Stability and Stabilization with Additive Freedom for Delayed Takagi-Sugeno Fuzzy Systems by Intermediary Polynomial-based Functions

Zhichen Li, Huaicheng Yan, Hao Zhang, Jun Sun, and Hak-Keung Lam

***Abstract*—This paper is devoted to the stability and stabilization for Takagi-Sugeno (T-S) fuzzy systems with time-varying delays. Firstly, an improved matrix inequality is presented to bound both strictly and non-strictly proper rational functions, which is more general than the existing versions of reciprocally convex lemmas. Secondly, by suitable operations on parameter-dependent polynomial multiplied by state rate, a couple of novel intermediary polynomial-based functions (IPFs) are developed in delay-product types. Benefitting from slack matrices of IPFs, a certain degree of flexibility is furnished. More importantly than that, by feat of adjustment of variable parameter, the resulting conditions will be further endowed with additive freedom, which relaxes the feasible space in a distinctive manner. Thirdly, by utilizing IPFs along with triple integrals, the stability criteria and controller design approach are derived by some advanced integral inequalities. Resorting to elaborate construction of IPFs, the strengths of bounding techniques are sufficiently exploited, and the information on delay derivative is adequately reflected. Consequently, more desirable performances are achieved, while without excessive computational complexity. Finally, the effectiveness of the proposed methods is verified by numerical examples.**

***Index Terms*—Delayed T-S fuzzy systems, stability, stabilization, intermediary polynomial-based functions, variable parameter.**

I. INTRODUCTION

IN real-world applications, a wide range of practical systems suffer severe nonlinearities [1-5], which impose formidable obstacles for analyzing and synthesizing systems [6-8]. With advent of fuzzy modeling, Takagi-Sugeno (T-S) fuzzy models have shown promising performance in approximation of nonlinear systems to any accuracy via combination of rigorous linear system theory and flexible fuzzy logic theory [6]. Time delays are frequently encountered in almost all of the industrial processes, which are attributed as the root causes of performance degradation or instability [9-14]. Accordingly, the investigations on T-S fuzzy systems with time delays are of both theoretical significance and practical meaning [15-18].

For analysis and synthesis of delayed systems, rational construction of Lyapunov-Krasovskii functional (LKF) and precise estimation of its derivative are perceived as primary procedures for preferable stability region [19, 20]. As regard to the first trend, the quadratic form of system state $x^\top(t)\mathcal{Q}x(t)$ is usually augmented by integrals of state $\int_{t-\tau(t)}^{t} x(s)ds$ and $\int_{t-h}^{t} x(s)ds$ to establish extensive relations among various extra-states [9, 12] ($0 \leq \tau(t) \leq h$ is time-varying delay). In [13], triple integral $\int_{t-h}^{t}\int_{u}^{t}\int_{\theta}^{t} \dot{x}^\top(s)\mathcal{R}\dot{x}(s)dsd\theta du$ are introduced in full consideration of delay information. By refining the Lyapunov matrix with slack variables, the matrix-refined-functions (MRFs) are developed to provide more feasibility [21]. Correspondingly, the pattern of augmented term equipped with a unitary matrix is thoroughly transformed to exert impressive contribution. However, the MRFs only involve single integrals, and the extension with double integrals will lead to higher-order time delays due to inherent formation, inducing difficulty in finding solution. Moreover, the slack matrices of MRFs are restrained by positive definiteness of a holistic matrix, and thus the adjusting room is enormously limited. An interesting question arises from this observation: how to utilize more system information and improve flexibility of slack variables, which is the first motivation of this paper.

As to the second trend, bounding integral terms and disposing treated delay-related terms are required for estimation task. For the first step, Jensen inequality (JI) occupies the mainstream in the early studies [14, 15, 17, 19, 20], although at the sacrifice of conservatism. By employing the quadratic with single integral, the estimating gap of JI is evidently narrowed by Wirtinger-based inequality (WBI) [22]. In [23], Bessel-Legendre inequality (BLI) achieves more and more accurate bounds as the degree of Legendre polynomial grows. Meanwhile, generalized double integral inequalities (GDIIs) [24] afford improvements over Jensen double integral inequalities (JDIs) [13]. For the second step, reciprocally convex lemma (RCL) has once played dominating role in handling $-\frac{1}{\tau(t)}$- and $-\frac{1}{h-\tau(t)}$-related terms [13, 19]. Furthermore, delay-dependent

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Z. Li and H. Yan are with the Key Laboratory of Advanced Control and Optimization for Chemical Process of Ministry of Education, East China University of Science and Technology, Shanghai 200237, China (e-mail: zcli@ecust.edu.cn; hcyan@ecust.edu.cn).

H. Zhang is with Department of Control Science and Engineering, Tongji University, Shanghai 200092, China (e-mail: zhang_hao@tongji.edu.cn).

J. Sun is with the Shanghai Key Laboratory of Aerospace Intelligence Control Technology, and also with Shanghai Aerospace Control Technology Institute, Shanghai 201100, China (e-mail: sjlovedh@hotmail.com).

H.-K. Lam is with the Department of Informatics, King's College London, London, WC2R 2LS, U.K. (e-mail: hak-keung.lam@kcl.ac.uk).

RCL is suggested to enhance RCL by requiring four variables [25]. Recently, extended RCL [26] offers identical estimation gap, but needs fewer decision variables compared with delay-dependent RCL. However, if adding triple integrals, not only the above strictly proper rational functions, but also non-strictly proper rational ones $-\frac{h-\tau(t)}{\tau(t)}$ and $-\frac{\tau(t)}{h-\tau(t)}$ are produced, to which little attention is paid. Therefore, how to develop improved matrix inequality suitable for LKF with triple integral terms still remains as a challenging task, which is the second motivation of this paper.

The above-mentioned directions are interactive for reducing conservatism. In [27], the relationships between LKF construction and integral estimation is discussed, and it is demonstrated that by a non-augmented LKF, the stability criterion by WBI is are equivalent to that by JI in the sense of conservatism, despite high accuracy of the former. From the unique perspective, the delay-product functions (DPFs) comprising single integral forms of augmented vectors [27, 28] are formulated, which aims to reveal the advantages of WBI. However, the DPFs still remain confined to extension of simple LKF with restricted adjusting space. Therefrom, an important issue is raised naturally: in order to fit improved inequalities and provide additional freedom, how to promote DPFs by reasonable utilization of slack variables, which is the third motivation of this paper.

Inspired by the above discussions, this paper focuses on stability and stabilization for T-S fuzzy systems with time-varying delays to solve these problems. The main contributions of this paper are summarized as follows:

1) An improved matrix inequality that can be conveniently combined with integral inequalities is derived for LKF with triple integrals. The proposed inequality is more general than the existing ones for estimating both strictly and non-strictly proper rational functions without improper approximation, while avoiding superfluous matrices.

2) By establishing polynomial with respect to variable parameter, the innovative intermediary polynomial-based functions (IPFs) are developed. By appropriate introduction of slack matrices, extra flexibility is explored than the commonly-used augmented LKFs and DPFs to some extent. Profiting from deliberate structure of delay-product type of IBFs, the information on delay change rate is taken into full consideration, the merits of advanced bounding techniques are effectually reflected, and their derivatives will be cast into linear matrix inequalities (LMIs).

3) Combing those two techniques with improved integral inequalities, the stability conditions and stabilization control approach for delayed T-S fuzzy system are designed with less conservatism. By coordinating the variable parameter, the adjusting room of slack matrices with parameter-dependent functions is enlarged to arbitrary degree, which is recognized as the substantial superiority over the recently reported studies. This is the first time to investigate stability and stabilization for T-S fuzzy system from the perspective of additive freedom.

The remainder of this paper is briefly outlined as follows. In Section II, the stability and stabilization problems for delayed T-S fuzzy systems, and some useful lemmas are formulated. The improved matrix inequality is also presented. In Section III, delay-product type of IPFs are developed. The stability criteria and controller design method are established in Section IV. In Section V, three numerical examples are provided to verify the effectiveness of the proposed approaches. The conclusion is drawn in Section VI.

Notations. $\mathbb{R}^n$ is n-dimensional Euclidean space. The superscripts T and -1 stand for transpose and inverse of matrix, respectively. $diag\{\cdot\}$ denotes a block diagonal matrix. $\mathcal{S} > 0$ (< 0) means $\mathcal{S}$ being a symmetric positive (negative) definite matrix. 0 and I are zero and identity matrices of appropriate dimensions, respectively. $*$ represents a term induced by symmetry. $\mathcal{S} \otimes \mathcal{R}$ is the Kronecker product of matrices $\mathcal{S}$ and $\mathcal{R}$. $\mathrm{sym}\{\mathcal{S}\}$ is defined as $\mathcal{S} + \mathcal{S}^\top$. $\mathrm{col}\{\cdot\}$ is a column vector.

II. Preliminaries

A. Problem Formulation

Suppose a class of nonlinear systems with time-varying delays, which can be described by the following T-S fuzzy model composed of r plant rules:

Plant Rule i: IF $\theta_1(t)$ is κ_{i1} and $\ldots$ and $\theta_p(t)$ is κ_{ip}, THEN

$$\begin{cases} \dot{x}(t) = \mathcal{A}_i x(t) + \mathcal{A}_{di} x(t-\tau(t)) + \mathcal{B}_i u(t) \\ x(t) = \phi(t), t \in [-h, 0], \quad i = 1, 2, \ldots, r \end{cases} \tag{1}$$

where $\theta_1(t), \ldots, \theta_p(t)$ denote the premise variables; $\kappa_{ij}(i = 1, \ldots, r; j = 1, \ldots, p)$ represent fuzzy sets; $x(t) \in \mathbb{R}^n$ is the state vector; $u(t) \in \mathbb{R}^m$ is the control input vector; $\phi(t)$ is the initial condition; $\mathcal{A}_i$, $\mathcal{A}_{di}$ and $\mathcal{B}_i$ are system matrices of compatible dimensions; $\tau(t)$ is time-varying delay satisfying

$$0 \le \tau(t) \le h, \qquad \mu_1 \le \dot{\tau}(t) \le \mu_2 \tag{2}$$

where h, μ_1 and μ_2 are known constant scalars. For notational simplicity, τ, $\dot{\tau}$ and $\widetilde{\tau}$ stand for $\tau(t)$, $\dot{\tau}(t)$ and $1 - \dot{\tau}(t)$ in the subsequent parts, respectively.

Employing the singleton fuzzifier, product inference, and center-average defuzzifier, the global dynamics of the delayed fuzzy model is inferred as a convex sum form:

$$\dot{x}(t) = \sum_{i=1}^{r} \lambda_i(\theta(t)) \left(\mathcal{A}_i x(t) + \mathcal{A}_{di} x(t-\tau) + \mathcal{B}_i u(t) \right) \tag{3}$$

where

$$\lambda_i(\theta(t)) = \frac{w_i(\theta(t))}{\sum_{i=1}^{r} w_i(\theta(t))} \ge 0, \ w_i(\theta(t)) = \prod_{j=1}^{p} \kappa_{ij}\left(\theta_j(t)\right)$$

with $\kappa_{ij}(\theta_j(t))$ representing the grade of membership of $\theta_j(t)$ in κ_{ij}. Some basic properties are obeyed:

$$w_i(\theta(t)) \ge 0, \quad \sum_{i=1}^{r} w_i(\theta(t)) > 0, \quad \sum_{i=1}^{r} \lambda_i(\theta(t)) = 1$$

Inspired by the parallel distribution compensation scheme, in which the same fuzzy sets with the plant are shared by the controller's premise variables, consider the fuzzy state feedback controller in the form of

Plant Rule i: IF $\theta_1(t)$ is κ_{i1} and $\ldots$ and $\theta_p(t)$ is κ_{ip}, THEN

$$u(t) = \mathcal{K}_i x(t), \quad i = 1, 2, \ldots, r \tag{4}$$

where $\mathcal{K}_i$ is the local gain matrix. Then, the overall controller is presented as

$$u(t) = \sum_{i=1}^{r} \lambda_i(\theta(t))\mathcal{K}_i x(t), \quad i = 1, 2, \ldots, r \tag{5}$$

Therefore, the closed-loop delayed T-S fuzzy system can be expressed as

$$\begin{aligned}\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\theta(t))\lambda_j(\theta(t))\big(\mathcal{A}_i x(t) + \mathcal{B}_i\mathcal{K}_j x(t) \\ + \mathcal{A}_{di}x(t-\tau)\big)\end{aligned} \tag{6}$$

with compact form

$$\dot{x}(t) = (\mathcal{A}(t) + \mathcal{B}(t)\mathcal{K}(t))\, x(t) + \mathcal{A}_d(t)(t-\tau) \tag{7}$$

where

$$\mathcal{A}(t) = \sum_{i=1}^{r} \lambda_i(\theta(t))\mathcal{A}_i, \quad \mathcal{B}(t) = \sum_{i=1}^{r} \lambda_i(\theta(t))\mathcal{B}_i$$
$$\mathcal{A}_d(t) = \sum_{i=1}^{r} \lambda_i(\theta(t))\mathcal{A}_{di}, \quad \mathcal{K}(t) = \sum_{i=1}^{r} \lambda_i(\theta(t))\mathcal{K}_i$$

The main aims of this paper are 1) deriving stability criteria for the system (3) with $u(t) = 0$, and the system (7) with known $\mathcal{K}(t)$; 2) developing controller design approach for the system (7). For this purpose, some indispensable lemmas for derivation process are recalled.

B. Related Lemmas

Lemma 1 [28]: For a quadratic function $F(s) = a_2 s^2 + a_1 s + a_0$, where $a_0, a_1, a_2 \in \mathbb{R}$, $F(s) < 0$, $\forall s \in [0, h]$, if

$$(i)\ F(h) < 0;\ (ii)\ F(0) < 0;\ (iii)\ -h^2 a_2 + F(0) < 0$$

Lemma 2 [23, 24]: Defining $\xi_1(a,b) = \frac{1}{b-a}\int_a^b x(s)ds$ and $\xi_2(a,b) = \frac{1}{(b-a)^2}\int_a^b\int_\theta^b x(s)dsd\theta$, for any continuously differentiable function $x : [a,b] \to \mathbb{R}^n$, and a given symmetric matrix $\mathcal{Q} > 0$, the following inequalities hold

$$\int_a^b \dot{x}^\top(s)\mathcal{Q}\dot{x}(s)ds \geq \frac{1}{b-a}\varrho_1^\top diag\{\mathcal{Q}, 3\mathcal{Q}, 5\mathcal{Q}\}\,\varrho_1 \tag{8}$$

$$\int_a^b\int_\theta^b \dot{x}^\top(s)\mathcal{Q}\dot{x}(s)dsd\theta \geq \varrho_2^\top diag\{2\mathcal{Q}, 4\mathcal{Q}\}\,\varrho_2 \tag{9}$$

$$\int_a^b\int_a^\theta \dot{x}^\top(s)\mathcal{Q}\dot{x}(s)dsd\theta \geq \varrho_3^\top diag\{2\mathcal{Q}, 4\mathcal{Q}\}\,\varrho_3 \tag{10}$$

where

$$\begin{aligned}\varrho_1 &= \mathrm{col}\{x(b)-x(a), x(b)+x(a)-2\xi_1(a,b),\\ &\qquad x(b)-x(a)+6\xi_1(a,b)-12\xi_2(a,b)\},\\ \varrho_2 &= \mathrm{col}\{x(b)-\xi_1(a,b), x(b)+2\xi_1(a,b)-6\xi_2(a,b)\},\\ \varrho_3 &= \mathrm{col}\{x(a)-\xi_1(a,b), x(a)-4\xi_1(a,b)+6\xi_2(a,b)\}.\end{aligned}$$

Remark 1: Owning to the inclusion of the quadratic forms with respect to $\xi_2(a,b)$, the estimation values of WBI and JDIs are analytically improved by the second-order BLI (SOBLI) (8) and GDIIs (9)-(10), respectively. While, along with the similar line of [27], it can be predicted that if there exist no $\xi_2(a,b)$-related cross terms induced by LKF, the stability conditions by the two sets of inequalities are equivalent. Actually, the augmented vectors caused by the former set are linearly independent of the double integral terms created by the latter one, and then the theoretical improvements are futile for reducing conservatism.

