Seismic design optimization of multi-storey steel-concrete composite buildings

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Abstract

This work presents a structural optimization framework for the seismic design of multi-storey composite buildings, which have steel HEB-columns fully encased in concrete, steel IPE-beams and steel L-bracings. The objective function minimized is the total cost of materials (steel, concrete) used in the structure. Based on Eurocodes 3 and 4, capacity checks are specified for individual members. Seismic system behavior is controlled through lateral deflection and fundamental period constraints, which are evaluated using nonlinear pushover and eigenvalue analyses. The optimization problem is solved with a discrete Evolution Strategies algorithm, which delivers cost-effective solutions and reveals attributes of optimal structural designs.

Keywords:

Structural optimization; Discrete optimization; Evolution Strategies; Earthquake-resistant; Pushover analysis; Frequency constraints

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1. Introduction

Steel-concrete composite elements are intended to fill the gap between reinforced concrete elements and pure steel elements. The utilization of steel-concrete composite elements is not a new concept, since they have gradually gained popularity during the course of the 20th century mainly in North America, Japan and Europe, while early applications of such elements at the end of the 19th century have been recorded. Over the past few decades, numerous steel-concrete composite structures have been erected worldwide. This form of construction is seen as an alternative mainly to constructing pure steel structures. The increasing preference in composite elements can be primarily attributed to the fact that concrete, a significantly less expensive material compared to steel, is utilized in an effort to cost-effectively replace a percentage of the required steel sections area. This way, overall material cost in a structure can be reduced and, at the same time, better lateral support and fire protection of the steel elements can be achieved, since concrete (which usually covers steel elements) offers a much better performance at high temperatures than structural steel. However, although the incorporation of steel-concrete composite elements in a structure is nowadays regarded as established design and construction practice, the investigations conducted on how such practice can be exploited in the most cost-effective way are rather limited.

Structural optimization is widely recognized as a valuable computational tool that aids engineers in identifying cost-effective designs. Numerous seismic design optimization applications for steel structures (e.g. [1-12]) and reinforced concrete structures (e.g. [13-15]) are presented in the literature. For composite elements and structures, the available publications are much less and are mostly dealing with the design optimization of composite floors [16-18] and beams [19-22]. The publications on the design optimization of composite buildings are rather few [23-25] and do not fully and explicitly take into account the complete set of design requirements that should be normally specified for composite buildings. In fact, these works concentrate on achieving adequate system performance to lateral (wind or earthquake) loading and actually ignore member capacity checks. This way, however, requirements on withstanding vertical (gravitational) loads are neglected and especially the beams are most probably under-designed. Moreover, in the aforementioned existing works, there is no control over the composite structures' eigenperiods, which means that designs with unrealistic vibration properties are not excluded from being selected as feasible optimal solutions. Thus, a more complete design optimization framework for composite buildings is needed.

The present paper is concerned with the design optimization of earthquake-resistant multi-storey composite buildings with steel-concrete columns. In these buildings, the composite columns consist of steel members with standard I-shaped sections fully encased in concrete; steel beams with standard I-shaped sections and (optional) steel bracings with standard L-shaped sections are considered. The aim of the developed optimization procedure is to minimize the total materials cost in a composite building under explicit constraints imposed based on member capacity checks of formal design codes. In particular, individual composite and pure steel members of the building assessed are required to satisfy the provisions of respective Eurocodes. Overall seismic resistance is controlled through additional constraints on interstorey drifts and top-storey displacements, which are evaluated using nonlinear static pushover analyses. Moreover, an upper allowable limit for constraining the fundamental period of the building is specified. The optimization problem is solved with a discrete Evolution Strategies algorithm, which can effectively handle the standard options available in the market for steel members. The optimizer is linked with a powerful structural analysis software (OpenSees [26]) to automatically obtain the structural response results needed for the evaluation of constraints. Hence, the contribution of this work is that it comprehensively presents and assesses a complete and well-organized framework for seismic design optimization of composite buildings. In an effort to enrich the available knowledge on the behavior of composite structures and facilitate the cost-effective use of composite elements, the developed optimization procedure is exploited to identify attributes of optimally designed composite buildings.

The remainder of this paper is organized as follows. Section 2 describes the structural design requirements specified for composite buildings in this work. Details on the structural configuration of the analyzed buildings, as well as on their numerical modeling and analysis, are given in section 3. The implemented design optimization procedure is explained in section 4. Design optimization results for composite buildings are reported and discussed in section 5. Section 6 concludes the paper with some final remarks.

2. Structural design requirements

In the framework of the optimization procedure implemented in the present work, each solution evaluated as a candidate optimum design of a composite building needs to be checked with respect to pre-specified feasibility constraints. These constraints represent the design requirements imposed by the adopted design codes, guidelines, etc. and include both individual member capacity checks and seismic system performance checks.

The design of the structural members of the buildings considered is performed according to the provisions of Eurocode 4 (EN 1994-1-1 [27]) for composite column members with concrete-encased steel HEB sections and Eurocode 3 (EN 1993-1-1 [28]) for pure steel beam members with IPE sections. The capacities of columns are checked with respect to axial force (EN 1994-1-1, §6.7.3.5), shear force (EN 1993-1-1, §6.2.6), bending moment (EN 1994-1-1, §6.7.3.3), combined axial force and biaxial bending moment (EN 1994-1-1, §6.7.3.6 and §6.7.3.7) and the respective types of local and global buckling (EN 1994-1-1, §6.7.3). The capacities of beams are checked for shear force (EN 1993-1-1, §6.2.6), bending moment and interaction with shear force (EN 1993-1-1, §6.2.5 and §6.2.8), as well as the respective types of local and global buckling (EN 1993-1-1, §6.3). The bracings are not considered to participate in the transference of the gravitational loads to the foundation, so their pure steel L-sections are determined based on the structural system performance.

