DOI:
10.1016/j.fss.2020.06.014

Document Version
Peer reviewed version

Citation for published version (APA):
Liu, G., Pan, Y., Lam, H. K., & Liang, H. (2021). Event-triggered fuzzy adaptive quantized control for nonlinear multi-agent systems in nonaffine pure-feedback form. *Fuzzy Sets and Systems*. https://doi.org/10.1016/j.fss.2020.06.014

Event-Triggered Fuzzy Adaptive Quantized Control for Nonlinear Multi-Agent Systems in Nonaffine Pure-Feedback Form

Guangliang Liu[a], Yingnan Pan[a,*], Hak-Keung Lam[b], Hongjing Liang[a]

[a] *College of Engineering, Bohai University, Jinzhou 121013, China*
[b] *Department of Engineering, King's College London, Bush House, Strand campus, London, WC2R 2LS , United Kingdom*

Abstract

In this paper, we address the problem of event-triggered fuzzy adaptive quantized control for s-tochastic nonlinear non-affine pure-feedback multi-agent systems. The fuzzy logic system is used to estimate the stochastic disturbance term and unknown nonlinear functions. A nonlinearity decomposition method of asymmetric hysteresis quantizer is proposed by applying sector bound property. Moreover, to reduce the communication burden, an adaptive event-triggered protocol with a varying threshold is constructed. Based on the backstepping technique and stochastic Lyapunov function method, a novel adaptive event-triggered fuzzy control protocol and adaptive laws are constructed. By using stochastic Lyapunov stability theory, it is demonstrated that all signals are bounded in the closed-loop systems in probability and all the outputs of followers converge to the neighborhood of the leader output. Simulation results illustrate the effectiveness of our proposed scheme.

Keywords: Asymmetric hysteretic quantizer, event-triggered adaptive control, non-affine pure-feedback, multi-agent systems.

1. Introduction

Consensus control of multi-agent systems has received a great amount of attentions due to its applications in various areas, such as cooperative control of nonidentical networks [1, 2], unmanned air vehicles [3, 4], formation control of mobile robots [5, 6]. Generally speaking, consensus control of multi-agent systems is defined as all agents synchronizing to a common state by a control protocol based on the neighbor agents' information. In practical systems, the stochastic noise is one of factors that affects systems performance. At present, due to the complexity in theoretical analysis, the stochastic nonlinear multi-agent systems have not been fully researched. Wen *et al.* in [7] researched the consensus ability of stochastic nonlinear multi-agent systems subject to repairable actuator failures. The authors in [8] studied the multi-agent systems subject to unknown nonlinear dynamics, and the uncertain part of systems was approximated by applying a fuzzy logic system. In practice, some pure-feedback multi-agent systems are in non-affine structure. This makes the

*Corresponding author: Yingnan Pan
Email addresses: LiuGuangliang17@163.com (Guangliang Liu), panyingnan0803@gmail.com (Yingnan Pan), hak-keung.lam@kcl.ac.uk (Hak-Keung Lam), lianghongjing99@163.com (Hongjing Liang)

controller design of pure-feedback systems more difficult, and some significant results have been reported in recent years [9–14]. However, since the complexity of theoretical analysis and calculation of non-affine functions, to the best of our knowledge, to date, there are few studies on stochastic pure-feedback nonlinear multi-agent systems with non-affine form. This further stimulated our research interest.

Recently, event-triggered control has been researched as an alternative to time-triggered control in the field of network systems [15–21]. Compared to the time-triggered scheme [22, 23], the event-triggered control algorithm typically requires less information transmission. Motivated by this fact, the event-triggered controller has been used to the consensus control of multi-agent systems in [24–30]. The authors in [24] developed an adaptive event-triggered control protocol to realize consensus control of the first-order multi-agent systems subject to undirected graph. An adaptive distributed event-triggered consensus control protocol was proposed for multi-agent systems with sensor faults and input saturation in [25]. Guo *et al.* in [26] developed a novel distributed adaptive event-triggered sampled-data transmission controller for multi-agent systems with directed graph. An event-triggered control scheme was proposed in [28] to deal with the cooperative output regulation problem of multi-agent systems. Li *et al.* in [30] considered the consensus control problem of multi-agent systems against false data-injection attacks, and the Zeno behaviour was avoided.

In modern control systems, quantization is not only inevitable, but also helpful. In reality, due to the bandwidth constraints in network, the data should be quantized before transmission. Recently, some effective quantized control methods have also been proposed [31–38]. Specifically, in [32], the authors gave the upper bound of consensus errors and analyzed the quantization effect on the average consensus. Hayakawaa *et al.* in [34] addressed an adaptive quantized control method for nonlinear systems, and the results were established by using the sector bounded property of quantization errors. In [35], by applying the linear matrix inequality method and the sector bound property, the quantized state feedback stabilization problem was researched for MIMO systems. Liu *et al.* in [37] considered quantized consensus problem of single-integrator multi-agent systems subject to balanced communication topology by using the method of sampling data. Chen *et al.* proposed a distributed adaptive control protocol in [38] to solve the leader-following consensus problem of multi-agent systems based on binary quantizers. However, the above literatures have not solved the problem of co-design of the asymmetric hysteretic quantization and event-triggered technology in stochastic multi-agent systems, which motivates our current research work.

The purpose of this paper is to design a novel event-triggered adaptive control protocol for stochastic nonlinear non-affine multi-agent systems. In the design progress, control signals are quantized, which satisfy system performance and practical requirement. The main contributions are listed as follows: First, Compared with [39] all agents are modeled by stochastic nonlinear systems in non-affine pure-feedback form, and the stochastic disturbances and nonlinear terms are completely unknown, which is more reasonable in practical systems. Moreover, mean value theorems and implicit function are applied to overcome the difficulty in controlling the non-affine pure-feedback systems. Second, unlike [40] and [41] a novel control scheme, which co-designs the asymmetric hysteretic quantization and event-triggered mechanism, is constructed to well integrate into backstepping control, thus the transmission burden can be decreased effectively, and the cooperative control problem for non-affine nonlinear stochastic multi-agent systems can be solved. Finally, simulation results have illustrated the effectiveness of the proposed theoretical results.

The rest of the paper is organized as follows. Section II is devoted to the introduction of basic graph theory and systems formulation. The event-triggered controller design and stability analysis are derived in Section III. A numerical simulation is presented in Section IV. Finally, Section V draws our conclusion.

2. Graph theory and problem formulation

2.1. Graph theory

Based on a digraph, we research the consensus tracking control problem of multi-agent systems. The digraph is usually represented by the symbol $\mathcal{G} = (\mathfrak{Z}, \mathcal{E}, A)$, where $\mathfrak{Z} = \{\mathfrak{Z}_1, \ldots, \mathfrak{Z}_M\}$ is a nonempty node set, $\mathcal{E} \in \mathfrak{Z} \times \mathfrak{Z}$ is the set of edges and $A = [a_{i,j}] \in R^{M\times M}$ is an adjacency matrix. $(\mathfrak{Z}_j, \mathfrak{Z}_i) \in \mathcal{E}$ implies that agent i is able to acquire information from agent j. If agent i can obtain information from agent j, $a_{i,j} > 0$, otherwise $a_{i,j} = 0$. We suppose that $a_{i,i} = 0$ for all $i = 1, \ldots, M$. The set of neighbors of the vertex $\mathfrak{Z}_i$ is denoted by $N_i = \{\mathfrak{Z}_j | (\mathfrak{Z}_i, \mathfrak{Z}_j) \in \mathcal{E}\}$. The degree matrix D denotes the diagonal matrix $D = \mathrm{diag}(d_1 \ldots, d_M) \in R^{M\times M}$ with $d_i = \sum_{j=1}^{m} a_{i,j}$ for $i = 1, \ldots, M$. Define the Laplacian matrix as $\mathfrak{L} = D - A$. A directed graph contains a directed spanning tree, if there exists a directed path from the root node to all other agents.

In this paper, the multi-agent systems consist of M followers and one leader. Without loss of generality, we mark the leader as 0 and follower as $1, \ldots, M$.

The following assumptions are given to support the handling of the aforementioned cooperative control problem.

Assumption 1. *The n-order time derivatives of leader output $y_r(t)$ is continuous,and it only can be obtained by the i-th agents satisfying $0 \in N_i\,(i = 1, \ldots M)$.*

Assumption 2.

1) Define the augmented graph $\bar{\mathcal{G}} = \left(\bar{\mathfrak{Z}}, \bar{\mathcal{E}}, \bar{A}\right)$ with $\bar{\mathfrak{Z}} = \{\mathfrak{Z}_0, \mathfrak{Z}_1, \ldots, \mathfrak{Z}_M\}$, $\bar{\mathcal{E}} \in \bar{\mathfrak{Z}} \times \bar{\mathfrak{Z}}$ and $\bar{A} = [a_{i,j}] \in R^{(M+1)\times(M+1)}$, where the leader node 0 is the root node of $\bar{\mathcal{G}}$.

2) The jth follower only accepts state information from the i follower that satisfies $j \in N_i$, $i = 1, \ldots, M$, and $i \neq j$.

Lemma 1. *[42] Define $\Delta = diag(b_{1,0}, \ldots, b_{M,0})$ with $b_{i,0}\,(1 \le i \le M)$ being the weight between the leader and the followers. If there exists a spanning tree in the communication graph $\mathcal{G}$ and root agent has chance to get information from the leader, then $\mathfrak{L} + \Delta$ is nonsingular.*

2.2. Problem formulation

Consider the stochastic non-affine uncertain nonlinear multi-agent systems with asymmetric hysteretic quantization being described by the following model

$$dv_{i,j} = \left(h_{i,j}\left(\bar{v}_{i,j}, v_{i,j+1}\right)\right) dt + \Psi_{i,j}\left(\bar{v}_{i,j}\right) dw, \quad 1 \le j \le n_i - 1$$

$$
\begin{aligned}
dv_{i,n_i} &= \left(h_{i,n_i}\left(\bar{v}_{i,n_i}, q\left(u_i\right)\right)\right) dt + \Psi_{i,n_i}\left(\bar{v}_{i,j}\right) dw, \\
y_i &= v_{i,1}
\end{aligned} \tag{1}
$$

where $v_{i,j} = \left[v_{i,1}, \ldots, v_{i,n_i}\right]^T \in R^{n_i}$ $(i = 1, \ldots, M)$ denotes the system state vector; u_i denotes the control input, and y_i is the output; $\bar{v}_{i,j} = \left[v_{i,1}, \ldots, v_{i,j}\right]^T \in R^j$ and $h_{i,j}(\cdot)$ are unknown and continuously differentiable non-affine functions; $\Psi_{i,j}(\cdot)$ is the unknown smooth functions; w is an r-dimension standard Brownian motion; $q\left(u_i\right) \in \mathbb{R}^{n_i}$ represents the hysteresis quantizer output.

