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Chance Constrained Robust Downlink Beamforming in Multicell Networks

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Abstract—We introduce a downlink robust optimization approach that minimizes a combination of total transmit power by a multiple antenna base station (BS) within a cell and the resulting aggregate inter-cell interference (ICI) power on the users of the other cells. This optimization is constrained to assure that a set of signal-to-interference-plus-noise ratio (SINR) targets are met at user terminals with certain outage probabilities. The outages are due to the uncertainties that naturally emerge in the estimation of channel covariance matrices between a BS and its intra-cell local users as well as the other users of the other cells. We model these uncertainties using random matrices, analyze their statistical behaviour and formulate a tractable probabilistic approach to the design of optimal robust downlink beamforming vectors. The proposed approach reformulates the original intractable non-convex problem in a semidefinite programming (SDP) form with linear matrix inequality (LMI) constraints. The resulting SDP formulation is convex and numerically tractable under the standard rank relaxation. We compare the proposed chance-constrained approach against two different robust design schemes as well as the worst-case robustness. The simulation results confirm better power efficiency and higher resilience against channel uncertainties of the proposed approach in realistic scenarios.

Index Terms—Robust; probabilistic optimization; channel uncertainty; inter-cell interference; linear matrix inequality.

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1 INTRODUCTION

Joint signal processing across the base stations (BSs) with multiple antennas for coordinated downlink beamforming has shown promising results in enhancing spectral efficiency and providing a uniform capacity coverage in cellular networks, e.g., [1]–[6]. An effective downlink beamforming requires the availability of an accurate channel state information (CSI) at BSs. However, the assumption that the CSI are accurately and globally available to all BSs via an ideal backhaul network is not a realistic one. In many practical scenarios, the available CSI at BSs is imperfect due to several reasons, e.g., estimation error, delay and the quantization error that may arise as a result of limited feedback from a user terminal to a BS. Ignoring the effect of CSI uncertainties in forming optimization models for cellular networks can lead to optimal solutions that may violate critical constraints and results in a poor outcome in realistic channel conditions [7]. These practical considerations have recently motivated a growing interest towards robust design of cellular networks.

Commonly, there are two methods of deterministic and stochastic modeling of imperfect CSI. In the former, the imperfection in the CSI is assumed to be bounded within an uncertainty region and the objective is to provide worst-case guarantees for the performance of the network. More specifically, robust designs based on the deterministic model are conservative, make no assumptions on the distribution of error and optimize the worst-case performance of the system, e.g., see the works in [8]–[14] for worst-case CSI modeling examples. Although, the deterministic optimization approaches provide robustness against CSI imperfections, the actual worst-case may occur with a very slim chance in practice. Hence, a deterministic design may lead to an inefficient design, as most system resources could be dedicated to provide guarantees for the worst-case scenarios. In order to provide less conservative solutions in favor of improved resource-efficient design, in the second approach, the perturbations in CSI are modeled to be statistically unbounded according to some known distributions. In the designs based on the stochastic modeling, the beamforming vectors are designed such that the quality-of service (QoS) requirements are met with a high probability, e.g., [15]. In [15], weighted variable-penalty alternating direction method of multipliers is used for a chance-constrained robust multicell beamforming problem to minimize the sum power of all BSs subject to SINR constraints at user terminals in a distributed fashion. In this approach, the probabilistic constraints are upper-bounded by tractable convex approximating functions. The transmit power minimization subject to probabilistic SINR constraints in a single-cell beamforming scenario is considered in [16] and [17]. The authors used conservative methods based on Bernstein inequality in [16] and relaxation-restriction approach in [17] to approximate the probabilistic constraints. The probability (chance)-constrained problems are known to be difficult to solve because the probabilistic SINR constraints in general do not have closed-form expression and are not convex. Transceiver design with QoS guarantee in the presence of uncertain CSI at the transmitter is studied for a broadcast scenario with a multi-antenna BS and single antenna user terminals in [18]. In this study, the scenario is formulated as an optimization problem and conservative approaches that yield deterministic convex approximation for randomly perturbed second order cone

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constraints are used to guarantee the satisfaction of the probabilistic constraints. In a similar broadcast scenario, [19] studies power allocation strategies to satisfy QoS targets at user terminals in the presence of channel estimation error with Gaussian distribution. The authors in [19] use Vysochanskii-Petunin inequality in combination with the theory of interference functions to find conservative solutions to the problem. The majority of available algorithms mainly rely on deriving analytical convex upper bounds for the probabilistic constrains and only find a feasible worst-case solution without any optimality guarantee.

In this paper, we introduce a chance-constraint downlink beamforming approach that minimizes a linear combination of total transmit power at individual BSs and the resulting overall interference on the other users of the other cells, subject to satisfying outage-based probabilistic QoS requirements (i.e., in terms of SINR) at user terminals in the presence of channel uncertainties. The outage-based constraints are motivated by the fact that most wireless systems can tolerate occasional outages in the QoS requirements [20]–[22]. While the proposed objective function maintains the local users' QoS demands in a robust and power efficient way, it balances the inter-cell interference (ICI) in an optimal way across the multiple cells under imperfect CSI and frequency reuse of one. In the following, we summarize the contributions of this paper.

- We generalize the analysis of our chance-constraint resource allocation problem in [23] for more realistic cases where the channel error is modeled as a random matrix with different variances for its various entries. In contrast to the methods that approximate the probabilistic constraints with their convex upper-bounds and effectively find a feasible worst-case solution without any optimality guarantee (e.g., [15], [19]), the proposed approach directly characterizes the statistical behavior of the random error matrix with no approximation and obtains the optimal solutions. The optimality of the proposed approach is evidenced by a comparative inspection of the simulation results of our scheme and the scheme in [15].

- We find a relationship between the Frobenius norm of the random matrix, modeling the radius of a hyper-spherical uncertainty region in the worst-case approach, and the outage parameter controlling the probability of the satisfaction of QoS requirement at users in the chance-constraint approach. This relation reveals and quantifies the implicit outage in the worst-case approach, i.e., due to the fact that the uncertainties in practical scenarios are statistically unbounded, and helps to compare it with the chance-constraint approach, fairly.

The rest of this paper is organized as follows. System model and problem formulation are given in Section II. The proposed beamforming problem is formulated as a probability constrained stochastic optimization problem in Section III. In Section IV, we develop a technique based on the outage probability and show its relationship to the worst-case based approach [14]. Simulation results are presented and discussed in Section V. Finally, Section VI concludes the paper.

Notation: Throughout the paper, w (or W), $\mathbf{w}$ and $\mathbf{W}$ are used to present a scalar w (or W), a column vector $\mathbf{w}$ and a matrix $\mathbf{W}$, respectively. $[\mathbf{W}]_{ij}$ indicates the entry in row i and column j of $\mathbf{W}$, $|\mathbf{W}|$ defines a matrix with real entries such that $[|\mathbf{W}|]_{ij} = |[\mathbf{W}]_{ij}|$, where $|\cdot|$ indicates the absolute value of the complex number $[\mathbf{W}]_{ij}$, and $\mathbf{W} \succeq 0$ denotes the positive semi-definiteness of $\mathbf{W}$. The notation $(\cdot)^H$ indicates conjugate transpose. $\|\cdot\|_F$ and $\|\cdot\|$ represent matrix Frobenius norm and the vector Euclidean norm, respectively. $\text{vec}(\mathbf{W})$ stands for the vector obtained by stacking the column vectors of $\mathbf{W}$. The notation Pr $(\cdot)$ denotes the probability operator. We characterize real-valued and complex Gaussian random variables as $\mathcal{N}(\cdot, \cdot)$ and $\mathcal{CN}(\cdot, \cdot)$, respectively.