C. Improved Matrix Inequality

Lemma 3: For given positive scalars r and w with $r+w=1$, symmetric matrices $\mathcal{G}_j > 0$ $(j = 1,2,3,4)$, and any matrices $\mathcal{X}_n$ $(n = 1,2)$, it holds that

$$\begin{bmatrix} \frac{1}{r}\mathcal{L}_1 & 0 \\ * & \frac{1}{w}\mathcal{L}_2 \end{bmatrix} \geq \begin{bmatrix} \mathcal{G}_1 + w\mathcal{Y}_1 & w\mathcal{X}_1 + r\mathcal{X}_2 \\ * & \mathcal{G}_2 + r\mathcal{Y}_2 \end{bmatrix} \tag{11}$$

where $\mathcal{L}_1 = \mathcal{G}_1 + w\mathcal{G}_3, \mathcal{L}_2 = \mathcal{G}_2 + r\mathcal{G}_4, \mathcal{Y}_1 = \mathcal{G}_1 + \mathcal{G}_3 - \mathcal{X}_2(\mathcal{G}_2+\mathcal{G}_4)^{-1}\mathcal{X}_2^\top$ and $\mathcal{Y}_2 = \mathcal{G}_2 + \mathcal{G}_4 - \mathcal{X}_1^\top(\mathcal{G}_1+\mathcal{G}_3)^{-1}\mathcal{X}_1$.

Proof: Since $\mathcal{G}_j > 0$, by the Schur complement, it can be deduced that

$$\Xi_1 = \begin{bmatrix} \mathcal{G}_{13} - \mathcal{G}_{13} + \mathcal{X}_2\mathcal{G}_{24}^{-1}\mathcal{X}_2^\top & -\mathcal{X}_2 \\ * & \mathcal{G}_{24} \end{bmatrix} \geq 0, \tag{12}$$

$$\Xi_2 = \begin{bmatrix} \mathcal{G}_{13} & -\mathcal{X}_1 \\ * & \mathcal{G}_{24} - \mathcal{G}_{24} + \mathcal{X}_1^\top\mathcal{G}_{13}^{-1}\mathcal{X}_1 \end{bmatrix} \geq 0 \tag{13}$$

with $\mathcal{G}_{13} = \mathcal{G}_1 + \mathcal{G}_3$ and $\mathcal{G}_{24} = \mathcal{G}_2 + \mathcal{G}_4$.

Due to $r + w = 1$, a convex combination $r\Xi_1 + w\Xi_2$ is nonnegative definite. Then, one has

$$\begin{bmatrix} \mathcal{G}_{13} - r\mathcal{Y}_1 & -w\mathcal{X}_1 - r\mathcal{X}_2 \\ * & \mathcal{G}_{24} - w\mathcal{Y}_2 \end{bmatrix} \geq 0. \tag{14}$$

Furthermore, pre- and post-multiplying (14) by the matrix $diag\left\{\sqrt{\frac{w}{r}}I, \sqrt{\frac{r}{w}}I\right\}$ gives rise to

$$\begin{bmatrix} \frac{w}{r}\mathcal{G}_{13} & 0 \\ * & \frac{r}{w}\mathcal{G}_{24} \end{bmatrix} \geq \begin{bmatrix} w\mathcal{Y}_1 & w\mathcal{X}_1 + r\mathcal{X}_2 \\ * & r\mathcal{Y}_2 \end{bmatrix}. \tag{15}$$

Finally, adding $diag\{\mathcal{G}_1, \mathcal{G}_2\}$ into both sides of (15), the matrix inequality (11) is derived.

Remark 2: It is worth noting that by eliminating $\mathcal{G}_3$ and $\mathcal{G}_4$, (11) is equivalent to the extended RCL [26] as

$$\begin{bmatrix} \frac{1}{r}\mathcal{G}_1 & 0 \\ * & \frac{1}{w}\mathcal{G}_2 \end{bmatrix} \geq \begin{bmatrix} \mathcal{G}_1 + w\mathcal{Z}_1 & w\mathcal{X}_1 + r\mathcal{X}_2 \\ * & \mathcal{G}_2 + r\mathcal{Z}_2 \end{bmatrix} \tag{16}$$

where $\mathcal{Z}_1 = \mathcal{G}_1 - \mathcal{X}_2\mathcal{G}_2^{-1}\mathcal{X}_2^\top$ and $\mathcal{Z}_2 = \mathcal{G}_2 - \mathcal{X}_1^\top\mathcal{G}_1^{-1}\mathcal{X}_1$. Next, from the combination of (11) by setting $\mathcal{X}_1 = \mathcal{X}_2$, and WBI and SOBLI, respectively, Lemma 4 and Lemma 6 of [29] are given, which means (11) is less conservative due to getting rid of restraint on variables.

For the inequality-processed derivatives of double and triple integrals, both $-\frac{1}{\tau}$-, $-\frac{1}{h-\tau}$-, and $-\frac{h-\tau}{\tau}$- and $-\frac{\tau}{h-\tau}$-dependent functions are caused. The approaches of [25, 26] fail to deal with such case directly. In [30], the latter functions are roughly enlarged as $-\frac{h-\tau}{h}$ and $-\frac{\tau}{h}$ with considerable conservatism. Similar to the treatment of [31] with $\frac{w}{r} = \frac{1}{r} - 1$ and $\frac{r}{w} = \frac{1}{w} - 1$, for any matrix $\mathcal{X}$, the following inequality is derived based on Lemma 3 of [32]:

$$\begin{bmatrix} \frac{1}{r}\mathcal{L}_1 & 0 \\ * & \frac{1}{w}\mathcal{L}_2 \end{bmatrix} \geq \begin{bmatrix} \mathcal{G}_1 + w\mathcal{Y}_1 & \mathcal{X} \\ * & \mathcal{G}_2 + r\mathcal{Y}_2 \end{bmatrix}. \tag{17}$$

Since the requirement $\mathcal{X}_1 = \mathcal{X}_2 = \mathcal{X}$ is waived, it is obvious that (17) is relaxed by (11).

From these discussions, it is found that, on the one hand, the matrix inequality (11) is more general than [25, 26, 32] for accommodating triple integrals. On the other hand, the estimation gaps of a series of results in [29] can be gradually reduced by the special case of (11) in association with BLI with incremental orders.

III. Intermediary polynomial-based functions

In this section, the intermediary polynomial-based functions (IPFs) are developed.

Lemma 4: Given a pair of positive scalars a and b with $a \leq b$ and $v = b - a$, any scalars σ_1, σ_2, and a continuously differentiable function $x : [a, b] \to \mathbb{R}^n$, if there exists symmetric matrix $\mathcal{U} = [\mathcal{U}_{pq}]_{3\times 3} > 0$ with appropriate dimension, the function $V_{\mathcal{P}}(x)$ defined as follows is positive definite.

$$V_{\mathcal{P}}(x) = \varsigma^\top(a,b)\bar{\mathcal{U}}\varsigma(a,b) + \int_a^b\int_\theta^b \dot{x}^\top(s)\mathcal{U}_{33}\dot{x}(s)dsd\theta$$

where

$$\begin{aligned}
&\varsigma(a,b) = \mathrm{col}\{x(b), \xi_1(a,b), \xi_2(a,b)\},\ \Im_1(\mathcal{D}) = [\mathcal{D} \quad -\mathcal{D} \quad 0],\\
&\Im_2(\sigma_1,\sigma_2,\mathcal{D}) = [(\sigma_1+\sigma_2)\mathcal{D} \quad -\sigma_2\mathcal{D} \quad -2\sigma_1\mathcal{D}],\\
&\bar{\mathcal{U}} = \widehat{\mathcal{U}} + \mathrm{sym}\{v\Im_1(\mathcal{U}_{13}) + v^2\Im_2(\sigma_1,\sigma_2,\mathcal{U}_{23})\}\\
&\qquad + \mathrm{sym}\Big\{\left(\frac{\sigma_1}{3}+\frac{\sigma_2}{2}\right)v^3\mathcal{U}_{12}\Big\},\\
&\widehat{\mathcal{U}} = \frac{v^2}{2}\mathcal{U}_{11} + \left(\frac{\sigma_1^2}{4}+\frac{\sigma_2^2}{2}+\frac{2\sigma_1\sigma_2}{3}\right)v^4\mathcal{U}_{22}.
\end{aligned}$$

Proof: The major purpose for construction of intermediary polynomial is to make use of information on $\xi_1(a,b)$ and $\xi_2(a,b)$. From integration by parts, one can obtain

$$\int_a^b\int_\theta^b (s-a)\dot{x}(s)dsd\theta = v^2x(b) - 2\int_a^b\int_\theta^b x(s)dsd\theta, \quad (18)$$

$$\int_a^b\int_\theta^b v\dot{x}(s)dsd\theta = v^2x(b) - v\int_a^b x(s)ds. \quad (19)$$

It is noticed that double integrals of state rate multiplied by the zero- and first-order polynomials are capable of producing single and double integrals of state. Hence, without generality, define the intermediary polynomial in the formation of

$$\psi(\sigma_1,\sigma_2,s) = \sigma_1(s-a) + \sigma_2 v.$$

On the one hand, it follows from (18)-(19) that

$$\int_a^b\int_\theta^b \varsigma^\top(a,b)\mathcal{U}_{13}\dot{x}(s)dsd\theta = v\varsigma^\top(a,b)\Im_1(\mathcal{U}_{13})\varsigma(a,b), \quad (20)$$

$$\begin{aligned}\int_a^b\int_\theta^b \psi(\sigma_1,\sigma_2,s)\varsigma^\top(a,b)\mathcal{U}_{23}\dot{x}(s)dsd\theta =\\ v^2\varsigma^\top(a,b)\Im_2(\sigma_1,\sigma_2,\mathcal{U}_{23})\varsigma(a,b).\end{aligned} \quad (21)$$

Moreover, by computation, it holds that

$$\begin{aligned}\int_a^b\int_\theta^b \varsigma^\top(a,b)\mathcal{U}_{12}\psi(\sigma_1,\sigma_2,s)\varsigma(a,b)dsd\theta =\\ v^3\left(\frac{\sigma_1}{3}+\frac{\sigma_2}{2}\right)\varsigma^\top(a,b)\mathcal{U}_{12}\varsigma(a,b).\end{aligned} \quad (22)$$

On the other hand, it is direct to acquire

$$\int_a^b\int_\theta^b \varsigma^\top(a,b)\mathcal{U}_{11}\varsigma(a,b)dsd\theta = \frac{v^2}{2}\varsigma^\top(a,b)\mathcal{U}_{11}\varsigma(a,b), \quad (23)$$

$$\begin{aligned}\int_a^b\int_\theta^b \psi^2(\sigma_1,\sigma_2,s)\varsigma^\top(a,b)\mathcal{U}_{22}\varsigma(a,b)dsd\theta =\\ v^4\left(\frac{\sigma_1^2}{4}+\frac{\sigma_2^2}{2}+\frac{2\sigma_1\sigma_2}{3}\right)\varsigma^\top(a,b)\mathcal{U}_{22}\varsigma(a,b).\end{aligned} \quad (24)$$

By algebraic calculation, one has

$$\begin{aligned}\varsigma^\top(a,b)\widehat{\mathcal{U}}\varsigma(a,b) + \int_a^b\int_\theta^b \dot{x}^\top(s)\mathcal{U}_{33}\dot{x}(s)dsd\theta =\\ \int_a^b\int_\theta^b \chi^\top(s)diag\{\mathcal{U}_{11},\mathcal{U}_{22},\mathcal{U}_{33}\}\chi(s)dsd\theta\end{aligned} \quad (25)$$

where $\chi(s) = \mathrm{col}\{\varsigma(a,b), \psi(\sigma_1,\sigma_2,s)\varsigma(a,b), \dot{x}(s)\}$.

Combining (20)-(25), it can be found that

$$V_{\mathcal{P}}(x) = \int_a^b\int_\theta^b \chi^\top(s)\mathcal{U}\chi(s)dsd\theta. \quad (26)$$

On the account of $\mathcal{U} > 0$, $V_{\mathcal{P}}(x) > 0$ is obtained, which ends the proof.

Remark 3: On the basis of polynomial $\psi(\sigma_1,\sigma_2,s)$, positive definitiveness of $V_{\mathcal{P}}(x)$ is formulated. Taking double integral on intermediary polynomial multiplied by state rate, $\int_a^b x(s)ds$ and $\int_a^b\int_\theta^b x(s)dsd\theta$ are raised simultaneously, which incorporate the full features of inequalities (8)-(10). For time-varying delay case, $V_{\mathcal{P}}(x)$ is refined to the delay-product versions with numerical solvability as follows.