Definition 1. *[43] For the stochastic nonlinear system*

$$dv = h\left(\bar{v}\right) dt + \Psi\left(\bar{v}\right) dw$$

where $v \in R^{n_i}$ denotes the state of the system, w is an r-dimension standard Brownian motion. $h(\cdot)$ and $\Psi(\cdot)$ are are locally Lipschitz functions in x and satisfy $h(0) = 0$ and $\Psi(0) = 0$. Given $V(v,t)$ a twice continuously differentiable function, we define a differential operator L for the above stochastic systems, as follows

$$LV = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial v} f + \frac{1}{2} Tr\left\{\Psi^T \frac{\partial^2 V}{\partial v^2} \Psi\right\} \tag{2}$$

with Tr being the matrix trace.

Similar to [40], we give the asymmetric hysteresis quantizer as follows

$$
q\left(u_i\right) = \begin{cases}
u_i^+, & \frac{u_i^+}{1+\delta_+} < u_i \leq u_i^+, \dot{u}_i < 0 \text{ or}, \\
 & u_i^+ < u_i \leq \frac{u_i^+}{1-\delta_+}, \dot{u}_i > 0; \\
u_i^-, & u_i^- \leq u_i < \frac{u_i^-}{1+\delta_-}, \dot{u}_i > 0 \text{ or}, \\
 & \frac{u_i^-}{1-\delta_-} \leq u_i < u_i^-, \dot{u}_i < 0 \\
u_i^+\left(1+\delta_+\right), & u_i^+ < u_i \leq \frac{u_i^+}{1-\delta_+}, \dot{u}_i < 0 \text{ or}, \\
 & \frac{u_i^+}{1-\delta_+} < u_i \leq \frac{1+\delta_+}{1-\delta_+} u_i^+, \dot{u}_i > 0 \\
u_i^+\left(1-\delta_-\right), & \frac{u_i^-}{1-\delta_-} \leq u_i < u_i^-, \dot{u}_i > 0 \text{ or}, \\
 & \frac{1+\delta_-}{1-\delta_-} u_i^- \leq u_i < \frac{u_i^-}{1-\delta_-}, \dot{u}_i < 0 \\
0, & 0 \leq u_i < \frac{u_{i,\min}^+}{1+\delta_+} \text{ or}, \\
 & \frac{u_{i,\min}^+}{1+\delta_+} \leq u_i \leq u_{i,\min}^+, \ \dot{u}_i > 0, \\
 & \frac{u_{i,\min}^-}{1+\delta_-} < u_i \leq 0 \text{ or}, \\
 & u_{i,\min}^- \leq u_i \leq \frac{u_{i,\min}^-}{1+\delta_-}, \dot{u}_i < 0 \\
q\left(u_i\left(t^-\right)\right), & \dot{u}_i = 0
\end{cases} \tag{3}
$$

with $q(u_i(t^-))$ being the state prior to $q(u_i(t))$. u_i^+ and u_i^- denote quantized values to be defined as

$$u_i^+ = \left(\frac{1-\delta_+}{1+\delta_+}\right)^{1-i} u_{i,\min}^+, \ u_i^- = \left(\frac{1-\delta_-}{1+\delta_-}\right)^{1-i} u_{i,\min}^- \tag{4}$$

where $u_{i,\min}^+$ and $u_{i,\min}^-$ $(i = 1, \ldots, M.)$ denote the dead-zone parameters as shown in Fig. 1 [40]. The constants δ_+ and $\delta_- \in (0,1)$ decide the coarseness of quantizer. The larger the chosen δ_+ and δ_- are, the coarser the quantizer is.

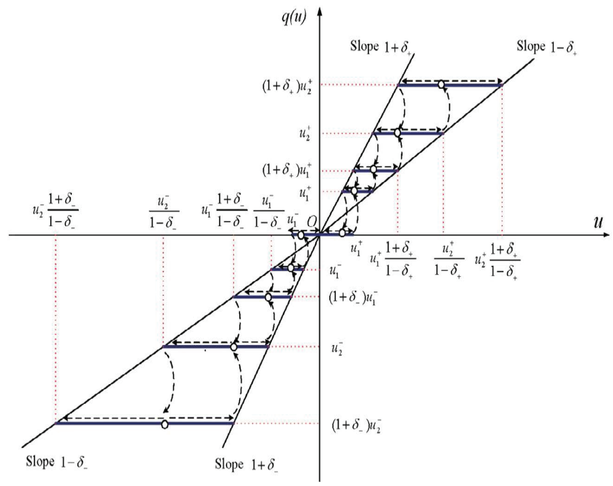

Figure 1: The asymmetric hysteresis quantizer

According to [40], in this paper, the asymmetric hysteresis quantizer is decomposed by using the following method.

Lemma 2. *[40] The quantizer (3) can be nonlinearly decomposed as*

$$q(u_i) = c(u_i)\, u_i + h(u_i) \tag{5}$$

where

$$c(u_i) = \begin{cases} \frac{q(u_i)}{u_i}, q(u_i) \neq 0 \\ 1, \quad q(u_i) = 0 \end{cases}, \tag{6}$$

$$h(u_i) = \begin{cases} 0, \quad q(u_i) \neq 0 \\ -u_i, \ q(u_i) = 0 \end{cases}. \tag{7}$$

Lemma 3. *[40] In decomposition (5), the disturbance-like term $h(u_i)$ and control coefficient $c(u_i)$ satisfy*

$$\begin{aligned} 1 - \delta_m &\leq c(u_i) \leq 1 + \delta_m, \quad (8) \\ |h(u_i)| &\leq \varepsilon. \quad (9) \end{aligned}$$

To lower the communication burden, the event-triggered control protocol is designed as

$$
\begin{aligned}
u_i &= \xi_i\left(t_k\right), \quad \forall t \in [t_k, t_{k+1}), \qquad (10)\\
t_{k+1} &= \inf\{t > t_k | \left|e_i\left(t\right)\right| \geq \varrho \left|u_i\left(t\right)\right| + \mathfrak{d}\} \qquad (11)
\end{aligned}
$$

with $e_i\left(t\right) = \xi_i\left(t\right) - u_i\left(t\right)$ being the error between the intermediate control $\xi_i\left(t\right)$ and control input $u_i\left(t\right)$. $0 < \varrho < 1$ and $\mathfrak{d}$ are positive design parameters.

Similar to [41], we are able to get $\xi_i\left(t\right) = \left(\chi_1\left(t\right)\varrho + 1\right)u_i\left(t\right) + \chi_2\left(t\right)\mathfrak{d}$ in the interval $[t_k, t_{k+1})$ with $\chi_m\left(t\right)\left(m = 1, 2\right)$ being time-varying parameters and satisfy $\left|\chi_m\left(t\right)\right| \leq 1$. Thus, one has

$$
u_i\left(t\right) = \frac{\xi_i\left(t\right)}{1 + \chi_1\left(t\right)\varrho} - \frac{\chi_2\left(t\right)\mathfrak{d}}{1 + \chi_1\left(t\right)\varrho}. \qquad (12)
$$

The control objective is to develop an adaptive event-triggered tracking control protocol for the system (1) subject to asymmetric hysteresis quantizer to achieve the following goals:

1) all the signals are bounded in the system (1);

2) all the outputs of followers $y_i(t)$ can track the leader's output $y_r(t)$.

To facilitate the controller design, the following assumptions are required.

For the control of stochastic pure-feedback system (1), define

$$
\phi_{i,j}\left(v_{i,j}, v_{i,j+1}\right) = \frac{\partial h_{i,j}\left(v_{i,j}, v_{i,j+1}\right)}{\partial v_{i,j+1}} \qquad (13)
$$

where $i = 1, \ldots M$, $j = 1, \ldots, n_i$. $v_{i,n_i+1} = q\left(u_i\right)$.

In this paper, we give the following assumption for $\phi_{i,j}\left(v_{i,j}, v_{i,j+1}\right)$.

Assumption 3. *There exist unknown constants b_m and b_M such that*

$$
0 < b_m \leq \left|\phi_{i,j}\left(v_i, v_{i,j+1}\right)\right| \leq b_M < \infty, \forall v_i, v_{i,j+1} \in R^i \times R.
$$

In addition, the sign of $\phi_{i,n_i}\left(v_{n_i}, q\left(u_i\right)\right)$ is unknown, and for $1 \leq j \leq n_i - 1$, the signs of $\phi_{i,j}\left(v_i, v_{i,j+1}\right)$ are known. Without loss of generality, it is further assumed that $\phi_{i,j}\left(v_i, v_{i,j+1}\right) \geq b_m > 0$. By applying the mean value theorem, the functions $h_{i,j}\left(\cdot,\cdot\right)$ in (1) can be written as

$$
h_{i,j}\left(v_{i,j}, v_{i,j+1}\right) = h_{i,j}\left(v_{i,j}, v^0_{i,j+1}\right) + \phi_{i,j}\left(v_{i,j}, \tau_{i,j+1}\right)\left(v_{i,j+1} - v^0_{i,j+1}\right) \qquad (14)
$$

where $\left(v^0_{i,1}, v^0_{i,2}, \ldots, v^0_{i,n_i}, u^0_i\right)^T$ is an equilibrium or operating point of interest, $v_{i,n_i+1} = q\left(u_i\right)$, $v^0_{i,n_i+1} = q\left(u^0_i\right)$ and $\tau_{i,j}$ are some point between $v_{i,j+1}$ and $v^0_{i,j+1}$. Substituting (14) into (1) and choosing $v^0_{i,j+1} = 0$ and $q\left(u^0_i\right) = 0$, the system dynamics can be rewritten as

$$
dv_{i,j} = \left(h_{i,j}\left(v_{i,j}, 0\right) + \phi_{i,j}\left(v_{i,j}, \tau_{i,j}\right)v_{i,j+1}\right)dt + \Psi_{i,j}\left(v_{i,j}\right)dw,
$$

$$
\begin{aligned}
dv_{i,n_i} &= \left(h_{i,n_i}\left(v_{i,n_i},0\right)+\phi_{i,n_i}\left(v_{i,n_i},\tau_{i,n_i}\right)q\left(u_i\right)\right)dt+\Psi_{i,n_i}\left(v_{i,j}\right)dw, \\
y_i &= v_{i,1}, \qquad 1\le j\le n-1.
\end{aligned} \tag{15}
$$

Definition 2. *[39] The consensus tracking errors between the leader and followers under the directed graph are cooperatively semiglobally uniformly ultimately bounded (CSUUB) in probability, if for* $\forall\epsilon>0$, $E\left|y_i(t)-y_r(t)\right|^4\le\epsilon$ *when* $t\to\infty$, *where* E *denotes the expectation operator,* $i=1,\dots,M$.

Define synchronization error as

$$
s_{i,1}=\sum_{j=1}^{m}a_{i,j}\left(y_i-y_j\right)+b_i\left(y_i-y_r\right) \tag{16}
$$

with $b_i\ge 0$ being the pinning gains, and $b_i>0$ if and only if follower agent i can get information from the leader.

To facilitate our stability analysis, the following lemmas are needed.