2 SYSTEM MODEL AND PROBLEM FORMULATION

Consider a downlink multicell network where each cell consists of a single BS with M transmit antennas and U single-antenna users. Let the set of indices of BSs in the network be denoted as $\mathcal{S}_b = \{1, \cdots, N\}$ and the set of active users in each cell as $\mathcal{S}_l = \{1, \cdots, U\}$, where index $i(q), i \in \mathcal{S}_l$ and $q \in \mathcal{S}_b$, indicates the ith user in cell q. Each BS communicates with its intra-cell users over the same frequency band as the adjacent BSs via the corresponding downlink beamforming vectors. Assume that $\mathbf{w}_{i(q)} \in \mathbb{C}^{M \times 1}$ and $\mathbf{h}_{i(q)}(q) \in \mathbb{C}^{M \times 1}$ are, respectively, the beamforming vector and the vector of channel coefficients of user $i(q)$ as seen by the BS of cell q. Hence, the received signal at user $i(q)$ can be written as

$$y_{i(q)} = \mathbf{h}_{i(q)}^H(q)\mathbf{w}_{i(q)}s_{i(q)} + \sum_{j \in \mathcal{S}_l, j \neq i} \mathbf{h}_{i(q)}^H(q)\mathbf{w}_{j(q)}s_{j(q)} + \zeta_{i(q)} + n_{i(q)}, \tag{1}$$

where $s_{i(q)}$ represents data symbol intended for user $i(q)$ and $n_{i(q)} \sim \mathcal{CN}(0, \sigma_n^2)$ is assumed to be zero mean circularly symmetric complex Gaussian (ZMCSCG) noise. In (1), $\zeta_{i(q)}$ denotes the induced ICI on user $i(q)$ due to the transmissions of all BSs, other than cell q, in the network. Let $\widetilde{\mathbf{R}}_{i(q)}(q) = \mathbb{E}\left(\mathbf{h}_{i(q)}(q)\mathbf{h}_{i(q)}^H(q)\right)$ and $\widetilde{\mathbf{R}}_{t(k)}(q) = \mathbb{E}\left(\mathbf{h}_{t(k)}(q)\mathbf{h}_{t(k)}(q)^H\right)$ indicate, respectively, the channel covariance matrix of user $i(q)$ and the cross-channel (i.e., the ICI channel) covariance matrix of user t of cell k, as seen by the BS in cell q. We assume that only an imperfect knowledge of $\widetilde{\mathbf{R}}_{i(q)}(q)$ and $\widetilde{\mathbf{R}}_{t(k)}(q)$, i.e., $\mathbf{R}_{i(q)}(q)$ and $\mathbf{R}_{t(k)}(q)$, respectively, are available to the BS q, such that

$$\widetilde{\mathbf{R}}_{i(q)}(q) = \mathbf{R}_{i(q)}(q) + \mathbf{\Delta}_{i(q)}, \tag{2}$$

$$\widetilde{\mathbf{R}}_{t(k)}(q) = \mathbf{R}_{t(k)}(q) + \mathbf{\Delta}_{t(k)}, \tag{3}$$

where $\mathbf{\Delta}_{i(q)}$ and $\mathbf{\Delta}_{t(k)}$ are random error matrices with respective rd-entries of $[\mathbf{\Delta}_{i(q)}]_{rd}$ and $[\mathbf{\Delta}_{t(k)}]_{rd}$, independently distributed as $[\mathbf{\Delta}_{i(q)}]_{rd} \sim \mathcal{CN}(0, \sigma_{rd}^2)$ and $[\mathbf{\Delta}_{i(q)}]_{rd} \sim \mathcal{CN}(0, \sigma_{rd}^2)$.

Whilst optimizing the total transmit power at any BS q in the presence of channel uncertainties and in a distributed manner, our aim is to optimally concentrate the transmitted power towards the intended intracell users by minimizing the leaking portion of the emitted power imposed on the unintended users. For this purpose, we consider the problem of

$$\min_{\mathbf{w}_{i(q)}} \quad \sum_{i \in \mathcal{S}_l} \mathbf{w}_{i(q)}^H \mathbf{w}_{i(q)} + \sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \sum_{i \in \mathcal{S}_l} \mathbf{w}_{i(q)}^H \left(\mathbf{R}_{t(k)}(q) + \boldsymbol{\Delta}_{t(k)} \right) \mathbf{w}_{i(q)}$$

$$\text{subject to} \quad \text{SINR}_{i(q)} = \frac{\mathbf{w}_{i(q)}^H \left(\mathbf{R}_{i(q)}(q) + \boldsymbol{\Delta}_{i(q)} \right) \mathbf{w}_{i(q)}}{\sum_{j \in \mathcal{S}_l, j \neq i} \mathbf{w}_{j(q)}^H \left(\mathbf{R}_{i(q)}(q) + \boldsymbol{\Delta}_{i(q)} \right) \mathbf{w}_{j(q)} + \xi_{i(q)} + \sigma_n^2} \geq \gamma_{i(q)}, \quad \forall i \in \mathcal{S}_l, q \in \mathcal{S}_b. \tag{4}$$

joint minimization of total transmitting power of any BS q, delivering signal to the intended users of the corresponding cell q, and its resulting total ICI power inflicted on the users of the other cells in a downlink multicell network and in the presence of channel uncertainties. This problem is constrained on satisfying the desired SINR levels at all individual users across the network. In our formulation in (4), shown at the top of the current page, we incorporate the total ICI power as a statistical regularization term into the objective function of a standard single cell power minimization problem. The regularization term, that is the second term in the objective function of (4), is a statistical quantity due to the incorporation of the channel uncertainties presented by a set of random matrices $\{\boldsymbol{\Delta}_{t(k)}\}_{t \in \mathcal{S}_l}^{k \in \mathcal{S}_b}$ in the proposed formulation. It is also noteworthy to mention that minimizing the total ICI power as a part of the proposed objective function has the following advantage over enforcing the ICI power levels under certain thresholds within the constraints of a sum-power minimization problem. The latter approach requires setting tolerable levels of ICI power on unintended users, which are not necessarily known in advance due to the turbulent channel conditions. Whereas in the former approach, these ICI thresholds are optimally adjusted at any given channel conditions.

Furthermore in (4), $\gamma_{i(q)}$ is the target SINR level required by an active local user $i \in \mathcal{S}_l$ in cell q and $\xi_{i(q)} = \mathbb{E}\left(\left|\zeta_{i(q)}\right|^2\right)$ is the total ICI power imposed on user $i(q)$. Notice, that we have normalized the average energy for transmitting the $i(q)$th symbol, i.e., $s_{i(q)}$, to unity, i.e., $\mathbb{E}_{s_{i(q)}}\left(\left|s_{i(q)}\right|^2\right) = 1$, in our formulations. In this setup, we have assumed a Gaussian model for the ICI and that each user $i(q) \in \mathcal{S}_l$ can estimate the arrived total ICI power $\xi_i(q)$, i.e., using the MMSE approach described in [24], and feed it back to its local BS. The BSs use the received information, i.e., $\xi_i(q)$, to design their beamforming vectors towards their intended users. Interested readers are also referred to [24] and [25] for more topics on ICI modeling. In cellular systems where users cannot directly communicate with their neighboring BSs, they report their CSI via their corresponding BSs to the neighboring ones. These systems are distributed in a sense that each BS designs its beamformers independently, i.e., in a decentralized manner. In either case, the channel parameters are prone to imperfection and the robustness of the designed downlink beamforming vectors against uncertainties in channel statistics is a critical task from practical point of view.

