $z_{i,1}$ is transformed as error $e_{i,1}$ with $\rho_{i,1} \le e_{i,1} < \bar{\rho}_{i,1}$ by some prescribed constraint conditions, the "singularity" cannot be caused from the terms $e_{i,1}^{2\ell-1}$ and $(\bar{\rho}_{i,1}^2 - e_{i,1}^2)^{\ell-2}$ in (30) and the time derivatives of them in the next step. To avoid such problem, this PPFs-based error transformation method is utilized throughout this paper.

Remark 3: In addition, the values of ℓ in [29]–[35] are always defined as $\ell = (2s-1)/(2s+1)$ $(s \ge 2,\ s \in \mathcal{N}^+)$ to keep the "singularity" problem of controllers from occurring. In this paper, the restrictive condition of ℓ is removed via the combination of new error transformation technique, ℓ can be taken any in the interval $(0,1)$ theoretically. In the subsequent simulation, the value of ℓ can be chosen as $\ell = 0.99$, which is different from [29].

Step j $(j = 2, ..., n_i - 1)$. Based on the coordinate transformation $z_{i,j} = \hat{x}_{i,j} - \alpha_{i,j-1}$ and (10), the time derivative of $z_{i,j}$ can be obtained

$$\dot{z}_{i,j} = z_{i,j+1} + \alpha_{i,j} + l_{i,j}\varrho_{i,1} - \dot{\alpha}_{i,j-1} \tag{32}$$

where

$$\begin{aligned}\dot{\alpha}_{i,j-1} &= \Upsilon_{i,j-1} + \sum_{m=2}^{j-1}\frac{\partial \alpha_{i,j-1}}{\partial \hat{x}_{i,m}} l_{i,m}\varrho_{i,1} + \frac{\partial \alpha_{i,j-1}}{\partial \hat{\Theta}_i}\dot{\hat{\Theta}}_i \\ &\quad + \frac{\partial \alpha_{i,j-1}}{\partial y_i}[\varrho_{i,2} + \mathcal{H}_{i,1}(\breve{y}) + d_{i,1}],\end{aligned} \tag{33}$$

$$\begin{aligned}\Upsilon_{i,j-1} &= \sum_{m=1}^{j-1}\frac{\partial \alpha_{i,j-1}}{\partial \hat{x}_{i,m}}\hat{x}_{i,m+1} + \frac{\partial \alpha_{i,j-1}}{\partial y_i}(\hat{x}_{i,2} + \phi_{i,1}) \\ &\quad \frac{\partial \alpha_{i,j-1}}{\partial \hat{\beta}_i}\dot{\hat{\beta}} + \sum_{m=1}^{j}\frac{\partial \alpha_{i,j-1}}{\partial y_{i,r}^{(m-1)}} y_{i,r}^{(m)} \\ &\quad + \sum_{m=1}^{j}\frac{\partial \alpha_{i,j-1}}{\partial \eta_{i,1}^{(m-1)}}\eta_{i,1}^{(m)} + \sum_{m=1}^{j-1}\frac{\partial \alpha_{i,j-1}}{\partial \eta_{i,2}^{(m-1)}}\eta_{i,2}^{(m)} \\ &\quad + ... + \sum_{m=1}^{2}\frac{\partial \alpha_{i,j-1}}{\partial \eta_{i,j-1}^{(m-1)}}\eta_{i,j-1}^{(m)}.\end{aligned}$$

Consider the following Lyapunov function

$$\mathcal{V}_j = \mathcal{V}_{j-1} + \sum_{i=1}^{\mathcal{N}}\left(\frac{1}{2}\frac{e_{i,j}^2}{\bar{\rho}_{i,j}^2 - e_{i,j}^2}\right). \tag{34}$$

The time derivative of $\mathcal{V}_j$ yields

$$\begin{aligned}\dot{\mathcal{V}}_j &= \dot{\mathcal{V}}_{j-1} + \sum_{i=1}^{\mathcal{N}}\{s_{i,j}[z_{i,j+1} + l_{i,j}\varrho_{i,1} - (e_{i,j} - \rho_{i,j})\dot{\eta}_{i,j} \\ &\quad + \alpha_{i,j} - \sum_{m=2}^{j-1}\frac{\partial \alpha_{i,j-1}}{\partial \hat{x}_{i,m}} l_{i,m}\varrho_{i,1} - \frac{\partial \alpha_{i,j-1}}{\partial \hat{\Theta}_i}\dot{\hat{\Theta}}_i \\ &\quad - \Upsilon_{i,j-1} - \frac{\partial \alpha_{i,j-1}}{\partial y_i}(\varrho_{i,2} + \mathcal{H}_{i,1}(\breve{y}) + d_{i,1})\Big]\Big\}\end{aligned} \tag{35}$$

where $s_{i,j} = \bar{\rho}_{i,j}^2 e_{i,j}/(\bar{\rho}_{i,j}^2 - e_{i,j}^2)^2\eta_{i,j}$. In the light of Young's inequality, one gets

$$\begin{aligned}& s_{i,j}(z_{i,j+1} + l_{i,j}\varrho_{i,1}) - s_{i,j}\frac{\partial \alpha_{i,j-1}}{\partial y_i}(\varrho_{i,2} + d_{i,1}) \\ &\le s_{i,j}^2 l_{i,j}^2 + \frac{s_{i,j}^2 + z_{i,j+1}^2}{2} + \frac{||\varrho_i||^2}{4} \\ &\quad + s_{i,j}^2\left(\frac{\partial \alpha_{i,j-1}}{\partial y_i}\right)^2 + \frac{||\varrho_i||^2 + \bar{d}_{i,1}^2}{2},\end{aligned} \tag{36}$$

$$\begin{aligned}& -s_{i,j}\frac{\partial \alpha_{i,j-1}}{\partial y_i}\mathcal{H}_{i,1} - s_{i,j}\sum_{m=2}^{j-1}\frac{\partial \alpha_{i,j-1}}{\partial \hat{x}_{i,m}} l_{i,m}\varrho_{i,1} \\ &\le \sum_{k=1}^{p}\lambda_{1,i,k}(|y_{i,r}|^{2k} + |z_{i,1}|^{2k}) + \frac{s_{i,j}^2}{4}\left(\frac{\partial \alpha_{i,j-1}}{\partial y_i}\right)^2 \\ &\quad s_{i,j}^2\left(\sum_{m=2}^{j-1}\frac{\partial \alpha_{i,j-1}}{\partial \hat{x}_{i,m}} l_{i,m}\right)^2 + \frac{||\varrho_i||^2}{4}.\end{aligned} \tag{37}$$