Proposition 1: Assuming there exist a given scalar σ, and matrices $\mathcal{G} = [\mathcal{G}_{pq}]_{3\times 3} > 0$ and $\mathcal{H} = [\mathcal{H}_{pq}]_{3\times 3} > 0$ with $\mathcal{G}_{12} = \mathcal{H}_{12} = 0$ of compatible dimensions, for a continuously differentiable function $x : [a,b] \to \mathbb{R}^n$, the following functions can be applied to constructing LKF for stability analysis of delayed T-S fuzzy systems (3).

$$V_{\mathcal{P}1}(x_t) = \varsigma_1^\top(t)\mathcal{G}_{[\tau,\sigma]}\varsigma_1(t) + \int_{t-\tau}^t\int_\theta^t \dot{x}^\top(s)\mathcal{G}_{33}\dot{x}(s)dsd\theta$$

$$V_{\mathcal{P}2}(x_t) = \varsigma_2^\top(t)\mathcal{H}_{[\tau,\sigma]}\varsigma_2(t) + \int_{t-h}^{t-\tau}\int_\theta^{t-\tau} \dot{x}^\top(s)\mathcal{H}_{33}\dot{x}(s)dsd\theta$$

where

$$\begin{aligned}
&\varsigma_1(t) = \varsigma(t-\tau,t),\ \varsigma_2(t) = \varsigma(t-h,t-\tau),\\
&\mathcal{G}_{[\tau,\sigma]} = \tau\left(\widehat{\mathcal{G}} + \mathrm{sym}\{\wp_1(\mathcal{G}_{13}) + \tau\wp_2(\sigma,\mathcal{G}_{23})\}\right),\\
&\mathcal{H}_{[\tau,\sigma]} = (h-\tau)\left(\widehat{\mathcal{H}} + \mathrm{sym}\{\wp_1(\mathcal{H}_{13}) + (h-\tau)\wp_2(\sigma,\mathcal{H}_{23})\}\right),\\
&\wp_1(\mathcal{D}) = \Im_1(\mathcal{D}),\ \wp_2(\sigma,\mathcal{D}) = \Im_2(-3\sigma,2\sigma,\mathcal{D}),\\
&\widehat{\mathcal{G}} = \sum_{n=1}^2 \frac{\sigma^{2(n-1)}}{2n}h^{2n-1}\mathcal{G}_{nn},\ \widehat{\mathcal{H}} = \sum_{n=1}^2 \frac{\sigma^{2(n-1)}}{2n}h^{2n-1}\mathcal{H}_{nn}.
\end{aligned}$$

Proof: In view of the relationship between time delay and delay variation range, the interval $[t-h,t]$ is decomposed into $[t-\tau,t]\cup[t-h,t-\tau]$. Considering the first subinterval, the intermediary polynomial is chosen as

$$\psi_1(\sigma_1,\sigma_2,s) = \sigma_1(s-t+\tau) + \sigma_2\tau.$$

From (25) and $\tau \leq h$, one has

$$\varsigma_1^\top(t)\aleph\varsigma_1(t) \geq \int_{t-\tau}^{t}\int_{\theta}^{t} \chi_1^\top(s)diag\{\mathcal{G}_{11},\mathcal{G}_{22},0\}\chi_1(s)dsd\theta \tag{27}$$

where $\chi_1(s) = \mathrm{col}\{\varsigma_1(t), \psi_1(\sigma_1,\sigma_2,s)\varsigma_1(t), \dot{x}(s)\}$ and

$$\aleph = \frac{\tau h}{2}\mathcal{G}_{11} + \left(\frac{\sigma_1^2}{4} + \frac{\sigma_2^2}{2} + \frac{2\sigma_1\sigma_2}{3}\right)\tau h^3\mathcal{G}_{22}.$$

Then, replacing $[a,b]$ and $\psi(\sigma_1,\sigma_2,s)$ by $[t-\tau,t]$ and $\psi_1(\sigma_1,\sigma_2,s)$, respectively, and implementing the similar way as Lemma 4, it can be derived that

$$\begin{aligned} V_{\mathcal{O}1}(x_t) &= \varsigma_1^\top(t)\left(\aleph + \mathrm{sym}\Big\{\Big(\frac{\sigma_1}{3}+\frac{\sigma_2}{2}\Big)\tau^3\mathcal{G}_{12}\Big\}\right)\varsigma_1(t) \\ &+ \varsigma_1^\top(t)\left(\mathrm{sym}\big\{\tau\Im_1(\mathcal{G}_{13}) + \tau^2\Im_2(\sigma_1,\sigma_2,\mathcal{G}_{23})\big\}\right)\varsigma_1(t) \\ &+ \int_{t-\tau}^{t}\int_{\theta}^{t}\dot{x}^\top(s)\mathcal{G}_{33}\dot{x}(s)dsd\theta \geq \int_{t-\tau}^{t}\int_{\theta}^{t}\chi_1^\top(s)\mathcal{G}\chi_1(s)dsd\theta. \end{aligned}$$

It is noteworthy that the condition including time-varying delay with order larger than 2 cannot be transformed into LMIs. For the sake of overcoming the difficulties in numerical solution, the assumption of orthogonality for polynomial sequel $\{1, \psi_1(\sigma_1,\sigma_2,s)\}$ is considered in the integral inner space:

$$\int_{t-\tau}^{t}\int_{\theta}^{t}\psi_1(\sigma_1,\sigma_2,s)dsd\theta = 0. \tag{28}$$

From (28), one has $\sigma_1/\sigma_2 = -3/2$, and setting $\sigma_1 = -3\sigma$ and $\sigma_2 = 2\sigma$, $V_{\mathcal{P}1}(x_t) = V_{\mathcal{O}1}(x_t)_{[\sigma_1=-3\sigma,\sigma_2=2\sigma]}$. Since $\mathcal{G} > 0$, $V_{\mathcal{P}1}(x_t) > 0$ holds. Moreover, $\mathcal{G}_{12}$ can be removed for easing the computational burden.

For the second subinterval, define the intermediary polynomial as

$$\psi_2(\sigma_1,\sigma_2,s) = \sigma_1(s-t+h) + \sigma_2(h-\tau).$$

Substituting $[t-h, t-\tau]$ into $[a,b]$, and executing parallel manner to the above procedure leads to $V_{\mathcal{P}2}(x_t) > 0$.

As a result, the positive definiteness of $V_{\mathcal{P}n}(x_t)(n=1,2)$ is proved, which means that $V_{\mathcal{P}n}(x_t)$ can be utilized to form LKF. This completes the proof.

Remark 4: In the literatures [6, 9, 12, 23-26], the augmented functions $V_{\mathcal{S}}(x_t) = \beta^\top(t)\mathcal{Q}\beta(t)$ are extensively selected as LKF elements, where the information on delay derivative is under-utilized. In IPFs, all constituent parts of $\mathcal{G}_{[\tau,\sigma]}$ and $\mathcal{H}_{[\tau,\sigma]}$ are accompanied by time-varying delay, and thus traditional model of augmented vectors with constant matrix is radically reformed. Accordingly, when differentiating IPFs, the delay derivative range is taken into full consideration.

When specifically assigning some matrices in $\mathcal{G}_{[\tau,\sigma]}$ and $\mathcal{H}_{[\tau,\sigma]}$, IPFs will reduce to DPFs-like ones [27, 28] in the form of $V_{\mathcal{D}}(x_t) = \tau\beta_1^\top(t)\mathcal{Q}_1\beta_1(t) + (h-\tau)\beta_2^\top(t)\mathcal{Q}_2\beta_2(t)$. Conversely, by proper augmentation of $\beta_n(t)$ and extension of $\mathcal{Q}_n$ $(n=1,2)$, it is difficult to evolve DPFs into quasi-IPFs free of parameter σ, since some intricate conditions are imperative for guaranteeing positive definiteness of functions, which may be unreachable. For IBFs, the relationships among the marginally delayed states, single and double integrals are strengthened by efficacious introduction of slack variables, through coordinating which, more flexibility is gained. Besides, $V_{\mathcal{P}_n}(x_t) > 0$ are ensured by requiring the sum of all terms $\mathcal{G} > 0$ and $\mathcal{H} > 0$, instead of the individuals of $\mathcal{G}_{[\tau,\sigma]}$ and $\mathcal{H}_{[\tau,\sigma]}$ to be positive definite, which weakens the restriction on stability condition.

Remark 5: With help of the single integral form of augmentation, MRFs are instrumental for highlighting the efficacy of WBI. For cooperating with improved bounding techniques, a natural choice for MRFs will resort to involvement of $\int_{t-\tau}^{t}\int_{\theta}^{t}x(s)dsd\theta$ and $\int_{t-h}^{t-\tau}\int_{\theta}^{t-\tau}x(s)dsd\theta$, while in the aftermath, time delays of fourth-order will be presented, which fails to fall into numerically tractable LMIs. By advisable manipulations on $\psi_n(\sigma_1,\sigma_2,s)\dot{x}(s)$ of IPFs, $\xi_2(t-\tau,t)$ and $\xi_2(t-h,t-\tau)$ concerned with (8)-(10) are completely incorporated into $\varsigma_n(t)$, which are conducive to revealing the advantages of SOBLI and GDIIs for time-varying delays. In addition, it is worth noting that the Proposition 1 is based on the following relationship:

$$V_{\mathcal{O}1}(x_t) \geq \int_{t-\tau}^{t}\int_{\theta}^{t}\chi_1^\top(s)\mathcal{G}\chi_1(s)dsd\theta.$$

If the sub-matrix $\mathcal{G}_{12} \neq 0$, it is obviously found that the term $\mathcal{G}(t) = \int_{t-\tau}^{t}\int_{\theta}^{t}\varsigma_1^\top(s)\mathcal{G}_{12}\psi_1(\sigma_1,\sigma_2,s)\varsigma_1(t)dsd\theta$ will appear in $V_{\mathcal{O}1}(x_t)$. When differentiating $\mathcal{G}(t)$, the 3rd-order time-varying delay will be produced, which cannot be converted into the LMI form. In order to find the numerical solution, $\mathcal{G}_{12}$ and its counterpart $\mathcal{H}_{12}$ are set as zeros. Thanks to delay-product structure with refined relation between σ_1 and σ_2, the derivatives of IPFs can be expressed in terms of LMI, which will be described in the next section.

Remark 6: It can be observed that the parameter σ exhibits complicated distribution with intensive impact on condition feasibility. Removing σ, similar to MRFs, a certain degree of flexibility is achieved by utilizing slack variables $\mathcal{G}_{13}$, $\mathcal{G}_{23}$, $\mathcal{H}_{13}$ and $\mathcal{H}_{23}$. However, these matrices only have extremely limited adjustment space due to the restriction from $\mathcal{G} = [\mathcal{G}_{ij}]_{3\times3} > 0$ and $\mathcal{H} = [\mathcal{H}_{ij}]_{3\times3} > 0$, which are the essential foundation for application of $V_{\mathcal{P}n}(x_t)$ as the LKF components. While, in Proposition 1, $V_{\mathcal{P}n}(x_t) > 0$ is irrelevant to the value of σ. With aid of arbitrarily adjusted parameter, the variation scopes of slack matrices coupled with σ-related functions are amplified to any level, and it corresponds to elimination of constraints on $\mathcal{G}$ and $\mathcal{H}$. For different delay derivative intervals, multiple choice of σ will yield superior performance, and for specific bound on delay derivative, one can seek the maximum delay range by adjusting σ. For this reason, additive freedom is imparted to the resulting criteria, which is the fundamental outperformance of IPFs over the previous LKF types, including augmented LKFs, DPFs and MRFs.

IV. Stability and Stabilization for Delayed T-S Fuzzy Systems

In this section, the proposed IPFs are applied to stability and stabilization for delayed T-S fuzzy systems.

A. Stability Analysis

For convenience of presentation, define

$$\zeta(t)=\mathrm{col}\{x(t),x(t-h),x(t-\tau),\dot{x}(t-h),\dot{x}(t-\tau),\\ \xi_1(t-\tau,t),\xi_2(t-\tau,t),\xi_1(t-h,t-\tau),\xi_2(t-h,t-\tau)\}$$
$$e_j=[0_{n\times(j-1)n}\quad I_{n\times n}\quad 0_{n\times(9-j)n}]\ (j=1,2,...,9)$$
$$e_{\mathcal{F}i}=\mathcal{A}_ie_1+\mathcal{A}_{di}e_3,\ \alpha_1=\mathrm{col}\{e_1,e_3\},\ \alpha_{2i}=\mathrm{col}\{e_{\mathcal{F}i},\widetilde{\tau}e_5\}$$
$$\delta_{n1}=e_{2n-1}-e_{4-n},\ \delta_{n2}=e_{2n-1}+e_{4-n}-2e_{2n+4}$$
$$\delta_{n3}=e_{2n-1}-e_{4-n}+6e_{2n+4}-12e_{2n+5}\ (n=1,2)$$
$$\Theta_1=\mathcal{Q}+\mathcal{R}_1,\ \Theta_2=\mathcal{Q}+\mathcal{R}_2,\ \varrho=diag\left\{1,3^{-1},5^{-1}\right\}$$
$$\delta=\mathrm{col}\{e_1-e_6,e_1+2e_6-6e_7,e_3-e_8,e_3+2e_8-6e_9,\\ e_3-e_6,e_3-4e_6+6e_7,e_2-e_8,e_2-4e_8+6e_9\}$$
$$\eta_1=\mathrm{col}\{e_1,e_6,e_7\},\ \widetilde{\eta}_i=\tau\eta_{2i}+\eta_3,\ \eta_{2i}=\mathrm{col}\{e_{\mathcal{F}i},0,0\}$$
$$\eta_3=\mathrm{col}\{0,e_1-\widetilde{\tau}e_3-\dot{\tau}e_6,e_1-\widetilde{\tau}e_6-2\dot{\tau}e_7\}$$
$$\varpi_1=\mathrm{col}\{e_3,e_8,e_9\},\ \varpi_2=\mathrm{col}\{\widetilde{\tau}e_5,0,0\}$$
$$\varpi_3=\mathrm{col}\{0,\dot{\tau}e_8+\widetilde{\tau}e_3-e_2,\widetilde{\tau}e_3-e_8+2\dot{\tau}e_9\}$$
$$\widetilde{\varpi}=(h-\tau)\varpi_2+\varpi_3.$$