3 OUTAGE BASED PROBABILISTIC OPTIMIZATION

In this section, we reformulate the optimization problem in (4) with chance-constrained settings. Defining $\mathbf{A} = \sum_{i \in \mathcal{S}_l} \mathbf{F}_{i(q)}$, where $\mathbf{F}_{i(q)} = \mathbf{w}_{i(q)} \mathbf{w}_{i(q)}^H$ and using $\mathbf{x}^H \mathbf{Y} \mathbf{x} =$

$\text{Tr}\left(\mathbf{Y} \mathbf{x} \mathbf{x}^H\right)$, we can rewrite the problem in (4) as (5), shown at the top of the next page, where ν is a slack variable and ρ is the probability of outage. The first and the second constraints in (5), respectively, ensure that the events $\text{SINR}_{i(q)} \geq \gamma_{i(q)}$ and $\sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \text{Tr}\left\{\left(\mathbf{R}_{t(k)}(q) + \boldsymbol{\Delta}_{t(k)}\right) \mathbf{A}\right\} \leq \nu$ hold with a minimum probability of $1 - \rho$ for every instantiation of the random matrices $\boldsymbol{\Delta}_{i(q)}$ and $\boldsymbol{\Delta}_{t(k)}$. Solving the optimization problem in (5) that involves probabilistic constraints is NP-hard, because the solutions should be feasible in the intersection of an infinite number of constraints. In the sequel, we overcome this problem by transforming the probabilistic constraints to more convenient and equivalent forms.

Lemma 1: Let $\mathbf{X}$ be a $M \times M$ random matrix with independently distributed ZMCSCG entries characterized as $[\mathbf{X}]_{ij} \sim \mathcal{CN}(0, \sigma_{ij}^2)$. Then, for any $\mathbf{L}, \mathbf{L} \in \mathcal{C}^{M \times M}$,

$$\text{Tr}\left(\mathbf{L}\mathbf{X}\right) \sim \mathcal{N}\left(0, \||\mathbf{L}| \odot \boldsymbol{\Sigma}_\mathbf{X}\|_F^2\right), \tag{6}$$

where $|\mathbf{L}|$ is a real-valued $M \times M$ matrix with entries $[|\mathbf{L}|]_{ij} = |[\mathbf{L}]_{ij}|$, i.e., equal to the absolute values of the entries of $\mathbf{L} \in \mathcal{C}^{M \times M}$, $\boldsymbol{\Sigma}$ is a real-valued $M \times M$ matrix with entries $[\boldsymbol{\Sigma}_\mathbf{X}]_{ij} = \sigma_{ij}$ and $\odot$ defines the Hadamard product, i.e., the element-wise product of two matrices.

Proof. : We can write $\text{Tr}\left(\mathbf{L}\mathbf{X}\right) = \left(\text{vec}(\mathbf{L}^H)\right)^H \text{vec}(\mathbf{X})$. Note that $\text{Tr}\left(\mathbf{L}\mathbf{X}\right)$ is also a ZMCSCG random variable, because it can be written as a weighted sum of independently distributed ZMCSCG random variables. Hence, the random variable $\text{Tr}\left(\mathbf{L}\mathbf{X}\right)$ can be characterized as $\text{Tr}\left(\mathbf{L}\mathbf{X}\right) \sim \mathcal{CN}(0, \sigma_{\mathbf{L}\mathbf{X}}^2)$. The variance $\sigma_{\mathbf{L}\mathbf{X}}^2$ can be calculated as

$$
\begin{aligned}
\sigma_{\mathbf{L}\mathbf{X}}^2 &= E\left[\left(\text{vec}(\mathbf{L}^H)\right)^H \text{vec}(\mathbf{X})\text{vec}(\mathbf{X})^H\text{vec}(\mathbf{L}^H)\right] \\
&= \left(\text{vec}(\mathbf{L}^H)\right)^H E[\text{vec}(\mathbf{X})\text{vec}(\mathbf{X})^H]\text{vec}(\mathbf{L}^H) \\
&= \left(\text{vec}(\mathbf{L}^H)\right)^H \text{diag}[\text{vec}(\boldsymbol{\Sigma}_\mathbf{X})]\text{diag}[\text{vec}(\boldsymbol{\Sigma}_\mathbf{X})]\text{vec}(\mathbf{L}^H) \\
&= \left(\text{diag}[\text{vec}(\boldsymbol{\Sigma}_\mathbf{X})]\text{vec}(\mathbf{L}^H)\right)^H \text{diag}[\text{vec}(\boldsymbol{\Sigma}_\mathbf{X})]\text{vec}(\mathbf{L}^H) \\
&= \||\mathbf{L}| \odot (\boldsymbol{\Sigma}_\mathbf{X})\|_F^2. \tag{7}
\end{aligned}
$$

$\square$

Corollary 1: Let $U \sim \mathcal{N}(0, 1)$ be a standard normal random variable. Then, the random variable $\text{Tr}\left(\mathbf{L}\mathbf{X}\right)$ in (6) can be expressed as $\text{Tr}\left(\mathbf{L}\mathbf{X}\right) = \||\mathbf{L}| \odot \boldsymbol{\Sigma}_\mathbf{X}\|_F U$.

In the sequel, we expand the event $\text{SINR}_{i(q)} \geq \gamma_{i(q)}$ in the optimization problem (5) as

$$\text{Tr}\left(\left(\mathbf{R}_{i(q)}(q) + \boldsymbol{\Delta}_{i(q)}\right) \mathbf{F}_{i(q)}\right) \geq$$
$$\gamma_{i(q)} \sum_{j \in \mathcal{S}_l, j \neq i} \text{Tr}\left(\left(\mathbf{R}_{i(q)}(q) + \boldsymbol{\Delta}_{i(q)}\right) \mathbf{F}_{j(q)}\right) + \gamma_{i(q)}(\xi_{i(q)} + \sigma_n^2), \tag{8}$$

$$\min_{\mathbf{F}_{i(q)},\nu} \quad \sum_{i \in \mathcal{S}_l} \mathrm{Tr}\left(\mathbf{F}_{i(q)}\right) + \nu$$

$$\text{subject to} \quad \mathrm{Pr}\left(\mathrm{SINR}_{i(q)} \geq \gamma_{i(q)}\right) \geq 1 - \rho,$$

$$\mathrm{Pr}\left(\sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \mathrm{Tr}\left\{\left(\mathbf{R}_{t(k)}(q) + \boldsymbol{\Delta}_{t(k)}\right)\mathbf{A}\right\} \leq \nu\right) \geq 1 - \rho,$$

$$\mathbf{F}_{i(q)} = \mathbf{F}_{i(q)}^H \succeq 0, \quad \mathrm{rank}\left(\mathbf{F}_{i(q)}\right) = 1, \quad \forall i \in \mathcal{S}_l, q \in \mathcal{S}_b. \tag{5}$$

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and compactly rewrite it as

$$\mathrm{Tr}\left(\mathbf{B}_{i(q)} \boldsymbol{\Delta}_{i(q)}\right) \leq \tau, \tag{9}$$

where $\mathbf{B}_{i(q)} = \gamma_{i(q)} \sum_{j \in \mathcal{S}_l, j \neq i} \mathbf{F}_{j(q)} - \mathbf{F}_{i(q)}$ and $\tau = -\mathrm{Tr}\left(\mathbf{B}_{i(q)} \mathbf{R}_{i(q)}(q)\right) - \gamma_{i(q)}\left(\xi_{i(q)} + \sigma_n^2\right)$. Using *Lemma 1* and *corollary 1*, we can write (9) as

$$U \leq \frac{\tau}{\left\||\mathbf{B}_{i(q)}| \odot \boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{i(q)}}\right\|_F}. \tag{10}$$