The overall seismic resistance of a structure is controlled through lateral deflection constraints. Following the provisions of FEMA 440 [29] and ASCE/SEI 41-06 [30], the structure's seismic capacity for the collapse prevention performance level can be assessed by performing a displacement-controlled nonlinear pushover analysis up to a pre-specified displacement. More specifically, a node at the roof level of the structural model is required to be able to reach a target displacement Δ_{target}, which is estimated as:

$$\Delta_{target} = C_0 C_1 C_2 C_3 S_a \frac{T^2}{4\pi^2} . \tag{1}$$

In this equation, C_0, C_1, C_2 and C_3 are factors defined in [29] and S_a is the design pseudo-acceleration of the structure with fundamental period T. Moreover, the maximum interstorey drift is constrained to be less than 4% of the storey height. This drift-limit is suggested in [30] for concrete frames. As there is no provision specifically for steel-concrete composite frames, the 4% limit is preferred over the 5% limit suggested for pure steel frames. It is noted that the internal forces developed in structural elements during the pushover analysis due to the combination of horizontal and gravitational loads are not checked with respect to the above mentioned provisions of Eurocodes 3 and 4 for steel and composite members. Enforcing the satisfaction of such provisions under this load combination and analysis would reduce the cost-effectiveness of the optimized designs achieved, since their intended seismic performance does not preclude the failure of individual structural elements, provided that partial or full system collapse is not triggered.

Preliminary test runs using all aforementioned design requirements of this section revealed the tendency of the implemented optimizer to select structural designs with high fundamental periods (even over 2s in some cases). Such structures generally attract relatively small earthquake-induced forces, but are also associated with increased potential for damage to non-structural components and building contents, as well as for discomfort of occupants, during seismic events. In order to avoid these undesirable long-period buildings, an additional design requirement is employed in this work, according to which the fundamental period of a structure is not allowed to exceed a threshold value $T_{\max}$. Period/frequency-information is incorporated also in a number of other optimization applications in structural mechanics (e.g. [31-34]). As no data on specifying $T_{\max}$ for composite buildings were found, the formula proposed in [35] for limiting the fundamental period of steel buildings is adopted herein:

$$T_{\max} = 0.045 H^{0.80}, \tag{2}$$

where H is the building height (in feet) above the base.

3. Structural configuration, modeling and analysis of composite buildings

3.1. Structural configuration

The steel-concrete columns of the composite buildings assessed in the present work are designed as fully encased I-shaped (HEB) sections (Fig. 1(a)). A concrete layer of 5cm around the steel section's edges is assumed, in which longitudinal (bars of 10mm diameter) and transversal (stirrups of 8mm diameter) reinforcement is installed. For small steel section sizes (up to HE 180 B), 3 longitudinal bars per side are used; for larger steel section sizes, 5 longitudinal bars per side are installed. Stirrups are placed with 10cm spacing around the longitudinal bars. The external concrete cover is fixed to 2.5cm. Thus, a composite column section is fully defined just by specifying the encased HEB-section; once the HEB-dimensions are known, the amount and layout of concrete and its reinforcement in the composite section can be deduced based on the section description given in this paragraph. The steel HEB-sections have a common orientation across all columns of a building. Specifically, all HEB-members are placed with their cross-sections' major axes parallel to the global horizontal *x*-axis of the building.

The beams and bracings are designed as pure steel elements (Figs. 1(b,c)). For the building's floors, corrugated composite slabs and secondary beams are installed. The columns at the base of the building are assumed to be fixed, while all beam-column connections are considered to be rigid. The design of connections is not within the scope of this paper.

3.2. Material models

OpenSees, which is the software utilized in the present work to perform all structural analyses, has the capability to include a variety of different materials in each structural analysis [26]. For the purposes of the present work, 3 different material models are utilized to simulate the stress-strain behavior of structural steel, concrete and reinforcing steel.

The bilinear steel material type 'Steel01' of OpenSees with hardening is used for all structural steel members (Fig. 2(a)). The yield stress and the elasticity modulus are taken 235MPa and 210GPa, respectively, while hardening is taken into account by defining the post-yielding stiffness to be 5‰ of the initial one. Although an ultimate strain capacity is not specified in this material model, strains do not exceed the threshold value of 20% at any structural design presented in this work.

As regards concrete, two distinct areas are defined for a column section: (a) the external concrete cover of 2.5cm, which is modelled as unconfined concrete, and (b) the remaining concrete area surrounded by the reinforcement, which is considered to be confined concrete with enhanced capacity and ductility properties. The concrete area between the flanges of the HEB-section can be considered as 'super-confined', because lateral deformations at 3 of its sides are fully restricted by HEB-parts, while on the 4th side a thick layer of confined concrete creates similar boundary conditions. However, due to lack of experimental data formally justifying a better performance of this 'super-confined' area, it is modeled as 'normally' confined concrete.

The 'Concrete01' material type is employed for all concrete regions of the composite columns (Fig. 2(b)). The compressive strength of confined concrete is set to 20MPa (no tensile strength is assumed), while its cracking and crushing strains are 2‰ and 3.5‰, respectively. Unconfined concrete is defined as a similar 'Concrete01' material, with reduced compressive strength (20% lower than that of confined concrete). This significant reduction in concrete strength is justified not only by the lack of confinement, but also by the relatively low active cover thickness of 2.5cm adopted in this work.