Define the other unknown auxiliary function $\breve{F}_{i,j}(\Lambda_{i,j})$ as

$$\begin{aligned}\breve{F}_{i,j}(\Lambda_{i,j}) &= -\Upsilon_{i,j-1} - (e_{i,j} - \rho_{i,j})\dot{\eta}_{i,j} + s_{i,j}(l_{i,j}^2 + 1) \\ &\quad + \frac{5s_{i,j}}{4}\left(\frac{\partial \alpha_{i,j-1}}{2\partial y_i}\right)^2 + \frac{1}{2s_{i,j}}z_{i,j}^2 \\ &\quad + s_{i,j}\left(\sum_{m=1}^{j-1}\frac{\partial \alpha_{i,j-1}}{\partial \hat{x}_{i,m}} l_{i,m}\right)^2 - \Psi_{i,j},\end{aligned} \tag{38}$$

$$\begin{aligned}\Psi_{i,j} &= \sum_{m=1}^{j-1}\frac{\partial \alpha_{i,j-1}}{\partial \hat{\Theta}_i}\frac{r_{i,b}}{2a_{i,m}^2}s_{i,m}^2\Xi_{i,m}^T\Xi_{i,m} \\ &\quad - \sum_{m=2}^{j}\frac{r_{i,b}}{2a_{i,j}^2}s_{i,j}\left|s_{i,m}\frac{\partial \alpha_{i,m-1}}{\partial \hat{\Theta}_i}\right| \\ &\quad - r_{i,0}\hat{\Theta}_i\frac{\partial \alpha_{i,j-1}}{\partial \hat{\Theta}_i}\end{aligned}$$

where $\Lambda_{i,j} = [\bar{\eta}_{i,1}^{(j)},\ \bar{\eta}_{i,2}^{(j-1)}, ...,\ \bar{\eta}_{i,j}^{(1)},\ \hat{\Theta}_i,\ \breve{x}_{i,j}^T,\ \bar{y}_{i,r}^{(j)T}]^T$ with $\bar{y}_{i,r}^{(j)} = [y_{i,r}, y_{i,r}^{(1)}, ..., y_{i,r}^{(j)}]^T$, $\bar{\eta}_{i,1}^{(j)} = [\eta_{i,1}, \eta_{i,1}^{(1)}, ..., \eta_{i,1}^{(j)}]^T$ and $\bar{\eta}_{i,2}^{(j-1)} = [\eta_{i,2}, \eta_{i,2}^{(1)}, ..., \eta_{i,2}^{(j-1)}]^T, ..., \bar{\eta}_{i,j}^{(1)} = [\eta_{i,j}, \eta_{i,j}^{(1)}]^T$.

From (36)-(38), (35) can be rewritten as

$$\begin{aligned}\dot{\mathcal{V}}_j &\le \dot{\mathcal{V}}_{j-1} + \sum_{i=1}^{\mathcal{N}}\Big[s_{i,j}(\breve{F}_{i,j} + \alpha_{i,j}) - \frac{s_{i,j}^2 + z_{i,j+1}^2}{2} \\ &\quad + \sum_{k=1}^{p}\lambda_{1,i,k}(|y_{i,r}|^{2k} + |z_{i,1}|^{2k}) + \frac{\bar{d}_{i,1}^2}{2} \\ &\quad + ||\varrho_i||^2 + s_{i,j}\left(\Psi_{i,j} - \frac{\partial \alpha_{i,j-1}}{\partial \hat{\Theta}_i}\dot{\hat{\Theta}}_i\right)\Big].\end{aligned} \tag{39}$$

On the basis of Lemma 1, there exists a FLS $W_{i,j}^T\Xi_{i,j}(\Lambda_{i,j})$ for given parameter $\varepsilon_{i,j} > 0$ such that

$$\breve{F}_{i,j}(\Lambda_{i,j}) = W_{i,j}^T\Xi_{i,j}(\Lambda_{i,j}) + \delta_{i,j}(\Lambda_{i,j}),\ |\delta_{i,j}| \le \varepsilon_{i,j}. \tag{40}$$

Then, it follows that

$$\begin{aligned}s_{i,j}\breve{F}_{i,j} &\le \frac{1}{2a_{i,j}^2}s_{i,j}^2\Theta_i\Xi_{i,j}^T\Xi_{i,j} + \frac{1}{2}a_{i,j}^2 \\ &\quad + \frac{1}{2}s_{i,j}^2 + \frac{1}{2}\varepsilon_{i,j}^2\end{aligned} \tag{41}$$

where $a_{i,j} > 0$ is the design parameter. Establish the virtual control signal $\alpha_{i,j}$ as

$$\alpha_{i,j} = -c_{i,j}\frac{\eta_{i,j}e_{i,j}^{2\ell-1}}{\bar{\rho}_{i,j}^2(\bar{\rho}_{i,j}^2 - e_{i,j}^2)^{\ell-2}} - \frac{1}{2a_{i,j}^2}s_{i,j}\hat{\Theta}_i\Xi_{i,j}^T\Xi_{i,j} \tag{42}$$

where $c_{i,j} > 0$ is the design parameter. In the light of the formulas (41) and (42), (39) can be changed as

$$\begin{aligned}\dot{\mathcal{V}}_j \leq & \sum_{i=1}^{\mathcal{N}}\Bigg[-\pi_{i,j}||\varrho_i||^2 - \sum_{m=1}^{j} c_{i,m}\frac{e_{i,m}^{2\ell}}{(\bar{\rho}_{i,j}^2 - e_{i,m}^2)^{\ell}} \\ & + \frac{1}{r_{i,b}}\tilde{\Theta}_i\left(\sum_{m=1}^{j}\frac{r_{i,b}}{2a_{i,m}^2}s_{i,m}^2\Xi_{i,m}^T\Xi_{i,m} - \dot{\hat{\Theta}}_i\right) \\ & + \sum_{m=2}^{j} s_{i,m}\left(\Psi_{i,m} - \frac{\partial\alpha_{i,m-1}}{\partial\hat{\Theta}_i}\dot{\hat{\Theta}}_i\right) + \frac{z_{i,j+1}^2}{2} \\ & + \sum_{k=1}^{p}(\lambda_{i,k} + j\lambda_{1,i,k})(|y_{i,r}|^{2k} + |z_{i,1}|^{2k}) \\ & + \frac{1}{r_{i,a}}\tilde{\beta}_i\left(r_{i,a}\sum_{k=1}^{p}|z_{i,1}|^{2k} - \dot{\hat{\beta}}_i\right) \\ & - \beta_i\sum_{k=1}^{p}|z_{i,1}|^{2k} + \sigma_{i,0} + \bar{D}_{i,j}\Bigg] \end{aligned} \tag{43}$$

where $\pi_{i,j} = \pi_{i,j-1} - 1$ and $\bar{D}_{i,j} = \bar{D}_{i,j-1} + \frac{1}{2}(\varepsilon_{i,j}^2 + \bar{d}_{i,j}^2 + a_{i,j}^2)$.