Hence, the left-hand-side (LHS) of the first constraint in problem (5) is evaluated as

$$\mathrm{Pr}(U \leq \frac{\tau}{\left\||\mathbf{B}_{i(q)}| \odot \boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{i(q)}}\right\|_F}) = \Phi(\frac{\tau}{\left\||\mathbf{B}_{i(q)}| \odot \boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{i(q)}}\right\|_F}), \tag{11}$$

where $\Phi(u) = \mathrm{Pr}(U \leq u) = \frac{1}{2}[1+\mathrm{erf}(\frac{u}{\sqrt{2}})]$ is the cumulative distribution function (CDF) of a standard normal random variable U, and $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x exp\left(-t^2\right) dt$. Using (11), we can rewrite the LHS of the first constraint in problem (5) as

$$\mathrm{Pr}\left(\mathrm{SINR}_{i(q)} \geq \gamma_{i(q)}\right) = \begin{cases} \frac{1}{2} + \frac{1}{2}\mathrm{erf}\left(\frac{\tau}{\sqrt{2}\left\||\mathbf{B}_{i(q)}|\odot\boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{i(q)}}\right\|_F}\right), \\ \quad \text{if} \quad \tau \geq 0, \\ \frac{1}{2} - \frac{1}{2}\mathrm{erf}\left(\frac{-\tau}{\sqrt{2}\left\||\mathbf{B}_{i(q)}|\odot\boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{i(q)}}\right\|_F}\right), \\ \quad \text{if} \quad \tau \leq 0. \end{cases} \tag{12}$$

To ensure that the first constraint in (5) is satisfied with an outage probability of no more than %50, i.e., $\rho < 0.5$ for reliable communications purposes, we enforce (12) with $\tau > 0$. Notice that designs based on using (12) with $\tau \leq 0$ lead to $\rho > 0.5$. Hence, the first probabilistic constraint in (5) can be written as

$$\frac{1}{2} + \frac{1}{2}\mathrm{erf}\left(\frac{\tau}{\sqrt{2}\left\||\mathbf{B}_{i(q)}| \odot \boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{i(q)}}\right\|_F}\right) \geq 1 - \rho, \tag{13}$$

or equivalently as

$$\tau \geq c\left\|\mathrm{vec}(|\mathbf{B}_{i(q)}| \odot \boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{i(q)}})\right\|, \tag{14}$$

where $c \triangleq \sqrt{2}\mathrm{erf}^{-1}\left(1 - 2\rho\right)$. Finally, using the Schur complement [26], we can write (14) in linear matrix inequality (LMI) form as

$$\begin{bmatrix} \frac{1}{c}\tau & \mathrm{vec}^H\left(|\mathbf{B}_{i(q)}| \odot \boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{i(q)}}\right) \\ \mathrm{vec}\left(|\mathbf{B}_{i(q)}| \odot \boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{i(q)}}\right) & \frac{1}{c}\tau\mathbf{I} \end{bmatrix} \succeq 0,$$
$$\forall i \in \mathcal{S}_l. \tag{15}$$

Similarly, we consider the second constraint in (5) and expand the event $\sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \mathrm{Tr}\{\left(\mathbf{R}_{t(k)}(q) + \boldsymbol{\Delta}_{t(k)}\right)\mathbf{A}\} \leq \nu$ as

$$\sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \mathrm{Tr}\left(\mathbf{A}\boldsymbol{\Delta}_{t(k)}\right) \leq \nu - \sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \mathrm{Tr}\left(\mathbf{A}\mathbf{R}_{t(k)}(q)\right). \tag{16}$$

It follows from *Lemma 1* that the LHS of (16), which is sum of normally distributed random variables, is distributed as $\sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \mathrm{Tr}\left(\mathbf{A}\boldsymbol{\Delta}_{t(k)}\right) \sim \mathcal{N}(0, \sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \left\||\mathbf{A}| \odot \boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{t(k)}}\right\|_F^2)$. Hence, according to *corollary 1*, we can express (16) in terms of standard normal variable U, as

$$U \leq \frac{\nu - b}{\sqrt{\sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \left\||\mathbf{A}| \odot \boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{t(k)}}\right\|_F^2}}, \tag{17}$$

where $b = \sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \mathrm{Tr}\left(\mathbf{A}\mathbf{R}_{t(k)}(q)\right)$. Consequently, the LHS of the second constraint in problem (5) can be expressed as

$$\Phi(\frac{\nu - b}{\sqrt{\sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \left\||\mathbf{A}| \odot \boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{t(k)}}\right\|_F^2}})$$
$$= \begin{cases} \frac{1}{2} + \frac{1}{2}\mathrm{erf}\left(\frac{\nu-b}{\sqrt{2}a}\right), \nu > b, \\ \frac{1}{2} - \frac{1}{2}\mathrm{erf}\left(\frac{b-\nu}{\sqrt{2}a}\right), \nu \leq b, \end{cases} \tag{18}$$

where $a = \sqrt{\sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \left\||\mathbf{A}| \odot \boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{t(k)}}\right\|_F^2}$. For the same reason of ensuring a reliable communications as in (12), we enforce (18) with $v > b$. Hence the second probabilistic constraint in (5) can be substituted by

$$\frac{1}{2} + \frac{1}{2}\mathrm{erf}\left(\frac{\nu - b}{\sqrt{2\sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \left\||\mathbf{A}| \odot \boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{t(k)}}\right\|_F^2}}\right) \geq 1-\rho, \tag{19}$$

or equivalently by

$$\nu - b \geq \beta \sqrt{\sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \left\||\mathbf{A}| \odot \boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{t(k)}}\right\|_F^2}, \tag{20}$$

where $\beta = \sqrt{2}\mathrm{erf}^{-1}\left(1 - 2\rho\right)$. Let ϱ be defined as

$$\varrho = \left[\mathrm{vec}^H(|\mathbf{A}| \odot \boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{1(1)}}) \cdots \mathrm{vec}^H(|\mathbf{A}| \odot \boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{U(q-1)}}), \right.$$
$$\left. \mathrm{vec}^H(|\mathbf{A}| \odot \boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{1(q+1)}}) \cdots \mathrm{vec}^H(|\mathbf{A}| \odot \boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{U(N)}})\right]^H. \tag{21}$$

Then (20) can be written as $\nu - b \geq \beta\|\boldsymbol{\varrho}\|$ and by applying the Schur complement is finally expressed in form as

$$\begin{bmatrix} \frac{1}{\beta}\left(\nu - b\right) & \boldsymbol{\varrho}^H \\ \boldsymbol{\varrho} & \frac{1}{\beta}\left(\nu - b\right)\mathbf{I} \end{bmatrix} \succeq 0. \tag{22}$$

Hence, the optimization problem in (5) with probabilistic constrains can be rewritten with equivalent LMI constraints as

$$\min_{\mathbf{F}_{i(q)},\nu} \qquad \sum_{i\in\mathcal{S}_l}\mathrm{Tr}\left(\mathbf{F}_{i(q)}\right) + \nu \tag{23}$$

$$\text{subject to} \qquad \text{LMIs in } (15) \text{ and } (22),$$
$$\mathbf{F}_{i(q)} = \mathbf{F}_{i(q)}^H \succeq 0, \quad \mathrm{rank}\left(\mathbf{F}_{i(q)}\right) = 1,$$
$$\forall i \in \mathcal{S}_l, q \in \mathcal{S}_b.$$

The optimization problem in (23) is a convex semidefinite programming (SDP) problem if the non-convex rank-one constraint is relaxed. The resulting SDP problem can be efficiently solved in $\mathbf{F}_{i(q)} = \mathbf{w}_{i(q)}\mathbf{w}_{i(q)}^H$ using the CVX [27]. In cases where the solution $\mathbf{F}_{i(q)}$ is not of rank-one, standard randomization techniques [28] can be applied to approximate $\mathbf{F}_{i(q)}$ by a rank-one matrix with sufficient accuracy. Finally, the optimal solution $\mathbf{w}_{i(q)}$ is determined as the principal eigenvector of the rank-one $\mathbf{F}_{i(q)}$ solution.