Lemma 4. *[39] Suppose there is an existing* $C^{2,1}$ *function* $V(v,t)$, *two constants* $p>0$ *and* $Q>0$, *class* κ_∞*-functions* κ_1 *and* κ_2, *such that*

$$
\begin{cases}
\kappa_1\left(\|v\|\right)\le V(v,t)\le\kappa_2\left(\|v\|\right)\\
LV\le -pV(v,t)+Q
\end{cases}
$$

where $t>0$ *and* $v\in R^n$, *there exists an unique strong solution for system (2).*

Lemma 5. *[39] Define* $s_{\bullet,1}=\left(s_{1,1},\dots,s_{m,1}\right)^T$, $y=\left(y_1,\dots,y_m\right)^T$, $\mathbf{y}_r=\left(y_r,\dots,y_r\right)$, *one gets*

$$
\|y-\mathbf{y}_r\|\le\|s_{\bullet,1}\|/\Delta\left(\mathfrak{L}+\varphi\right)
$$

where $\Delta\left(\mathfrak{L}+\varphi\right)$ *is the minimum singular value of* $\mathfrak{L}+\varphi$.

3. FLSs

A Fuzzy Logic System (FLS) consists of four parts, that is, the singleton fuzzifier, the knowledge base, the center-average defuzzifier, and the fuzzy inference engine [44]. The knowledge base of FLS is composed of a series of fuzzy IF-THEN inference rules of the following form:

$$
R^i:\ \text{IF } \mathfrak{N}_1 \text{ is } F_1^i \text{ and } \dots \text{ and } \mathfrak{N}_n \text{ is } F_n^i,
$$

$$
\text{then } Y \text{ is } B^i,\ i=1,\dots,K.
$$

where $\mathfrak{N}=\left(\mathfrak{N}_1,\dots,\mathfrak{N}_n\right)^T$, and Y are input and output of the FLS, respectively. K is the rules number. The FLS with the singleton fuzzifier, product inference and centre average defuzzifier can be expressed as

$$Y(\mathfrak{N}) = \frac{\sum_{i=1}^{N} \mathfrak{I}_i \prod_{j=1}^{n} P_{F_j^i}(\mathfrak{N}_j)}{\sum_{i=1}^{N} \left[\prod_{j=1}^{n} P_{F_j^i}(\mathfrak{N}_j)\right]}$$

where $\mathfrak{N} = [\mathfrak{N}_1, \mathfrak{N}_2, \ldots, \mathfrak{N}_n]^T \in R^n$, Fuzzy sets B^i and F_j^i are associated with the fuzzy membership functions $P_{B_i}(Y)$ and $P_{F_j^i}$, and $\mathfrak{I}_i = \max_{Y \in R}[P_{B_i}(Y)]$. Let

$$\psi_i(\mathfrak{N}) = \frac{\prod_{j=1}^{n} P_{F_j^i}(\mathfrak{N}_j)}{\sum_{i=1}^{N} \left[\prod_{j=1}^{n} P_{F_j^i}(\mathfrak{N}_j)\right]}, i = 1, \ldots, M.$$

Define $\psi(\mathfrak{N}) = [\psi_1(\mathfrak{N}), \ldots, \psi_K(\mathfrak{N})]^T$ and $W = [\mathfrak{I}_1, \mathfrak{I}_2, \ldots, \mathfrak{I}_K]^T = [W_1, W_2, \ldots, W_K]^T$. Then, the FLS can be rewritten as the following form

$$Y(\mathfrak{N}) = W^T \psi(\mathfrak{N}). \tag{17}$$

Lemma 6. *[45] Let $F(\mathfrak{N})$ be a continuous function defined on a compact set $\Omega_{\mathfrak{N}}$. Then, for any constant $\epsilon > 0$, there exists a FLS such as*

$$\sup_{\mathfrak{N} \in \Omega_{\mathfrak{N}}} = \left|F(\mathfrak{N}) - W^T \psi(\mathfrak{N})\right| < \epsilon.$$

Define the optimal parameter vector W^* as

$$W^* = \arg \min_{W \in \Omega_W} \sup_{\mathfrak{N} \in \Omega_{\mathfrak{N}}} \left|(\mathfrak{N}) - W^T \psi(\mathfrak{N})\right|$$

where $\Omega_{\mathfrak{N}}$ and Ω_W denote compact regions for $\mathfrak{N}$ and W, respectively, and the FLS minimum approximation error is given as follows

$$\epsilon = F(\mathfrak{N}) - W^{*T} \psi(\mathfrak{N}). \tag{18}$$

4. Main Results

4.1. Adaptive Event-Triggered Controller Derivation

In this section, by using the FLSs approximation property and the backstepping technique, an adaptive fuzzy event-triggered control protocol will be constructed. The backstepping approach design procedure contains n_i steps. Let $W_{i,j}^{*T} \psi(\mathfrak{N}_{i,j})$ be the FLSs which is used to model unknown nonlinear function at the kth step.

Then, define $\theta_{i,j}$ as

$$\theta_{i,j} = \max\left\{\frac{1}{b_M}\left\|W_{i,j}^*\right\|^2\right\}$$

and introduce $\hat{\theta}_{i,j}$ as the estimate of $\theta_{i,j}$. $\tilde{\theta}_{i,j} = \theta_{i,j} - \hat{\theta}_{i,j}$ $(i = 1, \ldots, M,\ j = 1, \ldots, n_i)$ denotes the estimation error. In the following design, we define the following coordinate transformation

$$s_{i,h} = v_{i,h} - \alpha_{i,h-1}, h = 2, \ldots, n_i \tag{19}$$

where $\alpha_{i,h-1}$ denotes the virtual control signal at the $(h-1)$th step to be designed.

Step 1 : According to Itô formula and (16), one gets

$$\begin{aligned} ds_{i,1} \;=\; & ((d_i + b_i)(\phi_{i,1}(v_{i,1}, \tau_{i,1}) v_{i,2} + h_{i,1}(v_{i,1}, v_{i,2}^0)) \\ & - \sum_{j=1}^{m} a_{i,j}(\phi_{j,1}(v_{j,1}, \tau_{j,1}) v_{j,2} + h_{j,1}(v_{j,1}, v_{j,2}^0)) - b_i \dot{y}_r) dt \\ & + ((d_i + b_i) \Psi_{i,1}(v_{i,1}) - \sum_{j=1}^{m} a_{i,j} \Psi_{j,1}(v_{j,1})) dw. \end{aligned} \tag{20}$$

Define the Lyapunov function candidate as

$$V_{i,1} = \frac{1}{4} s_{i,1}^4 + \frac{b_M}{2\zeta_{i,1}} \tilde{\theta}_{i,1}^2 \tag{21}$$

where $\zeta_{i,1} > 0$ is a design constant. By (2), (16) and (20), one obtains

$$\begin{aligned} LV_{i,1} \;=\; & s_{i,1}^3((d_i + b_i)(\phi_{i,1}(v_{i,1}, \tau_{i,1})(s_{i,2} + \alpha_{i,1}) + h_{i,1}(v_{i,1}, 0)) \\ & - \sum_{j=1}^{m} a_{i,j}(\phi_{j,1}(v_{j,1}, \tau_{j,1}) v_{j,2} + h_{j,1}(v_{j,1}, 0)) - b_i \dot{y}_r) \\ & + \frac{3}{2} s_{i,1}^2 \Phi_{i,1}^T \Phi_{i,1} - \frac{b_M}{\zeta_{i,1}} \tilde{\theta}_{i,1} \dot{\hat{\theta}}_{i,1} \end{aligned} \tag{22}$$

where

$$\Phi_{i,1} = (d_i + b_i) \Psi_{i,1}(v_{i,1}) - \sum_{j=1}^{m} a_{i,j} \Psi_{j,1}(v_{j,1}).$$

According to Assumption 3 and Young's inequality, one have

$$s_{i,1}^3 (d_i + b_i) \phi_{i,1}(v_{i,1}, \tau_{i,1}) s_{i,2} \;\leq\; \frac{3}{4}(d_i + b_i) b_M s_{i,1}^4 + \frac{1}{4}(d_i + b_i) b_M s_{i,2}^4 \tag{23}$$

$$\frac{3}{2} s_{i,1}^2 \Phi_{i,1}^T \Phi_{i,1} \;\leq\; \frac{3}{4} l_{i,1}^{-2} s_{i,1}^4 \|\Psi_{i,1}\|^4 + \frac{3}{4} l_{i,1}^2. \tag{24}$$

Substituting (23)-(24) into (20), it yields

$$\begin{aligned} LV_{i,1} \;\leq\; & s_{i,1}^3((d_i + b_i)(b_M \alpha_{i,1} + F_{i,1}(\mathfrak{N}_{i,1})) - \frac{b_M}{\zeta_{i,1}} \tilde{\theta}_{i,1} \dot{\hat{\theta}}_{i,1} \\ & + \frac{3}{4} l_{i,1}^2 + \frac{1}{4}(d_i + b_i) b_M s_{i,2}^4 \end{aligned} \tag{25}$$

where

$$\begin{aligned} F_{i,1}\left(\mathfrak{N}_{i,1}\right) &= \left(d_i+b_i\right) h_{i,1}\left(v_{i,1},0\right)+\frac{3}{4}\left(d_i+b_i\right) b_M s_{i,1} \\ &\quad -\sum_{j=1}^{m} a_{i,j}\left(h_{j,1}\left(v_{j,1},0\right)+\phi_{j,1}\left(v_{j,1},\tau_{j,1}\right) v_{j,2}\right) \\ &\quad -b_i \dot{y}_r+\frac{3}{4} l_{i,1}^{-2} s_{i,1}^{4}\left\|\Psi_{i,1}\right\|^{4}. \end{aligned}$$

Since $F_{i,1}\left(\mathfrak{N}_{i,1}\right)$ contains the unknown function $h_{i,1}$, $\phi_{j,1}$ and $\Psi_{i,1}$, $F_{i,1}\left(\mathfrak{N}_{i,1}\right)$ cannot be handled in a real system. For any constant $\epsilon_{i,1}>0$, based on Lemma 6, there exists a FLS $W_{i,1}^{*T}\psi\left(\mathfrak{N}_{i,1}\right)$ such as

$$F_{i,1}\left(\mathfrak{N}_{i,1}\right)=W_{i,1}^{*T}\psi\left(\mathfrak{N}_{i,1}\right)+\epsilon_{i,1}$$

where $\mathfrak{N}_{i,1}=\left(v_i,v_j,y_r,\dot{y}_r\right)^T$.

According to Young's inequality, it follows

$$s_{i,1}^{3} F_{i,1}\left(\mathfrak{N}_{i,1}\right) \leq \frac{b_M \theta_{i,1}}{2\sigma_{i,1}^{2}} s_{i,1}^{6} \psi^{T}\left(\mathfrak{N}_{i,1}\right) \psi\left(\mathfrak{N}_{i,1}\right)+\frac{\sigma_{i,1}^{2}}{2}+\frac{3 b_M s_{i,1}^{4}}{4}+\frac{\epsilon_{i,1}^{4}}{4 b_M}. \tag{26}$$

Construct the virtual control signal and adaptive law as follows

$$\alpha_{i,1} = \frac{1}{\left(d_i+b_i\right)}\left[-c_{i,1} s_{i,1}-\frac{3}{4} s_{i,1}-\frac{\hat{\theta}_{i,1}}{2\sigma_{i,1}^{2}} s_{i,1}^{3} \psi^{T}\left(\mathfrak{N}_{i,1}\right) \psi\left(X_{i,1}\right)\right] \tag{27}$$

$$\dot{\hat{\theta}}_{i,1} = \frac{\zeta_{i,1}}{2\sigma_{i,1}^{2}} s_{i,1}^{6} \psi^{T}\left(\mathfrak{N}_{i,1}\right) \psi\left(\mathfrak{N}_{i,1}\right)-\hat{\theta}_{i,1} \tag{28}$$

where $c_{i,1}>0$ denotes a design constant.