Step n_i. The triggering event mechanism is utilized to update the control input. When the preconceived condition is triggered, the obtained control input of the relative threshold strategy largely reduces the communication burden. First, we define the intermediate control function and virtual control signal α_{i,n_i} as

$$\begin{aligned}\breve{u}_i(t) = & -(1+\xi_{iu_i})\left[\alpha_{i,n_i}\tanh\left(\frac{s_{i,n_i}\alpha_{i,n_i}}{\epsilon_{iu_i}}\right)\right. \\ & \left. + \bar{\zeta}_i\tanh\left(\frac{s_{i,n_i}\bar{\zeta}_i}{\epsilon_{iu_i}}\right)\right], \end{aligned} \tag{44}$$

$$\begin{aligned}\alpha_{i,n_i} = & -c_{i,n_i}\frac{\eta_{i,n_i}e_{i,n_i}^{2\ell-1}}{\bar{\rho}_{i,n_i}^2(\bar{\rho}_{i,n_i}^2 - e_{i,n_i}^2)^{\ell-2}} \\ & - \frac{1}{2a_{i,n_i}^2}s_{i,n_i}\hat{\Theta}_i\Xi_{i,n_i}^T\Xi_{i,n_i}, \end{aligned} \tag{45}$$

where $s_{i,n_i} = \bar{\rho}_{i,n_i}^2 e_{i,n_i}/(1-e_{i,n_i}^2)^2\eta_{i,n_i}$, and $c_{i,n_i} > 0$ is a design parameter. The event-triggering mechanism is

$$u_i(t) = \breve{u}_i(t_{i,k}), \forall t \in [t_{i,k}, t_{i,k+1}), \tag{46}$$

$$t_{i,k+1} = \inf\{t > t_{i,k+1}||e_{iu_i}(t)| \geq \xi_{iu_i}|u_i(t)| + \zeta_i\}, \tag{47}$$

where $e_{iu_i} = \breve{u}_i - u_i$ is the measurement error, $0 < \xi_{iu_i} < 1$, $\zeta_i > 0$ is a constant satisfying $\bar{\zeta}_i \geq \zeta_i/(1-\xi_{iu_i})$, and the control input update time is defined as $t_{i,k}$, $k \in \mathbb{Z}^+$. When $t \in [t_{i,k}, t_{i,k+1})$, the control input holds as $\breve{u}_i(t_{i,k})$. When (47) is triggered, the control signal will be updated and it is marked as $\breve{u}_i(t_{i,k+1})$. Thus, there exist two functions $\varpi_{i,a}(t)$ and $\varpi_{i,b}(t)$ with $|\varpi_{i,a}(t)| \leq 1$ and $|\varpi_{i,b}(t)| \leq 1$ such that

$$\breve{u}_i(t) = (1+\varpi_{i,a}(t)\xi_{iu_i})u_i(t) + \varpi_{i,b}(t)\zeta_i. \tag{48}$$

Based on the above descriptions, the coordinate transformation $z_{i,n_i} = \hat{x}_{i,n_i} - \alpha_{i,n_i-1}$ and (10), we can get

$$\dot{z}_{i,n_i} = \frac{\breve{u}_i(t) - \varpi_{i,b}\zeta_i}{(1+\varpi_{i,a}\xi_{iu_i})} + l_{i,n_i}\varrho_{i.1} - \dot{\alpha}_{i,n_i-1} \tag{49}$$

where $\dot{\alpha}_{i,n_i-1}$ is defined as (33) with $j = n_i$. Take the total Lyapunov function $\mathcal{V}$ as

$$\mathcal{V} = \mathcal{V}_{n_i} = \mathcal{V}_{n_i-1} + \sum_{i=1}^{\mathcal{N}}\left(\frac{1}{2}\frac{e_{i,n_i}^2}{\bar{\rho}_{i,n_i}^2 - e_{i,n_i}^2}\right). \tag{50}$$

By using (49), the time derivative of $\mathcal{V}_{n_i}$ is

$$\begin{aligned}\dot{\mathcal{V}}_{n_i} = & \dot{\mathcal{V}}_{n_i-1} + \sum_{i=1}^{\mathcal{N}}\Bigg\{s_{i,n_i}\Bigg[\frac{\breve{u}_i(t) - \varpi_{i,b}\zeta_i}{(1+\varpi_{i,a}\xi_{iu_i})} - \alpha_{i,n_i} \\ & + \alpha_{i,n_i} - (e_{i,n_i} - \rho_{i,n_i})\dot{\eta}_{i,n_i} + l_{i,n_i}\varrho_{i.1} \\ & - \Upsilon_{i,n_i-1} - \frac{\partial\alpha_{i,n_i-1}}{\partial\hat{\Theta}_i}\dot{\hat{\Theta}}_i - \sum_{m=1}^{j-1}\frac{\partial\alpha_{i,n_i-1}}{\partial\hat{x}_{i,m}}l_{i,m}\varrho_{i,1} \\ & - \frac{\partial\alpha_{i,n_i-1}}{\partial y_i}(\varrho_{i,2} + \mathcal{H}_{i,1}(\breve{y}) + d_{i,1})\Bigg]\Bigg\}. \end{aligned} \tag{51}$$

By invoking the Young's inequality, one has

$$\begin{aligned} & -s_{i,n_i}\left[\frac{\partial\alpha_{i,n_i-1}}{\partial y_i}(\varrho_{i,2} + \mathcal{H}_{i,1}(\breve{y}) + d_{i,1}) + l_{i,n_i}\varrho_{i,1}\right] \\ \leq & \frac{5s_{i,n_i}^2}{4}\left(\frac{\partial\alpha_{i,n_i-1}}{\partial y_i}\right)^2 + \frac{||\varrho_i||^2}{2} + \frac{\bar{d}_{i,1}^2}{2} + \sum_{k=1}^{p}\lambda_{1,i,k} \\ & \times(|y_{i,r}|^{2k} + |z_{i,1}|^{2k}) + s_{i,n_i}^2 l_{i,n_i}^2 + \frac{||\varrho_i||^2}{4}, \end{aligned} \tag{52}$$

$$\begin{aligned} & -s_{i,j}\sum_{m=1}^{n_i-1}\frac{\partial\alpha_{i,n_i-1}}{\partial\hat{x}_{i,m}}l_{i,m}\varrho_{i,1} \\ \leq & s_{i,n_i}^2\left(\sum_{m=1}^{n_i-1}\frac{\partial\alpha_{i,n_i-1}}{\partial\hat{x}_{i,m}}l_{i,m}\right)^2 + \frac{||\varrho_i||^2}{4}. \end{aligned} \tag{53}$$

Due to the fact that $s_{i,n_i}\breve{u}_i(t) < 0$, $|\varpi_{i,b}\zeta_i/(1+\varpi_{i,a}\xi_{iu_i})| \leq \zeta_i/(1-\xi_{iu_i})$, $\bar{\zeta}_i \geq \zeta_i/(1-\xi_{iu_i})$, $|\varpi_{i,a}(t)| \leq 1$, and $|\varpi_{i,b}(t)| \leq 1$, then, defining $\bar{\epsilon}_{iu_i} = 0.2785\epsilon_{iu_i}$ and applying the Lemma 2 existing in [53], we have

$$\frac{s_{i,n_i}\breve{u}_i(t)}{1+\varpi_{i,a}\xi_{iu_i}} \leq \frac{s_{i,n_i}\breve{u}_i(t)}{1+\xi_{iu_i}}, \tag{54}$$

$$-\frac{s_{i,n_i}\varpi_{i,b}(t)\zeta_i}{(1+\varpi_{i,a}\xi_{iu_i})} \leq s_{i,n_i}\bar{\zeta}_i\tanh\left(\frac{s_{i,n_i}\bar{\zeta}_i}{\epsilon_{iu_i}}\right) + \bar{\epsilon}_{iu_i}, \tag{55}$$

$$-s_{i,n_i}\alpha_{i,n_i} \leq s_{i,n_i}\alpha_{i,n_i}\tanh\left(\frac{s_{i,n_i}\alpha_{i,n_i}}{\epsilon_{iu_i}}\right) + \bar{\epsilon}_{iu_i}. \tag{56}$$