Theorem 1: For given scalars h, μ_1, μ_2 and σ, the T-S fuzzy system (3) with $u(t)=0$ is asymptotically stable, if there exist symmetric matrices $\mathcal{P}=[\mathcal{P}_{mn}]_{2\times2}>0$, $\mathcal{Q}>0$, $\mathcal{R}_n>0$, $\mathcal{G}=[\mathcal{G}_{pq}]_{3\times3}>0$, $\mathcal{H}=[\mathcal{H}_{pq}]_{3\times3}>0$ with $\mathcal{G}_{12}=\mathcal{H}_{12}=0$, and any matrices $\mathcal{X}_n,\mathcal{S}_n$ $(n=1,2)$ of appropriate dimensions such that the following inequalities are feasible for $i=1,\dots,r$:

$$\mathscr{S}_{1[\dot{\tau},\sigma]i}=\begin{bmatrix}\Xi_{[0,\dot{\tau},\sigma]i}-\Pi_1 & \Phi_{[0]i}\\ * & -\Lambda_1\end{bmatrix}<0 \tag{29}$$
$$\mathscr{S}_{2[\dot{\tau},\sigma]i}=\begin{bmatrix}h^2\Omega_{[\dot{\tau},\sigma]i}+\Xi_{[h,\dot{\tau},\sigma]i}-\Pi_2 & \Phi_{[h]i}\\ * & -\Lambda_2\end{bmatrix}<0 \tag{30}$$
$$\mathscr{S}_{3[\dot{\tau},\sigma]i}=\begin{bmatrix}\Xi_{[0,\dot{\tau},\sigma]i}-h^2\Omega_{[\dot{\tau},\sigma]i}-\Pi_1 & \Phi_{[0]i}\\ * & -\Lambda_1\end{bmatrix}<0 \tag{31}$$

where

$$\Xi_{[\tau,\dot{\tau},\sigma]i}=\mathrm{sym}\left\{\alpha_1^\top\mathcal{P}\alpha_{2i}\right\}-\Psi+\Omega_{1[\tau,\dot{\tau},\sigma]i}+\Omega_{2[\tau,\dot{\tau},\sigma]}$$
$$\Pi_1=\sum_{m=1}^{3}(2m-1)\begin{bmatrix}\delta_{1m}\\ \delta_{2m}\end{bmatrix}^\top\begin{bmatrix}\mathcal{J}_1 & \frac{\mathcal{S}_1}{h}+\mathcal{X}_1\\ * & \frac{\mathcal{H}_{33}}{h}\end{bmatrix}\begin{bmatrix}\delta_{1m}\\ \delta_{2m}\end{bmatrix}$$
$$\Pi_2=\sum_{m=1}^{3}(2m-1)\begin{bmatrix}\delta_{1m}\\ \delta_{2m}\end{bmatrix}^\top\begin{bmatrix}\mathcal{J}_2 & \mathcal{X}_2+\frac{\mathcal{S}_2}{h}\\ * & \mathcal{J}_3\end{bmatrix}\begin{bmatrix}\delta_{1m}\\ \delta_{2m}\end{bmatrix}$$
$$\Phi_{[0]i}=\begin{bmatrix}\Phi_{2[0]i} & \Delta_1\\ -\Phi_{1[0]} & 0\end{bmatrix},\ \Phi_{[h]i}=\begin{bmatrix}\Phi_{2[h]i} & \Delta_2\\ -\Phi_{1[h]} & 0\end{bmatrix}$$
$$\Omega_{[\dot{\tau},\sigma]i}=\Omega_{1[\dot{\tau},\sigma]i}+\Omega_{2[\dot{\tau},\sigma]},\Lambda_1=diag\left\{\varrho\otimes\Theta_2,h\varrho\otimes\mathcal{H}_{33}\right\}$$
$$\Lambda_2=diag\left\{\varrho\otimes\Theta_1,h\varrho\otimes\widetilde{\tau}\mathcal{G}_{33}\right\}$$

with

$$\Omega_{1[\dot{\tau},\sigma]i}=\mathrm{sym}\left\{\eta_1^\top\mathrm{sym}\left\{\wp_2(\sigma,\mathcal{G}_{23})\right\}\eta_{2i}\right\}$$
$$\Omega_{2[\dot{\tau},\sigma]}=\mathrm{sym}\left\{\varpi_1^\top\mathrm{sym}\left\{\wp_2(\sigma,\mathcal{H}_{23})\right\}\varpi_2\right\}$$
$$\Omega_{1[\tau,\dot{\tau},\sigma]i}=\dot{\tau}\eta_1^\top\big(\widehat{\mathcal{G}}+\mathrm{sym}\left\{\wp_1(\mathcal{G}_{13})+2\tau\wp_2(\sigma,\mathcal{G}_{23})\right\}\big)\eta_1\\ +\tau\mathrm{sym}\left\{\eta_1^\top\mathrm{sym}\left\{\wp_2(\sigma,\mathcal{G}_{23})\right\}\eta_3\right\}\\ +\mathrm{sym}\left\{\eta_1^\top\left(\widehat{\mathcal{G}}+\mathrm{sym}\left\{\wp_1(\mathcal{G}_{13})\right\}\right)\widetilde{\eta}_i\right\}$$
$$\Omega_{2[\tau,\dot{\tau},\sigma]}=-\dot{\tau}\varpi_1^\top\big(\mathrm{sym}\{\wp_1(\mathcal{H}_{13})+2(h-\tau)\wp_2(\sigma,\mathcal{H}_{23})\}\\ +\widehat{\mathcal{H}}\big)\varpi_1+\widetilde{\tau}(h-\tau)e_5^\top\mathcal{H}_{33}e_5\\ +\mathrm{sym}\left\{\varpi_1^\top\left(\widehat{\mathcal{H}}+\mathrm{sym}\left\{\wp_1(\mathcal{H}_{13})\right\}\right)\widetilde{\varpi}\right\}\\ +(h-\tau)\mathrm{sym}\left\{\varpi_1^\top\mathrm{sym}\left\{\wp_2(\sigma,\mathcal{H}_{23})\right\}\varpi_3\right\}\\ +(h^2-2\tau h)\mathrm{sym}\left\{\varpi_1^\top\mathrm{sym}\left\{\wp_2(\sigma,\mathcal{H}_{23})\right\}\varpi_2\right\}$$
$$\Phi_{1[\tau]}=diag\left\{\mathcal{Q},\ \frac{\mathcal{R}_1}{2},\ \frac{\mathcal{R}_2}{2},\ \tau\mathcal{G}_{33}\right\}$$
$$\Phi_{2[\tau]i}=\left[he_{\mathcal{F}i}^\top\mathcal{Q}\ \ \frac{h}{2}e_{\mathcal{F}i}^\top\mathcal{R}_1\ \ \frac{h}{2}e_{\mathcal{F}i}^\top\mathcal{R}_2\ \ \tau e_{\mathcal{F}i}^\top\mathcal{G}_{33}\right]$$
$$\Psi=\delta^\top\left[diag\left\{2,4,2,4\right\}\otimes\mathcal{R}_1,diag\left\{2,4,2,4\right\}\otimes\mathcal{R}_2\right]\delta$$
$$\Delta_1=\left[\left[\delta_{11}^\top\ \ \delta_{12}^\top\ \ \delta_{13}^\top\right]\mathcal{X}_2\ \ \left[\delta_{11}^\top\ \ \delta_{12}^\top\ \ \delta_{13}^\top\right]\mathcal{S}_2\right]$$
$$\Delta_2=\left[\left[\delta_{21}^\top\ \ \delta_{22}^\top\ \ \delta_{23}^\top\right]\mathcal{X}_1^\top\ \ \left[\delta_{21}^\top\ \ \delta_{22}^\top\ \ \delta_{23}^\top\right]\mathcal{S}_1^\top\right]$$
$$\mathcal{J}_1=\mathcal{Q}+\Theta_1+(2\widetilde{\tau}/h)\mathcal{G}_{33},\mathcal{J}_2=\mathcal{Q}+(\widetilde{\tau}/h)\mathcal{G}_{33}$$
$$\mathcal{J}_3=\mathcal{Q}+\Theta_2+(2/h)\mathcal{H}_{33}.$$

Proof: Incorporating IPFs, the LKF candidate is chosen as

$$V(x_t)=\sum_{n=1}^{2}\left(V_n(x_t)+V_{\mathcal{P}n}(x_t)\right) \tag{32}$$

where

$$V_1(x_t)=\gamma^\top(t)\mathcal{P}\gamma(t)+h\int_{t-h}^{t}\int_{\theta}^{t}\dot{x}^\top(s)\mathcal{Q}\dot{x}(s)dsd\theta$$
$$V_2(x_t)=\int_{t-h}^{t}\int_{\varphi}^{t}\int_{\theta}^{t}\dot{x}^\top(s)\mathcal{R}_1\dot{x}(s)dsd\theta d\varphi\\ +\int_{t-h}^{t}\int_{t-h}^{\varphi}\int_{\theta}^{t}\dot{x}^\top(s)\mathcal{R}_2\dot{x}(s)dsd\theta d\varphi$$

with $\gamma(t)=\mathrm{col}\{x(t),x(t-\tau)\}$.

Firstly, the derivatives of individual LKFs along the trajectory of (3) are computed as

$$\dot{V}_1(x_t)=\mathrm{sym}\left\{\begin{bmatrix}x(t)\\ x(t-\tau)\end{bmatrix}^\top\mathcal{P}\begin{bmatrix}\dot{x}(t)\\ \widetilde{\tau}\dot{x}(t-\tau)\end{bmatrix}\right\}\\ +h^2\dot{x}^\top(t)\mathcal{Q}\dot{x}(t)-h\int_{t-h}^{t}\dot{x}^\top(s)\mathcal{Q}\dot{x}(s)ds\\ =\sum_{i=1}^{r}\lambda_i(\theta(t))\zeta^\top(t)\left(\mathrm{sym}\left\{\alpha_1^\top\mathcal{P}\alpha_{2i}\right\}\right)\zeta(t)-\Upsilon_1\\ +h^2\zeta^\top(t)\left(\sum_{i=1}^{r}\lambda_i(\theta(t))e_{\mathcal{F}i}^\top\right)\mathcal{Q}\left(\sum_{i=1}^{r}\lambda_i(\theta(t))e_{\mathcal{F}i}\right)\zeta(t) \tag{33}$$

$$\dot{V}_2(x_t)=\frac{h^2}{2}\dot{x}^\top(t)\left(\mathcal{R}_1+\mathcal{R}_2\right)\dot{x}(t)\\ -\int_{t-h}^{t}\left(\int_{\theta}^{t}\dot{x}^\top(s)\mathcal{R}_1\dot{x}(s)+\int_{t-h}^{\theta}\dot{x}^\top(s)\mathcal{R}_2\dot{x}(s)\right)dsd\theta\\ =\frac{h^2}{2}\zeta^\top(t)\left(\sum_{i=1}^{r}\lambda_i(\theta(t))e_{\mathcal{F}i}^\top\right)\sum_{n=1}^{2}\mathcal{R}_n\left(\sum_{i=1}^{r}\lambda_i(\theta(t))e_{\mathcal{F}i}\right)\zeta(t)\\ -\Upsilon_2-\Upsilon_3 \tag{34}$$

where

$$\Upsilon_1=h\int_{t-\tau}^{t}\dot{x}^\top(s)\mathcal{Q}\dot{x}(s)ds+h\int_{t-h}^{t-\tau}\dot{x}^\top(s)\mathcal{Q}\dot{x}(s)ds$$
$$\Upsilon_2=(h-\tau)\int_{t-\tau}^{t}\dot{x}^\top(s)\mathcal{R}_1\dot{x}(s)ds+\tau\int_{t-h}^{t-\tau}\dot{x}^\top(s)\mathcal{R}_2\dot{x}(s)ds$$

$$\varUpsilon_3=\int_{t-\tau}^{t}\left(\int_{\theta}^{t}\dot{x}^\top(s)\mathcal{R}_1\dot{x}(s)+\int_{t-\tau}^{\theta}\dot{x}^\top(s)\mathcal{R}_2\dot{x}(s)\right)dsd\theta$$
$$+\int_{t-h}^{t-\tau}\left(\int_{\theta}^{t-\tau}\dot{x}^\top(s)\mathcal{R}_1\dot{x}(s)+\int_{t-h}^{\theta}\dot{x}^\top(s)\mathcal{R}_2\dot{x}(s)\right)dsd\theta.$$

For any matrices $\mathcal{X}_1$ and $\mathcal{X}_2$, it follows from (8) and (11) that

$$-\varUpsilon_1-\varUpsilon_2\leq-\sum_{i=1}^{r}\lambda_i(\theta(t))\zeta^\top(t)\varPsi_{[\tau]}\zeta(t) \tag{35}$$

where

$$\varPsi_{[\tau]}=\sum_{m=1}^{3}(2m-1)\Bigg(\delta_{1m}^\top\mathcal{Q}\delta_{1m}+\delta_{2m}^\top\mathcal{Q}\delta_{2m}$$
$$+\begin{bmatrix}\delta_{1m}\\ \delta_{2m}\end{bmatrix}^\top\Bigg(\frac{\tau}{h}\begin{bmatrix}0 & \mathcal{X}_2\\ * & \varTheta_2-\mathcal{X}_1^\top\varTheta_1^{-1}\mathcal{X}_1\end{bmatrix}$$
$$+\frac{h-\tau}{h}\begin{bmatrix}\varTheta_1-\mathcal{X}_2\varTheta_2^{-1}\mathcal{X}_2^\top & \mathcal{X}_1\\ * & 0\end{bmatrix}\Bigg)\begin{bmatrix}\delta_{1m}\\ \delta_{2m}\end{bmatrix}\Bigg).$$

By use of (9)-(10), the double integrals can be estimated as

$$-\varUpsilon_3\leq-\sum_{i=1}^{r}\lambda_i(\theta(t))\zeta^\top(t)\varPsi\zeta(t). \tag{36}$$

Secondly, when differentiating $V_{\mathcal{P}1}(x_t)$, some terms related to $\varsigma_1(t)$ and $\dot{\varsigma}_1(t)$ are engendered. $\varsigma_1(t)$ can be presented as $\eta_1\zeta(t)$, while $\dot{\varsigma}_1(t)$ cannot be expressed by $\zeta(t)$ linearly due to the existence of the last two components in $\dot{\varsigma}_1(t)$, which are shown as

$$\dot{\xi}_1(t-\tau,t)=\frac{x(t)-\widetilde{\tau}x(t-\tau)}{\tau}+\int_{t-\tau}^{t}\frac{\partial}{\partial t}\left(\frac{x(s)}{\tau}\right)ds$$
$$=\frac{e_1-\widetilde{\tau}e_3-\dot{\tau}e_6}{\tau}\zeta(t),$$
$$\dot{\xi}_2(t-\tau,t)=\int_{t-\tau}^{t}\frac{\partial}{\partial t}\left(\int_{\theta}^{t}\frac{x(s)}{\tau^2}ds\right)d\theta-\widetilde{\tau}\int_{t-\tau}^{t}\frac{x(s)}{\tau^2}ds$$
$$=\frac{e_1-\widetilde{\tau}e_6-2\dot{\tau}e_7}{\tau}\zeta(t).$$

In view of $\mathcal{G}_{[\tau,\sigma]}$ with delay-product terms, $\dot{V}_{\mathcal{P}1}(x_t)$ can be written as

$$\dot{V}_{\mathcal{P}1}(x_t)=\varsigma_1^\top(t)\dot{\mathcal{G}}_{[\tau,\sigma]}\varsigma_1(t)+\mathrm{sym}\{\varsigma_1^\top(t)\mathcal{G}_{[\tau,\sigma]}\dot{\varsigma}_1(t)\}$$
$$+\frac{d}{dt}\left(\int_{t-\tau}^{t}\int_{\theta}^{t}\dot{x}^\top(s)\mathcal{G}_{33}\dot{x}(s)dsd\theta\right)$$
$$=\sum_{i=1}^{r}\lambda_i(\theta(t))\zeta^\top(t)\left(\tau^2\varOmega_{1[\dot{\tau},\sigma]i}+\varOmega_{1[\tau,\dot{\tau},\sigma]i}\right)\zeta(t) \tag{37}$$
$$+\tau\left(\sum_{i=1}^{r}\lambda_i(\theta(t))e_{\mathcal{F}i}^\top\right)\mathcal{G}_{33}\left(\sum_{i=1}^{r}\lambda_i(\theta(t))e_{\mathcal{F}i}\right)-\varUpsilon_4.$$