Substituting (26)-(28) into (25), it yields

$$\begin{aligned} LV_{i,1} &\leq -c_{i,1} b_M s_{i,1}^{4}+\frac{\sigma_{i,1}^{2}}{2}+\frac{\epsilon_{i,1}^{4}}{4}+\frac{3}{4} l_{i,1}^{2} \\ &\quad +\frac{1}{4}\left(d_i+b_i\right) b_M s_{i,2}^{4}+\frac{b_M}{\zeta_{i,1}} \tilde{\theta}_{i,1} \hat{\theta}_{i,1}. \end{aligned} \tag{29}$$

Notice that

$$\frac{b_M}{\zeta_{i,1}} \tilde{\theta}_{i,1} \hat{\theta}_{i,1} \leq -\frac{b_M}{2\zeta_{i,1}} \tilde{\theta}_{i,1}^{2}+\frac{b_M}{2\zeta_{i,1}} \theta_{i,1}^{2}.$$

Inequation (29) can be rewritten as

$$LV_{i,1} \leq -c_{i,1} b_M s_{i,1}^{4}+\frac{1}{4}\left(d_i+b_i\right) b_M s_{i,2}^{4}-\frac{b_M}{2\zeta_{i,1}} \tilde{\theta}_{i,1}^{2}+\Gamma_{i,1} \tag{30}$$

where

$$\Gamma_{i,1}=\frac{b_M}{2\zeta_{i,1}} \theta_{i,1}^{2}+\frac{\sigma_{i,1}^{2}}{2}+\frac{\epsilon_{i,1}^{4}}{4 b_M}+\frac{3}{4} l_{i,1}^{2}$$

Step $h\,(2 \le h \le n_i - 1)$: Based on Itô formula and (19), one obtains

$$\begin{aligned} ds_{i,h} &= (\phi_{i,h}(v_{i,h}, \tau_{i,h})\, v_{i,h+1} + h_{i,h}(v_{i,h}, 0) - \sum_{j=1}^{h-1} \frac{\partial \alpha_{i,h-1}}{\partial v_{i,j}} (\phi_{i,j}(v_{i,j}, \tau_{i,j})\, v_{i,j+1} \\ &\quad + h_{i,j}(v_{i,j}, 0)) - \sum_{j=0}^{h-1} \frac{\partial \alpha_{i,h-1}}{\partial y_r^{(j)}} y_r^{(j+1)} - \sum_{j=1}^{h-1} \frac{\partial \alpha_{i,h-1}}{\partial \hat{\theta}_{i,j}} \dot{\hat{\theta}}_{i,j} \\ &\quad - \frac{1}{2} \sum_{t,o=1}^{h-1} \frac{\partial^2 \alpha_{i,h-1}}{\partial v_{i,t} \partial v_{i,o}} \Psi_{i,t}^T \Psi_{i,o}) dt + (\Psi_{i,h}(v_{i,h}) - \sum_{j=1}^{h-1} \frac{\partial \alpha_{i,h-1}}{\partial v_{i,j}} \Psi_{i,j}(v_{i,j}))^T dw. \end{aligned} \tag{31}$$

Choose stochastic Lyapunov function as

$$V_{i,h} = V_{i,h-1} + \frac{1}{4} s_{i,h}^4 + \frac{b_M}{2\zeta_{i,h}} \tilde{\theta}_{i,h}^2. \tag{32}$$

Then, by applying (2) and (31), one has

$$\begin{aligned} LV_{i,h} &\le LV_{i,h-1} + s_{i,h}^3 (h_{i,h}(v_{i,h}, 0) + \phi_{i,h}(v_{i,h}, \tau_{i,h})(\alpha_{i,h} + s_{i,h+1}) \\ &\quad - \sum_{j=1}^{h-1} \frac{\partial \alpha_{i,h-1}}{\partial \hat{\theta}_{i,j}} \dot{\hat{\theta}}_{i,j} - \sum_{j=0}^{h-1} \frac{\partial \alpha_{i,h-1}}{\partial y_r^{(j)}} y_r^{(j+1)} - \frac{1}{2} \sum_{t,o=1}^{h-1} \frac{\partial^2 \alpha_{i,h-1}}{\partial v_{i,t} \partial v_{i,o}} \Psi_{i,t}^T \Psi_{i,o} \\ &\quad - \sum_{j=1}^{h-1} \frac{\partial \alpha_{i,h-1}}{\partial v_{i,j}} (\phi_{i,j}(v_{i,j}, \tau_{i,j})\, v_{i,j+1} + h_{i,j}(v_{i,j}, 0)) + \frac{3}{2} s_{i,h}^2 \Phi_{i,h}^T \Phi_{i,h} - \frac{b_M}{\zeta_{i,h}} \tilde{\theta}_{i,h} \dot{\hat{\theta}}_{i,h} \end{aligned} \tag{33}$$

where

$$\Phi_{i,h} = \Psi_{i,h}(v_{i,h}) - \sum_{j=1}^{h} \frac{\partial \alpha_{i,h-1}}{\partial v_{i,j}} \Psi_{i,j}(v_{i,j}).$$

140 According to Young's inequality, one have

$$\phi_{i,h}(v_{i,h}, \tau_{i,h})\, s_{i,h}^3 s_{i,h+1} \le \frac{3}{4} b_M s_{i,h}^4 + \frac{1}{4} b_M s_{i,h+1}^4 \tag{34}$$

$$\frac{3}{2} s_{i,h}^2 \Phi_{i,h}^T \Phi_{i,h} \le \frac{3}{4} l_{i,h}^{-2} s_{i,h}^4 \|\Phi_{i,h}\|^4 + \frac{1}{4} l_{i,h}^2. \tag{35}$$

Substituting (34) and (35) into (33), it yields

$$\begin{aligned} LV_{i,h} &\le LV_{i,h-1} + s_{i,h}^3 (F_{i,h}(\mathfrak{N}_{i,h}) + b_M \alpha_{i,h}) \\ &\quad - \frac{b_M}{\zeta_{i,h}} \tilde{\theta}_{i,h} \dot{\hat{\theta}}_{i,h} + \frac{1}{4} b_M s_{i,h+1}^4 + \frac{1}{4} l_{i,h}^2 \end{aligned} \tag{36}$$

where

$$F_{i,h}(\mathfrak{N}_{i,h}) = h_{i,h}(v_{i,h}, 0) - \sum_{j=1}^{h-1} \frac{\partial \alpha_{i,h-1}}{\partial \hat{\theta}_{i,j}} \dot{\hat{\theta}}_{i,j} - \sum_{j=0}^{h-1} \frac{\partial \alpha_{i,h-1}}{\partial y_r^{(j)}} y_r^{(j+1)}$$

$$-\frac{1}{2}\sum_{t,o=1}^{h-1}\frac{\partial^2\alpha_{i,h-1}}{\partial v_{i,t}\partial v_{i,o}}\Psi_{i,t}^T\Psi_{i,o}-\sum_{j=1}^{h-1}\frac{\partial\alpha_{i,h-1}}{\partial v_{i,j}}(\phi_{i,j}\left(v_{i,j},\tau_{i,j}\right)v_{i,j+1}$$
$$+h_{i,j}\left(v_{i,j},0\right))+\frac{1}{4}\left(\breve{d}_i+\breve{b}_i\right)b_M s_{i,h}+\frac{3}{4}b_M s_{i,h}+\frac{3}{4}l_{i,h}^{-2}s_{i,h}\left\|\Phi_{i,h}\right\|^4$$

where for $h=2$, take $\breve{d}_i+\breve{b}_i=d_i+b_i$, and for $3\leq h\leq n_i-1$, take $\breve{d}_i+\breve{b}_i=1$.

According to Young's inequality, one has

$$s_{i,h}^3F_{i,h}\left(\mathfrak{N}_{i,h}\right)\leq\frac{b_M\theta_{i,h}}{2\sigma_{i,h}^2}s_{i,h}^6\psi^T\left(\mathfrak{N}_{i,h}\right)\psi\left(\mathfrak{N}_{i,h}\right)+\frac{\sigma_{i,h}^2}{2}+\frac{3b_Ms_{i,h}^4}{4}+\frac{\epsilon_{i,h}^4}{4b_M}\tag{37}$$

where $\mathfrak{N}_{i,h}=\left[v_{i,h},y_r^{(h)},\hat{\theta}_{i,h}^T\right]$.

Construct the virtual controller and adaptive law as follows

$$\begin{aligned}\alpha_{i,h}&=[-c_{i,h}s_{i,h}-\frac{3}{4}s_{i,h}-\frac{\hat{\theta}_{i,h}}{2\sigma_{i,h}^2}s_{i,h}^3\psi^T\left(\mathfrak{N}_{i,h}\right)\psi\left(\mathfrak{N}_{i,h}\right)],&(38)\\ \dot{\hat{\theta}}_{i,h}&=\frac{\zeta_{i,h}}{2\sigma_{i,h}^2}s_{i,h}^6\psi^T\left(\mathfrak{N}_{i,h}\right)\psi\left(\mathfrak{N}_{i,h}\right)-\hat{\theta}_{i,h}.&(39)\end{aligned}$$

Meanwhile, it is noticed that

$$\frac{b_M}{\zeta_{i,h}}\tilde{\theta}_{i,h}\hat{\theta}_{i,h}\leq-\frac{b_M}{2\zeta_{i,h}}\tilde{\theta}_{i,h}^2+\frac{b_M}{2\zeta_{i,h}}\theta_{i,h}^2.\tag{40}$$

Therefore, one has

$$LV_{i,h}\leq-\sum_{j=1}^{h}c_{i,j}s_{i,j}^4-\sum_{j=1}^{h}\frac{b_M}{2\zeta_{i,j}}\tilde{\theta}_{i,j}^2+\frac{1}{4}b_Ms_{i,h+1}^4+\Gamma_{i,h}\tag{41}$$

where

$$\Gamma_{i,h}=\Gamma_{i,h-1}+\frac{1}{4}l_{i,h}^2+\frac{\sigma_{i,h}^2}{2}+\frac{\epsilon_{i,h}^4}{4b_M}+\frac{b_M}{2\zeta_{i,h}}\theta_{i,h}^2$$