Finally, the 'ReinforcingSteel' material type is used for the longitudinal and transversal reinforcement bars of the composite columns (Fig. 2(c)). The elastic behaviour of this material type is similar to the one of 'Steel01', while its post-yield behaviour includes both strain hardening and softening. The 'ReinforcingSteel' material is implemented with a yield stress equal to 500MPa, an ultimate stress of 600MPa, a yield strain of 2.5‰ and an ultimate strain of 20%.

3.3. Modeling of structural components

Fiber section elements are used to represent all structural members, in order to adequately capture the locations of plastic hinge formation. Each section is first divided into sub-sections, which correspond to the section's regions with different material properties. Then, each sub-section is further divided into an adequate number of fibers.

The columns and beams of the composite building are modelled in OpenSees as 'nonlinearBeamColumn' elements, which can simulate the spread of plasticity along each element. In the column elements, second order effects are taken into account. Moreover, perfect anchorage and splicing of the reinforcement bars is assumed in the composite columns (possible anchorage slip or bond failure is not taken into account in the structural model). As regards the connections, no additional 'zeroLength' element is used to model the behavior of any beam-column joint. This implies that: (i) the beam-column joints are capable of transferring the full moment, shear and axial force they receive, (ii) the beam-column joints are not deformable and the angle of each connection between the beam and the column remains unaltered (columns and beams remain perpendicular to each other) and (iii) all columns and beams are allowed to deform inelastically along their full body, as no rigid zones are defined (plastic hinges may develop adjacent to joints).

The bracings are modelled as 'truss' elements, which are nonlinear fiber elements providing accuracy analogous to that of 'nonlinearBeamColumn' elements with hinged ends. A 'truss' element is restricted from developing shear forces or bending moments.

All sections defined are divided into quadrilateral patches. Preliminary analyses revealed that, because OpenSees assembles stiffness matrices by calculating the stiffness of the fiber sections, the fundamental period of the structure is underestimated for small numbers of fibers.

Hence, fine section discretizations are generated, in order to achieve high analysis accuracy for the modeling assumptions made. Specifically, each quadrilateral concrete patch consists of 100 fibers (10 fibers along each of the local *y*- and *z*-directions), as illustrated in Fig. 1. The steel sections are divided into quadrilateral patches consisting of 10 fibers along the larger side (length) and 3 fibers along the smaller side (thickness) (Fig. 1). A smaller number of fibers is used along the thickness of steel patches, in order to reduce the computational cost, as a larger number of fibers was not found to significantly increase analysis accuracy. Moreover, 4 integration points are defined along each element.

The corrugated composite slabs and secondary beams of the building's floors are not included in the structural model. Their design depends only on the gravitational loads, therefore they are designed a-priori and are not treated in the framework of the optimization procedure presented in this work. However, their contribution is simulated by transferring their loads to the beams and by considering all slabs to perform as rigid diaphragms (using the 'rigidDiaphragm' command, all nodes at a floor level are constrained to move together horizontally). The characteristic values of the dead and live loads of the slabs are g=9.85kN/m^2 and q=2kN/m^2 (residential building), respectively.

3.4. Structural analyses

Five analyses are conducted for each structural design using the software OpenSees, in order to evaluate its adequacy with respect to the design requirements of section 2: (a) a force-controlled linear static analysis under gravitational loads, in order to perform member capacity checks according to Eurocodes 3 and 4, (b) two displacement-controlled non-linear static pushover analyses (one for each horizontal direction), in order to assess the nonlinear response of the structure under seismic action, and (c) two eigenvalue analyses (one for each horizontal direction), in order to check the fundamental periods of the structure along both directions and define the targeted top displacement used in each pushover analysis. The loads utilized at each analysis are combined according to Eurocode 0 (EN 1990, §6.4 [36]).

When a design fails any of the member capacity checks based on the results of the linear static analysis, which means that the design is infeasible irrespective of the outcome of other checks, its seismic performance is still evaluated, i.e. all 5 structural analyses are conducted anyway. The reason for fully evaluating infeasible solutions is that designs with relatively weak beams and strong columns might fail under gravitational loads, but could perform well under horizontal ones. Respectively, designs with relatively strong beams and weak columns might be found unsuitable for seismic loads, but adequate for gravitational ones. In both cases, the evaluated designs are infeasible and are rejected as a final solution, as the optimum design should have adequate sections both for beams and columns to withstand vertical and horizontal loads. However, any infeasible design may have desirable properties, which can be exploited during an optimization run (e.g. through crossover operations in the framework of an evolutionary optimizer) to accelerate convergence and increase the probability of detecting a high-quality final solution. Therefore, as described in the next section, a penalty function is used for infeasible designs and they are not immediately discarded from the current population of the evolutionary optimization procedure.

4. Structural design optimization

Optimization methods based on probabilistic search of the design space (genetic algorithms, evolution strategies, differential evolution, etc.) have been found to be very effective for structural optimization problems (e.g. [37,38]). The Evolution Strategies (ES) optimization algorithm [37] is used in this work to determine the most cost-effective design for each test case considered. The aim of this non-deterministic optimization algorithm is to minimize an objective function by selecting combinations of the decision variables in a systematic manner and checking the feasibility or infeasibility of each candidate optimum solution through the defined constraints. Its basic concept is to imitate the evolution from generation to generation of a population (i.e. a group of structural designs) under the imposed constraints.