Performing a similar operation to $\dot{V}_{\mathcal{P}1}(x_t)$ yields

$$\dot{V}_{\mathcal{P}2}(x_t)=\varsigma_2^\top(t)\dot{\mathcal{H}}_{[\tau,\sigma]}\varsigma_2(t)+\mathrm{sym}\left\{\varsigma_2^\top(\mathrm{t})\mathcal{H}_{[\tau,\sigma]}\dot{\varsigma}_2(\mathrm{t})\right\}$$
$$+\frac{d}{dt}\left(\int_{t-h}^{t-\tau}\int_{\theta}^{t-\tau}\dot{x}^\top(s)\mathcal{H}_{33}\dot{x}(s)dsd\theta\right)$$
$$=\sum_{i=1}^{r}\lambda_i(\theta(t))\zeta^\top(t)\left(\tau^2\varOmega_{2[\dot{\tau},\sigma]}+\varOmega_{2[\tau,\dot{\tau},\sigma]}\right)\zeta(t)-\varUpsilon_5 \tag{38}$$

where

$$\varUpsilon_4+\varUpsilon_5=\widetilde{\tau}\int_{t-\tau}^{t}\dot{x}^\top(s)\mathcal{G}_{33}\dot{x}(s)ds+\int_{t-h}^{t-\tau}\dot{x}^\top(s)\mathcal{H}_{33}\dot{x}(s)ds.$$

Then, for any matrices $\mathcal{S}_1$ and $\mathcal{S}_2$, by special case of Lemma 3 with $\mathcal{G}_3=\mathcal{G}_4=0$, it can be found that

$$-\varUpsilon_4-\varUpsilon_5\leq-\sum_{i=1}^{r}\lambda_i(\theta(t))\zeta^\top(t)\varPsi_{[\tau,\dot{\tau}]}\zeta(t) \tag{39}$$

where

$$\varPsi_{[\tau,\dot{\tau}]}=\sum_{m=1}^{3}\frac{2m-1}{h}\Bigg(\widetilde{\tau}\delta_{1m}^\top\mathcal{G}_{33}\delta_{1m}+\delta_{2m}^\top\mathcal{H}_{33}\delta_{2m}$$
$$+\begin{bmatrix}\delta_{1m}\\ \delta_{2m}\end{bmatrix}^\top\Bigg(\frac{\tau}{h}\begin{bmatrix}0 & \mathcal{S}_2\\ * & \mathcal{H}_{33}-\widetilde{\tau}^{-1}\mathcal{S}_1^\top\mathcal{G}_{33}^{-1}\mathcal{S}_1\end{bmatrix}$$
$$+\frac{h-\tau}{h}\begin{bmatrix}\widetilde{\tau}\mathcal{G}_{33}-\mathcal{S}_2\mathcal{H}_{33}^{-1}\mathcal{S}_2^\top & \mathcal{S}_1\\ * & 0\end{bmatrix}\Bigg)\begin{bmatrix}\delta_{1m}\\ \delta_{2m}\end{bmatrix}\Bigg).$$

Summing up the above analysis, $\dot{V}(x_t)$ can be bounded as

$$\dot{V}(x_t)\leq\zeta^\top(t)\left(\sum_{i=1}^{r}\lambda_i(\theta(t))\varGamma_{[\tau,\dot{\tau},\sigma]i}+\varGamma_{[\tau]}\right)\zeta(t) \tag{40}$$

where

$$\varGamma_{[\tau]}=\left(\sum_{i=1}^{r}\lambda_i(\theta(t))e_{\mathcal{F}i}^\top\right)\left(\tau\mathcal{G}_{33}+h^2\left(\mathcal{Q}+\frac{\sum_{n=1}^{2}\mathcal{R}_n}{2}\right)\right)$$
$$\times\left(\sum_{i=1}^{r}\lambda_i(\theta(t))e_{\mathcal{F}i}\right)$$
$$\varGamma_{[\tau,\dot{\tau},\sigma]i}=\mathrm{sym}\left\{\alpha_1^\top\mathcal{P}\alpha_{2i}\right\}-\varPsi_{[\tau]}-\varPsi-\varPsi_{[\tau,\dot{\tau}]}+\varOmega_{1[\tau,\dot{\tau},\sigma]i}$$
$$+\varOmega_{2[\tau,\dot{\tau},\sigma]}+\tau^2\left(\varOmega_{1[\dot{\tau},\sigma]i}+\varOmega_{2[\dot{\tau},\sigma]}\right).$$

Apparently, if $\sum_{i=1}^{r}\lambda_i(\theta(t))\varGamma_{[\tau,\dot{\tau},\sigma]i}+\varGamma_{[\tau]}<0$ holds, $\dot{V}(x_t)<-\epsilon\|x(t)\|^2$ for a sufficiently small scalar ϵ. From Schur complement, $\sum_{i=1}^{r}\lambda_i(\theta(t))\varGamma_{[\tau,\dot{\tau},\sigma]i}+\varGamma_{[\tau]}<0$ is equivalent to

$$\sum_{i=1}^{r}\lambda_i(\theta(t))\begin{bmatrix}\varGamma_{[\tau,\dot{\tau},\sigma]i} & \varPhi_{2[\tau]i}\\ * & -\varPhi_{1[\tau]}\end{bmatrix}<0. \tag{41}$$

All the elements constituting $\varGamma_{[\tau,\dot{\tau},\sigma]i},\varPhi_{1[\tau]}$ and $\varPhi_{2[\tau]i}$ except for $\tau^2\left(\varOmega_{1[\dot{\tau},\sigma]i}+\varOmega_{2[\dot{\tau},\sigma]}\right)$ are first-order functions of τ and $\dot{\tau}$. From Lemma 1, by Schur complement once again, (41) is true, if (29)-(31) hold at the vertices of $\dot{\tau}\in[\mu_1,\mu_2]$, which implies that the T-S fuzzy system (3) with $u(t)=0$ is asymptotically stable. Then, the proof is completed.

Remark 7: When differentiating $V_{\mathcal{P}1}(x_t)$ and $V_{\mathcal{P}2}(x_t)$, it is found that $\dot{\varsigma}_1(t)$ and $\dot{\varsigma}_2(t)$ are dependent on the combinations of $\zeta(t)$ and $\frac{1}{\tau}\zeta(t)$, and $\frac{1}{h-\tau}\zeta(t)$, respectively. Due to the elaborate framework of IPFs, τ and $h-\tau$ in denominators are eliminated by the delay-product matrices $\mathcal{G}_{[\tau,\sigma]}$ and $\mathcal{H}_{[\tau,\sigma]}$. Consequently, the extension difficulties of MRFs limited to the numerical intractability are well addressed by effectively disposing the delay-product terms.

Remark 8: The conspicuous features of the newly established stability criterion principally lie in three aspects: 1) Taking derivatives of $\varsigma_1(t)$ and $\varsigma_2(t)$, a great many of cross terms emerge in $\Omega_{1[\dot{\tau},\sigma]i}, \Omega_{2[\dot{\tau},\sigma]}, \Omega_{1[\tau,\dot{\tau},\sigma]i}$ and $\Omega_{2[\tau,\dot{\tau},\sigma]}$, and thus extensive relationships of extra states are formulated. 2) Single and double integrals are estimated by SOBLI combined with improved matrix inequality and GDIIs, respectively, with prominent progress. Driven by double integral types of augmented vectors, the capacities of advanced inequalities are brought into full play for reduction of conservatism. 3) Above all, in light of slack variables with parameter σ, further freedom is offered for finding feasible solution, and the links of various system information are intensified, which will take pivotal role in relaxing criterion. Hence, in virtue of the merits of IPFs and their facilitation for bounding techniques, more desirable stability range can be expected by Theorem 1.

Next, the stability condition for T-S fuzzy system (7) with known $\mathcal{K}(t)$ is presented as follows.

Theorem 2: Consider given scalars h, μ_1 μ_2 and σ, and suppose that the gain matrix $\mathcal{K}(t)$ is known. If there exist symmetric matrices $\mathcal{P} = [\mathcal{P}_{mn}]_{2\times 2} > 0$, $\mathcal{Q} > 0$, $\mathcal{R}_n > 0$, $\mathcal{G} = [\mathcal{G}_{pq}]_{3\times 3} > 0$, $\mathcal{H} = [\mathcal{H}_{pq}]_{3\times 3} > 0$ with $\mathcal{G}_{12} = \mathcal{H}_{12} = 0$, and any matrices $\mathcal{X}_n, \mathcal{S}_n$ $(n = 1, 2)$, and $\mathcal{Z}(t) = \sum_{i=1}^{r} \lambda(\theta(t))\mathcal{Z}_i$ of appropriate dimensions such that the following inequalities hold for $p = 1, 2, 3$ and $i, j, k = 1, \ldots, r$:

$$\mathscr{J}_{p[\dot{\tau},\sigma]iik} < 0 \tag{42}$$

$$\mathscr{J}_{p[\dot{\tau},\sigma]ijk} + \mathscr{J}_{p[\dot{\tau},\sigma]jik} < 0,\ 1 \le i \le j \le r \tag{43}$$

where

$$\mathscr{J}_{1[\dot{\tau},\sigma]ijk} = \begin{bmatrix} \bar{\Xi}_{[0,\dot{\tau},\sigma]ijk} - \Pi_1 & \bar{\Phi}_{[0]} \\ * & -\Lambda_1 \end{bmatrix}$$

$$\mathscr{J}_{2[\dot{\tau},\sigma]ijk} = \begin{bmatrix} h^2\bar{\Omega}_{[\dot{\tau},\sigma]} + \bar{\Xi}_{[h,\dot{\tau},\sigma]ijk} - \Pi_2 & \bar{\Phi}_{[h]} \\ * & -\Lambda_2 \end{bmatrix}$$

$$\mathscr{J}_{3[\dot{\tau},\sigma]ijk} = \begin{bmatrix} \bar{\Xi}_{[0,\dot{\tau},\sigma]ijk} - h^2\bar{\Omega}_{[\dot{\tau},\sigma]} - \Pi_1 & \bar{\Phi}_{[0]} \\ * & -\Lambda_1 \end{bmatrix}$$

with

$$\begin{aligned}
\bar{\Xi}_{[\tau,\dot{\tau},\sigma]ijk} &= \mathrm{sym}\left\{\alpha_1^\top \mathcal{P}\bar{\alpha}_2\right\} - \Psi + \bar{\Omega}_{1[\tau,\dot{\tau},\sigma]} + \Omega_{2[\tau,\dot{\tau},\sigma]} \\
&\quad + \mathrm{sym}\left\{\mathcal{Z}_i \mathcal{Z}_{\mathcal{S}jk}\right\} \\
\bar{\Omega}_{1[\tau,\dot{\tau},\sigma]} &= \dot{\tau}\eta_1^\top\left(\widehat{\mathcal{G}} + \mathrm{sym}\left\{\wp_1(\mathcal{G}_{13}) + 2\tau\wp_2(\sigma,\mathcal{G}_{23})\right\}\right)\eta_1 \\
&\quad + \tau\mathrm{sym}\left\{\eta_1^\top \mathrm{sym}\left\{\wp_2(\sigma,\mathcal{G}_{23})\right\}\eta_3\right\} \\
&\quad + \mathrm{sym}\left\{\eta_1^\top\left(\widehat{\mathcal{G}} + \mathrm{sym}\left\{\wp_1(\mathcal{G}_{13})\right\}\right)(\tau\bar{\eta}_2 + \eta_3)\right\} \\
\bar{\Omega}_{[\dot{\tau},\sigma]} &= \bar{\Omega}_{1[\dot{\tau},\sigma]} + \Omega_{2[\dot{\tau},\sigma]} \\
\bar{\Omega}_{1[\dot{\tau},\sigma]} &= \mathrm{sym}\left\{\eta_1^\top \mathrm{sym}\left\{\wp_2(\sigma,\mathcal{G}_{23})\right\}\bar{\eta}_2\right\} \\
\bar{\alpha}_2 &= \mathrm{col}\{e_{10}, \widetilde{\tau}e_5\},\ \bar{\eta}_2 = \mathrm{col}\{e_{10}, 0, 0\} \\
\bar{\Phi}_{[0]} &= \begin{bmatrix} \bar{\Phi}_{2[0]} & \Delta_1 \\ -\Phi_{1[0]} & 0 \end{bmatrix},\ \bar{\Phi}_{[h]} = \begin{bmatrix} \bar{\Phi}_{2[h]} & \Delta_2 \\ -\Phi_{1[h]} & 0 \end{bmatrix} \\
\bar{\Phi}_{2[\tau]} &= \left[he_{10}^\top\mathcal{Q} \quad \frac{h}{2}e_{10}^\top\mathcal{R}_1 \quad \frac{h}{2}e_{10}^\top\mathcal{R}_2 \quad \tau e_{10}^\top\mathcal{G}_{33}\right] \\
\mathcal{Z}_{\mathcal{S}jk} &= \left[\mathcal{A}_j + \mathcal{B}_j\mathcal{K}_k \quad 0 \quad \mathcal{A}_{dj} \quad \underbrace{0 \quad \ldots \quad 0}_{6} \quad -I\right],
\end{aligned}$$

then the T-S fuzzy system (7) is asymptotically stable.

