Copula-based Performance Analysis for Fluid Antenna Systems under Arbitrary Fading Channels

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***Abstract*—In this letter, we study the performance of a single-user fluid antenna system (FAS) under arbitrary fading distributions, in which the fading channel coefficients over the ports are correlated. We adopt copula theory to model the structure of dependency between fading coefficients. Specifically, we first derive an exact closed-from expression for the outage probability in the most general case, i.e., for any arbitrary choice of fading distribution and copula. Afterwards, for an important specific case, we analyze the performance of the outage probability under correlated Nakagami-m fading channels by exploiting popular Archimedean copulas, namely, Frank, Clayton, and Gumbel. The results confirm that the spatial correlation dependency structure for the FAS is a key factor to determine its performance, which is natively captured through the choice of copula.**

***Index Terms*—Fluid antenna system, arbitrary fading, correlation, outage probability, Archimedean copulas.**

I. Introduction

Multiple-input multiple-output (MIMO) systems are one of the most popular wireless technologies in the last decades, providing significant capacity by exploiting diversity over multiple signals affected by fading. However, to ensure full diversity gain in MIMO systems, the antennas need to be separated by at least half the radiation wavelength, which is not always practical for mobile devices due to physical space limitations. To overcome such an issue, a novel fluid antenna system (FAS) has been recently proposed in [1], in which a single antenna has the ability to switch its location (i.e., ports) in a small space. This concept was greatly motivated by the recent advances in mechanically flexible antennas such as liquid metal antennas or ionized solutions as well as reconfigurable pixel-like antennas, e.g., [2]–[4].

Several works have been recently conducted to investigate the performance of FAS from various viewpoints, e.g., [5], [6], [8]–[10]. The ergodic capacity for a FAS under correlated Rayleigh fading channels was analyzed in [5], providing approximate expressions of the capacity lower bound. Integral-form expressions for the outage probability for a point-to-point FAS under correlated Nakagami-m and α-μ fading were derived in [6], [7], respectively. Moreover, by exploiting stochastic geometry, [8] derived an analytical expression of the outage probability in FAS large-scale cellular networks. Quite rightly, the performance of FAS is highly dependent on the spatial correlation model used for studying the performance of FAS. In [9], it was revealed that previous contributions may not accurately capture the correlation between the FAS ports. For this purpose, [10] proposed an eigenvalue-based model to approximate the spatial correlation given by Jakes' model, where they illustrated that, under such model, the FAS has limited performance gain as the number of ports increases. Multiuser communications exploiting FAS have also been proposed recently in [11]–[13].

One of the key challenges in FAS is to correctly characterize the spatial correlation of the channel ports without sacrificing tractability. Despite the previous efforts, there is lack of an accurate procedure to model the inherent channel correlation between ports. Specifically, generating the true multivariate distributions of correlated channels in FAS is hard due to mathematical and statistical limitations. To overcome this, one flexible approach to describe the structure of dependency between two or more random variables (RVs) is copula theory which has recently received significant attention in the performance analysis of wireless communication systems [14]–[20] and wired communication systems [21], [22]. Generally speaking, copulas are mainly described with a dependence parameter which can measure the degree of dependence between RVs beyond linear correlation. Copulas can accurately generate the multivariate distributions of correlated RVs by only knowing the marginal distribution of each, and the copula parameter capturing the degree of dependence.

Motivated by the above, we analyze the performance of FAS under arbitrary fading channels through copula theory. By doing so, we solve the key drawbacks of state-of-the-art approaches: (*i*) they are either restricted to specific correlation structures or underlying fading distributions; and (*ii*) they lack tractability, providing at best integral-form expressions for key performance metrics. Instead, our approach is valid for *any* arbitrary choice of dependence structure (i.e., copula) and *any* underlying choice of fading distribution, and provides closed-form expressions for the multivariate distributions of interest, and for the outage probability. Our framework is exemplified for the relevant case of correlated Nakagami-m fading, and

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exploiting popular Archimedean[1] copulas including Frank, Clayton, and Gumbel. We aim to evaluate how changing the dependence structure might help to improve the performance of FAS, so that it can be further used as a guideline to potentially implement FAS with such a dependence structure.