Remark 1: The regularization term in (4), i.e., $\sum_{k\in\mathcal{S}_b,k\neq q}\sum_{t\in\mathcal{S}_l}\sum_{i\in\mathcal{S}_l}\mathbf{w}_{i(q)}^H(\mathbf{R}_{t(k)}(q) + \boldsymbol{\Delta}_{t(k)})\mathbf{w}_{i(q)}$ can be written as (24), shown at the top of the next page, and the term $\sum_{k\in\mathcal{S}_b,k\neq q}\sum_{t\in\mathcal{S}_l}\left\|\,|\mathbf{A}|\odot\boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{t(k)}}\right\|_F^2$ in (24) quantifies the total effect of channel uncertainty in terms of power.

4 IMPLICIT OUTAGE IN WORST-CASE SETTING

In this section, we establish a connection between the proposed probability-constrained stochastic optimization problem in (4) and the worst-case optimization problem in [14]. In [14] it is assumed that the imperfections in CSI is bounded within a hyper-spherical region. In a worst-case approach, one can express the problem in (4) as (25), given at the top of the next page, where δ_t and δ_i indicate the radii of the hyper-spheres corresponding to the uncertainties in the crosstalk and the local channel knowledge, respectively, at a given BS. For simplicity and without loss of generality, it is assumed $\delta_t = \delta_i = \delta$. The detailed solution can be found in [14].

In practical scenarios, the entries of $\boldsymbol{\Delta}_{t(k)}$ and $\boldsymbol{\Delta}_{i(q)}$ are unbounded random variables. Then, indeed their Frobenius norms, i.e., $\|\boldsymbol{\Delta}_{t(k)}\|_F$ and $\|\boldsymbol{\Delta}_{i(q)}\|_F$, become unbounded random variables. Hence, confining the CSI imperfections within a bounded uncertainty region in the worst-case approach would naturally imply that with a certain probability the uncertain CSI may fall outside of the considered uncertainty region. Thus, with certain outage probabilities the norm constraints in problem (25) may not hold in a realistic scenario and, hence, their corresponding optimal solutions may no longer be feasible. In this section, we find a metric that enables us to illustrate a link between the worst case-based and probabilistically constrained robust designs. We provides an explicit relationship between the probability ρ and the uncertainty parameter δ and, therefore, provide a practical rule for choosing δ based on the QoS requirements.

Lemma 2: Let $\boldsymbol{\Delta}$ be a $n\times n$ random matrix with ZMCSCG entries defined as $[\boldsymbol{\Delta}]_{ij} \sim \mathcal{CN}(0,\sigma^2)$. Then $\Pr(\|\boldsymbol{\Delta}\|_F^2 \leq \delta^2) = 1 - \rho$, where

$$\delta = \sqrt{\frac{\sigma^2\Psi_{\mathcal{X}^2(2n^2)}^{-1}(1-\rho)}{2}}, \tag{26}$$

$0 \leq \rho \leq 1$ is the outage, i.e., the probability that $\|\boldsymbol{\Delta}\|_F^2 > \delta^2$ and $\Psi_{\mathcal{X}^2(2n^2)}^{-1}(.)$ is the inverse CDF of a standard chi-square random variable with $2n^2$ degrees of freedom.

Proof. : We can write

$$\|\boldsymbol{\Delta}\|_F^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}|[\boldsymbol{\Delta}]_{ij}|^2, \tag{27}$$

where $|[\boldsymbol{\Delta}]_{ij}|^2 = \Re\{[\boldsymbol{\Delta}]_{ij}\}^2 + \Im\{[\boldsymbol{\Delta}]_{ij}\}^2$. Since $[\boldsymbol{\Delta}]_{ij}$ is ZM-CSCG, then its real and imaginary parts can be expressed in terms of standard normal random variables $U_k \sim \mathcal{N}(0,1)$ as $\Re\{[\boldsymbol{\Delta}]_{ij}\} = \frac{\sigma}{\sqrt{2}}U_k$ and $\Im\{[\boldsymbol{\Delta}]_{ij}\} = \frac{\sigma}{\sqrt{2}}U_{k+1}$, respectively. Hence, (27) can be rewritten as

$$\|\boldsymbol{\Delta}\|_F^2 = \frac{\sigma^2}{2}Q, \tag{28}$$

where $Q = \sum_{k=1}^{2n^2} U_k^2$ is distributed as a standard chi-square random variable with $2n^2$ degrees of freedom, i.e., $Q \sim \mathcal{X}^2(2n^2)$. Hence, $\Pr(\|\boldsymbol{\Delta}\|_F^2 \leq \delta^2) = \Pr(Q \leq \frac{2\delta^2}{\sigma^2}) = \Psi_{\mathcal{X}^2(2n^2)}(\frac{2\delta^2}{\sigma^2})$, where $\Psi_{\mathcal{X}^2(2n^2)}(\cdot)$ indicates the CDF of a standard chi-square random variable with $2n^2$ degrees of freedom. By setting $\Psi_{\mathcal{X}^2(2n^2)}(\frac{2\delta^2}{\sigma^2}) = 1 - \rho$ and calculating δ in terms of the outage probability ρ, we obtain (26). $\qquad\square$

5 SIMULATION RESULTS

5.1 Simulation setup

In this section, computer simulations are carried out to evaluate the performance of our proposed approach compared to a cooridnated beamforming method with perfect CSI in [29], a conventional downlink beamforming, an outage-based probabilistic approach in [30], and worst-case robust approaches in [14] and [15]. We obtain the conventional beamforming by removing the regularization term, i.e., $\sum_{k\in\mathcal{S}_b,k\neq q}\sum_{t\in\mathcal{S}_l}\sum_{i\in\mathcal{S}_l}\mathbf{w}_{i(q)}^H(\mathbf{R}_{t(k)}(q) + \boldsymbol{\Delta}_{t(k)})\mathbf{w}_{i(q)}$ in the proposed objective function in (4).