Proof: Differentiating (32) along the solution of system (7), and executing deriving process similar to Theorem 1 yield

$$\dot{V}(x_t) \le \bar{\zeta}^\top(t)\left(\bar{\Gamma}_{[\tau,\dot{\tau},\sigma]} + \bar{\Gamma}_{[\tau]}\right)\bar{\zeta}(t) \tag{44}$$

where $\bar{\zeta}(t) = \mathrm{col}\{\zeta(t), \dot{x}(t)\}$, and

$$\begin{aligned}
\bar{\Gamma}_{[\tau]} &= e_{10}^\top\left(\tau\mathcal{G}_{33} + h^2\left(\mathcal{Q} + \frac{\sum_{n=1}^{2}\mathcal{R}_n}{2}\right)\right)e_{10} \\
\bar{\Gamma}_{[\tau,\dot{\tau},\sigma]} &= \mathrm{sym}\left\{\alpha_1^\top\mathcal{P}\bar{\alpha}_2\right\} - \Psi_{[\tau]} - \Psi - \Psi_{[\tau,\dot{\tau}]} \\
&\quad + \tau^2\left(\bar{\Omega}_{1[\dot{\tau},\sigma]} + \Omega_{2[\dot{\tau},\sigma]}\right) + \bar{\Omega}_{1[\tau,\dot{\tau},\sigma]} + \Omega_{2[\tau,\dot{\tau},\sigma]}.
\end{aligned}$$

From (7), for any matrices $\mathcal{Z}_i$, one has

$$\begin{aligned}
\Delta = 2\bar{\zeta}^\top(t)\mathcal{Z}(t)((\mathcal{A}(t) + \mathcal{B}(t)\mathcal{K}(t))x(t) + \mathcal{A}_d(t)x(t-\tau) \\
- \dot{x}(t)) = 0.
\end{aligned}$$

Hence, it is easy to get

$$\dot{V}(x_t) = \dot{V}(x_t) + \Delta \le \bar{\zeta}^\top(t)\bar{\Gamma}_{[\tau,\dot{\tau},\sigma]}(t)\bar{\zeta}(t) \tag{45}$$

where $\bar{\Gamma}_{[\tau,\dot{\tau},\sigma]}(t) = \bar{\Gamma}_{[\tau,\dot{\tau},\sigma]} + \bar{\Gamma}_{[\tau]} + \mathrm{sym}\left\{\mathcal{Z}(t)\mathcal{Z}_{\mathcal{S}}(t)\right\}$ and

$$\mathcal{Z}_{\mathcal{S}}(t) = \left[\mathcal{A}(t) + \mathcal{B}(t)\mathcal{K}(t) \quad 0 \quad \mathcal{A}_d(t) \quad \underbrace{0 \quad \ldots \quad 0}_{6} \quad -I\right].$$

It is notable that $\bar{\Gamma}_{[\tau,\dot{\tau},\sigma]}(t)$ can be expressed as

$$\begin{aligned}
&\sum_{i=1}^{r}\lambda_i(\theta(t))\sum_{j=1}^{r}\lambda_j(\theta(t))\sum_{k=1}^{r}\lambda_k(\theta(t))\bar{\Gamma}_{[\tau,\dot{\tau},\sigma]ijk} \\
&= \sum_{i=1}^{r}\lambda_i^2(\theta(t))\sum_{k=1}^{r}\lambda_k(\theta(t))\bar{\Gamma}_{[\tau,\dot{\tau},\sigma]iik} + 2\sum_{i=1}^{r-1}\lambda_i(\theta(t)) \\
&\times \sum_{j>i}^{r}\lambda_j(\theta(t))\sum_{k=1}^{r}\lambda_k(\theta(t))\left(\frac{\bar{\Gamma}_{[\tau,\dot{\tau},\sigma]ijk} + \bar{\Gamma}_{[\tau,\dot{\tau},\sigma]jik}}{2}\right)
\end{aligned}$$

where $\bar{\Gamma}_{[\tau,\dot{\tau},\sigma]ijk} = \bar{\Gamma}_{[\tau,\dot{\tau},\sigma]} + \bar{\Gamma}_{[\tau]} + \mathrm{sym}\left\{\mathcal{Z}_i\mathcal{Z}_{\mathcal{S}jk}\right\}$.

As a consequence, for $i, j, k = 1, \ldots, r$, $\bar{\Gamma}_{[\tau,\dot{\tau},\sigma]}(t) < 0$, if the following inequalities hold

$$\bar{\Gamma}_{[\tau,\dot{\tau},\sigma]iik} < 0, \tag{46}$$

$$\bar{\Gamma}_{[\tau,\dot{\tau},\sigma]ijk} + \bar{\Gamma}_{[\tau,\dot{\tau},\sigma]jik} < 0, \quad 1 \le i < j \le r. \tag{47}$$

By applying Schur complement and Lemma 1, it can be deduced that if (42)-(43) are feasible, $\bar{\Gamma}_{[\tau,\dot{\tau},\sigma]}(t) < 0$ holds, which indicates that the closed-loop system (7) with known control gain matrix is asymptotically stable. Thus, the proof is completed.

B. Controller Design

On the basis of Theorem 2, the controller design method for the system (7) is provided in the Theorem 3.

Theorem 3: For given scalars h, μ_1, μ_2, σ, and $\varphi_{i1}, \varphi_{i2}, \ldots, \varphi_{i10}$, the closed-loop fuzzy system (7) is asymptotically stable under the controller (5), if there exist symmetric matrices $\widetilde{\mathcal{P}} = [\widetilde{\mathcal{P}}_{mn}]_{2\times 2} > 0$, $\widetilde{\mathcal{Q}} > 0$, $\widetilde{\mathcal{R}}_n > 0$, $\widetilde{\mathcal{G}} = [\widetilde{\mathcal{G}}_{pq}]_{3\times 3} > 0$, $\widetilde{\mathcal{H}} = [\widetilde{\mathcal{H}}_{pq}]_{3\times 3} > 0$ with $\widetilde{\mathcal{G}}_{12} = \widetilde{\mathcal{H}}_{12} = 0$, and any matrices $\widetilde{\mathcal{X}}_n, \widetilde{\mathcal{S}}_n$ $(n = 1, 2), \mathcal{M}_i$ and symmetric matrix $\mathcal{Y}$ of appropriate dimensions such that the following inequalities hold for $p = 1, 2, 3$ and $i, j = 1, \ldots, r$:

$$\widetilde{\mathscr{J}}_{p[\dot{\tau},\sigma]ii} < 0 \tag{48}$$

$$\widetilde{\mathscr{J}}_{p[\dot{\tau},\sigma]ij} + \widetilde{\mathscr{J}}_{p[\dot{\tau},\sigma]ji} < 0,\ 1 \le i \le j \le r \tag{49}$$

where

$$\widetilde{\mathscr{J}}_{1[\dot{\tau},\sigma]ij} = \begin{bmatrix} \widetilde{\Xi}_{[0,\dot{\tau},\sigma]ij} - \widetilde{\Pi}_1 & \widetilde{\Phi}_{[0]} \\ * & -\widetilde{\Lambda}_1 \end{bmatrix}$$
$$\widetilde{\mathscr{J}}_{2[\dot{\tau},\sigma]ij} = \begin{bmatrix} h^2\widetilde{\Omega}_{[\dot{\tau},\sigma]} + \widetilde{\Xi}_{[h,\dot{\tau},\sigma]ij} - \widetilde{\Pi}_2 & \widetilde{\Phi}_{[h]} \\ * & -\widetilde{\Lambda}_2 \end{bmatrix}$$
$$\widetilde{\mathscr{J}}_{3[\dot{\tau},\sigma]ij} = \begin{bmatrix} \widetilde{\Xi}_{[0,\dot{\tau},\sigma]ij} - h^2\widetilde{\Omega}_{[\dot{\tau},\sigma]} - \widetilde{\Pi}_1 & \widetilde{\Phi}_{[0]} \\ * & -\widetilde{\Lambda}_1 \end{bmatrix}$$
$$\widetilde{\Xi}_{[\tau,\dot{\tau},\sigma]ij} = \mathrm{sym}\left\{\alpha_1^\top \widetilde{\mathcal{P}}\bar{\alpha}_2\right\} - \widetilde{\Psi} + \widetilde{\Omega}_{1[\tau,\dot{\tau},\sigma]} + \widetilde{\Omega}_{2[\tau,\dot{\tau},\sigma]} + \mathrm{sym}\left\{\widetilde{\mathcal{Z}}_i\widetilde{\mathcal{Z}}_{\mathcal{S}ij}\right\}$$
$$\widetilde{\Pi}_1 = \sum_{m=1}^{3}(2m-1)\begin{bmatrix}\delta_{1m}\\ \delta_{2m}\end{bmatrix}^\top \begin{bmatrix}\widetilde{\mathcal{J}}_1 & \frac{\widetilde{\mathcal{S}}_1}{h}+\widetilde{\mathcal{X}}_1 \\ * & \frac{\widetilde{\mathcal{H}}_{33}}{h}\end{bmatrix}\begin{bmatrix}\delta_{1m}\\ \delta_{2m}\end{bmatrix}$$
$$\widetilde{\Pi}_2 = \sum_{m=1}^{3}(2m-1)\begin{bmatrix}\delta_{1m}\\ \delta_{2m}\end{bmatrix}^\top \begin{bmatrix}\widetilde{\mathcal{J}}_2 & \widetilde{\mathcal{X}}_2+\frac{\widetilde{\mathcal{S}}_2}{h} \\ * & \widetilde{\mathcal{J}}_3\end{bmatrix}\begin{bmatrix}\delta_{1m}\\ \delta_{2m}\end{bmatrix}$$
$$\widetilde{\Phi}_{[0]} = \begin{bmatrix}\widetilde{\Phi}_{2[0]} & \widetilde{\Delta}_1 \\ -\widetilde{\Phi}_{1[0]} & 0\end{bmatrix},\ \widetilde{\Phi}_{[h]} = \begin{bmatrix}\widetilde{\Phi}_{2[h]} & \widetilde{\Delta}_2 \\ -\widetilde{\Phi}_{1[h]} & 0\end{bmatrix}$$
$$\widetilde{\Omega}_{[\dot{\tau},\sigma]} = \widetilde{\Omega}_{1[\dot{\tau},\sigma]} + \widetilde{\Omega}_{2[\dot{\tau},\sigma]}$$
$$\widetilde{\Lambda}_1 = diag\left\{\varrho\otimes\left(\widetilde{\mathcal{Q}}+\widetilde{\mathcal{R}}_2\right), h\varrho\otimes\widetilde{\mathcal{H}}_{33}\right\}$$
$$\widetilde{\Lambda}_2 = diag\left\{\varrho\otimes\left(\widetilde{\mathcal{Q}}+\widetilde{\mathcal{R}}_1\right), h\varrho\otimes\tilde{\tau}\widetilde{\mathcal{G}}_{33}\right\}$$

with

$$\begin{aligned}\widetilde{\Omega}_{1[\tau,\dot{\tau},\sigma]} &= \dot{\tau}\eta_1^\top\left(\acute{\mathcal{G}} + \mathrm{sym}\left\{\wp_1(\widetilde{\mathcal{G}}_{13}) + 2\tau\wp_2(\sigma,\widetilde{\mathcal{G}}_{23})\right\}\right)\eta_1 \\ &\quad + \tau\mathrm{sym}\left\{\eta_1^\top\mathrm{sym}\left\{\wp_2(\sigma,\widetilde{\mathcal{G}}_{23})\right\}\eta_3\right\} \\ &\quad + \mathrm{sym}\left\{\eta_1^\top\left(\acute{\mathcal{G}} + \mathrm{sym}\left\{\wp_1(\widetilde{\mathcal{G}}_{13})\right\}\right)(\tau\bar{\eta}_2+\eta_3)\right\}\end{aligned}$$
$$\begin{aligned}\widetilde{\Omega}_{2[\tau,\dot{\tau},\sigma]} &= -\dot{\tau}\varpi_1^\top\Big(\mathrm{sym}\left\{\wp_1(\widetilde{\mathcal{H}}_{13}) + 2(h-\tau)\wp_2(\sigma,\widetilde{\mathcal{H}}_{23})\right\} \\ &\quad + \acute{\mathcal{H}}\Big)\varpi_1 + \tilde{\tau}(h-\tau)e_5^\top\widetilde{\mathcal{H}}_{33}e_5 \\ &\quad + \mathrm{sym}\left\{\varpi_1^\top\left(\acute{\mathcal{H}} + \mathrm{sym}\left\{\wp_1(\widetilde{\mathcal{H}}_{13})\right\}\right)\tilde{\varpi}\right\} \\ &\quad + (h-\tau)\mathrm{sym}\left\{\varpi_1^\top\mathrm{sym}\left\{\wp_2(\sigma,\widetilde{\mathcal{H}}_{23})\right\}\varpi_3\right\} \\ &\quad + (h^2-2\tau h)\mathrm{sym}\left\{\varpi_1^\top\mathrm{sym}\left\{\wp_2(\sigma,\widetilde{\mathcal{H}}_{23})\right\}\varpi_2\right\}\end{aligned}$$
$$\acute{\mathcal{G}} = \sum_{n=1}^{2}\frac{\sigma^{2(n-1)}}{2n}h^{2n-1}\widetilde{\mathcal{G}}_{nn},\ \acute{\mathcal{H}} = \sum_{n=1}^{2}\frac{\sigma^{2(n-1)}}{2n}h^{2n-1}\widetilde{\mathcal{H}}_{nn}$$
$$\widetilde{\mathcal{J}}_1 = 2\widetilde{\mathcal{Q}} + \widetilde{\mathcal{R}}_1 + (2\tilde{\tau}/h)\widetilde{\mathcal{G}}_{33}, \widetilde{\mathcal{J}}_2 = \widetilde{\mathcal{Q}} + (\tilde{\tau}/h)\widetilde{\mathcal{G}}_{33}$$
$$\widetilde{\mathcal{J}}_3 = 2\widetilde{\mathcal{Q}} + \widetilde{\mathcal{R}}_2 + (2/h)\widetilde{\mathcal{H}}_{33}$$
$$\widetilde{\Delta}_1 = \left[\left[\delta_{11}^\top\ \ \delta_{12}^\top\ \ \delta_{13}^\top\right]\widetilde{\mathcal{X}}_2\ \ \ \left[\delta_{11}^\top\ \ \delta_{12}^\top\ \ \delta_{13}^\top\right]\widetilde{\mathcal{S}}_2\right]$$
$$\widetilde{\Delta}_2 = \left[\left[\delta_{21}^\top\ \ \delta_{22}^\top\ \ \delta_{23}^\top\right]\widetilde{\mathcal{X}}_1^\top\ \ \ \left[\delta_{21}^\top\ \ \delta_{22}^\top\ \ \delta_{23}^\top\right]\widetilde{\mathcal{S}}_1^\top\right]$$
$$\widetilde{\Psi} = \delta^\top\left[diag\{2,4,2,4\}\otimes\widetilde{\mathcal{R}}_1, diag\{2,4,2,4\}\otimes\widetilde{\mathcal{R}}_2\right]\delta$$
$$\widetilde{\Omega}_{1[\dot{\tau},\sigma]} = \mathrm{sym}\left\{\eta_1^\top\mathrm{sym}\left\{\wp_2(\sigma,\widetilde{\mathcal{G}}_{23})\right\}\bar{\eta}_2\right\}$$
$$\widetilde{\Omega}_{2[\dot{\tau},\sigma]} = \mathrm{sym}\left\{\varpi_1^\top\mathrm{sym}\left\{\wp_2(\sigma,\widetilde{\mathcal{H}}_{23})\right\}\varpi_2\right\}$$
$$\widetilde{\Phi}_{1[\tau]} = diag\left\{\widetilde{\mathcal{Q}},\ \frac{\widetilde{\mathcal{R}}_1}{2},\ \frac{\widetilde{\mathcal{R}}_2}{2},\ \tau\widetilde{\mathcal{G}}_{33}\right\}$$
$$\widetilde{\Phi}_{2[\tau]} = \left[he_{10}^\top\widetilde{\mathcal{Q}}\ \ \frac{h}{2}e_{10}^\top\widetilde{\mathcal{R}}_1\ \ \frac{h}{2}e_{10}^\top\widetilde{\mathcal{R}}_2\ \ \tau e_{10}^\top\widetilde{\mathcal{G}}_{33}\right]$$
$$\widetilde{\mathcal{Z}}_i = \left[\varphi_{i1}I\ \ \varphi_{i2}I\ \ \ldots\ \ \varphi_{in}I\right]^\top$$
$$\widetilde{\mathcal{Z}}_{\mathcal{S}ij} = \left[\mathcal{A}_i\mathcal{Y} + \mathcal{B}_i\mathcal{M}_j\ \ 0\ \ \mathcal{A}_{di}\mathcal{Y}\ \ \underbrace{0\ \ \ldots\ \ 0}_{6}\ \ -\mathcal{Y}\right].$$

The corresponding state feedback control gain matrices are given by $\mathcal{K}_i = \mathcal{M}_i\mathcal{Y}^{-1}$.