II. System Model

We consider a point-to-point communication system, where a single fixed-antenna transmitter sends an information signal X with transmit power P to a receiver equipped with a single fluid antenna. We assume that the fluid antenna can move freely along K equally distributed positions (i.e., ports) on a linear space. Assuming that there is only one RF chain in the FAS, only one port can be activated for communications, and the received signal at the k-th port can be expressed as

$$Y_k = \sqrt{P_{\rm T} L_{\rm T}} h_k X + Z_k, \tag{1}$$

with $P_{\rm T}$ is the equivalent transmit power, $L_{\rm T}$ the aggregate losses (including path loss and other propagation effects), h_k denotes the normalized fading channel coefficient of the k-th port, X the normalized transmitted symbol with $\mathbb{E}\{|X|^2\} = 1$, and Z_k is the independent identically distributed (i.i.d.) additive white Gaussian noise (AWGN) with zero mean and variance N at every port. We also assume that the FAS is able to select the best port with the strongest signal for communication, i.e., $h_{\rm FAS} = \max(|h_1|, |h_2|, \dots, |h_K|)$, in which the channel coefficients h_k for $k \in \{1, 2, ..., K\}$ are correlated since they can be arbitrarily close to each other. Therefore, the instantaneous received signal-to-noise ratio (SNR) for the FAS can be expressed as $\gamma = \frac{P_{\rm T} L_{\rm T} |X|^2 h_{\rm FAS}^2}{N} = \overline{\gamma} |X|^2 h_{\rm FAS}^2$, where $\overline{\gamma} = \frac{P_{\rm T} L_{\rm T}}{N}$ can be interpreted as the average SNR at the receiver in the single-antenna case.

III. Performance Analysis

We first aim to determine the distribution of $h_{\rm FAS}$ under arbitrary correlated fading coefficients. To proceed, we find it useful to briefly review some concepts of d-dimensional copula theory. Then, by exploiting the obtained distribution, we will be able to derive the closed-form expression of the outage probability in general and specific scenarios.

A. Copula Properties

Definition 1 (d-dimensional copula). *Let* $\mathbf{S} = (S_1, S_2, \dots, S_d)$ *be a vector of* d *RVs with marginal and joint CDFs* $F_{S_i}(s_i)$ *and* $F_{S_1, S_2, \dots, S_d}(s_1, s_2, \dots, s_d)$ *for* $i \in \{1, 2, \dots, d\}$, *respectively. Then, the copula function* $C(u_1, u_2, \dots, u_d)$ *of the random vector* $\mathbf{S}$ *defined on the unit hypercube* $[0,1]^d$ *with uniformly distributed RVs* $U_i := F_{S_i}(s_i)$ *over* $[0,1]$ *is given by*

$$C(u_1, u_2, \dots, u_d) = \Pr(U_1 \le u_1, U_2 \le u_2, \dots, U_d \le u_d), \tag{2}$$

where $u_i = F_{S_i}(s_i)$.

Now, under this assumption, there exists one Copula function C such that for all s_i in the extended real line domain $\overline{R}$ [23]

$$F_{S_1, \dots, S_d}(s_1, \dots, s_d) = C\left(F_{S_1}(s_1), \dots, F_{S_d}(s_d)\right), \tag{3}$$

where (3) is known as the Sklar's theorem.

B. General Model: Arbitrary Correlated Fading Distribution

Here, we derive the CDF and PDF of $h_{\rm FAS}$, as well as the outage probability for the most general case with any arbitrary choice of copula and fading distribution.