Step n_i : In this step, we will structure a real controller. Based on Itô formula and (19), one has

$$\begin{aligned}ds_{i,n_i}&=(\phi_{i,n_i}\left(v_{i,n_i},\tau_{i,n_i}\right)(\frac{c\left(u_i\right)\xi_i\left(t\right)}{1+\chi_1\left(t\right)\varrho}-\frac{c\left(u_i\right)\chi_2\left(t\right)d}{1+\chi_1\left(t\right)\varrho}+h\left(u_i\right))+h_{i,n_i}\left(v_{i,n_i},0\right)\\&\quad-\sum_{j=1}^{n_i-1}\frac{\partial\alpha_{i,n_i-1}}{\partial v_{i,j}}(\phi_{i,j}\left(v_{i,j},\tau_{i,j}\right)v_{i,j+1}+h_{i,j}\left(v_{i,j},0\right))-\sum_{j=0}^{n_i-1}\frac{\partial\alpha_{i,n_i-1}}{\partial y_r^{(j)}}y_r^{(j+1)}\\&\quad-\sum_{j=1}^{n_i-1}\frac{\partial\alpha_{i,h-1}}{\partial\hat{\theta}_{i,j}}\dot{\hat{\theta}}_{i,j}-\frac{1}{2}\sum_{t,o=1}^{n_i-1}\frac{\partial^2\alpha_{i,n_i-1}}{\partial v_{i,t}\partial v_{i,o}}\Psi_{i,t}^T\Psi_{i,o})dt\\&\quad+(\Psi_{i,n_i}\left(v_{i,n_i}\right)-\sum_{j=1}^{n_i-1}\frac{\partial\alpha_{i,n_i-1}}{\partial v_{i,j}}\Psi_{i,j}^T)^Tdw.\end{aligned}\tag{42}$$

Choose the Lyapunov function candidate as

$$V_{i,n_i} = V_{i,n_i-1} + \frac{s_{i,n_i}^4}{4} + \frac{1-\delta_m}{2r_{i,n_i}}\tilde{\eta}_{i,n_i}^2 + \frac{b_M}{2\zeta_{i,n_i}}\tilde{\theta}_{i,n_i}^2 \tag{43}$$

with r_{i,n_i} being a design constant, $\tilde{\eta}_{i,n_i} = \eta_{i,n_i} - \hat{\eta}_{i,n_i}$ denotes the approximation error between $\hat{\eta}_{i,n_i}$ and $\eta_{i,n_i} = \frac{1}{1-\delta_m}$. Following the event-triggered mechanism in (12), and by applying (2), (5) and (42), one has

$$\begin{aligned} LV_{i,n_i} \leq\ & LV_{i,n_i-1} + s_{i,n_i}^3 (h_{i,n_i}(v_{i,n_i},0) + b_M (\frac{c(u_i)\xi_i(t)}{1+\chi_1(t)\varrho} - \frac{c(u_i)\chi_2(t)d}{1+\chi_1(t)\varrho} \\ & + h(u_i)) - \sum_{j=1}^{n_i-1} \frac{\partial \alpha_{i,n_i-1}}{\partial v_{i,j}} (\phi_{i,j}(v_{i,j},\tau_{i,j}) v_{i,j+1} + h_{i,j}(v_{i,j},0)) \\ & - \sum_{j=0}^{n_i-1} \frac{\partial \alpha_{i,n_i-1}}{\partial y_r^{(j)}} y_r^{(j+1)} - \frac{1}{2}\sum_{t,o=1}^{n_i-1} \frac{\partial^2 \alpha_{i,n_i-1}}{\partial v_{i,t}\partial v_{i,o}} \Psi_{i,t}^T \Psi_{i,o} - \sum_{j=1}^{n_i-1} \frac{\partial \alpha_{i,h-1}}{\partial \hat{\theta}_{i,j}} \dot{\hat{\theta}}_{i,j} \\ & + \frac{3}{2} s_{i,n_i}^2 \Phi_{i,n_i}^T \Phi_{i,n_i} - \frac{(1-\delta_m)}{r_{i,n_i}} \tilde{\eta}_{i,n_i} \dot{\hat{\eta}}_{i,n_i} - \frac{b_M}{\zeta_{i,n_i}} \tilde{\theta}_{i,n_i} \dot{\hat{\theta}}_{i,n_i}. \end{aligned} \tag{44}$$

By applying Young's inequality, one has

$$\frac{3}{2} s_{i,n_i}^2 \Phi_{i,n_i}^T \Phi_{i,n_i} \leq \frac{3}{4} l_{i,n_i}^{-2} s_{i,n_i}^4 \left\| \Phi_{i,n_i} \right\|^4 + \frac{1}{4} l_{i,n_i}^2. \tag{45}$$

Design the adaptive controller as

$$\xi_i(t) = -(1+\varrho) D \tag{46}$$

where

$$D = \left(\frac{\mathfrak{d}}{1-\varrho} + \frac{\varepsilon}{1-\delta_m} \right) \frac{s_{i,n_i}^3}{\sqrt{s_{i,n_i}^6 + \varsigma^2(t)}} + \frac{s_{i,n_i}^3 \alpha_{i,n_i}^2 \hat{\eta}_{i,n_i}^2}{\sqrt{s_{i,n_i}^6 \alpha_{i,n_i}^2 \hat{\eta}_{i,n_i}^2 + \varsigma^2(t)}}. \tag{47}$$

$\varsigma(t)$ is chosen as a positive integrable function and satisfies $\int_0^\infty \varsigma(t)\,dt < +\infty$. Noting that many functions satisfy this condition such as $\varsigma(t) = \ell e^{-\ell t}$ with positive number ℓ.

From [40], we obtain the inequality

$$0 \leq |Z| - \frac{Z^2}{\sqrt{Z^2 + \epsilon^2}} \leq \epsilon \tag{48}$$

where $\epsilon \geq 0$. Further, one has

$$\begin{aligned} s_{i,n_i}^3 b_M \frac{c(u_i)\varpi_i(t)}{1+\chi_1(t)\varrho} \leq\ & -s_{i,n_i}^3 b_M \alpha_{i,n_i} + b_M(1-\delta_m)\varsigma(t) - \frac{b_M c(u_i)\mathfrak{d}}{1-\sigma} \frac{s_{i,n_i}^6}{\sqrt{s_{i,n_i}^6 + \varsigma^2(t)}} \\ & + b_M(1-\delta_m)\tilde{\eta}_{i,n_i} \left| s_{i,n_i}^3 \alpha_{i,n_i} \right| - \frac{b_M \varepsilon s_{i,n_i}^6}{\sqrt{s_{i,n_i}^6 + \varsigma^2(t)}}, \end{aligned} \tag{49}$$

$$
\begin{aligned}
-\frac{s_{i,n_i}^3 b_M c\left(u_i\right) \chi_2\left(t\right) \mathfrak{d}}{1+\chi_1\left(t\right) \varrho} &\leq \frac{b_M c\left(u_i\right) \mathfrak{d}}{1-\varrho}\left(\frac{s_{i,n_i}^6}{\sqrt{s_{i,n_i}^6+\varsigma^2\left(t\right)}}+\varsigma\left(t\right)\right), && (50)\\
b_M s_{i,n_i}^3 \varepsilon - \varepsilon \frac{b_M s_{i,n_i}^6}{\sqrt{s_{i,n_i}^6+\varsigma^2\left(t\right)}} &\leq b_M \varepsilon \varsigma\left(t\right). && (51)
\end{aligned}
$$

Substituting (49)-(51) into (44), it yields

$$
\begin{aligned}
LV_{i,n_i} \leq\; & LV_{i,n_i-1} - s_{i,n_i}^3 \alpha_{i,n_i} b_M + s_{i,n_i}^3 F_{i,n_i}\left(\mathfrak{N}_{i,n_i}\right) + b_M\left(1+\delta_m\right)\varsigma\left(t\right) + b_M \varepsilon \varsigma\left(t\right)\\
& + b_M\left(1-\delta_m\right)\tilde{\eta}_{i,n_i}\left|s_{i,n_i}^3 \alpha_{i,n_i}\right| + \frac{\mathfrak{d}}{1-\varrho} b_M\left(1+\delta_m\right)\varsigma\left(t\right)\\
& - \frac{\left(1-\delta_m\right)}{r_{i,n_i}}\tilde{\eta}_{i,n_i}\dot{\hat{\eta}}_{i,n_i} - \frac{b_M}{\zeta_{i,n_i}}\tilde{\theta}_{i,n_i}\dot{\hat{\theta}}_{i,n_i} \qquad (52)
\end{aligned}
$$

where

$$
\begin{aligned}
F_{i,n_i}\left(\mathfrak{N}_{i,n_i}\right) =\; & h_{i,n_i}\left(v_{i,n_i},0\right) - \sum_{j=1}^{n_i-1}\frac{\partial \alpha_{i,n_i-1}}{\partial \hat{\theta}_{i,n_i}}\dot{\hat{\theta}}_{i,n_i} - \sum_{j=0}^{n_i-1}\frac{\partial \alpha_{i,n_i-1}}{\partial y_r^{(j)}} y_r^{(j+1)}\\
& - \frac{1}{2}\sum_{t,o=1}^{n_i-1}\frac{\partial^2 \alpha_{i,n_i-1}}{\partial v_{i,t}\partial v_{i,o}}\Psi_{i,t}^T \Psi_{i,o} - \sum_{j=1}^{n_i-1}\frac{\partial \alpha_{i,n_i-1}}{\partial v_{i,j}}\left(\phi_{i,j}\left(v_{i,j},\tau_{i,j}\right) v_{i,j+1}\right.\\
& \left. + h_{i,j}\left(v_{i,j},0\right)\right) + b_M s_{i,n_i} + \frac{3}{4} l_{i,n_i}^{-2} s_{i,n_i}\left\|\Phi_{i,n_i}\right\|^4.
\end{aligned}
$$

By using Young's inequality, one has

$$
s_{i,n_i}^3 F_{i,n_i}\left(\mathfrak{N}_{i,n_i}\right) \leq \frac{b_M \theta_{i,n_i}}{2\sigma_{i,n_i}^2} s_{i,n_i}^6 \psi^T\left(\mathfrak{N}_{i,n_i}\right)\psi\left(\mathfrak{N}_{i,n_i}\right) + \frac{\epsilon_{i,n_i}^4}{4b_M} + \frac{3b_M s_{i,n_i}^4}{4} + \frac{\sigma_{i,n_i}^2}{2} \qquad (53)
$$

where $\mathfrak{N}_{i,n_i} = \left[v_{i,n_i}, y_r^{(h)}, \hat{\theta}_{i,n_i}^T\right]$.