Proof: Denote $\mathcal{F} = \mathcal{Y}^{-1}$, and introduce the matrices

$$\mathcal{E} \triangleq diag\{\underbrace{\mathcal{F},\ \mathcal{F},\ \ldots,\ \mathcal{F}}_{16}\},\ \mathcal{E}_1 \triangleq diag\{\mathcal{F},\ \mathcal{F}\}$$
$$\mathcal{E}_2 \triangleq diag\{\mathcal{F},\ \mathcal{F},\ \mathcal{F}\},\ \mathcal{E}_3 \triangleq diag\{\underbrace{\mathcal{F},\ \mathcal{F},\ \ldots,\ \mathcal{F}}_{10}\}.$$

Pre- and post-multiplying (48) and (49) by $\mathcal{E}$ and $\mathcal{E}^\top$, respectively, leads to

$$\mathcal{E}\widetilde{\mathscr{J}}_{p[\dot{\tau},\sigma]ii}\mathcal{E}^\top < 0,\quad i = 1,\ldots,r, \tag{50}$$
$$\mathcal{E}\left(\widetilde{\mathscr{J}}_{p[\dot{\tau},\sigma]ij} + \widetilde{\mathscr{J}}_{p[\dot{\tau},\sigma]ji}\right)\mathcal{E}^\top < 0,\ 1 \le i \le j \le r. \tag{51}$$

Define

$$\mathcal{P} \triangleq \mathcal{E}_1\widetilde{\mathcal{P}}\mathcal{E}_1^\top,\ \mathcal{Q} \triangleq \mathcal{F}\widetilde{\mathcal{Q}}\mathcal{F}^\top,\ \mathcal{R}_n \triangleq \mathcal{F}\widetilde{\mathcal{R}}_n\mathcal{F}^\top,\ \mathcal{X}_n \triangleq \mathcal{F}\widetilde{\mathcal{X}}_n\mathcal{F}^\top$$
$$\mathcal{S}_n \triangleq \mathcal{F}\widetilde{\mathcal{S}}_n\mathcal{F}^\top,\ \mathcal{G}_{nn} \triangleq \mathcal{E}_2\widetilde{\mathcal{G}}_{nn}\mathcal{E}_2^\top,\ \mathcal{G}_{n3} \triangleq \mathcal{E}_2\widetilde{\mathcal{G}}_{n3}\mathcal{F}^\top$$
$$\mathcal{H}_{nn} \triangleq \mathcal{E}_2\widetilde{\mathcal{H}}_{nn}\mathcal{E}_2^\top,\ \mathcal{H}_{n3} \triangleq \mathcal{E}_2\widetilde{\mathcal{H}}_{n3}\mathcal{F}^\top\ (n = 1,2)$$
$$\mathcal{G}_{33} \triangleq \mathcal{F}\widetilde{\mathcal{G}}_{33}\mathcal{F}^\top,\ \mathcal{H}_{33} \triangleq \mathcal{F}\widetilde{\mathcal{H}}_{33}\mathcal{F}^\top,\ \mathcal{Z}_i \triangleq \mathcal{E}_3\widetilde{\mathcal{Z}}_i.$$

Then, one has

$$\begin{aligned}\mathcal{E}_3\widetilde{\mathcal{Z}}_i\widetilde{\mathcal{Z}}_{\mathcal{S}ij}\mathcal{E}_3^\top &= \left[\varphi_{i1}\mathcal{F}^\top\ \ \varphi_{i2}\mathcal{F}^\top\ \ \ldots\ \ \varphi_{i10}\mathcal{F}^\top\right]^\top\widetilde{\mathcal{Z}}_{\mathcal{S}ij}\mathcal{E}_3^\top \\ &= \mathcal{Z}_i\left[\mathcal{A}_j + \mathcal{B}_j\mathcal{K}_k\ \ 0\ \ \mathcal{A}_{dj}\ \ \underbrace{0\ \ \ldots\ \ 0}_{6}\ \ -I\right] \\ &= \mathcal{Z}_i\mathcal{Z}_{\mathcal{S}jk}.\end{aligned}$$

Therefore, from the above definition and transformation, it can be seen that (50) and (51) are equivalent to (42) and (43), respectively. Consequently, according to the Theorem 2, the T-S fuzzy system (7) is asymptotically stable under the control gains $\mathcal{K}_i = \mathcal{M}_i\mathcal{Y}^{-1}$, which ends the proof.

Remark 9: By means of the tensor product (TP) model transformation, the quasi-linear parameter varying (qLPV) can be effectively transformed into the T-S fuzzy model [33, 34]. In [33], the TP model transformation is introduced as a numerical transformation that has the advantages of readily accommodating models described by non-conventional modeling and identification approaches, such as neural networks and fuzzy rules. By discussing the relationship between TP models and T-S fuzzy models, it is found that the model manipulation and LMI design concepts in [34] can be utilized for fuzzy modeling and control design. In [35], it is proved that the LMI-based feasibility of controller and observer design is strongly influenced by the vertexes of TP model type polytopic representation for qLPV state-space model. By stability analysis for a qLPV state-space model, it is verified that the convex hull of the polytopic TP model representation have an impact on the feasibility of LMI-based stability analysis approach [36]. In [37], considering the time delay as a parameter, the TP model transformation is used to derive a polytopic representation for a LPV model. The TP model transformation can serve as a final step of identification to build a bridge to T-S fuzzy model-based control theories of analysis and design [42].

The primary purpose of this paper is to achieve stability and stabilization approaches for the well-established T-S fuzzy models, which can be achieved by any modeling techniques, such as sector nonlinearity method, and even the TP model transformation method. The main contribution is developing novel IPFs to give additive freedom. Thus, the widely utilized T-S fuzzy systems are presented as the numerical examples for fair comparison of the proposed approaches with the existing methods. It can be predicted that in the case of another T-S fuzzy model by a TP model transformation, the results by IPFs tend to be more desirable for the same T-S fuzzy model. In the future work, inspired by the innovative works [33-37, 42], the proposed approaches will be extended to analysis and synthesis for T-S fuzzy models by the TP model transformation, especially for the model difficult to find the feasible solution from LMI.

V. Numerical Examples

In this section, the advantages of the proposed stability criteria and stabilization approach are demonstrated by three numerical examples. For comparison, the conservatism and computational complexity of different methods are measured by the allowable upper delay bounds (AUDBs) guaranteeing stability of systems and the number of variables (NoVs), respectively.

Example 1: Consider the delayed T-S fuzzy system with $u(t) = 0$, which is in the form of (3) with two plant rules, where

$$\mathcal{A}_1 = \begin{bmatrix} -3.2 & 0.6 \\ 0.0 & -2.1 \end{bmatrix},\ \mathcal{A}_{d1} = \begin{bmatrix} 1.0 & 0.9 \\ 0.0 & 2.0 \end{bmatrix},$$
$$\mathcal{A}_2 = \begin{bmatrix} -1.0 & 0.0 \\ 1.0 & -3.0 \end{bmatrix},\ \mathcal{A}_{d2} = \begin{bmatrix} 0.9 & 0.0 \\ 1.0 & 1.6 \end{bmatrix},$$
$$\lambda_1(\theta(t)) = \frac{1}{1+\mathrm{e}^{-2x_1(t)}},\quad \lambda_2(\theta(t)) = 1-\lambda_1(\theta(t)).$$

For various $\mu = \mu_2 = -\mu_1$, the AUDBs derived by some recently reported approaches and Theorem 1 with different σ are tabulated in the Table I. In [5], taking advantage of delay-decomposition approach, the stability condition is derived by augmented LKF and free-matrix-based inequalities. By selecting the augmented vectors with scalar functions, the fuzzy-dependent matrices and the convex analysis approach with parameter α are combined to treat integral terms, while introducing a great many decision variables [7]. It is noted that in both of [5] and [7], the characteristics of bounding techniques are ignored when constructing LKF. In Theorem 1* of [28], the triple integral form of LKF is constructed, and the single and double integral terms are estimated by free-matrix-based integral inequality [38] and JDIs. The integrals with time-varying delays are treated by the Lemma 6 of [29]. In Theorem 1 of [28], the DPFs with single integrals are added. While, in Theorem 1 of this paper, the IPFs consisting of double integrals are tailored with slack variables to expand feasibility. Moreover, the SOBLI combined with improved matrix inequality and GDIIs encompassing the bounding techniques of [5, 7, 28] as special cases are applied for estimation purpose, the improvements of which are consolidated by IPFs. From Table I, it is apparent that Theorem 1 delivers significantly better performance than the existing results.

Applying Theorem 1 by optimizing parameter σ within $[-10, 10]$, the maximum AUDBs are achieved at $\sigma \in \{4.13, -8.92, -5.27, 0.64\}$ for $\mu \in \{0.03, 0.10, 0.50, 0.90\}$, respectively, which is described along with the NoVs of Theorem 1 in Table II. From Table II, one can see lucidly that more superior stability intervals are captured through coordinating variable parameter, which embodies the effectiveness of additive freedom given by the IPFs. Moreover, the NoVs of different criteria are listed in Table III, and it is found that the NoV required by Theorem 1 is much fewer than the those of [7, 28]. From these comparative results, it can be concluded that the conservatism is reduced signally with less computational complexity through Theorem 1. To further illustrate the applicability of the proposed approach, under the initial condition $\phi(t) = [\ 3 \quad -6\ ]^\top$, the state trajectories of Example 1 with $0 \le \tau \le 2.4291$ and $\mu = 0.03$ are depicted in Fig. 1, which converge to zero as time increases.

TABLE I: AUDBs with $\mu = \mu_2 = -\mu_1$ for various σ

Criteria	μ=0.03	μ=0.10	μ=0.50	μ=0.90
[5, Th.1]	0.8771	0.7687	0.7584	0.7524
[7, Th.1]($\alpha = 0$)	0.9281	0.8092	0.7671	0.7573
[7, Th.1]($\alpha = 0.5$)	0.9192	0.7985	0.7630	0.7541
[28, Th.1*]	1.8328	1.3857	1.2186	1.0820
[28, Th.1]	1.9137	1.4354	1.3123	1.2063
Th.1($\sigma = 1$)	2.2759	1.6016	1.4802	1.3661
Th.1($\sigma = 3$)	2.2810	1.6251	1.4797	1.3734
Th.1($\sigma = 5$)	2.2782	1.6065	1.4819	1.3686

⋆ Th. indicates Theorem.

TABLE II: Maximum AUDBs and NoVs for Theorem 1

Criteria	μ=0.03	μ=0.10	μ=0.50	μ=0.90	NoVs
Th.1	2.4291	1.7493	1.6355	1.4908	$38.5n^2 + 9.5n$

TABLE III: NoVs for different criteria

[5, Th.1]	[7, Th.1]	[28, Th.1*]	[28, Th.1]
$16.5n^2 + 6.5n$	$70.5n^2 + 7.5n$	$47.5n^2 + 7.5n$	$51.5n^2 + 9.5n$

Example 2: Consider the nonlinear Lorenz system with input term [39]:

$$\begin{cases} \dot{x}_1(t) = -ax_1(t) + ax_2(t) + u(t) \\ \dot{x}_2(t) = cx_1(t) - x_2(t) - x_1(t)x_3(t) \\ \dot{x}_3(t) = x_1(t)x_2(t) - bx_3(t) \end{cases} \tag{52}$$

For $-d \le x_1(t) \le d$, the Lorenz system (52) can be represented as:

$$\dot{x}(t) = \sum_{i=1}^{r} \lambda_i(\theta(t)) \left(\widetilde{\mathcal{A}}_i x(t) + \widetilde{\mathcal{B}}_i u(t) \right) \tag{53}$$

where

$$\widetilde{\mathcal{A}}_1 = \begin{bmatrix} -a & a & 0 \\ c & -1 & -d \\ 0 & d & -b \end{bmatrix},\ \widetilde{\mathcal{A}}_2 = \begin{bmatrix} -a & a & 0 \\ c & -1 & -d \\ 0 & -d & -b \end{bmatrix}$$
$$\widetilde{\mathcal{B}}_1 = [\ 1 \quad 0 \quad 0\ ]^\top,\ \widetilde{\mathcal{B}}_2 = [\ 1 \quad 0 \quad 0\ ]^\top$$

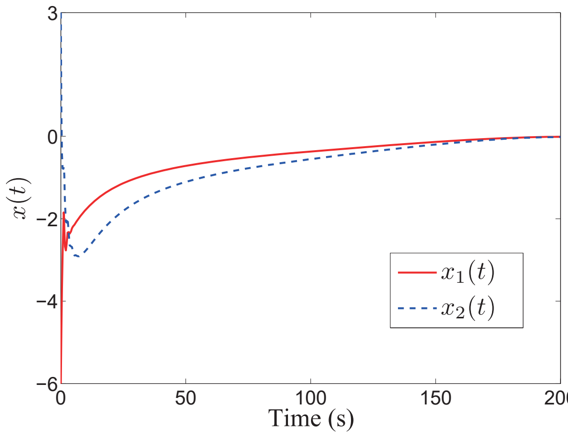

Fig. 1: State responses of the system (Example 1)

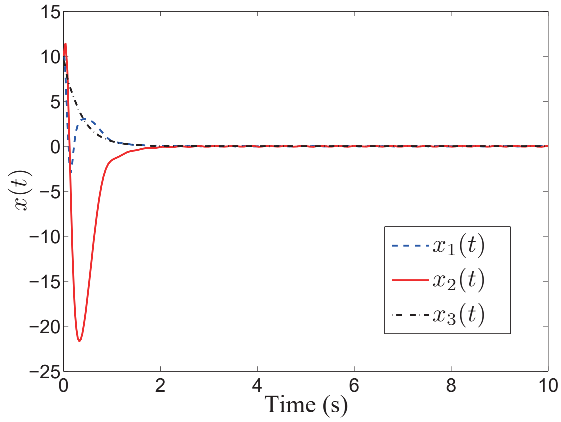

Fig. 2: State responses of the system (Example 2)

$$\lambda_1(\theta(t)) = \frac{1}{2}\left(1 + \frac{1}{d}x_1(t)\right), \quad \lambda_2(x(t)) = 1 - \lambda_1(\theta(t))$$

and the parameters are assumed to be $a = 10, b = \frac{8}{3}, c = 28, d = 25$.

In order to facilitate comparison with the result of [39], from the parallel distributed compensation idea, applying the following sampled-data controller as [39]:

$$u(t) = \sum_{i=1}^{r} \lambda_i(\theta(t_k))\mathcal{K}_i x(t_k), \quad i = 1, 2, \ldots, r, \quad t_k \le t \le t_{k+1}$$

where t_k denotes sampling instant with $t_{k+1} - t_k \le \widetilde{h}$ ($\widetilde{h}$ is the maximum allowable sampling period to be determined).