In order to define the optimization problem solved using ES for each structural design case, the formulation and handling of the design variables, the objective function and the constraints are described in this section. Moreover, some details are given on the ES implementation developed.

4.1. Design variables

The elements modified in the optimization procedure are the steel sections of structural members (columns, beams, bracings). The members of a building are first organized into groups and then a design variable is assigned to each group. Standardized steel sections are used for all structural elements, hence the search space consists only of discrete design options, which renders the investigation performed a discrete optimization problem. In particular, the design variables take values from the following 3 discrete databases: (a) HE 100 B to HE 1000

B for columns, (b) IPE 80 to IPE 600 for beams and (c) L 90×90×7 to L 250×250×28 for bracings. In order to give the optimizer the freedom to activate bracings only when they are needed, a ‘zero’ option (no bracing section) is included as the first option in the database with L-shaped sections. Thus, the optimization result may be a moment resisting (unbraced) frame or a braced frame, depending on the relative cost-effectiveness of these two design approaches for the particular case considered. Hence, the developed design formulation is a mixed sizing-topology optimization problem for the determination of a composite structure’s optimal steel sections and bracings topology. It is noted that no design variables are defined for controlling the amount of concrete and its reinforcement to encase the HEB-sections of composite columns, because basically the same configuration is always used, as described in subsection 3.1. The amount of concrete required is dictated by the size of the HEB-section it encases.

The proper sorting of the steel sections included in the 3 databases is an essential task that needs to be performed prior to the optimization runs. In order to achieve a well-functioning optimization process, it has to be ensured that, for any two sections i and j with $j>i$ in a database, the objective function has a higher value with section j than with section i. In simpler words, a higher selection from the database has to lead to higher materials cost. Moreover, a higher selection from the database has to lead also to improved capacity of the affected structural member(s). For the members under axial forces only (bracings), sorting the respective database according to the areas of the L-sections satisfies both material cost and member capacity requirements. For the members under bending (columns, beams), in addition to the area of each I-section, its stiffness about the axis of bending needs to be taken into account. In beams, the section stiffness only about the major axis is of interest, thus sorting the database with IPE sections is simple. Columns, however, are under biaxial bending, therefore each section’s stiffness about both the major and the minor axis has to be considered when sorting the HEB-database. Additional difficulty poses the fact that the column sections are composite.

In order to verify the proper sorting of the HEB-database, the effective stiffness of the resulting composite sections is compared to the corresponding equivalent section areas in Fig. 3. The effective stiffness $(EI)_{\text{eff}}$ of each composite column cross-section with respect to its two local axes is calculated according to EN 1994-1-1 (§6.7.3) [27], which takes into account the stiffness contributions of the structural steel section, the concrete section and the reinforcement. The total cross-sectional areas $A_{s,\text{tot}}$ given in Fig. 3 are equivalent steel areas, which are calculated using the cost ratio CR (defined in the next subsection) for the conversion of concrete areas to equivalent steel areas. Indeed, according to the graphs of Fig. 3, the HEB-database is properly sorted.

It is also interesting to visualize the contribution of each material to the total stiffness $(EI)_{\mathrm{eff}}$ of each composite section considered in the present work. Fig. 4 illustrates the percentage of the total section stiffness about the major and minor axes provided by the steel core of each section and by the surrounding concrete part together with the relevant reinforcement. It can be clearly seen that, in all composite sections, the stiffness about the minor axis is mainly provided by the concrete and the reinforcement. Their contribution in the total section stiffness is up to about 85% and at no case below 60%, while the respective maximum contribution of the steel core is of the order of 40%. Significant contribution by concrete and reinforcement is also observed in the stiffness about the major axis of the composite sections. This contribution can be almost 70% for a small-size section; contributions are lower for larger sections, but at no case below 27%. These observations highlight the large impact of concrete and its reinforcement on the structural performance of steel-concrete composite sections. Thus, for designing composite buildings, we cannot injudiciously rely on available methods and experience regarding the design optimization of pure steel structures (e.g. [1-12]); the explicit treatment of composite buildings within a specially developed design optimization framework, such as the one presented in the present work, is therefore justified.

It should be also noted that the proper handling of design variables is not ensured just by carefully sorting the section databases. The stochastic selection of design variable values in the framework of the ES optimization algorithm employed may yield designs with incompatibilities among different member sections. Two cases of such incompatibilities require special treatment in the ES implementation of the present work. The first case is associated with the realization of beam-column connections. When the width of the beam flange exceeds the space available on the column web (between the column flanges) for connecting the two members, then the corresponding design is infeasible. This incompatibility is eliminated by increasing the column section, in order to provide the web with a height that can accommodate the connection with the given beam section. The second case of incompatible member sections may arise when the section of a column is allowed to change along the height of the building. In engineering practice, the column section at a storey is not allowed to be larger than the column section at the storey directly below. When this practice is violated, the larger section (i.e. the one of the column at the higher storey) is assigned also to the column at the lower storey. The checks for such column section incompatibilities start from the columns at the top storey and proceed towards the building's base, until the columns at all storeys are processed. In both aforementioned incompatibility cases, column sections are automatically increased by the ES procedure before performing structural analyses to evaluate the design requirements of section 2.