Theorem 1. *The CDF of* $h_{\rm FAS} = \max(|h_1|, |h_2|, \dots, |h_K|)$ *in the general dependence structure of arbitrary fading coefficients* $|h_k|$ *for* $k \in \{1, 2, \dots, K\}$ *is given by*

$$F_{h_{\rm FAS}}(r) = C\left(F_{|h_1|}(r), F_{|h_2|}(r), \dots, F_{|h_K|}(r)\right), \tag{4}$$

where $C(\cdot)$ *is the copula function and* $F_{|h_k|}(r)$ *denotes the CDF of fading coefficient* $|h_k|$ *with an arbitrary distribution.*

Proof. By exploiting the definition of the CDF, $F_{h_{\rm FAS}}(r)$ can be mathematically defined as

$$\begin{aligned} F_{h_{\rm FAS}}(r) &= \Pr\left(h_{\rm FAS} \le r\right) \\ &= \Pr\left(\max\{|h_1|, |h_2|, \dots, |h_K|\} \le r\right) \\ &= \Pr\left(|h_1| \le r, |h_2| \le r, \dots, |h_K| \le r\right) \\ &= F_{|h_1|, |h_2|, \dots, |h_K|}\left(r, r, \dots, r\right) \\ &\overset{(a)}{=} C\left(F_{|h_1|}(r), F_{|h_2|}(r), \dots, F_{|h_K|}(r)\right), \end{aligned} \tag{5}$$

where (a) is derived from (3). ■

Theorem 2. *The PDF of* $h_{\rm FAS} = \max(|h_1|, |h_2|, \dots, |h_K|)$ *in the general dependence structure of arbitrary fading coefficients* $|h_k|$ *for* $k \in \{1, 2, \dots, K\}$ *is given by*

$$\begin{aligned} f_{h_{\rm FAS}}(r) = &f_{|h_1|}(r) f_{|h_2|}(r) \cdots f_{|h_K|}(r) \\ &\times c\left(F_{|h_1|}(r), F_{|h_2|}(r), \dots, F_{|h_K|}(r)\right), \end{aligned} \tag{6}$$

where $f_{|h_k|}(r)$ *denotes the marginal PDF of fading coefficient* $|h_k|$ *with an arbitrary distribution and* $c(\cdot)$ *is the copula density function which can be determined as*

$$\begin{aligned} &c\left(F_{|h_1|}(r), F_{|h_2|}(r), \dots, F_{|h_K|}(r)\right) \\ &\quad = \frac{\partial^d C\left(F_{|h_1|}(r), F_{|h_2|}(r), \dots, F_{|h_K|}(r)\right)}{\partial F_{|h_1|}(r) \partial F_{|h_2|}(r) \cdots \partial F_{|h_K|}(r)}. \end{aligned} \tag{7}$$

Proof. The result is obtained by applying the chain rule to Theorem 1. ■

Theorem 3. *The outage probability of the considered FAS in the general dependence structure under an arbitrary fading distribution is given by*

$$P_{\rm out} = C\left(F_{|h_1|}(\hat{\gamma}), F_{|h_2|}(\hat{\gamma}), \dots, F_{|h_K|}(\hat{\gamma})\right), \tag{8}$$

where $\hat{\gamma} = \sqrt{\frac{\gamma_{\rm th}}{\overline{\gamma}}}$ *and* $\gamma_{\rm th}$ *is the SNR threshold.*

Proof. Outage probability is an appropriate metric to evaluate the performance of FAS, which is defined as the probability

[1] We use Archimedean copulas for several reasons: (i) the ease with which they can be constructed; (ii) the great variety of families of copulas which belong to this class; and (iii) the many nice properties possessed by the members of this class [23]. While other copulas such as FGM have appealing analytical properties, they can only model weak dependence structures, which seems insufficient in the FAS context.