For $t_k \le t \le t_{k+1}$, by input delay approach with $0 \le \tau = t - t_k \le \widetilde{h}$, the closed-loop system (53) can be rewritten as

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\theta(t))\lambda_j(\theta(t_k))\left(\widetilde{\mathcal{A}}_i x(t) + \widetilde{\mathcal{B}}_i\widetilde{\mathcal{K}}_j x(t-\tau)\right). \tag{54}$$

For given maximum sampling period $\widetilde{h} = 0.05$, by using the Theorem 2 of [39], the control gain matrices are given as

$$\widetilde{\mathcal{K}}_1 = \widetilde{\mathcal{K}}_2 = \widetilde{\mathcal{K}} = \begin{bmatrix} -8.7663 & -23.1521 & 0.0000 \end{bmatrix}.$$

Since $\lim_{t\to t_k^-} V(x_t) = \lim_{t\to t_k^+} V(x_t)$, $V(x_t)$ (32) does not increase at the jumping instants t_k. Besides, the derivative of input delay $\dot{\tau} = 1$ except for $t = t_k$. Then, when $\widetilde{\mathcal{K}}_1 = \widetilde{\mathcal{K}}_2$, the Theorem 2 with μ_1 and μ_2 approaching 1 can be applied for stability analysis of the system (54). Selecting the identical control gains as [39] and regarding $\widetilde{\mathcal{A}}_i$ and $\widetilde{\mathcal{B}}_i\widetilde{\mathcal{K}}$ of (54) as $\mathcal{A}_i$ and $\mathcal{A}_{di}$ of (6), respectively, $\widetilde{h}$ are derived as 0.0593 and 0.0580 at $\sigma = -7.62$ and $\sigma = -4.19$ via Theorem 2 with $-10 \le \sigma \le 10$, respectively. Thus, the distinct outperformance of the presented method is indicated compared with that of [39]. For initial condition $\phi(t) = \begin{bmatrix} 10 & 10 & 10 \end{bmatrix}^\top$, the dynamic behaviors of the system with $0 \le \tau \le 0.0593$ under $\widetilde{\mathcal{K}}_1$ and $\widetilde{\mathcal{K}}_2$ are shown in Fig. 2, which go to equilibrium points.

Example 3: Consider nonlinear model of truck-trailer system with time delay formulated in [28, 40, 41]:

$$\begin{cases} \dot{x}_1(t) = -a\dfrac{v\bar{t}}{Lt_0}x_1(t) - b\dfrac{v\bar{t}}{Lt_0}x_1(t-\tau) + \dfrac{v\bar{t}}{lt_0}u(t) \\ \dot{x}_2(t) = a\dfrac{v\bar{t}}{Lt_0}x_1(t) + b\dfrac{v\bar{t}}{Lt_0}x_1(t-\tau) \\ \dot{x}_3(t) = \dfrac{v\bar{t}}{t_0}\sin\left(x_2(t) + a\dfrac{v\bar{t}}{2L} + b\dfrac{v\bar{t}}{2L}x_1(t-\tau)\right) \end{cases} \tag{55}$$

where $x_1(t)$ is the angular difference between the truck and trailer, $x_2(t)$ is the angle of trailer, and $x_3(t)$ is the vertical position of rear end of trail; l and L are the lengths of truck and trailer, and v is constant speed of backing up.

Defining the state as $x(t) = [x_1(t) \quad x_2(t) \quad x_3(t)]^\top$, and $\theta(t) = x_2(t) + a(v\bar{t})/(2L)x_1(t) + b(v\bar{t})/(2L)x_1(t-\tau)$, (55) can be represented as the T-S fuzzy system [40, 41]:

Plant Rule 1: IF $\theta(t)$ is about 0 rad,

THEN $\dot{x}(t) = \mathcal{A}_1 x(t) + \mathcal{A}_{d1}x(t-\tau) + \mathcal{B}_1 u(t)$.

Plant Rule 2: IF $\theta(t)$ is about π rad or $-\pi$ rad,

THEN $\dot{x}(t) = \mathcal{A}_2 x(t) + \mathcal{A}_{d2}x(t-\tau) + \mathcal{B}_2 u(t)$.

where

$$\mathcal{A}_1 = \begin{bmatrix} -a\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\dfrac{v^2\bar{t}^2}{2Lt_0} & \dfrac{v\bar{t}}{t_0} & 0 \end{bmatrix}, \quad \mathcal{A}_{d1} = \begin{bmatrix} -b\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ b\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ b\dfrac{v^2\bar{t}^2}{2Lt_0} & 0 & 0 \end{bmatrix}$$

$$\mathcal{A}_2 = \begin{bmatrix} -a\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\dfrac{gv^2\bar{t}^2}{2Lt_0} & \dfrac{gv\bar{t}}{t_0} & 0 \end{bmatrix}, \quad \mathcal{A}_{d2} = \begin{bmatrix} -b\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ b\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ b\dfrac{gv^2\bar{t}^2}{2Lt_0} & 0 & 0 \end{bmatrix}$$

$$\mathcal{B}_1 = \begin{bmatrix} \dfrac{v\bar{t}}{lt_0} & 0 & 0 \end{bmatrix}^\top, \quad \mathcal{B}_2 = \begin{bmatrix} \dfrac{v\bar{t}}{lt_0} & 0 & 0 \end{bmatrix}^\top$$

with the membership functions

$$\lambda_1(\theta(t)) = \begin{cases} \dfrac{\sin(\theta(t)) - \bar{g}\theta(t)}{\theta(t)(1-\bar{g})}, & \theta(t) \neq 0 \\ 1, & \theta(t) = 0 \end{cases}$$

$$\lambda_2(\theta(t)) = \begin{cases} \dfrac{-\sin(\theta(t)) + \theta(t)}{\theta(t)(1-\bar{g})}, & \theta(t) \neq 0 \\ 0, & \theta(t) = 0 \end{cases}$$

Let the model parameters be $a = 0.7$, $b = 1 - a$, $l = 2.8$, $L = 5.5$, $v = -1.0$, $\bar{t} = 2.0$, $t_0 = 0.5$, $g = 10t_0/\pi$ and $\bar{g} = 10^{-2}/\pi$ [40, 41].

Considering the constant delay case, the AUBD is given as 114 by utilizing the approach of [40] with the following control gains

$$\widetilde{\mathcal{K}}_1 = \left[\begin{array}{ccc} 12.9598 & -11.0199 & 0.7596 \end{array}\right]$$
$$\widetilde{\mathcal{K}}_2 = \left[\begin{array}{ccc} 12.9818 & -11.0313 & 0.7622 \end{array}\right]$$

It is noted that the constant delay can be regraded as the time-varying delay with equal lower and upper bounds, and zero derivative. Therefore, the proposed approach can be applied to constant delay systems by setting $\tau = h$ and $\mu_1 = \mu_2 = 0$. By the Theorem 2 with $-5 < \sigma < 5$, the AUDB is obtained as 153 with the controllers of [40] at $\sigma = 3.41$, which shows the advantage of the IPFs.

In [28], choosing the controller gains as [41]

$$\widetilde{\mathcal{K}}_1 = \left[\begin{array}{ccc} 20.6936 & -51.9608 & 0.5275 \end{array}\right],$$
$$\widetilde{\mathcal{K}}_2 = \left[\begin{array}{ccc} 20.6902 & -51.9170 & 0.5256 \end{array}\right],$$

the AUDB can reach 8.1951 with $\mu = \mu_2 = -\mu_1 = 0.9$. By use of Theorem 2 with $\widetilde{\mathcal{K}}_1$, $\widetilde{\mathcal{K}}_2$, and $-5 \leq \sigma \leq 5$, one can compute the maximum AUDB as 8.4277 when $\sigma = 2.36$, which validates the superiorities of the proposed approach for less conservatism. With unknown control gains, the maximum AUDB is given as 8.9406 by Theorem 3 with $\sigma = -4.52$, and the corresponding controller gains are calculated as

$$\mathcal{K}_1 = \left[\begin{array}{ccc} 2.5667 & -2.3915 & 0.0254 \end{array}\right],$$
$$\mathcal{K}_2 = \left[\begin{array}{ccc} 2.4071 & -2.1416 & 0.0360 \end{array}\right].$$

Under the fuzzy controller (5) with gain matrices $\mathcal{K}_1$ and $\mathcal{K}_2$, Fig. 3 describes the state responses of the system with initial condition $\phi(t) = \left[\begin{array}{ccc} -0.5\pi & 0.75\pi & -10 \end{array}\right]^\top$, $\mu = 0.9$, and $0 \leq \tau \leq 8.9406$. From Fig.3, it is plainly visible that the system is asymptotically stable at its equilibrium points.

Remark 10: Duo to the theoretical importance and practical significance, the research on T-S fuzzy systems has attracted a great deal of attention. In the paper, by suitable operations on parameter-dependent polynomial, some novel IPFs with variable parameter are developed in delay-product types, which introduce obvious improvements on system performance, while avoiding excessive computational complexity. It is noted that the proposed IPFs has more advantages than the enlargement on stable delay regions. In essence, in the framework of Lyapunov theory, the fundamental superiority of the IPFs lies in providing additional flexibility for choice of decision variables. It is found that the system state $x(t)$ and exactly delayed state $x(t-\tau)$ are involved into the IPFs. When differentiating IPFs, $\dot{x}(t)$ is produced, and thus more

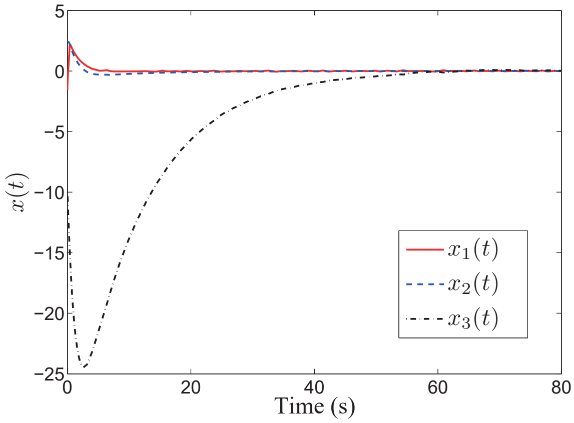

Fig. 3: State responses of the system (Example 3)

information closely related to the concerned T-S fuzzy model (6), including that on local subsystems and memberships, is taken into consideration. Moreover, the relationship among the various states of T-S fuzzy system is consolidated by slack matrices. By adjusting variable parameters, the resulting conditions are further endowed with the more freedom. The significance for reducing conservatism by the IPFs reflects on both of the desirable performance for system analysis criterion, and the superior feasibility for controller design approach. As thus, on the one hand, referring to various analysis issues of delayed T-S fuzzy systems, such as H_∞ analysis [16] and dissipative analysis [19], more preferable performance index will be achieved by means of IPFs with the given delay ranges. On the other hand, as to the control synthesis for delayed systems with diverse constraints, such as asynchronous grades of membership [8] and incomplete measurement [11], it tends to be easier to achieve feasible solutions for fuzzy control strategies taking advantages of IPFs. Thus, such characteristics of IPFs can make positive contributions to a variety of problems for T-S fuzzy systems, which will be investigated in the future work.

VI. Conclusion

In this paper, the stability analysis and controller design for delayed T-S fuzzy systems are investigated. In order to be suitable for triple integral terms, an improved matrix inequality is proposed to estimate both strictly and non-strictly rational proper functions. By appropriately defining polynomials with variable parameter, novel delay-product versions of IPFs are developed with slack matrices. Using the LKF with IPFs, the stability criteria and stabilization control method are derived via some advanced integral inequalities. By virtue of the IPFs, additional flexibility is achieved by introducing slack variables, and the relationships between various system information is enhanced, which is beneficial for highlighting the active affections of bounding techniques. Above all, taking advantages of adjusting parameter, the criteria are refined from the point of view of additive freedom. As a result, the conservatism is significantly reduced by the proposed approaches, but without requiring excessive computational cost. In the future work, the proposed IPFs will be extended to analysis and synthesis for T-S fuzzy systems subject to the constraints and T-S fuzzy systems by the TP model transformation.

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Zhichen Li received the B.S. degree in automation and the Ph.D. degree in pattern recognition and intelligent systems from North China Electric Power University, Beijing, China, in 2011 and 2017, respectively. From 2017 to 2019, Dr. Li is a Post-Doctoral Fellow with Key Laboratory of Advanced Control and Optimization for Chemical Process of Ministry of Education, East China University of Science and Technology, Shanghai, China. His research interests include time delay systems, networked control systems, and complex dynamical systems.

Huaicheng Yan received his B.Sc. degree in automatic control from Wuhan University of Technology, China, in 2001, and the Ph.D. degree in control theory and control engineering from Huazhong University of Science and Technology, China, in 2007. In 2011, he was a Research Fellow with the University of Hong Kong, Hong Kong, for three months, and also a Research Fellow with the City University of Hong Kong, Hong Kong, in 2012, for six months. Currently, he is a Professor with the School of Information Science and Engineering, East China University of Science and Technology, Shanghai, China. His research interests include networked control systems and multi-agent systems.

Hao Zhang received the B.Sc. degree in automatic control from Wuhan University of Technology, Wuhan, China, in 2001 and received Ph.D. degree in control theory and control engineering from Huazhong University of Science and Technology Wuhan, China, in 2007. From 2011 to 2013, she is a Post-Doctoral Fellow with the City University of Hong Kong, Hong Kong. Currently, she is a Professor with the School of Electronics and Information Engineering, Tongji University, Shanghai, China. Her research interests include network based control systems, multi-agent systems and complex networks.

Jun Sun received his B.Sc. degree in automation from Nanjing University of Aeronautics and Astronautics, China, in 2004, and the M.S. degree in navigation, guidance and control from Shanghai Institute of Space Technology ResearchChina, in 2007. He received his Ph.D. in aeronautical and astronautical science and technology from Harbin Institute of Technology, China, in 2017. From 2014, he is Senior Engineer, and from 2015, he is the dean of Research and Development Center in Shanghai Aerospace Control Technology Institute. His research interests include complex space system dynamics, integrated dynamics and control, nonlinear system and control.

Hak-Keung Lam (SM10) received the B.Eng. (Hons.) and Ph.D. degrees from the Department of Electronic and Information Engineering, The Hong Kong Polytechnic University, Hong Kong, in 1995 and 2000, respectively. During 2000-2005, he worked with the Department of Electronic and Information Engineering at The Hong Kong Polytechnic University as Post-Doctoral Fellow and Research Fellow respectively. He joined as a Lecturer at Kings College London in 2005 and is currently a Reader. He has served as a program committee member, international advisory board member, invited session chair and publication chair for various international conferences and a reviewer for various books, international journals and international conferences. He was an associate editor for IEEE Transactions on Fuzzy Systems (2009-2018) and is an associate editor for IEEE Transactions on Circuits and Systems II: Express Briefs, IET Control Theory and Applications, International Journal of Fuzzy Systems and Neorocomputing; and guest editor and on the editorial board for a number of international journals. His current research interests include intelligent control, computational intelligence and machine learning.