4.2. Objective function

The objective function used implicitly monitors the total materials cost of the structural elements in the composite building considered. The total structural cost actually depends on various factors, whose influence cannot be easily predicted and quantified, such as the labor cost, the availability of materials in the market, the soil characteristics, etc. In this work, the contribution of such factors is considered to be incorporated into the total unit material costs C_S and C_C of steel and concrete, respectively. All structural parts and details that can be designed separately, such as the slabs and secondary beams, the connections, the foundation, etc., are excluded from the total cost calculation. However, as already mentioned, their contribution to the structural performance is taken into account in the structural modeling process. Thus, in this work, the term total cost refers to the materials cost for columns, beams and bracings. Furthermore, because the beams and bracings in all designs are simulated using pure steel sections, the cost of concrete refers specifically to the steel-concrete composite columns.

The total materials cost C_{tot} of a structure, which is the objective to be minimized by the employed optimization procedure, can be simply calculated as:

$$C_{tot} = C_S \cdot M_S + C_C \cdot V_C , \tag{3}$$

where C_S (€/tn) and C_C (€/m^3) are average total unit costs for steel and concrete, respectively (in engineering practice, structural steel cost is evaluated based on steel mass and reinforced concrete cost is related to concrete volume), while M_S and V_C are the total steel mass (tn) and concrete volume (m^3), respectively, used in the structure. Similar expressions referring to the total materials cost of composite structures have been utilized also in other studies (e.g. [22]). In Eq. (3) the total cost C_{tot} is calculated in monetary units (€), so its value for a particular structural design needs update in order to be consistent with current prices. For instance, any changes in the prices of construction materials, the currency exchange rate or the labor costs can affect directly or indirectly the value of C_{tot} for a given design. Hence, the calculation of C_{tot} is not a straightforward task, as estimating current values for C_S and C_C (which are intended to incorporate contributions from various factors) is cumbersome in practical applications. However, it is not necessary to determine the exact costs C_S and C_C, in order to apply the optimization formulation of this work; a relative cost can be used instead, which is easier to estimate.

Following the above discussion, a more robust objective function equation is utilized, which calculates the total equivalent steel mass of all material quantities used for the structural elements in the building considered. In order to effectively handle the buildings with composite columns, a Cost Ratio *CR* of unit cost for concrete over unit cost for steel is introduced, which allows us to convert the total concrete volume in the structure to equivalent steel mass. Then, the total equivalent steel mass M_s^{tot} (tn of steel) in the structure is the sum of the actual steel mass and the converted concrete mass. Thus, the final form of the objective function implemented in this paper is given by the equation:

$$M_s^{tot} = M_S + CR \cdot V_C \, . \tag{4}$$

The cost ratio to convert from concrete volume to equivalent steel mass is defined as $CR=C_C/C_S$, although *CR* can be directly estimated without first specifying exact values for C_C and C_S. In any case, expression (4) is simpler and easier to implement in practice than the corresponding original expression (3).

The value specified for the cost ratio *CR* plays a significant role in the estimation of the total equivalent steel mass of a structure with composite columns and therefore has an effect in the optimum design identified by the optimization algorithm. The value of *CR* needs to be separately specified in each country (maybe even in specific regions within relatively large countries) and should be expected to vary with time. For the period the test runs of the present paper were conducted, CR=0.012 tn/m^3 was estimated for Cyprus, which corresponds to ‘cheap’ concrete and ‘expensive’ steel. It is noted that cement is locally produced in Cyprus, while steel members and reinforcing bars are imported. These facts certainly affect the prices offered in the local market for these construction materials and consequently influence the estimated value of the cost ratio *CR*. In order to derive this *CR*-value, apart from the material prices of structural steel and concrete, the following items contributing to cost were taken into account: (a) connections (beam-column, beam-beam and column-base), (b) steel reinforcement and shear connectors for the composite columns and (c) scaffolding boards for the wet concrete of composite columns.

4.3. Constraints

The Eurocode and earthquake-related design requirements described in Section 2 are imposed as constraints in the developed optimization procedure. Thus, structural member capacities, system resistance under seismic action and fundamental periods are checked for each candidate optimum design. Violation of at least one of these checks renders the evaluated design infeasible. In order to evaluate the constraints, 5 structural analyses are performed for each candidate optimum design (1 linear, 2 nonlinear pushover and 2 eigenvalue analyses).

Infeasible designs are not discarded from the parent population, but are eligible to be selected for the generation of offsprings, as already mentioned in subsection 3.4. In the case of constraint violation, the fitness of the design is penalized by adding a penalty term to the objective function (4). The penalty term is equal to the total equivalent steel mass of the same building as the one evaluated, but designed with the largest section available in the respective database for each structural member, rounded up to 100 tn. In other words, the imposed penalty refers to the heaviest design possible for the database options available. In order to apply this static penalty, all constraints are organized into 5 groups; each group is associated with one of the 5 structural analyses conducted for a candidate optimum design. Immediately after a structural analysis is completed, the constraints needing the results of the particular analysis are evaluated; if at least one of the constraints in this group is violated, then the penalty term is added to the objective function. This approach for handling constraints performs well for the applications considered in the present paper.

4.4. ES implementation

The optimization software developed in the framework of the present work implements the ES algorithm described in [37]. More specifically, at each ES-generation, a population of μ parent designs produces a population of λ offspring designs ($\lambda \geq \mu$) by means of recombination and mutation operations. Then, using the so-called (μ,λ)-ES version, μ individuals are selected from the λ offsprings to form the parent population of the next generation. Convergence to the optimum solution is assumed when the best value of the objective function achieved cannot be improved upon for κ consecutive ES-generations. The parameter values μ=30, λ=30 and κ=15 are adopted in the present work. A flowchart describing macroscopically the implemented optimization procedure is presented in Fig. 5.