Design the other unknown auxiliary function $\breve{F}_{i,n_i}(\Lambda_{i,n_i})$ as

$$\begin{aligned}\breve{F}_{i,n_i}(\Lambda_{i,n_i}) = & s_{i,n_i}(l_{i,n_i}^2 + 0.5) - (e_{i,n_i} - \rho_{i,n_i})\dot{\eta}_{i,n_i} \\ & - \Upsilon_{i,n_i-1} + \frac{5s_{i,n_i}}{4}\left(\frac{\partial\alpha_{i,j-1}}{2\partial y_i}\right)^2 + \frac{z_{i,n_i}^2}{2s_{i,n_i}} \\ & + s_{i,n_i}\left(\sum_{m=1}^{n_i-1}\frac{\partial\alpha_{i,n_i-1}}{\partial\hat{x}_{i,m}}l_{i,m}\right)^2 - \Psi_{i,n_i} \end{aligned} \tag{57}$$

where Ψ_{i,n_i} and Λ_{i,n_i} are defined as $\Psi_{i,j}$ and $\Lambda_{i,j}$ with $j = n_i$, respectively. Substituting (52)-(57) into (51), it results in

$$\begin{aligned}\dot{\mathcal{V}}_{n_i} \leq & \dot{\mathcal{V}}_{n_i-1} + \sum_{i=1}^{\mathcal{N}}\left[s_{i,n_i}(\breve{F}_{i,n_i} + \alpha_{i,n_i}) + ||\varrho_i||^2\right. \\ & + \sum_{k=1}^{p}\lambda_{1,i,k}(|y_{i,r}|^{2k} + |z_{i,1}|^{2k}) \end{aligned}$$

$$+s_{i,n_i}\left(\Psi_{i,n_i}-\frac{\partial\alpha_{i,n_i-1}}{\partial\hat{\Theta}_i}\dot{\hat{\Theta}}_i\right) +\frac{\bar{d}_{i,1}^2}{2}-\frac{s_{i,n_i}^2}{2}+0.2785\epsilon_{iu_i}\Bigg]. \quad (58)$$

From Lemma 1, there exists a FLS $W_{i,n_i}^T\Xi_{i,n_i}(\Lambda_{i,n_i})$ for a given positive constant ε_{i,n_i} meeting

$$\breve{F}_{i,n_i}=W_{i,n_i}^T\Xi_{i,n_i}+\delta_{i,n_i}(\Lambda_{i,n_i}),\ |\delta_{i,n_i}|\le\varepsilon_{i,n_i}. \quad (59)$$

Then, it follows that

$$s_{i,n_i}\breve{F}_{i,n_i} \le \frac{1}{2a_{i,n_i}^2}s_{i,n_i}^2\Theta_i\Xi_{i,n_i}^T\Xi_{i,n_i}+\frac{1}{2}a_{i,n_i}^2 +\frac{1}{2}s_{i,n_i}^2+\frac{1}{2}\varepsilon_{i,n_i}^2 \quad (60)$$

where $a_{i,n_i}>0$ is the design parameter. By combining with the formulas of α_{i,n_i} and (60), we can obtain

$$\begin{aligned}\dot{\mathcal{V}}_{n_i} \le \sum_{i=1}^{\mathcal{N}}\Bigg[&-\pi_{i,n_i}||\varrho_i||^2-\sum_{j=1}^{n_i}c_{i,j}\frac{e_{i,j}^{2\ell}}{(\bar{\rho}_{i,j}^2-e_{i,j}^2)^\ell} +\frac{1}{r_{i,b}}\tilde{\Theta}_i\left(\sum_{j=1}^{n_i}\frac{r_{i,b}}{2a_{i,j}^2}s_{i,j}^2\Xi_{i,j}^T\Xi_{i,j}-\dot{\hat{\Theta}}_i\right)\\ &+\sum_{j=2}^{n_i}s_{i,j}\left(\Psi_{i,j}-\frac{\partial\alpha_{i,j-1}}{\partial\hat{\Theta}_i}\dot{\hat{\Theta}}_i\right)+\sum_{j=1}^{n_i}\bar{D}_{i,j} +\frac{1}{r_{i,a}}\tilde{\beta}_i\left(r_{i,a}\sum_{k=1}^{p}|z_{i,1}|^{2k}-\dot{\hat{\beta}}_i\right)+\sigma_{i,0}\Bigg] \end{aligned} \quad (61)$$

where $\pi_{i,n_i}=\pi_{i,n_i-1}-1$, $\bar{D}_{i,n_i}=\bar{D}_{i,n_i-1}+\frac{1}{2}(\varepsilon_{i,n_i}^2+\bar{d}_{i,n_i}^2+a_{i,n_i}^2)+2\bar{\epsilon}_{iu_i}+\sum_{k=1}^{p}(\lambda_{i,k}+n_i\lambda_{1,i,k})(\max_{t\ge0}|y_{i,r}|^{2k})$.

According to the work in [54], we can conclude

$$\sum_{j=2}^{n_i}s_{i,j}\left(\Psi_{i,j}-\frac{\partial\alpha_{i,j-1}}{\partial\hat{\Theta}_i}\dot{\hat{\Theta}}_i\right)\le 0. \quad (62)$$

Construct the adaptive functions as

$$\dot{\hat{\Theta}}_i = \sum_{j=1}^{n_i}\frac{r_{i,b}}{2a_{i,j}^2}s_{i,j}^2\Xi_{i,j}^T\Xi_{i,j}-r_{i,0}\hat{\Theta}_i, \quad (63)$$

$$\dot{\hat{\beta}}_i = r_{i,a}\sum_{k=1}^{p}|z_{i,1}|^{2k}-r_{i,c}\hat{\beta}_i. \quad (64)$$

On the other hand, according to the definitions of $\hat{\Theta}_i$ and $\hat{\beta}_i$, we have

$$\frac{r_{i,0}}{r_{i,b}}\tilde{\Theta}_i\hat{\Theta}_i \le -\frac{r_{i,0}}{2r_{i,b}}\tilde{\Theta}_i^2+\frac{r_{i,0}}{2r_{i,b}}\Theta_i^2, \quad (65)$$

$$\frac{r_{i,c}}{r_{i,a}}\tilde{\beta}_i\hat{\beta}_i \le -\frac{r_{i,c}}{2r_{i,a}}\tilde{\beta}_i^2+\frac{r_{i,c}}{2r_{i,a}}\beta_i^2. \quad (66)$$