The major problem in designing beamforming vectors for multicell networks is the coupling effect amongst BSs due to intercell interference. This problem becomes more critical if the users are at cell borders where their received interference powers from the interfering BSs are comparable to their received signal powers from their corresponding local BSs. From the point of view of efficiency in energy and backhaul consumption, the authors of [31] and [32] have shown that the cell-edge users mostly benefit from the coordination of 3 BSs of the adjacent sectors. Hence in this paper, we consider a 3-cell cellular network where simultaneously active users per cell are randomly scheduled within their 3 adjacent sectors to reflect the severe impact of ICI on the network. Such a 3-cell scenario is also used in a number of other papers, e.g., [15], [30], [32], [33], for simulation purposes. Monte-Carlo simulations are carried

$$\sum_{k\in\mathcal{S}_b,k\neq q}\sum_{t\in\mathcal{S}_l}\mathrm{Tr}\left\{\left(\mathbf{R}_{t(k)}(q)+\boldsymbol{\Delta}_{t(k)}\right)\mathbf{A}\right\}\sim\mathcal{N}\big(\sum_{k\in\mathcal{S}_b,k\neq q}\sum_{t\in\mathcal{S}_l}\mathrm{Tr}\left(\mathbf{A}\mathbf{R}_{t(k)}(q)\right),\ \sum_{k\in\mathcal{S}_b,k\neq q}\sum_{t\in\mathcal{S}_l}\left\||\mathbf{A}|\odot\boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{t(k)}}\right\|_F^2\big)\tag{24}$$

$$\min_{\mathbf{w}_{i(q)}}\quad\max_{\|\boldsymbol{\Delta}_{t(k)}\|_F\leq\delta_t}\quad\sum_{i\in\mathcal{S}_l}\mathbf{w}_{i(q)}^H\mathbf{w}_{i(q)}+\sum_{k\in\mathcal{S}_b,k\neq q}\sum_{t\in\mathcal{S}_l}\sum_{i\in\mathcal{S}_l}\mathbf{w}_{i(q)}^H\left(\mathbf{R}_{t(k)}(q)+\boldsymbol{\Delta}_{t(k)}\right)\mathbf{w}_{i(q)}$$
$$\text{subject to}\quad\min_{\|\boldsymbol{\Delta}_{i(q)}\|_F\leq\delta_i}\quad\mathrm{SINR}_{i(q)}\geq\gamma_{i(q)},\quad\forall i\in\mathcal{S}_l,q\in\mathcal{S}_b,\tag{25}$$
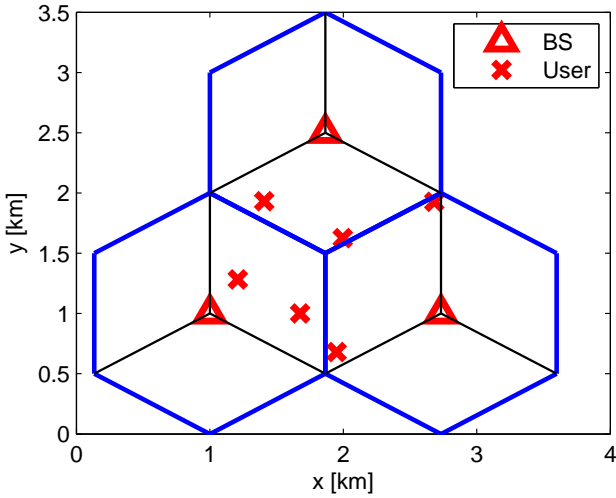

Fig. 1. An example of random set of user distribution.

out with 6 antenna elements per sectoral BSs and 30 independent random sets of 2-user-per-sector distribution, one of which is shown in Fig. 1. Notice that with 6 antennas per sectoral BSs, the proposed algorithm can simultaneously and effectively form beams and nulls towards 6 users, i.e., 2 users per-sector-per-cell. If the number of users is greater than the number of antenna elements, the performance of the proposed algorithm degrades. The distance between any two neighboring BSs is 0.5 km. The entries of $[\tilde{\mathbf{R}}_{i(q)}(q)]_{nm}$ and $[\tilde{\mathbf{R}}_{t(k)}(q)]_{nm}$ are modeled as [34]:

$$e^{\frac{j2\pi\Lambda}{\lambda}\left[(n-m)\sin\theta_{i(p)}\right]}e^{-2\left[\frac{\pi\Lambda\sigma_{as}}{\lambda}\left\{(n-m)\cos\theta_{i(p)}\right\}\right]^2},\tag{29}$$

where λ is the carrier wavelength, $\Lambda=\lambda/2$ is the antenna spacing at BSs, and $\theta_{i(p)}$ is the angle of departure for user $i(p)$ with respect to the broadside of the antenna array. Furthermore it is assumed that the resulting angle spread/offset due to the scatterers is distributed as normal with zero mean and standard deviation of $\sigma_{as}=2°$. In order to capture the effects of fading, path-loss and shadowing, we have scaled the channel covariance matrices by $L_{i(q)}(q)\sigma_F^2 e^{-0.5\frac{(\sigma_s\ln10)^2}{100}}$, where $L_{i(q)}(q)$ is the path loss coefficient between BS q and user $i(p)$ according to $34.53+38\log_{10}(\ell)$, i.e., where ℓ is the distance between the BS and the user, $\sigma_F^2=1$ is the variance of the complex Gaussian fading coefficient and $\sigma_s=10$ is the standard deviation of the log-normal shadow fading coefficient. In the following simulations, we have assumed that the entries of each one of the random

matrices $\boldsymbol{\Delta}_{i(q)}$ and $\boldsymbol{\Delta}_{t(k)}$ have the same variances, i.e., $[\boldsymbol{\Delta}_{i(q)}]_{rd}\sim\mathcal{CN}(0,\sigma_i^2)$ and $[\boldsymbol{\Delta}_{t(k)}]_{rd}\sim\mathcal{CN}(0,\sigma_t^2),\forall r,d,$ and furthermore $\sigma_i^2=\sigma_t^2=\sigma^2$.

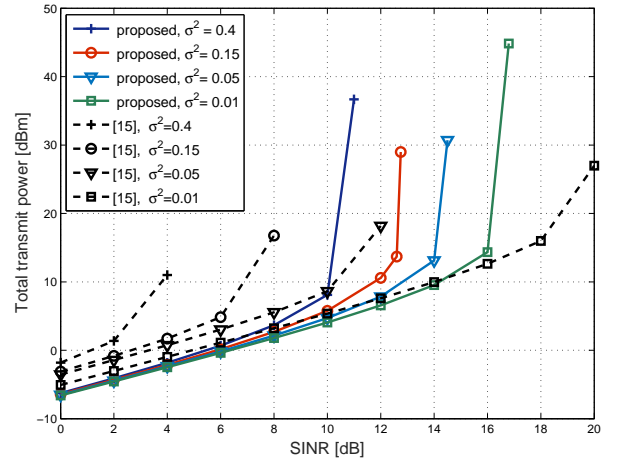### 5.2 Comparisons against instantaneous-CSI methods

Fig. 2. Performance comparison between the proposed approach and the scheme introduced in [15] with various variances of uncertainties at a fixed outage of $\rho=0.3$.
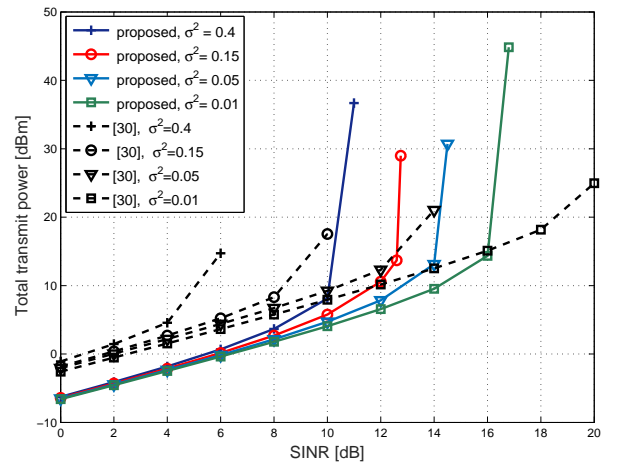

Fig. 3. Performance comparison between the proposed approach and the scheme described in [30] with various variances of uncertainties at a fixed outage of $\rho=0.3$.