Although the ES procedure is a probabilistic optimizer known to be very effective in globally searching the design space, it may be trapped in a local optimum. Therefore, in an effort to avoid suboptimal final solutions, the results of multiple optimization runs for each tested case are considered. More specifically, the developed ES software is invoked in a cascade manner, with each optimization run starting from the best design attained by the previous optimization run [39,40]. The design adopted finally for each test case is the one with the lowest cost among all feasible designs detected during the cascade runs.

Cascading is applied in the present work also to accelerate the parametric study performed in the next section, which considers several similar optimization cases. Usually, the ES optimization procedure is initiated with a randomly identified feasible solution or with the heaviest possible design and then it proceeds until convergence is achieved to the optimum or a near-optimum solution. This procedure is followed in this paper, when the first design optimization case ('reference case') is processed. For another optimization case (e.g. considering a building just with a different bay width compared to the reference one), first the optimum design identified for the reference case is adjusted by strategically increasing or decreasing the section sizes of certain member groups and then this adjusted design is used to initiate the ES run. This way, the ES procedure is provided with a starting point that typically is much nearer to the optimum solution than a randomly identified initial design or the heaviest possible design. Thus, the ES run is drastically accelerated and the effect from using a static penalty approach to handle infeasible solutions is diminished.

5. Design optimization results and discussion

5.1. Design optimization results for 6-storey 5×5-bay composite building

The reference building assessed in the present work is a composite steel-concrete 6-storey space frame with 5 bays per horizontal direction (Fig. 6). The locations of the (optional) bracings are either at the middle bay (Fig. 6) or at the two corner bays of each external side of the building. The height of each storey is 3.5m, thus the total height of the building is H=21m=68.90ft and the upper limit for the fundamental period in both x- and y-directions is calculated according to formula (2) as T_{max}=1.33s. In order to investigate the effect of the bay width (which is directly related to the total seismic mass of each storey) on the optimized designs attained, 4 different beam lengths L_B from 5m to 8m are considered, yielding altogether 8 different optimization cases.

A total number of 17 member groups, which are illustrated with different colors in Fig. 6, are defined for the 6-storey composite building; one discrete design variable is assigned to each member group. In particular, columns are organized every 2 storeys into 4 groups: (1) corner, (2) peripheral in *x*-direction, (3) peripheral in *y*-direction and (4) internal. Corner columns are separately grouped, because they receive the lowest axial force due to gravitational loads, as only two beams per storey are connected to them. The remaining peripheral columns receive double axial load compared to corner columns and half axial load compared to internal columns. Moreover, when bracings are activated (whether at the middle or at the corner bays), they are connected to peripheral columns and are expected to play a significant role in the selection of the sections of these columns. Peripheral columns are separately grouped in the two horizontal directions, because the steel sections of all columns have the same orientation, which results in higher overall stiffness of the structural system in the *y*-direction. The groups containing internal columns have the largest number of members. Consequently, they can have the largest impact on the overall stiffness of the structural system, as well as to the total material mass of the structure. A total number of 3×4=12 design variables are thus defined for the columns taking values from the HEB-database.

The definition of beam-groups is based on the results of a preliminary investigation, in which it was noticed that the required beam-sections were in fact defined mainly by the gravitational loads. Indeed, in most optimization cases considered in the present work, the compressive force capacity of beams designed for the vertical gravitational loads suffices for receiving the extra stresses due to the horizontal seismic action. Moreover, in order to provide the final design with the degree of uniformity usually encountered in engineering practice, it is avoided to organize beams into different groups within each storey. However, the optimizer is given the option to modify (if needed) the design of beams along the height of the building. Therefore, the steel beams of the building are organized into 3 groups; every 2 storeys, all beams belong to one group associated with one design variable taking values from the IPE-database. It should be noted that a different design variable configuration for beams may be needed to cost-effectively withstand more severe seismic actions.

The common orientation of the steel HEB-sections across all columns creates global 'major' and 'minor' axes of the structural system, about which overturning moments may develop in the building due to seismic action. In order to allow the optimization algorithm to compensate (if needed) for the reduced stiffness about the system's 'minor' axis, 2 groups of bracings are specified, one for each horizontal direction. As each of these two groups contains a small number of elements with sections of relatively small size, the bracings do not contribute much to the total materials cost, therefore bracings are not further divided into groups along the height of the building. Thus, 2 design variables are defined for the bracings taking values from the L-database.

The final structural designs achieved for the 8 optimization cases of the 6-storey 5×5-bay building are presented in Tables 1 and 2 for bracings installed at the middle (designs 1-4) and corner (designs 5-8) bays, respectively (notice the numbers assigned to designs in the tables). As expected, higher bay widths induce the need for larger amounts of structural materials in the buildings analyzed, not only because they imply larger floor plans (and therefore larger buildings overall), but also because they correspond to larger beam spans and create larger storey masses. It is also noticed that the fundamental period constraint is satisfied in all designs attained.

As regards columns, the optimized designs can be classified into two categories. The first category includes the optimized designs, in which the column sections could be determined by a design engineer through a ‘manual’ trial-and-error procedure based on engineering judgment, without resorting to an optimization algorithm. Design 4 is the most representative member of this category: all columns in a storey share the same section (with the only exception of internal columns at storeys 5-6). Designs 3 and 5 also fall into this category, although variations of column sections in a storey are observed, but these are not large. These designs are less regular than design 4, which means that extra effort would be required to manually identify such optimized solutions.