Substituting (62)-(66) into (61) and further applying Lemma 2 in [32], one has

$$\begin{aligned}\dot{\mathcal{V}}_{n_i} \le \sum_{i=1}^{\mathcal{N}}\Bigg[&-\gamma_i\varrho_i^T\Phi_i\varrho_i-\sum_{j=1}^{n_i}\breve{c}_i\left(\frac{e_{i,j}^2}{2(\bar{\rho}_{i,1}^2-e_{i,j}^2)}\right)^\ell -\frac{r_{i,0}}{2r_{i,b}}\tilde{\Theta}_i^2-\frac{r_{i,c}}{2r_{i,a}}\tilde{\beta}_i^2+\bar{D}_{i,n_i}+\sigma_{i,0}\\ &+\frac{r_{i,0}}{2r_{i,b}}\Theta_i^2+\frac{r_{i,c}}{2r_{i,a}}\beta_i^2\Bigg]\end{aligned} \quad (67)$$

where $\breve{c}_i=\min_{1\le j\le n_i}\{2^\ell c_{i,j}\}$ and $\gamma_i=\pi_{i,n_i}/\lambda_{\max}(\Phi_i)$ with π_{i,n_i} satisfying $\pi_{i,n_i}>0$. Moreover, applying Lemma 3 in [29], let $z=1$, $\mu=1-\ell$, $\alpha=\varrho_i^T\Phi_i\varrho_i$, $\theta=\ell$, $\iota=\ell^{\frac{\ell}{1-\ell}}$, then, the following inequality can be obtained

$$\left(\varrho_i^T\Phi_i\varrho_i\right)^\ell\le(1-\ell)\,\iota+\varrho_i^T\Phi_i\varrho_i. \quad (68)$$

Similarly, the following inequalities hold

$$\left(\frac{1}{2r_{i,b}}\tilde{\Theta}_i^2\right)^\ell \le (1-\ell)\,\iota+\frac{1}{2r_{i,b}}\tilde{\Theta}_i^2, \quad (69)$$

$$\left(\frac{1}{2r_{i,a}}\tilde{\beta}_i^2\right)^\ell \le (1-\ell)\,\iota+\frac{1}{2r_{i,a}}\tilde{\beta}_i^2. \quad (70)$$

By using the above inequalities, one gets

$$\begin{aligned}\dot{\mathcal{V}}_{n_i} \le \sum_{i=1}^{\mathcal{N}}\Bigg[&-\gamma_i\left(\varrho_i^T\Phi_i\varrho_i\right)^\ell-\sum_{j=1}^{n_i}\breve{c}_i\left(\frac{e_{i,j}^2}{2(\bar{\rho}_{i,j}^2-e_{i,j}^2)}\right)^\ell -r_{i,0}\left(\frac{1}{2r_{i,b}}\tilde{\Theta}_i^2\right)^\ell-r_{i,c}\left(\frac{1}{2r_{i,a}}\tilde{\beta}_i^2\right)^\ell\\ &+(1-\ell)\,\iota(\gamma_i+r_{i,0}+r_{i,c})+\frac{r_{i,0}}{2r_{i,b}}\Theta_i^2 +\bar{D}_{i,n_i}+\frac{r_{i,c}}{2r_{i,a}}\beta_i^2+\sigma_{i,0}\Bigg].\end{aligned} \quad (71)$$

Define $\mathcal{C}_1=\min_{1\le i\le\mathcal{N}}\{\gamma_i,\breve{c}_i,r_{i,0},r_{i,c}\}$ and $\mathcal{C}_2=\sum_{i=1}^{\mathcal{N}}[\bar{D}_{i,n_i}+\frac{r_{i,0}}{2r_{i,b}}\Theta_i^2+\sigma_{i,0}+\frac{r_{i,c}}{2r_{i,a}}\beta_i^2+(1-\ell)\,\iota(\gamma_i+r_{i,0}+r_{i,c})]$. Then, the inequality (71) can be further rewritten as

$$\dot{\mathcal{V}}\le-\mathcal{C}_1\mathcal{V}^\ell+\mathcal{C}_2. \quad (72)$$

Then, the above backstepping-based finite-time decentralized control design can be summarized by the following theorem.

Theorem 1: In terms of the controlled system (1) preceded by full-state tracking error constraints, if the initial condition satisfies $\rho_{i,j}\le e_{i,j}(0)<\bar{\rho}_{i,j}$, Assumptions 1 and 2 also hold, under the actions of the virtual controllers (30) and (42), the intermediate control function (44), and the adaptive laws (63) and (64), then, the researched closed-loop system will satisfy the following properties: 1) All the closed-loop signals are SGPFS, and the output tracking error reach to a small neighborhood of origin in finite time. 2) The full-state tracking errors are confined to the predefined boundaries during operation, i.e., $|z_{i,j}|<|\eta_{i,j}|$ holds.

Proof. In order to verify that all the closed-loop signals are SGPFS, we will start by defining $T^*=[1/((1-\ell)\kappa\mathcal{C}_1)]\times[\mathcal{V}^{1-\ell}(\varrho_i(0),e_i(0),\tilde{\beta}_i(0),\tilde{\Theta}_i(0))-(\mathcal{C}_2/(1-\kappa)\mathcal{C}_1))^{1-\ell/\ell}]$ with $\varrho_i(0)=[\varrho_{i,1}(0),...,\varrho_{i,n_i}(0)]^T$, $e_i(0)=[e_{i,1}(0),...,e_{i,n_i}(0)]^T$, and $0<\kappa<1$, $i=1,...,\mathcal{N}$. Thus, it follows from Lemma 2 that for $\forall t\ge T^*$, $\mathcal{V}^\ell\left(\varrho_i,e_i,\tilde{\beta}_i,\tilde{\Theta}_i\right)\le\mathcal{C}_2/\left((1-\kappa)\mathcal{C}_1\right)$ that implies all the signals in the closed-loop system are SGPFS.

From $\mathcal{V}^\ell\le\mathcal{C}_2/\left((1-\kappa)\mathcal{C}_1\right)$, the structure of $\mathcal{V}$ and the fact that $\rho_{i,j}\le e_{i,j}<\bar{\rho}_{i,j}$ $(j=1,...,n_i)$, it follows that

$$|z_{i,j}|\le|\eta_{i,j}|\sqrt{2(1+2\rho_{i,j})\left(\frac{\mathcal{C}_2}{(1-\kappa)\mathcal{C}_1}\right)^{1/\ell}}. \quad (73)$$

The above inequality when $j=1$ means that the output tracking error can converge to a small neighborhood of the origin and remains there after the finite time T^*.

Moreover, we can further obtain from (73)

$$|z_{i,1}| = |x_{i,1} - y_{i,r}| < |\eta_{i,1}|, \quad (74)$$
$$|z_{i,j}| = |\hat{x}_{i,j} - \alpha_{i,j-1}| < |\eta_{i,j}|, \ j = 2, ..., n_i. \quad (75)$$

Therefore, the full-state tracking errors are proved to remain within the prescribed bounds, i.e., the full-state tracking error constraints are satisfied. The proof of Theorem 1 is finished. ■

Remark 4: To obtain the relation of $|z_{i,j}| < |\eta_{i,j}|$, $j = 1, ..., n_i$, the advisable parameter is chosen in (73) such that inequality $\mathcal{C}_2 < \left[\frac{1}{2(1+2\rho_{i,j})}\right]^{\ell} (1-\kappa)\mathcal{C}_1$ holds.