that the random SNR γ is less than an SNR threshold $\gamma_{\rm th}$. Therefore, the outage probability is defined as

$$P_{\rm out} = \Pr\left(\gamma \le \gamma_{\rm th}\right) = \Pr\left(h_{\rm FAS} \le \sqrt{\frac{\gamma_{\rm th}}{\bar{\gamma}}}\right)$$
$$= \Pr\left(\max\left\{|h_1|, |h_2|, \ldots, |h_K|\right\} \le \sqrt{\frac{\gamma_{\rm th}}{\bar{\gamma}}}\right)$$
$$= F_{h_{\rm FAS}}(\hat{\gamma}), \tag{9}$$

where by exploiting the obtained CDF form (4), the proof is completed. ■

Remark 1. *In contrast to previous contributions, the results in (4), (6), and (8) indicate that there is no need to solve any complicated integral for deriving the outage probability and the joint distribution of channel coefficients in the FAS, provided that the copula function $C(\cdot)$ is given in closed-form.*

Remarkably, the results in Theorems 1–3 are valid for *any* arbitrary choice of fading distribution and copula function over the proposed FAS. Now, to analyze the system performance, we consider a special case in the following section.

C. Special Case: Correlated Nakagami-m Fading

Here, for exemplary purposes, we assume that the fading channel coefficients $|h_k|$ follow the Nakagami-m distribution, where the parameter $m \ge 0.5$ denotes the fading severity. In the special case $m = 1$, the Rayleigh fading with an exponentially distributed instantaneous power is recovered. Hence, the marginal distributions for the fading channel coefficient $|h_k|$ can be expressed as

$$f_{|h_k|}(r) = \frac{2m^m}{\Gamma(m)\mu^m} r^{2m-1} {\rm e}^{-\frac{m}{\mu}r^2}, \tag{10}$$

$$F_{|h_k|}(r) = \frac{\gamma\left(m, \frac{m}{\mu}r^2\right)}{\Gamma(m)}, \tag{11}$$

in which m and μ define the shape and spread parameters, respectively. The terms $\Gamma(\cdot)$ and $\gamma(\cdot,\cdot)$ denote gamma function and the lower incomplete gamma function, respectively.

In order to evaluate the structure of dependency between correlated fading channel coefficients, there are many types of copula functions that can be used. However, in this letter, we derive the analytical expressions by using the popular Archimedean copulas, namely, Frank, Clayton, and Gumbel. These flexible copula functions can efficiently describe weak/strong, negative/positive, and linear/non-linear dependence structures between correlated RVs.

Definition 2 (d-dimension Archimedean copula)**.** *The d-dimension Archimedean copula $C_{\rm AR}$ is defined as [24]*

$$C_{\rm AR}\left(u_1, u_2, \ldots, u_d\right) = \phi^{-1}\left(\phi\left(u_1\right) + \phi\left(u_2\right) + \cdots + \phi\left(u_d\right)\right), \tag{12}$$

where $\phi(t)$ denotes a generator function of the Archimedean and $\phi^{-1}(.)$ is the its inverse function.

Definition 3 (Popular Archimedean copulas)**.** *The d-dimension Frank, Clayton, and Gumbel copulas are respectively defined as [24]*

$$C_{\rm FR}(u_1, u_2, \ldots, u_d) = -\frac{1}{\alpha}\ln\left(1 + \frac{\prod_{j=1}^{d}\left({\rm e}^{-\alpha u_j} - 1\right)}{\left({\rm e}^{-\alpha} - 1\right)^{d-1}}\right), \tag{13}$$

$$C_{\rm CL}(u_1, u_2, \ldots, u_d) = \left[\sum_{j=1}^{d}\left(u_j^{-\beta} - 1\right) + 1\right]^{-\frac{1}{\beta}}, \tag{14}$$

$$C_{\rm GU}(u_1, u_2, \ldots, u_d) = \exp\left(-\left[\sum_{j=1}^{d}\left(-\ln u_j\right)^{\theta}\right]^{\frac{1}{\theta}}\right), \tag{15}$$

where $\alpha \in \mathbb{R}\backslash\{0\}$, $\beta \in [0, \infty)$, and $\theta \in [1, \infty)$ are the dependence structure parameters of the Frank, Clayton, and Gumbel copulas, respectively. The independent case is achieved when $\alpha \to 0$, $\beta \to 0$, and $\theta = 1$.