In Figs. 2 and 3, we compare the performance of our approach against those of the schemes proposed in [15] and [30], respectively. For fair comparisons in terms of antenna correlation, we have used the following instantaneous channel model [29], [35], [36],

$$\mathbf{h}^H = z\mathbf{h}_w^H \mathbf{R}^{\frac{1}{2}}, \tag{30}$$

where z captures the effects of path-loss, fading and shadowing, $\mathbf{h}_w^H$ is randomly generated zero mean circularly symmetric complex Gaussian variable with unit variance, and $\mathbf{R}$ is the spatial covariance matrix with its (n, m)th entry given by (29). It can be seen from those figures that the proposed approach outperforms the schemes introduced in [15] and [30] in the observed error levels and SINR ranges except for the cases when the error level is at $\sigma^2 = 0.01$ and the SINR requirements are greater than 14dB and 16dB, respectively. This exception can be explained on the basis of the degree of dominance of the transmit power over the uncertainty level, as follows. At an uncertainty level as low as $\sigma^2 = 0.01$ and for SINRs above 14 dB in Fig. 2 and above 16 dB in Fig. 3, the transmit power dominates the level of uncertainties and, hence, the effect of channel estimation error is less significant. Whereas, at the same level of uncertainty and for SINRs below 14 dB in Fig. 2 and below 16 dB in Fig. 3, the transmit power is not large enough to dominate the uncertainty level and, as a result, the impact of channel estimation error becomes more effective.

At higher error levels, i.e., $\sigma^2 > 0.01$, the proposed approach requires lower transmit power than the ones in [15] and [30] at any given SINR requirement and, furthermore, spans wider SINR dynamic range at any given transmit power and error levels. The results in Figs. 2 and 3 confirm that the higher is the error level, the larger is the performance gap between the proposed approach and its counterparts in [15] and [30]. The improved resilience of the proposed scheme against channel estimation errors with respect to those in [15] and [30] is due to the effect of the regularization term, as concluded in Remark 1. According to the result in Remark 1, an increase in the variance of the channel estimation error translates to an increase in the amount of power contributed by the regularization term into the objective function in (4), which encourages the optimization process to further decrease the effect of the overall ICI. This, in turn, leads to further reduction of transmit power at each BS, as each BS follows the same strategy.

Fig. 4 illustrates the impact of the proposed regularization term added to the objective function of the standard power minimization problem, e.g., in [17], in (4) on the convergence behavior of the overall multicell network. Whilst, all BSs in the network independently design their own beamforming vectors, the proposed regularization term into the objective function minimizes the coupling effects amongst the BSs, brings their transmission strategies into a balance and stabilizes the multicell network at an equilibrium point.

5.3 Comparisons against second-order-CSI methods

Fig. 5 shows that at a fix outage probability of $\rho = 0.3$, the total transmit power of 3 BSs increases as the variance, i.e.,
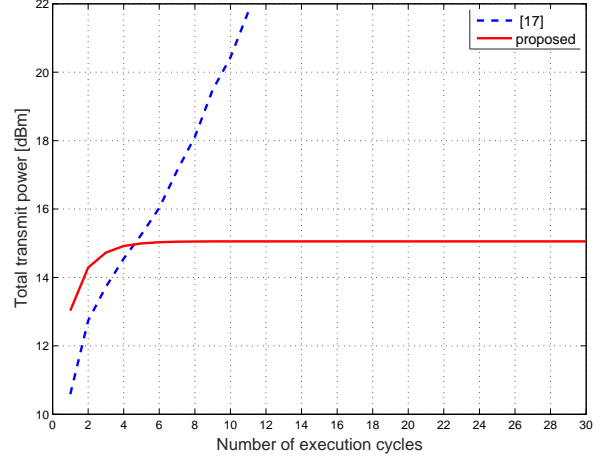

Fig. 4. Convergence comparison between the proposed approach and the single-cell scheme described in [17] with $\rho = 0.3$, $\sigma^2 = 0.1$ and SINR=14dB.
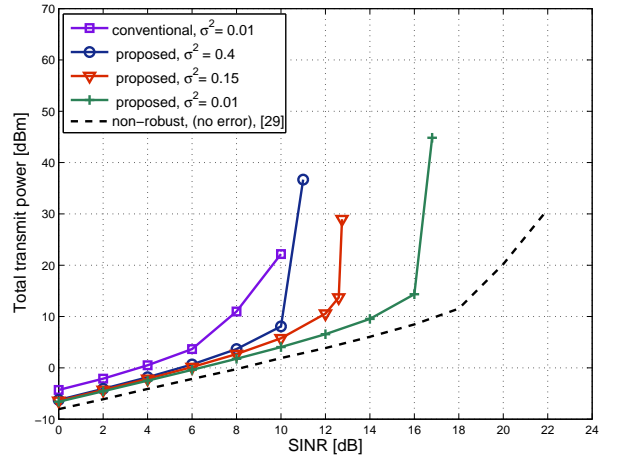

Fig. 5. The proposed probabilistic approach with various variances of uncertainties at a fixed outage of $\rho = 0.3$.

σ^2, of channel uncertainty increases, in the proposed probabilistic approach. To further verify the proposed approach and to illustrate the impact of the regularization term on the total transmit power of BSs, we have also shown in Fig. 5 the result for chance-constraint conventional beamforming. A comparison of the results confirm the effectiveness of the proposed regularization term in significant reduction of the total transmit power at BSs and in achieving higher SINR targets with affordable sum-power levels at BSs. Furthermore from Fig. 5, the non-robust approach in [29] appears to be more power efficient than the proposed probabilistic design. This increase in transmit power in the proposed design is the price to be paid to achieve robustness against channel uncertainties.

To investigate the power consumption of the proposed approach, we plot the total transmit power of BSs at a fixed statistical CSI uncertainty of $\sigma^2 = 0.2$ versus the required SINR threshold, with different outage probability constraints measured by ρ, in Fig. 6. From Fig. 6, it can
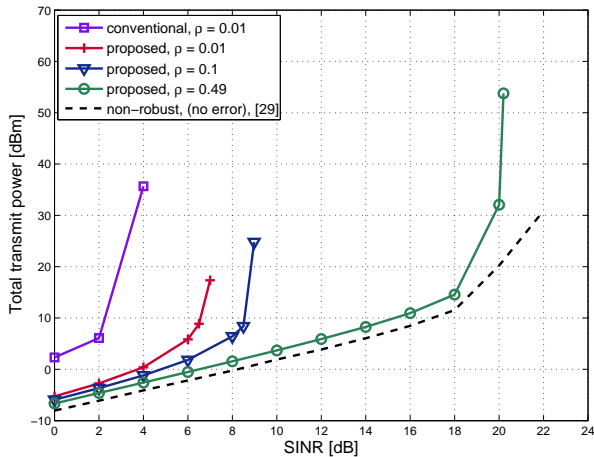
Fig. 6. The proposed probabilistic approach with various outages at fix error variance of $\sigma^2 = 0.2$.
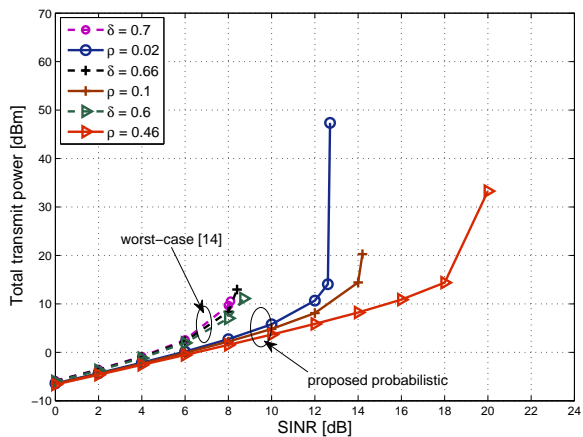