Remark 5: There is a time $t_i^* > 0$ such that $t_{i,k+1} - t_{i,k} \geq t_i^*$, $\forall k \in \mathbb{Z}^+$. Based on the equation e_{iu_i}, we can get

$$\frac{d}{dt}|e_{iu_i}| = \dot{e}_{iu_i}\text{sign}(e_{iu_i}) \leq |\dot{\breve{u}}_i(t)|. \quad (76)$$

From (44), one has

$$\begin{aligned} \dot{\breve{u}}_i(t) = & -(1+\xi_{iu_i})\left[\dot{\alpha}_{i,n_i}\tanh\left(\frac{s_{i,n_i}\alpha_{i,n_i}}{\epsilon_{iu_i}}\right)\right. \\ & +\alpha_{i,n_i}\frac{\dot{s}_{i,n_i}\alpha_{i,n_i} + s_{i,n_i}\dot{\alpha}_{i,n_i}}{\epsilon_{iu_i}\cosh^2\left(\frac{s_{i,n_i}\alpha_{i,n_i}}{\epsilon_{iu_i}}\right)} \\ & \left.+\frac{\bar{\zeta}_i^2\dot{s}_{i,n_i}}{\epsilon_{iu_i}\cosh^2\left(\frac{s_{i,n_i}\bar{\zeta}_i}{\epsilon_{iu_i}}\right)}\right]. \end{aligned} \quad (77)$$

Since all the signals are bounded, then, the inequality $|\dot{\breve{u}}_i(t)| \leq \mathcal{M}_i$ holds with $\mathcal{M}_i$ being a positive constant. Due to that $e_{iu_i}(t_{i,k}) = 0$ and $\lim_{t\to t_{i,k+1}} e_{iu_i}(t) = \xi_{iu_i}|u_i(t)| + \zeta_i$, thus, the Zeno behavior is avoided when the lower bound of the interexecution time t_i^* meets $t_i^* \geq (\xi_{iu_i}|u_i(t)| + \zeta_i)/\mathcal{M}_i$.

Remark 6: The authors in [29]–[35] devised the finite-time control strategies for different nonlinear systems, respectively. Noting that all the designed virtual control signals α_i $(i = 1, ..., n-1)$ contain the terms of $c_i z_i^{2\ell-1}$, where $0 < \ell < 1$ and $c_i > 0$ is the design parameter, z_i denotes the state tracking error. In backstepping control design, the virtual control signals α_i are needed to be differentiated repeatedly, which may cause the control "singularity" problem on account of that the value of z_i is uncertain and the time derivative of $c_i z_i^{2\ell-1}$ is $(2\ell-1)c_i z_i^{2\ell-2}$ $(2\ell-2<0)$. To overcome this issue, the new error-constrained control method is employed with the transformation error $e_{i,j}$ being restricted to $[\rho_{i,j}, \bar{\rho}_{i,j})$, so the "singularity" problem can be skilfully avoided and the restrictive condition of ℓ in [29]–[35] is eliminated.

IV. Simulation Results

In this section, two simulation examples are given to to test the availability of the proposed approach.

Example 1: The following numerical large-scale nonlinear system is considered.

$$\begin{cases} \dot{x}_{1,1} = \phi_{1,1}(\breve{x}_{1,1}) + x_{1,2} + \mathcal{H}_{1,1}(\breve{y}) + d_{1,1}(t), \\ \dot{x}_{1,2} = \phi_{1,2}(\breve{x}_{1,2}) + u_1 + \mathcal{H}_{1,2}(\breve{y}) + d_{1,2}(t), \\ y_1 = x_{1,1}, \\ \dot{x}_{2,1} = \phi_{2,1}(\breve{x}_{2,1}) + x_{2,2} + \mathcal{H}_{2,1}(\breve{y}) + d_{2,1}(t), \\ \dot{x}_{2,2} = \phi_{2,2}(\breve{x}_{2,2}) + u_2 + \mathcal{H}_{2,2}(\breve{y}) + d_{2,2}(t), \\ y_2 = x_{2,1} \end{cases} \quad (78)$$

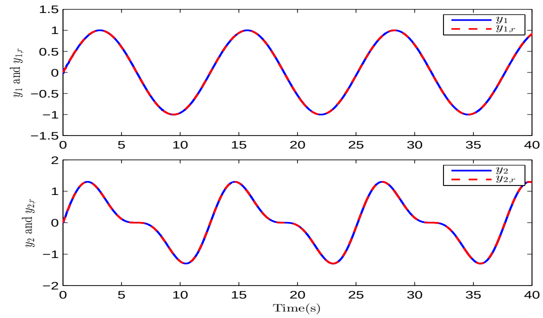

Fig. 1. System outputs y_i and reference signals $y_{i,r}$ for $i = 1, 2$.

where $\phi_{1,1} = 0.1\sin(0.1x_{1,1}^3)$, $d_{1,1}(t) = 0.01\sin(1.5t)$, $\mathcal{H}_{1,1}(\breve{y}) = -0.06y_1y_2$, $\phi_{1,2}(\breve{x}_{1,2}) = 0.37x_{1,1}x_{1,2}^3$, $d_{1,2}(t) = 0$, $\mathcal{H}_{1,2}(\breve{y}) = 0.1\sin(y_1 - y_2^2)$, $\phi_{2,1} = 0.11x_{2,1}\cos(0.5x_{2,1}^5)$, $d_{2,1}(t) = 0.2\cos(0.01t)$, $\mathcal{H}_{2,1}(\breve{y}) = -0.1(y_1^3 - y_2)$, $\phi_{2,2} = 0.5x_{2,1}x_{2,2}^3 + 0.01/\exp(x_{2,1} + x_{2,2})$, $d_{2,2}(t) = 0$, $\mathcal{H}_{2,2}(\breve{y}) = 0.1y_1y_2$.

In this simulation, the desired reference trajectories are defined as $y_{1,r} = \sin(0.5t)$ and $y_{2,r} = \sin(0.5t) + 0.5\sin(t)$. All the initial conditions are taken as $x_{i,j}(0) = \hat{x}_{i,j}(0) = 0.1$, $\hat{\Theta}_i(0) = \hat{\beta}_i(0) = 0$. The design parameters are $a_{i,1} = 3$, $a_{i,2} = 2$, $c_{i,j} = 90$, $r_{i,0} = 10$, $r_{i,a} = r_{i,c} = 1$, $r_{i,b} = 2.5$, $\rho_{i,j} = 0.01$, $\ell = 0.99$, $l_{i,1} = 6$, $l_{1,2} = 8$, $l_{2,2} = 12$ $\xi_{1u_1} = 0.3$, $\zeta_1 = 0.1$, $\xi_{2u_2} = 0.1$, $\zeta_2 = 0.01$, $\bar{\zeta}_i = 1.2$, and $\epsilon_{iu_i} = 16$ $(i = 1, 2,\ j = 1, 2)$. The performance functions are $\mu_{i,1}(t) = 1.1\exp(-t) + 0.04$, $\mu_{1,2}(t) = 15\exp(-t) + 0.6$ and $\mu_{2,2}(t) = 10\exp(-t) + 0.4$, where the corresponding design parameters are $\varkappa_i = 1$, $H_{Li,1} = 0.5$, $H_{hi,1} = 0.55$, $H_{Li,2} = 0.95$, and $H_{hi,2} = 1$ $(i = 1, 2)$. The fuzzy membership functions are defined as $\mu_{\mathcal{F}_{i,1}^l}(\check{\mathcal{X}}_{i,1}) = \exp[-\left(\check{\mathcal{X}}_{i,1} + \varsigma_l\right)^2/2]$ and $\mu_{\mathcal{F}_{i,2}^l}(\check{\mathcal{X}}_{i,2}) = \exp[-\left(\check{\mathcal{X}}_{i,2} + \varsigma_l\right)^2/2]$, where $i = 1, 2$, $l = 1, 2, ..., 5$, $\check{\mathcal{X}}_{i,1}$ can be chosen as $x_{i,1}$, $\bar{y}_{i,r}^{(1)}$ and $\bar{\eta}_{i,1}^{(1)}$, $\check{\mathcal{X}}_{i,2}$ is about variables $x_{i,1}$, $\hat{x}_{i,2}$, $\bar{y}_{i,r}^{(2)}$, $\bar{\eta}_{i,1}^{(2)}$, $\bar{\eta}_{i,2}^{(1)}$ and Θ_i, and $\varsigma_1 = 2$, $\varsigma_2 = 1$, $\varsigma_3 = 0$, $\varsigma_4 = -1$, $\varsigma_5 = -2$.