Design the intermediate control law and adaptive laws as follows

$$
\begin{aligned}
\alpha_{i,n_i} &= c_{i,n_i} s_{i,n_i} + \frac{3}{4} s_{i,n_i} + \frac{\hat{\theta}_{i,n_i}}{2\sigma_{i,n_i}^2} s_{i,n_i}^3 \psi^T\left(\mathfrak{N}_{i,n_i}\right)\psi\left(\mathfrak{N}_{i,n_i}\right), && (54)\\
\dot{\hat{\theta}}_{i,n_i} &= \frac{\zeta_{i,n_i}}{2\sigma_{i,n_i}^2} s_{i,n_i}^6 \psi^T\left(\mathfrak{N}_{i,n_i}\right)\psi\left(\mathfrak{N}_{i,n_i}\right) - \hat{\theta}_{i,n_i}, && (55)\\
\dot{\hat{\eta}}_{i,n_i} &= r_{i,n_i}\left|s_{i,n_i}^3 \alpha_{i,n_i}\right| b_M - \pi \hat{\eta}_{i,n_i}. && (56)
\end{aligned}
$$

Substituting (53)-(56) into (52), it yields

$$
\begin{aligned}
LV_{i,n_i} \leq\; & -\sum_{j=1}^{n_i} c_{i,j} s_{i,j}^4 + \frac{1-\delta_m}{r_{i,n_i}}\tilde{\eta}_{i,n_i}\pi\hat{\eta}_{i,n_i} + b_M \varepsilon \varsigma\left(t\right) + \frac{1}{4}\sum_{j=1}^{n_i} l_{i,j}^2\\
& - \frac{b_M}{\zeta_{i,n_i}}\tilde{\theta}_{i,n_i}\hat{\theta}_{i,n_i} + \left(1+\delta_m\right) b_M \varsigma\left(t\right)\left(1+\frac{\mathfrak{d}}{1-\varrho}\right).
\end{aligned}
$$

Applying the Young's inequality, we get

$$\begin{aligned}\frac{1-\delta_m}{r_{i,n_i}}\tilde{\eta}_{i,n_i}\pi\hat{\eta}_{i,n_i} &\leq -\frac{1-\delta_m}{2r_{i,n_i}}\pi\tilde{\eta}_{i,n_i}^2+\frac{1-\delta_m}{2r_{i,n_i}}\pi\eta_{i,n_i}^2 && (57)\\ \frac{b_M}{\zeta_{i,n_i}}\tilde{\theta}_{i,n_i}\hat{\theta}_{i,n_i} &\leq -\frac{b_M}{2\zeta_{i,n_i}}\tilde{\theta}_{i,n_i}^2+\frac{b_M}{2\zeta_{i,n_i}}\theta_{i,n_i}^2. && (58)\end{aligned}$$

Further, we obtain

$$LV_{i,n_i} \leq -\sum_{j=1}^{n_i} c_{i,j}s_{i,j}^4-\frac{1-\delta_m}{2r_{i,n_i}}\pi\tilde{\eta}_{i,n_i}^2-\sum_{j=1}^{n_i}\frac{b_M}{2\zeta_{i,n_i}}\tilde{\theta}_{i,n_i}^2+b_M\varepsilon\varsigma(t)+\Gamma_{i,n_i} \tag{59}$$

where

$$\begin{aligned}\Gamma_{i,n_i} &= \frac{1-\delta_m}{2r_{i,n_i}}\pi\eta_{i,n_i}^2+(1+\delta_m)\,b_M\varsigma(t)\left(1+\frac{\mathfrak{d}}{1-\varrho}\right)\\ &\quad+\sum_{j=1}^{n_i}\left(\frac{1}{4}l_{i,j}^2+\frac{b_M}{2\zeta_{i,j}}\theta_{i,j}^2+\frac{\sigma_{i,j}^2}{2}+\frac{\epsilon_{i,j}^4}{4b_M}\right).\end{aligned}$$

Let

$$\gamma_i=\min\{2c_{i,h},\beta_{i,h}\}>0,\ h=1,\ldots,n_i$$

then one gets

$$LV_{i,n_i}\leq-\gamma_i V_{i,n_i}+\Gamma_{i,n_i} \tag{60}$$

4.2. Stability Analysis

The following theorem shows the stability analysis of the proposed distributed control law.

Theorem 1. *Consider the closed-loop adaptive non-affine pure-feedback stochastic multi-agent systems consisting of M uncertain nonlinear subsystem (1) satisfying Assumptions 1–3, the event-triggered controller (10), the designed adaptive controller (46), the intermediate control law and the parameter adaptive laws (54)-(56). All followers' outputs are able to converge to a small neighborhood of the output of leader, and all the signals in the closed-loop system are bounded in probability.*

Proof 1. *Choose the following Lyapunov function V*

$$V=\sum_{i=1}^{m}V_{i,n_i}(t). \tag{61}$$

Based on (59), we have

$$LV\leq-\gamma V+\Gamma \tag{62}$$

where $\gamma=\min\{\gamma_i, i=1,\ldots,m\}$, $\Gamma=\sum_{i=1}^{m}\Gamma_{i,n_i}+\Theta$ with $\Theta=b_M\varepsilon\int_0^\infty\varsigma(t)\,dt$ being a positive constant since the parameter $\varsigma(t)$ in (47) is selected as the integrable function.

According to Lemma 4 and the definition of V, all the signals are CSUUB in probability in the closed-loop system.

Furthermore, according to [39] and (62), one gets

$$\frac{d\left(E\left[V\left(t\right)\right]\right)}{dt}=E\left[LV\left(t\right)\right]\leq-\gamma E\left(V\left(t\right)\right)+\Gamma,t\geq0. \tag{63}$$

We define $E\left[V\right]=b$ *and* $\gamma>\frac{\Gamma}{b}$. *It is obvious to obtain* $\frac{d(E[V])}{dt}\leq0$. *Then, it has* $E\left[V\left(t\right)\right]\leq b$ *when* $E\left[V\left(0\right)\right]\leq b$ *for all* $t\geq0$. *One has*

$$0\leq E\left[V\left(t\right)\right]\leq\left(V\left(0\right)-\frac{\Gamma}{\mu}\right)e^{-\mu t}+\frac{\Gamma}{\mu}. \tag{64}$$

Further

$$E\left[V\left(t\right)\right]\leq\frac{\Gamma}{\mu},t\rightarrow\infty. \tag{65}$$

According to $s_{i,1}=\left(s_{1,1},\ldots,s_{m,1}\right)^{T}$, *based on (65) and the definition of V, one gets*

$$\begin{aligned}E\left(\|s_{i,1}\|^{4}\right) &\leq E\left(s_{1,1}^{2}+s_{2,1}^{2}\ldots,s_{m,1}^{2}\right)^{2}\\ &\leq 2E\left(s_{1,1}^{4}+s_{2,1}^{4}\ldots,s_{m,1}^{4}\right)\\ &\leq \frac{8\Gamma}{\mu}.\end{aligned} \tag{66}$$

Theoretically, according to the definitions of γ *and* Γ, *for any* $\varepsilon>0$, *we select the appropriate design parameters* $c_{i,h}$, $\beta_{i,h}$ *to be sufficiently large, choosing* $a_{i,h}$ *to be sufficiently small, one can obtain*

$$\frac{\Gamma}{\mu}\leq\frac{\varepsilon}{8}\left(\Delta\left(L+\varphi\right)\right)^{4}. \tag{67}$$

Furthermore, when $t\rightarrow\infty$, *according to Lemma 5, the following inequality holds:*

$$E\left(\|y-\mathbf{y}_{r}\|\right)\leq E\left(\|s_{i,1}\|^{4}\right)/\Delta\left(L+\varphi\right)^{4}\leq\varepsilon \tag{68}$$

According to Definition 2, the tracking errors $e_i=y_i\left(t\right)-y_r\left(t\right)$ *are CSUUB in probability.* ■

Furthermore, according to [46], for $\forall k\in s^{+}$, we prove that there exists a constant $h^{*}>0$, $\{t_{k+1}-t_k\}\geq h^{*}$. According to $e_i\left(t\right)=\xi_i\left(t\right)-u_i\left(t\right),\forall t\in\left[t_k,t_{k+1}\right)$, we get

$$\frac{\mathfrak{d}}{dt}\left|e_i\right|=sign\left(e_i\right)de_i\leq\left|d\xi_i\right|.$$

From (46) and (59), one has

$$d\xi_i\left(t\right)=-\left(1+\varrho\right)\left(dD\right) \tag{69}$$

where

$$
\begin{aligned}
dD \;=\; & \left(\frac{\mathfrak{d}}{1-\varrho}+\frac{\varepsilon}{1-\delta_m}\right)\Big\{\left(ds_{i,n_i}^{3}\right)\sqrt{s_{i,n_i}^{6}+\varsigma^{2}\left(t\right)}\;-0.5s_{i,n_i}^{3}(s_{i,n_i}^{6}+\varsigma^{2}\left(t\right))^{-0.5}(ds_{i,n_i}^{6}+2\dot{\varsigma}\left(t\right))\Big\} \\
& /\left[s_{i,n_i}^{6}+\varsigma^{2}\left(t\right)\right]+\Big\{\Big(\left(ds_{i,n_i}^{3}\right)\alpha_{i,n_i}^{2}\hat{\eta}_{i,n_i}^{2}\;+2s_{i,n_i}^{3}\left(d\alpha_{i,n_i}\right)\hat{\eta}_{i,n_i}^{2}\;+2s_{i,n_i}^{3}\alpha_{i,n_i}^{2}\dot{\hat{\eta}}_{i,n_i}\Big) \\
& \times\sqrt{s_{i,n_i}^{6}\alpha_{i,n_i}^{2}\hat{\eta}_{i,n_i}^{2}+\varsigma^{2}\left(t\right)}-0.5s_{i,n_i}^{3}\alpha_{i,n_i}^{2}\hat{\eta}_{i,n_i}^{2}\left(s_{i,n_i}^{6}\alpha_{i,n_i}^{2}\hat{\eta}_{i,n_i}^{2}+\varsigma^{2}\left(t\right)\right)^{-0.5}\left(\left(ds_{i,n_i}^{3}\right)\alpha_{i,n_i}^{2}\hat{\eta}_{i,n_i}^{2}\right. \\
& \left.+2s_{i,n_i}^{3}\left(d\alpha_{i,n_i}\right)\hat{\eta}_{i,n_i}^{2}\;+2s_{i,n_i}^{3}\alpha_{i,n_i}^{2}\dot{\hat{\eta}}_{i,n_i}+2\dot{\varsigma}\left(t\right)\right)\Big\}/\left(s_{i,n_i}^{6}\alpha_{i,n_i}^{2}\hat{\eta}_{i,n_i}^{2}+\varsigma^{2}\right). \qquad (70)
\end{aligned}
$$

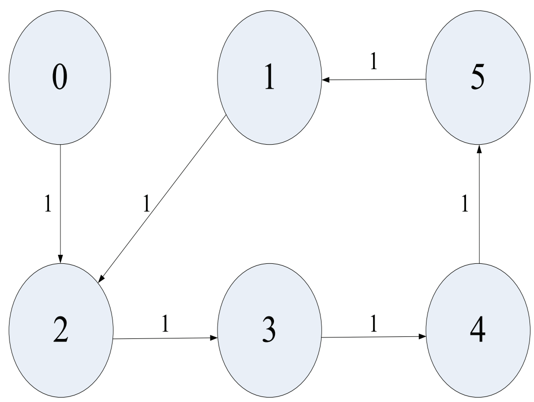

Figure 2: Graph $\bar{\mathcal{G}}$ used in the simulation

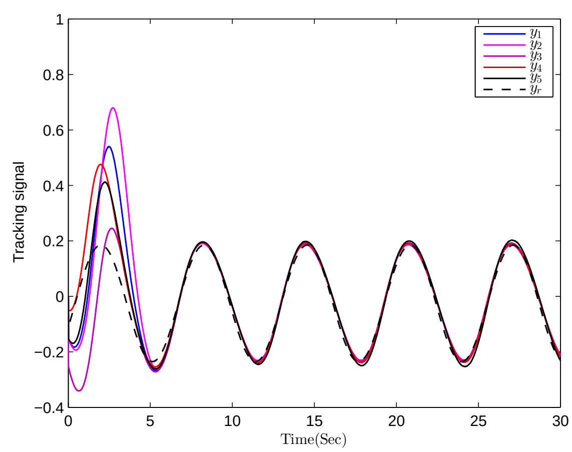

Figure 3: Output of the followers and leader.