The second category contains the optimized designs, in which the column sections are practically not detectable by a design engineer through a ‘manual’ procedure. In these designs, the optimizer employs rather complex design philosophies, which can actually be applied only by an automated procedure. Hence, asymmetries can be noticed in designs 1-2 and 6-8, which include various non-standard section combinations for the columns of each storey. It is thus evident that the optimizer operates in a rather non-predictable manner, as it is programmed to consider any section combination in the effort to identify an optimal solution. It should be however emphasized that, although the optimized designs of this category do not follow design philosophies commonly encountered in engineering practice, none of the finally achieved solutions violates any of the design constraints imposed.

As regards beams, their optimal sections do not differ or differ slightly among buildings with the same bay width regardless of the location of bracings (at corner or middle bays). Moreover, although 3 beam groups are defined along the building height, the same or about the same IPE-section is adopted for all beam groups in each building optimally designed. This regularity observed in optimal beam sections is due to the fact that the design of beams for the buildings investigated in the present work is governed in most cases by the Eurocode 3 member checks for gravitational loads. Satisfying these checks generally provides beam resistances to combined axial force and uniaxial bending moment that suffice to receive the earthquake-induced stresses. Slightly increased beam sections are dictated in a few cases by the seismic system resistance requirements, which happen to be more critical than the Eurocode 3 provisions for checking particular beams.

As regards bracings, they are contained in both x- and y-directions in all final designs yielded by the optimizer. Thus, although the ‘zero’ option available in the L-database to deactivate bracings (see subsection 4.1) allows for the selection of pure moment resisting frames in one or both directions, braced frames are consistently preferred by the optimizer in both directions. Various L-shaped bracing sections are selected by the optimizer for the finally achieved designs. For verification purposes, all optimal designs identified were reevaluated using smaller L-sections for bracings. All these reevaluations took place for reduced L-sections in one, as well as in both directions. None of the new designs failed under gravitational loads, as bracings are not supposed to participate in carrying such loads; however, the maximum interstorey drift specified was exceeded in all these designs. It should also be noted that, when building designs with the same bay width in Tables 1 and 2 are compared, the installation of bracings at the corner bays yields more cost-effective solutions than their installation at the middle bays. With the former bracings topology, a larger proportion of the required building stiffness is provided by the bracings, therefore smaller column sections can be used.

Selected optimization cases of the 6-storey 5×5-bay building are run also by deactivating the fundamental period constraint. The final structural designs attained (designs 9-11) are depicted in Table 3. Non-regular combinations of column sections in each storey are generally obtained. While beam sections are the same with corresponding cases in Tables 1 and 2, column and bracing sections are generally not the same. The designs of Table 3 have significantly lower total equivalent steel masses compared to corresponding cases in Tables 1 and 2. However, all designs of Table 3 have rather high fundamental periods (1.8-2.0s).

5.2. Design optimization results for 6-storey 8×8-bay composite building

In addition to the reference building of the previous subsection, a 6-storey 8×8-bay building is optimized with (optional) bracings installed only at the corner bays. As the number of bays is increased in this case compared to the 5×5-bay building (the height-to-plan-area ratio is significantly reduced), while the number of installed bracings remains the same, the bracings' percentage contribution to the total stiffness of the building is expected to be reduced. The optimization algorithm needs to compensate for this reduction by increasing significantly either the columns' sections or the bracings' sections or both. The beam length in the single optimization case considered for this building is L_B=6m. The upper limit for the fundamental period in both *x*- and *y*-directions is again $T_{\max}$=1.33s. The optimized design achieved (design 12) is presented in Table 4.

Particular attention needs to be paid to the design optimization of buildings with large floor plans without an adequate number of bracings to provide the required lateral stiffness. The large seismic mass per storey of such buildings leads to several candidate optimum designs processed by the optimizer that have high fundamental periods (much higher than 1s). Such high fundamental periods are related with large drifts and, consequently, infeasible designs. In the particular building considered in the present subsection, the number of such infeasible candidate solutions is rather high. This results in a cumbersome optimization process that greatly benefits from the cascade runs of the optimizer and finally yields a rather non-regular optimum design. Hence, in storeys 1-2, design 12 has the largest possible HEB-section for the peripheral columns parallel to *y*-axis, while in storeys 5-6 the same column-group has the smallest HEB-section in the building. Different attributes of sections along the building height are observed for the other column groups. Moreover, larger beam sections than those required for the gravitational loads only (IPE 270) are used, indicating that these structural elements need to contribute more to the system resistance against horizontal actions. Finally, larger bracings are installed in *x*-direction than in *y*-direction, in order to make up for the reduced overall stiffness of the structure about the *y*-axis due to the predefined orientation of the column sections. Such a design is a typical example of an optimum solution, the detection of which using a 'manual' procedure would be unlikely.

5.3. Design optimization results for 4-storey 5×5-bay composite building

Finally, a 4-storey 5×5-bay building with (optional) bracings at the corner bays is optimized. The particular building has the same floor plan configuration as the 6-storey reference building studied in subsection 5.1, but the height-to-plan-area ratio of the 4-storey building is 2/3 of the respective ratio of the 6-storey reference building. The total number of section groups and corresponding design variables for the 4-storey building is reduced to 12, as the 4 variables for the columns and the 1 variable for the beams of storeys 5-6 are deactivated. The building height is now H=14m=45.93ft and the upper limit for the fundamental period is calculated according to formula (2) as $T_{\max}$=0.96s. The optimized design attained for the 4-storey building with a beam length L_B=6m (design 13) is given in Table 5.