Now, by exploiting the definition of the above-mentioned copulas, the outage probability in the specific model can be determined in the following theorem.

Theorem 4. *The outage probability of the considered FAS under Nakagami-m fading channel, using the Frank, Clayton, and Gumbel copulas is, respectively, given by*

$$P_{\rm out}^{\rm FR} = -\frac{1}{\alpha}\ln\left(1 + \frac{\left[\exp\left(\frac{-\alpha\gamma\left(m, \frac{m}{\mu}\hat{\gamma}^2\right)}{\Gamma(m)}\right) - 1\right]^K}{\left({\rm e}^{-\alpha} - 1\right)^{K-1}}\right), \tag{16}$$

$$P_{\rm out}^{\rm CL} = \left[K\left(\left(\frac{\gamma\left(m, \frac{m}{\mu}\hat{\gamma}^2\right)}{\Gamma(m)}\right)^{-\beta} - 1\right) + 1\right]^{-\frac{1}{\beta}}, \tag{17}$$

and

$$P_{\rm out}^{\rm GU} = \exp\left(K^{\frac{1}{\theta}}\ln\frac{\gamma\left(m, \frac{m}{\mu}\hat{\gamma}^2\right)}{\Gamma(m)}\right). \tag{18}$$

Proof. By inserting (11) into (13), (14), and (15) for $u_j = F_{|h_k|}(\hat{\gamma})$ and then considering (8), the proof is completed. ■

Corollary 1. *As $K \to \infty$, the outage probability goes to 0 as long as $\alpha, \beta \neq 0$.*

Proof. It can be seen from (16) that the fraction term inside the logarithm is the product of K less-than-one values. If $K \to \infty$, it goes to 0. In (17), it is straightforward as $K \to \infty$, the outage probability reaches 0 due to the term $\frac{-1}{\beta}$. Finally, in (18), the logarithm value is always negative, thereby when $K \to \infty$, the outage probability achieves 0. ■

Remark 2. *The outage probability for the considered FAS can be accurately obtained in a closed-form expression according to Theorem 4. We can see that the outage probability highly depends on the number of ports K, dependence parameters α, β, and θ, and the fading parameter m, namely, the performance of the outage probability will improve as K and m increase and α, β, and θ decrease.*

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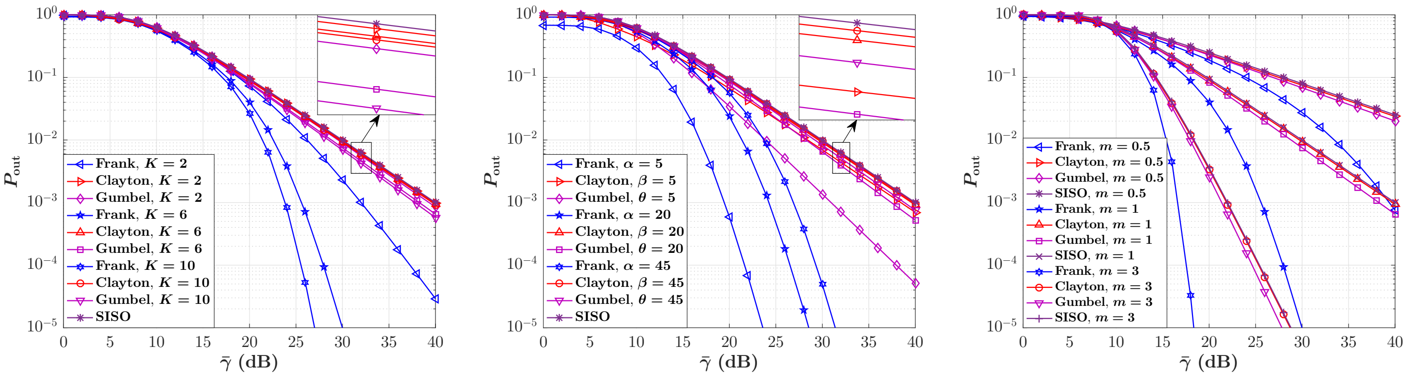