Fig. 7. Comparison of the proposed approach with $\sigma^2 = 0.01$ with the worst-case approach at various outages.
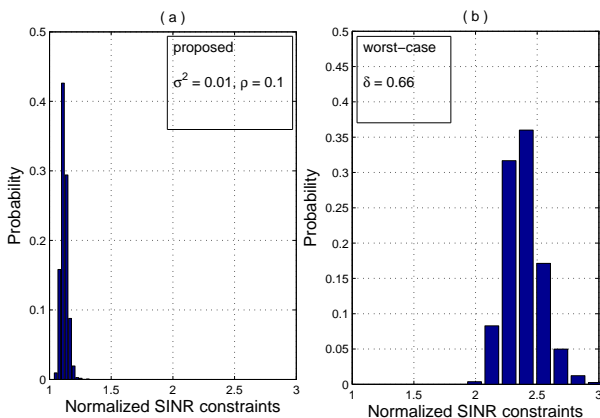

Fig. 8. Histogram of normalized SINR constraints for target SINR value of 8 dB.

be observed for a given SINR target the required transmit power increases as the outage probability decreases. For instance, the proposed method with $\rho = 0.1$ requires 4.87 dB more power than with $\rho = 0.49$ for SINR = 8 dB. This is due to the cost to be paid in terms of more power consumption to achieve the pay-off in terms of gaining more insurance level for robustness against channel uncertainties. When the outage constraint is becoming stricter (i.e., less values of ρ), the probability of non-outage, e.g., $1 - \rho$, which defines the probability of delivering the required quality of service to the users, increases, and therefore, more power is required to meet the requested targets by the users, as shown in the Fig. 6. Further, it can be observed that at $\rho = 0.1$ the proposed method cannot go beyond a critical SINR point of 8.96 dB with limited transmit power. It is noticed that higher SINR targets at lower transmit power can be achieved at higher outage probabilities, i.e., ρ, hence the critical SINR decreases with a smaller ρ value. This effect can be explained as follows:

At a given variance of error of σ^2, with a lower SINR outage constraint of ρ, it is more likely that the beams become wider in order to satisfy the SINR constraints with a higher probability of $1 - \rho$. Whereas, with a higher SINR outage constraint of ρ at the same given variance of error, the likelihood of satisfaction of the SINR constraints at (exactly) the desired levels falls to a lower level and hence, the likelihood of wider beams decreases. In other words, intuitively speaking, having a higher ρ at the same given variance of error is synonymous to lowering the desired SINR target which does not require as wider beam as the situation where ρ is lower. On the other hand, when the beams become wider, they eventually start to overlap over the different users and cause more interference on them. This effect can progressively increase the BSs' transmit power levels which may go up to infinity and, hence, result in an infeasible set of solutions to the optimization problem.
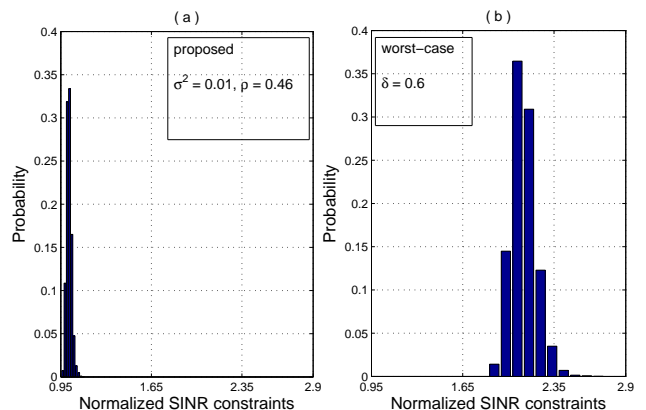

Fig. 9. Histogram of normalized SINR constraints for target SINR value of 8 dB.

In Fig. 7, we have compared the performance of the proposed approach with that of the worst-case approach in [14] at various outage probabilities of 0.46, 0.1 and 0.02 and a fixed statistical CSI uncertainty of $\sigma^2 = 0.01$. Using (26), we

calculate the deterministic upper bounds in the worst-case, i.e., conservative, approach in [14] as $\delta = 0.6$, 0.66 and 0.7, respectively, corresponding to $\rho = 0.46$, 0.1 and 0.02 in $\left\|\boldsymbol{\Delta}_{i(q)}\right\|_F \leq \delta$ and $\left\|\boldsymbol{\Delta}_{t(k)}\right\|_F \leq \delta$. A comparison of results in Fig. 7 shows that the proposed probabilistic approach is more power efficient than its conservative worst-case counterpart in [14]. In particular, this superiority in being more power efficient becomes even more significant at higher SINR targets, i.e., SINR> 8 dB. Furthermore, Fig. 7 also confirms that the results for conservative cases are less sensitive to variations in outage values than the results for the probabilistic cases. This is due to the characteristics of the inverse Gaussian CDF in (26) that maps a wider range of outages, i.e., $\%2 \leq \rho \leq \%46$, in the probabilistic case onto a narrower range of corresponding hyper-sphere uncertainty radius, i.e., $0.6 \leq \delta \leq 0.7$, in the conservative case. Figs. 8 and 9 demonstrate the corresponding histograms for the normalized SINR constraint at $\%10$ and $\%46$ outages, respectively, at a target SINR = 8 dB. Comparing Fig. 8(a) with Fig. 8(b) and Fig. 9(a) with Fig. 9(b), one can see that although, the worst-case approach in Fig. 8(b) and Fig. 9(b) fully satisfy the set SINR target, it consumes nearly $\%72$ more power at SINR = 8 dB than the proposed approach, i.e., see Fig. 7. Furthermore, a comparison of Fig. 8(a) and Fig. 9(a) reveals that although, the proposed approach at $\%46$ of outage ensures the satisfaction of SINR targets at above $\%95$ of beamforming instants, it perfectly meets, i.e., well above $\%100$, the target SINR at $\%10$ of outage.

6 CONCLUSION

We have proposed a probabilistic robust downlink beamforming approach to deliver users' desired SINR targets with certain adjustable outages. Users who are located within the adjacent cells of a cellular network communicate only with their own BSs and over a shared bandwidth. The proposed scheme is amenable to distributed implementation. This is due to accounting for the inter-BS coupling effect by minimizing the resulting inflicted aggregate ICI by each BS on the users of the other cells as an integral part of the proposed objective function of optimization. However, such an amenability to distributed implementation comes at the price of additional computational complexity at user terminals for estimating the incoming ICI from the other BSs in the adjacent cells and feeding it back to the local BSs. In this paper, we have relaxed the rank one constraints and solved the proposed optimization problem using the SDP method. Interestingly, our simulations by CVX always generate exact rank one solutions, such that we have never needed to use an additional randomization process to approximate the rank one solutions with an additional computational complexity. An interesting direction for future research is to attempt to prove analytically that the proposed optimization problem always generates rank one solutions by SDP approach. Comparison results with [15] and [30] confirm that the inclusion of the proposed stochastic regularization term in the objective function of the standard cellular power minimization problem leads to a more effective robustness against channel uncertainties, whilst it stabilizes the multicell network in an energy-efficient optimal equilibrium. Simulation results confirm that not only does the proposed approach outperforms the conventional scheme, but it also shows a significantly superior power saving performance at higher SINR targets, when compared with its conservative worst-case design counterpart.

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