A comparison of designs 6 and 13 reveals that the optimizer adopts quite different philosophies in the final (optimum) designs of the 6-storey and 4-storey buildings. In design 13, as regards columns, the optimizer mainly invests in the peripheral columns parallel to the *y*-axis, as these members consistently have the largest sections at any storey of the building. On the other hand, although design 6 is not regular, there is a more even distribution of strengths among columns at each storey. The beam sections for both designs are actually the ones defined based on standard Eurocode 3 provisions for gravitational loads using linear analysis results (IPE 270). Of particular interest are the optimal bracings' sections selected. In the *x*-direction, the 4-storey building has stronger bracings than the 6-storey building, while the opposite applies in the *y*-direction. This demonstrates the complex effect of the imposed constraints (especially of the fundamental period constraint, which seems to strongly influence the selection of bracing sections) on the optimum design for each optimization case.

5.4. Convergence and computational efficiency of the optimization procedure

The convergence history of a characteristic optimization run is depicted in Fig. 7. This figure displays the gradual decrease of the objective function value achieved as more candidate optimum designs are evaluated. The figure also shows the final plateau, which signifies convergence of the optimization process. Despite the re-invocation of the optimizer to continue searching the design space by performing a second ES run, the objective function value finally attained at the initial run cannot be improved upon, therefore the optimization process stops without conducting further cascade runs. More cascade optimization runs are required in a number of other optimization cases processed in this work.

It is also interesting to analyze the computing requirements for processing the optimization case of Fig. 7. Hence, 2940 candidate optimum designs were evaluated in about 76 hours during the initial ES run and another 480 designs in about 12 hours during the second (cascade) ES run (a HP Z400 workstation with Intel Xeon CPU W3520 at 2.67 GHz and 16GB

RAM was utilized). Thus, the total computing time required to process these designs, in order to reach the final optimum solution, was about 88 hours, i.e. more than 3.5 days. These characteristic timing results reveal the huge computing demands induced by the optimization framework presented in this work. However, such high computational workloads are expected when utilizing an evolutionary optimizer (especially when run in a cascade fashion) to assess a large number of candidate optimum solutions, with each candidate requiring several (linear, nonlinear, eigenvalue) analyses to be performed. This drawback can be alleviated by accelerating the optimization computations with the use of parallel processing, advanced solution techniques and metamodel-assisted analysis predictions (e.g. using neural networks [41]).

6. Concluding remarks

This work presents an optimization framework for designing three-dimensional steel-concrete composite frames. A discrete evolutionary optimization algorithm is employed to minimize the total materials cost of a composite building subject to constraints associated with: (a) Eurocode 4 provisions for safety of composite column-members, (b) Eurocode 3 provisions for safety of steel beam-members, (c) structural system resistance to seismic action, which is assessed through interstorey drifts and top-storey displacements calculated using nonlinear pushover analyses, and (d) the building's fundamental periods to mitigate the potential for discomfort of occupants and for damage to non-structural components and building contents. It is essential to concentrate on composite buildings, because they form a special category that has not been adequately explored yet from the viewpoint of structural optimization. The reinforced concrete that encases the columns' steel core has a significant contribution to the resistance capability of composite columns under lateral loading (see Fig. 4), therefore the existing approaches and related experience developed for the design optimization of pure steel buildings do not fully apply and cannot be straightforwardly adjusted to the case of structures with steel-concrete columns. The results obtained in the present paper demonstrate the effectiveness and usefulness of the proposed design optimization approach for composite buildings.

Based on the numerical experiments conducted, some conclusions on the attributes of optimally designed composite buildings can be drawn:

- The presented optimization procedure usually yields optimum designs having arbitrary combinations of composite column sections. Section variations are observed across the column groups of a single storey, as well as over the building height. In most cases, the optimum column sections of a composite building are practically impossible to predict without invoking an optimizer. In a few cases, however, favorable designs with more or less regular combinations of column sections are identifiable also by 'manually' conducted parametric analyses. Nevertheless, in such cases, we cannot know whether a better, less regular solution exists. In other words, we always have to invoke the optimization procedure, in order to be practically certain that the detected column section combination is actually optimal.
- Usually, the steel beam sections of a composite building are dictated by the Eurocode 3 requirements evaluated for gravitational loads using linear static analysis results. In certain optimum designs, however, beams are required to participate more actively in the development of the required system resistance to seismic loads, therefore the sections of particular beams may need to be a little larger than those obtained when relying only on Eurocode 3 provisions. In any case, the Eurocode 3 requirements define the smallest acceptable steel beam sections to use in an optimized composite building.
- Bracings are typically needed in steel buildings to provide adequate lateral resistance; it appears that optimal composite buildings have similar needs. Indeed, bracings are activated by the optimizer in both x- and y-directions in all optimum designs attained in the present work. Thus, although bracings are optional, they seem to be necessary, in order to cost-effectively provide the required lateral resistance with respect to the global 'major' and 'minor' axes of a composite structural system. It should be mentioned, however, that optimum bracing sections are difficult to identify manually. Actually, this means that the interplay between the pure moment resisting and the braced frame functions of a composite building can be quantitatively treated only with the aid of an automatic optimization procedure.
- The imposed fundamental period constraints strongly influence the design of an optimized composite building and the corresponding amounts of structural materials needed. When such constraints are neglected, rather inexpensive optimal designs are obtained, which have, however, unacceptable vibration properties. On the other hand, the satisfaction of these constraints induces a significant extra cost for structural materials.

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