Since all the signals are bounded, for any $\hbar \geq 0$, we can obtain $|d\xi_i\left(t\right)| \leq \hbar$. Based on (11) and (12), $e_i = 0$ and $\lim_{t\to t_{k+1}}\left(e_i\left(t\right)\right) = \varrho\left|u_i\left(t\right)\right| + \mathfrak{d}$ hold. We can obtain the lower bound of interexecution intervals h^* satisfying $h^* \geq \left(\varrho\left|u_i\left(t\right)\right| + \mathfrak{d}\right)/\hbar$, hence, the Zeno behavior has been successfully avoided.

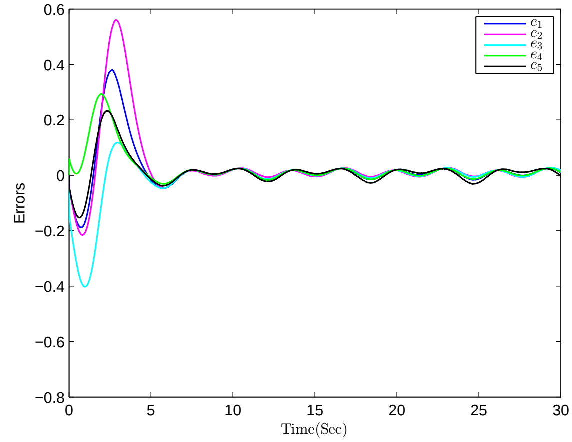

Figure 4: The tracking errors e_i

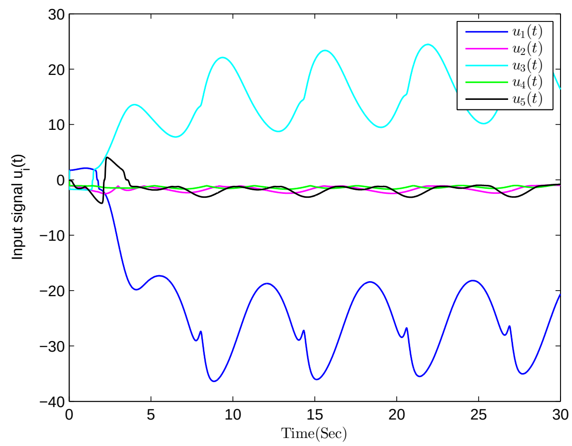

Figure 5: Event-triggered control signal

5. Simulation Study

We will illustrate the effectiveness of the proposed adaptive event-triggered control protocol in the section by using the following uncertain nonlinear non-affine pure-feedback stochastic multi-agent systems:

$$\begin{aligned} dv_{i,1} &= [v_{i,2} - v_{i,2}\sin(v_{i,2}) + 0.5\sin(t)]\, dt + [0.2\sin(v_{i,1})]\, dw \\ dv_{i,2} &= [q(u_i) + 0.8\sin(u_i) + 0.5\sin(t)]\, dt + [0.2\sin(v_{i,2})]\, dw \\ y_i &= v_{i,1} \end{aligned} \tag{71}$$

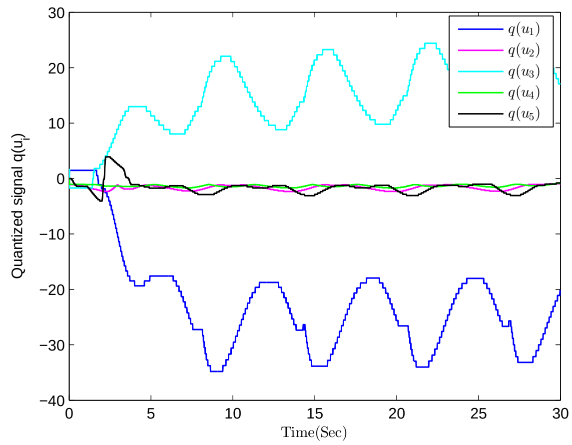

Figure 6: Quantized control signal

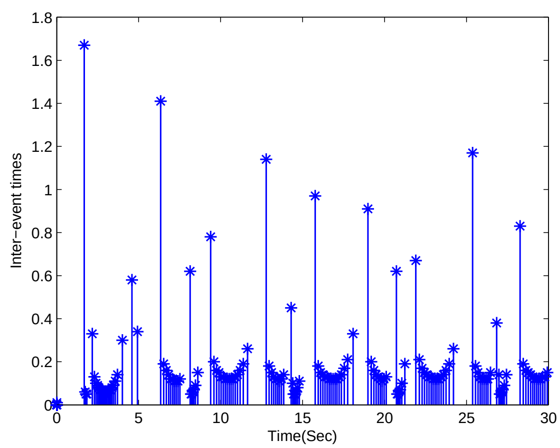

Figure 7: Interevent times of $\xi_1(t)$.

The initial conditions are selected as $v_1(0) = (-0.15, -0.05)^T$, $v_2(0) = (-0.15, -0.15)^T$, $v_3(0) = (-0.25, -0.01)^T$, $v_4(0) = (-0.05, 0.25)^T$, $v_5(0) = (-0.15, 0.13)^T$. The output of the leader is $y_r = -0.2\cos(t)$. In (71), $q(u_i)$ denotes the asymmetric hysteresis quantizer modelled as (3), and the quantization parameters $u_{i,\min}^{+} = 0.06$, $u_{i,\min}^{-} = -0.08$, $\delta_+ = 0.4$ and $\delta_- = 0.3$. The design parameters are chosen as $c_{i,j} = 0.5$, $r_{1,1} = 0.5$, $r_{1,2} = r_{2,1} = r_{2,2} = 0.1$, $r_{3,1} = r_{4,1} = r_{5,1} = 0.15$, $r_{3,2} = r_{4,2} = r_{5,2} = 0.25$. $\sigma_{i,j} = 0.01$, $\delta_m = 0.01$, $\varsigma(t) = 0.01e^{-0.01t}$. As shown in Fig. 3, the output signals of followers y_i can well follow the leader signal y_r. Fig. 5 - Fig. 6 show the event-triggered control signals and quantized control signals. The time of the released interval of triggered events is shown in Fig. 7 - Fig. 11.

According to these figures, we can see that the outputs of the followers follow the trajectory of the leader, and all the signals in the closed-loop system are bounded. These simulation results validate the effectiveness of the proposed control scheme.

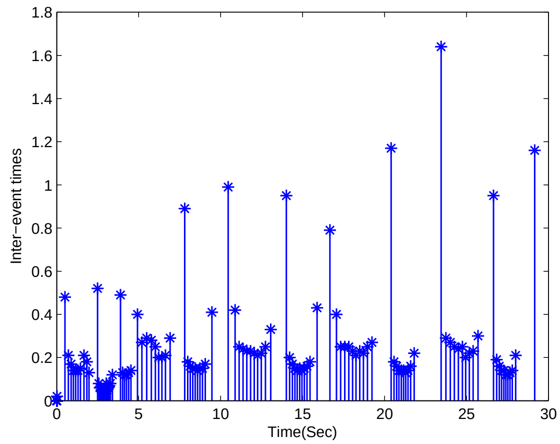

Figure 8: Interevent times of $\xi_2(t)$.

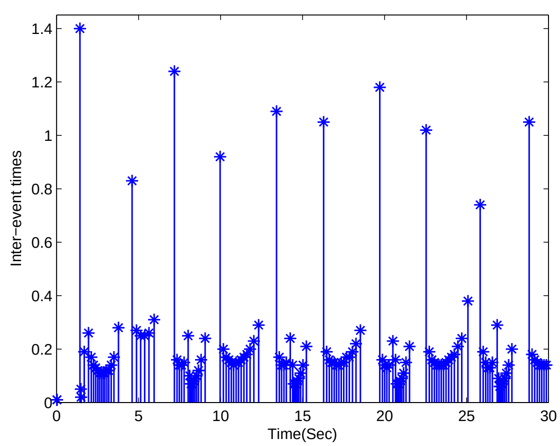

Figure 9: Interevent times of $\xi_3(t)$.

Remark 1. *The authors in [47] developed a novel event-triggered control protocol without requiring continuous communication among the follower agents, and an algorithm to actively adjust the leader adjacency matrix was proposed for the first time, which efficiently expands the application scope of existing methods. The authors in [48] presented an integrated sampled-data-based event-triggered communication scheme to improve the efficiency of data transmission, and a state-error-dependent delay system was designed to model the multi-agent systems. Finally, the efficiency of the proposed transmission scheme and consensus protocol are verified. Both of the above two event-triggered protocols are of great significance and research value. In this paper, however, we can not directly use the event-triggered protocol in [47] and [48], because we consider input quantification and apply synchronization error. The validity of the event-triggered protocol we designed is also validated in*

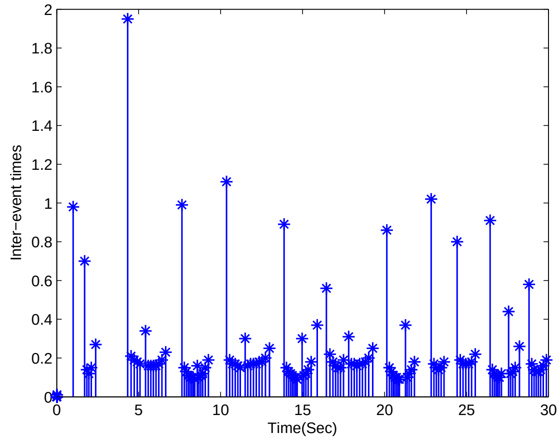

Figure 10: Interevent times of $\xi_4(t)$.

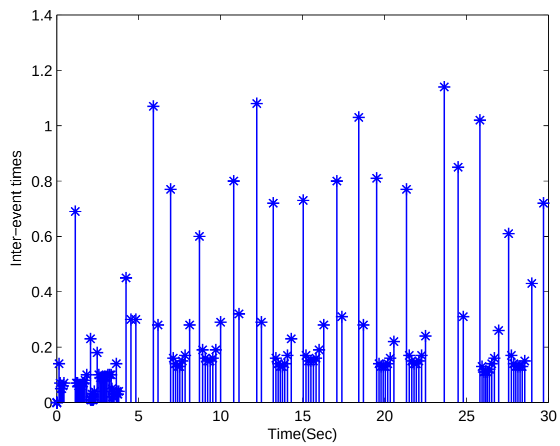

Figure 11: Interevent times of $\xi_5(t)$.

simulation. We will study and use the event-triggered protocol in [47] and [48] in the future work.

6. Conclusion

The problem of event-triggered fuzzy adaptive tracking control for non-affine pure-feedback stochastic nonlinear multi-agent systems with input quantization has been solved in this paper. With the designed asymmetrical hysteretic quantizer and event-triggered mechanism, the performance of the systems has been improved, and the burden of communication has been reduced. By applying the stochastic Lyapunov function method, it has been illustrated that all the followers' outputs converge to the neighborhood of the output of leader and all the signals are bounded in probability in the closed-loop system. Finally, the effectiveness of the approach has been illustrated through simulation example. In our future work, we will attempt to solve the fixed-time relay tracking control problem for agents in higher-dimensional space by the definition of the Voronoi diagram accordingly.