Figs. 1-5 show the simulation results. Fig. 1 plots the trajectories of reference signals $y_{i,r}$ and system outputs y_i $(i = 1, 2)$. The curves of states $x_{i,j}$ and state estimations $\hat{x}_{i,j}$ $(i = 1, 2, j = 1, 2)$ are shown in Fig. 2. The curves of the full-state tracking errors $z_{i,j}$ $(i = 1, 2,\ j = 1, 2)$ are described in Fig. 3. We know that the full-state tracking errors remain within the predefined bounds for all $t \geq 0$. Fig. 4 depicts the curves of control inputs u_1 and u_2 using event-triggered method or using time-triggered method, which shows the advantage of cost saving for the event-triggered controller. In addition, the relevant time intervals of triggering are depicted in Fig. 5. From these figures, it is obvious that the control aim can be achieved by the proposed control method.

Example 2: The tripled inverted pendulums [55] are introduced to further test the proposed control strategy. The system model is described by

$$\begin{aligned} \ddot{\Xi}_1 = & \tfrac{g}{l}\sin\Xi_1 + u_1 + d_{1,2}(t) \\ & + \tfrac{k_1a^2}{m_1l^2}\left(\sin\Xi_2\cos\Xi_2 - \sin\Xi_1\cos\Xi_1\right), \\ \ddot{\Xi}_2 = & \tfrac{g}{l}\sin\Xi_2 + u_2 + d_{2,2}(t) \end{aligned}$$

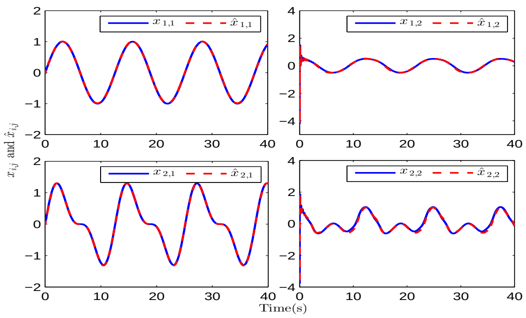

Fig. 2. System states $x_{i,j}$ and estimated states $\hat{x}_{i,j}$ for $i = j = 1, 2$.

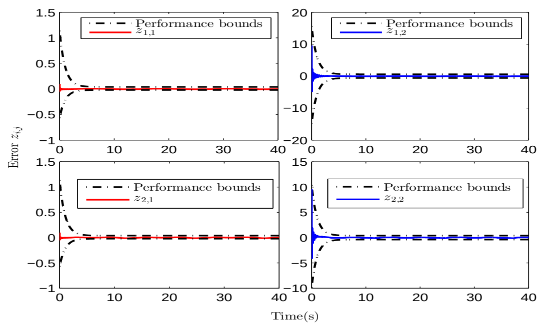

Fig. 3. Errors $z_{i,j}$ for $i = j = 1, 2$ and their performance bounds.

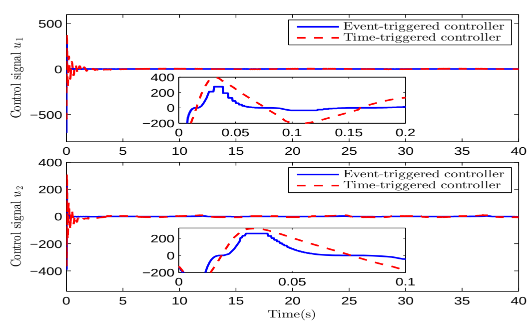

Fig. 4. Control inputs u_1 and u_2.

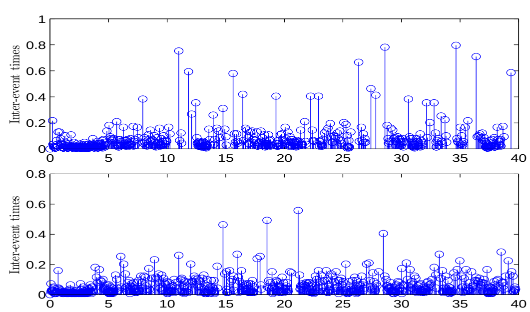

Fig. 5. Time interval of triggering.

$$
\begin{aligned}
&+ \frac{k_1 a^2}{m_2 l^2} \left(\sin \Xi_1 \cos \Xi_1 - \sin \Xi_2 \cos \Xi_2\right) \\
&+ \frac{k_2 a^2}{m_2 l^2} \left(\sin \Xi_3 \cos \Xi_3 - \sin \Xi_2 \cos \Xi_2\right), \\
\ddot{\Xi}_3 &= \frac{g}{l} \sin \Xi_3 + u_3 + d_{3,2}(t) \\
&+ \frac{k_2 a^2}{m_3 l^2} \left(\sin \Xi_2 \cos \Xi_2 - \sin \Xi_3 \cos \Xi_3\right)
\end{aligned}
$$

where $i = 1, 2, 3$, Ξ_i is the pendulum angle position, m_i is the rod mass, l is the rod length, g is the gravitational acceleration, $d_{i,2}$ is the external disturbance, k_i, $i = 1, 2$, are the connected spring constants. Define the state vectors as $(x_{i,1}, x_{i,2})^T = (\Xi_i, \dot{\Xi}_i)^T$ $(i = 1, 2, 3)$, then, the above system model can be expressed as

$$
\begin{cases}
\dot{x}_{i,1} = x_{i,2} + \mathcal{H}_{i,1}(y_1, y_2, y_3) + d_{i,1}(t), \\
\dot{x}_{i,2} = \frac{g}{l} \sin x_{i,1} + u_i + \mathcal{H}_{i,2}(y_1, y_2, y_3) + d_{i,2}(t), \\
y_i = x_{i,1}, i = 1, 2, 3
\end{cases} \tag{79}
$$

where

$$
\begin{aligned}
&\mathcal{H}_{i,1} = 0, \ d_{i,1}(t) = 0, \\
&\mathcal{H}_{1,2} = \frac{k_1 a^2}{m_1 l^2} (\sin y_2 \cos y_2 - \sin y_1 \cos y_1), \\
&\mathcal{H}_{2,2} = \frac{k_1 a^2}{m_2 l^2} (\sin y_1 \cos y_1 - \sin y_2 \cos y_2) \\
&\qquad + \frac{k_2 a^2}{m_2 l^2} (\sin y_3 \cos y_3 - \sin y_2 \cos y_2), \\
&\mathcal{H}_{3,2} = \frac{k_2 a^2}{m_3 l^2} (\sin y_2 \cos y_2 - \sin y_3 \cos y_3), \\
&d_{1,2}(t) = 0.01 \sin(1.5t), \\
&d_{2,2}(t) = 0.1 \sin(1.5t) - 0.1 \cos(t), \\
&d_{3,2}(t) = -0.1 \cos(1.5t).
\end{aligned}
$$