(a) (b) (c)

Fig. 1. Outage probability versus average SNR $\bar{\gamma}$ for selected values of: (a) the number of antenna ports K when $m = 1$, $\alpha = \beta = \theta = 30$, and $\mu = 1$; (b) the fading parameter m when $K = 6$, $\alpha = \beta = \theta = 30$, and $\mu = 1$; and (c) the dependence parameters when $m = 1$, $K = 6$, $\mu = 1$.

IV. Numerical Results

In this section, we present numerical results to evaluate the FAS performance in terms of the outage probability. First, we need to generate correlated RVs (e.g., h_k) in terms of Archimedean copulas. In this regard, the most frequently used approach is the conditional distribution which is not feasible in higher dimensions since it involves a differentiation step for each dimension of the problem. To this end, we exploit Marshall and Olkin's alternative method [25] as shown in Algorithm 1, which is expressed in terms of the Laplace transform and is computationally more straightforward than the conditional distribution approach.

Algorithm 1 Archimedean copula sampling

Step 1. *Generate* $J \sim \mathcal{L}^{-1}\left[\phi(t)\right]$*, where* $J \sim \log\left(1-\mathrm{e}^{\alpha}\right)$ *for Frank copula,* $J \sim \Gamma\left(\frac{1}{\beta}, \beta\right)$ *for Clayton copula, and* $J \sim \mathcal{S}\left(\frac{1}{\theta}, 1, \cos^{\theta}\left(\frac{\pi}{2\theta}\right), 0\right)$ *for Gumbel copula*
Step 2. *Generate* $\mathbf{S} = (S_1, \ldots, S_d)$*, such that* $S_i \sim \mathcal{U}(0,1)$ *for* $i = 1, \ldots, d$
Step 3. *Return* $U_i = \phi\left(-\frac{\log(S_i)}{J}\right)$*, where* $\phi(t) = -\ln t\left(\frac{\mathrm{e}^{-\alpha t}-1}{\mathrm{e}^{-\alpha}-1}\right)$ *for Frank copula,* $\phi(t) = \frac{t^{-\beta}-1}{\beta}$ *for Clayton copula, and* $\phi(t) = (-\ln t)^{\theta}$ *for Gumbel copula*

Fig. 1(a) indicates the behavior of the outage probability against the average SNR $\bar{\gamma}$ for different numbers of fluid antenna ports K under correlated fading channels. As expected, FAS outperforms the conventional single-input-single-output (SISO) system, so that for a fixed value of dependence parameters, the outage probability performance improves as K increases. By careful observation of the curves, we can also see that the Frank copula is more sensitive to even a small change of K, compared with the Clayton and Gumbel copulas. Frank copula also appears to give the least outage probability in FAS compared to others, which suggests that one should design FAS that resembles the correlation structure described by Frank copula.

The impact of fading channel correlation on the performance of the outage probability is illustrated in Fig. 1(b). It is clearly seen that fading correlation has destructive effects on the outage probability. As the dependence parameters of the Archimedean copulas grow, the outage probability increases. Even under a strong positive dependence structure between fading channel coefficients, FAS outperforms the SISO case. Moreover, it is found that the Frank copula also provides the best performance from a correlation perspective compared with two other considered copulas.