Acknowledgments

This work was partially supported by the Project of Liaoning Province Science and Technology Program(2019-KF-03-13).

References

[1] J. Zhang, X. Chen, G. Gu, State consensus for discrete-time multi-agent systems over time-varying graphs, IEEE Transactions on Automatic Control, DOI: 10.1109/TAC.2020.2979750.

[2] R. Lu, W. Yu, J. Lü, A. Xue, Synchronization on complex networks of networks, IEEE Transactions on Neural Networks and Learning Systems 25 (11) (2014) 2110–2118.

[3] J. A. Fax, R. M. Murray, Information flow and cooperative control of vehicle formations, IEEE Transactions on Automatic Control 49 (9) (2004) 1465–1476.

[4] J. Na, Y. Huang, X. Wu, G. Gao, G. Herrmann, J. Z. Jiang, Active adaptive estimation and control for vehicle suspensions with prescribed performance, IEEE Transactions on Control System Technology 11 (6) (2018) 2063–2077.

[5] Y. Pan, P. Du, H. Xue, H.-K. Lam, Singularity-free fixed-time fuzzy control for robotic systems with user-defined performance, IEEE Transactions on Fuzzy Systems, DOI: 10.1109/TFUZZ.2020.2999746.

[6] Z. Wang, Y. Xu, R. Lu, H. Peng, Finite-time state estimation for coupled markovian neural networks with sensor nonlinearities, IEEE Transactions on Neural Networks and Learning Systems 28 (3) (2017) 630–638.

[7] G. Wen, Z. Duan, Z. Li, G. Chen, Stochastic consensus in directed networks of agents with nonlinear dynamics and repairable actuator failures, IET Control Theory and Applications 6 (11) (2012) 1583–1593.

[8] C. E. Ren, L. Chen, C. L. P. Chen, Adaptive fuzzy leader-following consensus control for stochastic multi-agent systems with heterogeneous nonlinear dynamics, IEEE Transactions on Fuzzy Systems 25 (1) (2017) 181–190.

[9] Z. Lian, Y. He, C.-K. Zhang, M. Wu, Stability and stabilization of t-s fuzzy systems with time-varying delays via delay-product-type functional method, IEEE Transactions on Cybernetics 50 (6) (2020) 2580–2589.

[10] C.-K. Zhang, F. Long, Y. He, W. Yao, L. Jiang, M. Wu, A relaxed quadratic function negative-determination lemma and its application to time-delay systems, Automatica 113 (2020) 108764.

[11] X. Li, X. Yang, T. Huang, Persistence of delayed cooperative models: Impulsive control method, Applied Mathematics and Computation 342 (2019) 130–146.

[12] M. Chen, G. Tao, Adaptive fault-tolerant control of uncertain nonlinear large-scale systems with unknown dead-zone, IEEE Transactions on Cybernetics 46 (8) (2016) 1851–1862.

[13] X.-M. Li, B. Zhang, P. Li, Q. Zhou, R. Lu, Finite-horizon $h\infty$ state estimation for periodic neural networks over fading channels, IEEE transactions on neural networks and learning systemsDOI: 10.1109/TNNLS.2019.2920368.

[14] H. Liang, L. Zhang, Y. Sun, T. Huang, Containment control of semi-markovian multi-agent systems with switching topologies, IEEE Transactions on Systems, Man and Cybernetics: Systems, DOI: 10.1109/TSMC.2019.2946248.

[15] H. Li, Z. Zhang, H. Yan, X. Xie, Adaptive event-triggered fuzzy control for uncertain active suspension systems, IEEE Transactions on Cybernetics, DOI: 10.1109/TCYB.2018.2864776.

[16] X. M. Zhang, Q. L. Han, B. L. Zhang, An overview and deep investigation on sampled-data-based event-triggered control and filtering for networked systems, IEEE Transactions on Industrial Informatics 13 (1) (2017) 4–16.

[17] M. Chen, S. Shao, B. Jiang, Adaptive neural control of uncertain nonlinear systems using disturbance observer, IEEE Transactions on Cybernetics 47 (10) (2017) 3110–3123.

[18] T. Liu, Z. P. Jiang, A small-gain approach to robust event-triggered control of nonlinear systems, IEEE Transactions on Automatic Control 60 (8) (2015) 2072–2085.

[19] Z. Zhu, Y. Pan, Q. Zhou, C. Lu, Event-triggered adaptive fuzzy control for stochastic nonlinear systems with unmeasured states and unknown backlash-like hysteresis, IEEE Transactions on Fuzzy Systems, DOI: 10.1109/TFUZZ.2020.2973950.

[20] H. Liang, X. Guo, Y. Pan, T. Huang, Event-triggered fuzzy bipartite tracking control for network systems based on distributed reduced-order observers, IEEE Transactions on Fuzzy Systems, DOI: 10.1109/TFUZZ.2020.2982618.

[21] Y. Pan, G.-H. Yang, Event-driven fault detection for discrete-time interval type-2 fuzzy systems, IEEE Transactions on Systems, Man, and Cybernetics: Systems, DOI:10.1109/TSMC.2019.2945063.

[22] H.-K. Lam, L. D. Seneviratne, Stability analysis of interval type-2 fuzzy-model-based control systems,, IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics) 38 (3) (2008) 617–628.

[23] H.-K. Lam, H. Li, C. Deters, E. L. Secco, H. A. Wurdemann, K. Althoefer, Control design for interval type-2 fuzzy systems under imperfect premise matching, IEEE Transactions on Industrial Electronics 61 (2) (2013) 956–968.

[24] D. V. Dimarogonas, E. Frazzoli, K. H. Johansson, Distributed event-triggered control for multi-agent systems, IEEE Transactions on Automatic Control 57 (5) (2012) 1291–1297.

[25] L. Cao, H. Li, G. Dong, R. Lu, Event-triggered control for multiagent systems with sensor faults and input saturation, IEEE Transactions on Systems, Man, and Cybernetics: SystemsDOI: 10.1109/TSMC.2019.2938216.

[26] G. Guo, L. Ding, Q. L. Han, A distributed event-triggered transmission strategy for sampled-data consensus of multi-agent systems, Automatica 50 (5) (2014) 1489–1496.

[27] X. Li, J. Shen, H. Akca, R. Rakkiyappan, Lmi-based stability for singularly perturbed nonlinear impulsive differential systems with delays of small parameter, Applied Mathematics and Computation 250 (2015) 798–904.

[28] W. Hu, L. Liu, Cooperative output regulation of heterogeneous linear multi-agent systems by event-triggered control, IEEE Transactions on Automatic Control 47 (1) (2017) 105–116.

[29] J. Na, Y. Huang, Q. Pei, X. Wu, G. Gao, G. Li, Active suspension control of full-car systems without function approximation, IEEE Transactions on Cybernetics 50 (6) (2020) 2639–265.

[30] X.-M. Li, Q. Zhou, P. Li, H. Li, R. Lu, Event-triggered consensus control for multi-agent systems against false data-injection attacks, IEEE transactions on cybernetics, DOI: 10.1109/T-CYB.2019.2937951.

[31] Q. Zhou, W. Wang, H. Liang, M. Basin, B. Wang, Observer-based event-triggered fuzzy adaptive bipartite containment control of multi-agent systems with input quantization, IEEE Transactions on Fuzzy SystemsDOI: 10.1109/TFUZZ.2019.2953573.

[32] A. Nedic, A. Olshevsky, A. E. Ozdaglar, J. N. Tsitsiklis, On distributed averaging algorithms and quantization effects, IEEE Transactions on Automatic Control 54 (11) (2009) 2506–2517.

[33] R. Lu, Y. Xu, A. Xue, J. Zheng, Networked control with state reset and quantized measurements: observer-based case, IEEE Transactions on Industrial Electronics 60 (11) (2012) 5206–5213.

[34] T. Hayakawa, H. Ishii, K. Tsumura, Adaptive quantized control for linear uncertain discrete-time systems, Automatica 45 (3) (2009) 692–700.

[35] M. Fu, L. Xie, The sector bound approach to quantized feedback control, IEEE Transactions on Automatic Control 50 (11) (2005) 1698–1711.

[36] H. Liang, Y. Zhang, T. Huang, H. Ma, Prescribed performance cooperative control for multiagent systems with input quantization, IEEE Transactions on Cybernetics, DOI: 10.1109/TCY-B.2019.2893645.

[37] S. Liu, T. Li, L. Xie, M. Fu, J. F. Zhang, Continuous-time and sampled-data-based average consensus with logarithmic quantizers, Automatica 49 (11) (2013) 3329–3336.

[38] G. Chen, F. L. Lewis, L. Xie, Finite-time distributed consensus via binary control protocols, Automatica 47 (9) (2011) 1962–1968.

[39] F. Wang, B. Chen, C. Lin, X. Li, Distributed adaptive neural control for stochastic nonlinear multiagent systems, IEEE Transactions on Cybernetics 47 (7) (2017) 1795–1803.

[40] G. Lai, Z. Liu, C. L. P. Chen, Y. Zhang, Adaptive asymptotic tracking control of uncertain nonlinear system with input quantization, Systems and Control Letters 96 (2016) 23–29.

[41] L. Xing, C. Wen, Z. Liu, H. Su, J. Cai, Adaptive compensation for actuator failures with event-triggered input, Automatica 85 (2017) 129–136.

[42] H. Zhang, F. L. Lewis, Z. Qu, Lyapunov, adaptive, and optimal design techniques for cooperative systems on directed communication graphs, IEEE Transactions on Industrial Electronics 59 (7) (2012) 3026–3041.

[43] C. L. Chen, Y. J. Liu, G. X. Wen, Fuzzy neural network-based adaptive control for a class of uncertain nonlinear stochastic systems., IEEE Transactions on Cybernetics 44 (5) (2014) 583–593.

[44] L.-X. Wang, A course in fuzzy systems and control, Vol. 2, Prentice Hall PTR Upper Saddle River, NJ, 1997.

[45] F. Wang, B. Chen, Y. Sun, C. Lin, Finite time control of switched stochastic nonlinear systems, Fuzzy Sets and Systems, DOI: 10.1016/j.fss.2018.04.016.

[46] L. Wang, C. L. P. Chen, H. Li, Event-triggered adaptive control of saturated nonlinear systems with time-varying partial state constraints, IEEE Transactions on Cybernetics, DOI:10.1109/TCYB.2018.2865499.

[47] M. Zhao, C. Peng, W. He, Y. Song, Event-triggered communication for leader-following consensus of second-order multiagent systems, IEEE Transactions on Cybernetics 48 (6) (2018) 1888–1897.

[48] C. Peng, J. Zhang, Q.-L. Han, Consensus of multiagent systems with nonlinear dynamics using an integrated sampled-data-based event-triggered communication scheme, IEEE Transactions on Systems, Man, and Cybernetics: Systems 49 (3) (2018) 589–599.