Some system parameters are defined as $a = 3$m, $k_1 = 1$N/m, $k_2 = 1.2$N/m, $m_1 = 0.2$kg, $m_2 = 0.4$kg, $m_3 = 0.3$kg, $l = 9$m, and $g = 9.8\text{m/s}^2$. The desired reference trajectories are defined as $y_{1,r} = y_{3,r} = \sin(0.5t)$ and $y_{2,r} = \sin(0.5t) + 0.5\sin(t)$. All the initial conditions for $x_{i,j}$, $\hat{x}_{i,j}$, $\hat{\Theta}_i$ and $\hat{\beta}_i$ are taken as 0.1. The design parameters are $a_{3,1} = 3$, $a_{3,2} = 2$, $c_{1,1} = c_{3,1} = 80$, $c_{1,2} = c_{3,2} = 30$, $c_{2,1} = 60$, $c_{2,2} = 25$, $r_{3,0} = 10$, $r_{3,a} = r_{3,c} = 1$, $r_{3,b} = 2.5$, $\rho_{3,1} = \rho_{3,2} = 0.001$, $l_{1,1} = 3$, $l_{1,2} = 50$, $l_{2,1} = l_{3,1} = 6$, $l_{2,2} = l_{3,2} = 30$, $\xi_{1u_1} = \xi_{2u_2} = \xi_{3u_3} = 0.1$, $\zeta_1 = \zeta_3 = 0.01$, $\zeta_2 = 0.02$, $\bar{\zeta}_1 = 6$, $\bar{\zeta}_3 = 1.2$ and $\epsilon_{3u_3} = 16$. The performance functions are $\mu_{1,1}(t) = 1.1\exp(-t) + 0.05$, $\mu_{1,2}(t) = 15\exp(-t) + 0.3$, $\mu_{2,1}(t) = \mu_{3,1}(t) = 1.1\exp(-t) + 0.1$, $\mu_{2,2}(t) = 10\exp(-t) + 0.4$, and $\mu_{3,2}(t) = 15\exp(-t) + 0.7$, where the corresponding design parameters are $\varkappa_3 = 1$, $H_{L3,1} = 0.5$, $H_{h3,1} = 0.55$, $H_{L3,2} = 0.95$, and $H_{h3,2} = 1$. Besides, other parameters are the same as Example 1.

From Figs. 6-8, the availability of the proposed control strategy can be ensured for the practical system. Additionally, Fig. 9 shows the difference between the finite-time controller and infinite-time controller, from which it can be seen that the convergence rate of $z_{i,1}$ under the designed finite-time controller is faster than the traditional infinite-time one, i.e., the presented finite-time controller is effective.

V. CONCLUSIONS

In this paper, a novel observe-based decentralized adaptive fuzzy event-triggered finite-time control strategy has been designed for the large-scale nonlinear systems with full-state tracking error constraints and external disturbances. The unknown functions have been identified by FLSs. A PPFs-based error transformation method has been proposed, and

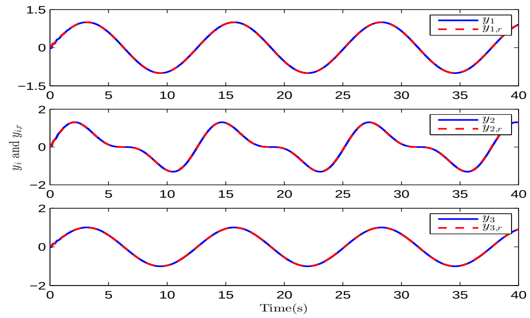

Fig. 6. System outputs y_i and reference signals $y_{i,r}$ for $i = 1, 2$.

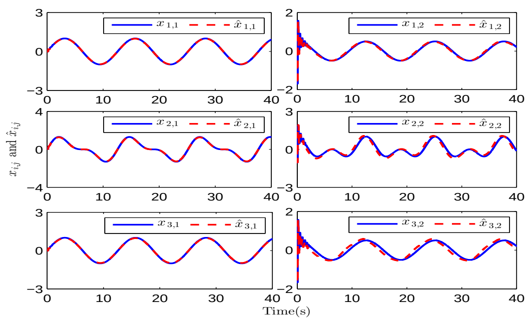

Fig. 7. System states $x_{i,j}$ and estimated states $\hat{x}_{i,j}$ for $i = j = 1, 2, 3$.

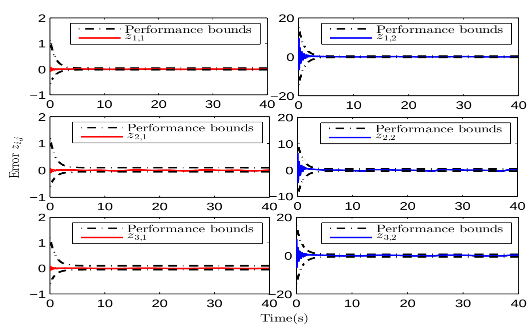

Fig. 8. Errors $z_{i,j}$ for $i = j = 1, 2, 3$ and their performance bounds.

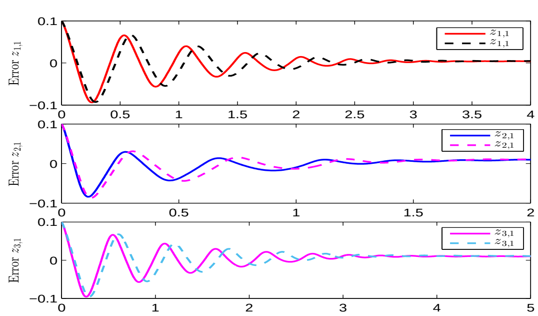

Fig. 9. Tracking error responses under the finite-time controller (solid line) and the infinite-time controller (dotted line).

the transformed state tracking errors have been restricted to a positive interval. Under the action of the new PPFs-based error transformation method and the SBLFs, the "singularity" problem caused by the combination of backstepping-based adaptive fuzzy control and finite time control has been eliminated, and the requirements of full-state tracking error constraints have been achieved. Additionally, the restrictive condition of exponential power term in finite-time controller has been removed. Integrating with event-triggered control technique and finite-time control method, the obtained control input signal of the relative threshold strategy has guaranteed that all the system signals are bounded, and the full-state tracking errors can remain within the predesigned performance regions in finite time. Eventually, the feasibility of the presented method has been validated via some simulation results.

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