Regarding the importance of fading severity on the performance of FAS, Fig. 1(c) shows the behavior of the outage probability versus the average SNR for different values of the fading parameter m under correlated Nakagami-m distribution. The results indicate that as the fading severity reduces (i.e., m increases), the outage probability performance ameliorates and such an improvement is more noticeable when Frank copula is considered to describe the fading correlation. To gain more insight into the impact of the number of ports on the FAS performance, Fig. 2 shows the behavior of the outage probability in terms of K for selected values of the average SNR. It can be seen that even under a strong correlation provided by the Frank copula (e.g., $\alpha = 30$), the outage probability significantly decreases as K becomes large. Fig. 3 also shows the impact of the dependence parameter on the outage probability performance for the considered FAS. The results reveal that when the correlation is weak (i.e., low dependence parameter), a lower outage probability is achieved. In addition, the Frank copula provides better performance for weak correlation compared with the Clayton and Gumbel copulas. However, as the correlation between channel coefficients becomes stronger (i.e., larger dependence parameter[2]), the Clayton and Gumbel copulas offer almost the same behavior as the Frank copula. Therefore, by comparing the results in Figs. 2 and 3, it can be found that the best performance for the outage probability occurs when K is sufficiently large and the dependence parameter is small enough.

V. Conclusion

In this letter, we studied the performance of a point-to-point FAS, where the correlated fading channels have arbitrary

[2]Note that while the dependence parameters for these copulas are qualitatively similar (i.e., dependence grows with the parameter), they are quantitatively different and may yield different values of correlation (often measured through rank correlation coefficients).

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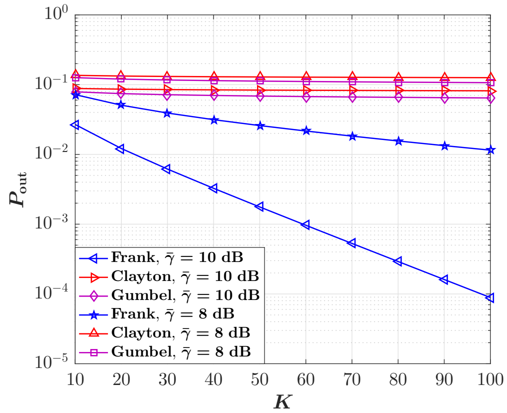

Fig. 2. Outage probability versus number of ports K for selected values of the average SNR, when $m = 1$, $\alpha = \beta = \theta = 30$, and $\mu = 1$.

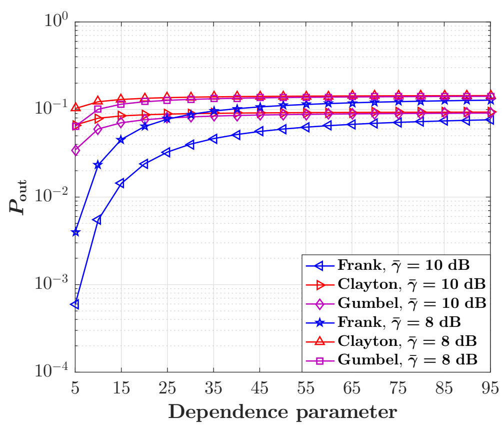

Fig. 3. Outage probability versus dependence parameters for selected values of the average SNR, when $m = 1$, $K = 6$, and $\mu = 1$.

distributions. By exploiting the copula approach to model the structure of dependency between correlated arbitrary fading channels, we derived the closed-form expression for the outage probability in the most general case. Then, as a specific scenario, we analyzed the outage probability for the considered FAS under correlated Nakagami-m fading channels by using popular Archimedean copulas. Eesults showed that the best performance is achieved by increasing the number of fluid antenna ports and the fading parameter as well as reducing the value of the dependence parameter.

The use of copula theory for the analysis of FAS has good potential due to its versatility and analytical tractability. A general design procedure that allows to design, analyze and engineer FAS systems using copula theory with a target dependency structure is desirable, but needs further developments. The extension to more sophisticated scenarios including a MIMO-FAS (i.e., FA also at the transmitter side), FA port selection and location optimization, the derivation of additional performance metrics, or the use of sample datasets to analyze the true dependence structures seem exciting lines for future research activities.

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