Constraints on anomalous dimensions from the positivity of the S matrix

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We show that the analyticity and crossing symmetry of the S matrix, together with the optical theorem, impose restrictions on the renormalization group evolution of dimension-8 operators in the Standard Model effective field theory. Moreover, in the appropriate basis of operators, the latter manifest as zeros in the anomalous dimension matrix that, to the best of our knowledge, have not been anticipated anywhere else in the literature. Our results can be trivially extended to other effective field theories.

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I. INTRODUCTION

One of the most studied aspects of quantum field theory (QFT) is the evolution of the scale-dependent parameters under renormalization group (RG) running. For renormalizable QFTs involving only scalars, fermions and gauge bosons, the explicit form of the RG equations (RGEs) is completely known up to two loops [1–11]. For QFTs involving operators of dimension larger than four, also known as effective field theories (EFTs) because they characterize only the low-energy behavior of renormalizable QFTs, this problem is significantly much more complicated. Besides the larger number of interactions, the reason is the ubiquitous mixing between different parameters, described by the anomalous dimensions matrix (ADM); see the Appendix for notation.

Since the last ten years or so, there has been significant progress toward the renormalization of EFTs, particularly in the Standard Model (SM) EFT (SMEFT) [12,13] at one loop and up to dimension 6 [14–19]. More recently, and in light of the numerous studies highlighting the insufficiency of dimension-6 terms to capture the richness of low-energy physics, either because experimental measurements are precise enough to be sensitive to higher corrections [20–23] or because certain selection rules force dimension-6 corrections to SM observables to appear at the same order in perturbation theory as dimension-8 ones [24] or simply because dimension-6 interactions do not arise in several well-motivated models of new physics [25–27], the challenge of renormalizing the dimension-8 SMEFT is being tackled from multiple angles, including off-shell approaches [28–31], amplitude methods [32] and geometry [33]. Software tools that automatize part of this process have been of enormous importance in this respect [34–36]. Still, the computations entail so many technical and conceptual challenges, that the full renormalization of the dimension-8 SMEFT, let alone arbitrary EFTs, is far from complete.

However, there are *aspects* of the RG flow of EFTs that can be understood without necessarily struggling with the explicit calculation of RGEs. One of these aspects is the existence of zeros of the ADM. These zeros indicate that certain EFT interactions are not affected by some quantum corrections, making classical predictions reliable. Approaches based on generalized unitarity [37,38], together with on-shell amplitude methods [39,40], have shown that certain classes of operators do not mix under RG running. For example, $\phi^6 D^2$ does not renormalize $B\phi^4 D^2$, or in other words $\gamma_{B\phi^4 D^2,\phi^6 D^2} = 0$. See also Refs. [33,41] for works that unveil certain nontrivial zeros in the ADM of the SMEFT, but from the perspectives of supersymmetry and of EFT geometry, respectively.

Yet, there exist other zeroes, representing the (vanishing) mixing of *certain* operators of some classes into *certain* operators of different ones, that are not explained by current methods. As a matter of example, let us consider the operator $\tilde{\mathcal{O}}^{(1)}_{e^2\phi^2 D^3} \equiv \mathcal{O}^{(1)}_{e^2\phi^2 D^3} - \mathcal{O}^{(2)}_{e^2\phi^2 D^3}$, with

$$\mathcal{O}^{(1)}_{e^2\phi^2 D^3} = \mathrm{i}(\bar{e}\gamma^\mu D^\nu e)(D_{(\mu} D_{\nu)}\phi^\dagger \phi) + \mathrm{H.c.}, \tag{1}$$

$$\mathcal{O}^{(2)}_{e^2\phi^2 D^3} = \mathrm{i}(\bar{e}\gamma^\mu D^\nu e)(\phi^\dagger D_{(\mu} D_{\nu)}\phi) + \mathrm{H.c.} \tag{2}$$

It can be shown that $\tilde{\mathcal{O}}^{(1)}_{e^2\phi^2 D^3}$ does not renormalize the following interaction,

$$\mathcal{O}^{(1)}_{B^2\phi^2 D^2} = (D^\mu \phi^\dagger D^\nu \phi) B_{\mu\rho} B^\rho_\nu, \tag{3}$$

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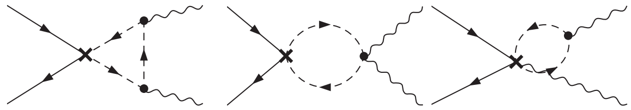

FIG. 1. Example diagrams for the renormalization of operators of type $B^2\phi^2D^2$ by interactions of the form $e^2\phi^2D^3$. The cross represents the EFT term, while the dot stands for SM couplings.

despite the fact that, obviously, there are diagrams that are separately nonvanishing; see Fig. 1.

A rough understanding of this result goes as follows. By simple dimensional analysis, the running of $c^{(1)}_{B^2\phi^2D^2}$ scales linearly with $\tilde{c}^{(1)}_{e^2\phi^2D^3}$: $\dot{c}^{(1)}_{B^2\phi^2D^2} \sim \#_1 \tilde{c}^{(1)}_{e^2\phi^2D^3}$, with $\#_1$ some arbitrary number. Moreover, from very fundamental QFT principles [42], we know that $\dot{c}^{(1)}_{B^2\phi^2D^2} \geq 0$, whereas $\tilde{c}^{(1)}_{e^2\phi^2D^3}$ can have arbitrary sign. Consequently, we must conclude that $\#_1 = 0$.

In this paper, we formulate this insight in general and precise terms, explaining and anticipating zeroes of this sort. Furthermore, we follow the same strategy to unravel the signs of certain (nonvanishing) anomalous dimensions, which dictate the increase or decrease of the EFT couplings under running.

II. DISPERSION RELATIONS AND RUNNING

Let us consider again the operators $\mathcal{O}^{(1,2)}_{e^2\phi^2D^3}$ of Eq. (1). Locality and unitarity in the UV teaches us that the Wilson coefficients of these operators must satisfy the following positivity conditions [30,43]:

$$-c^{(1)}_{e^2\phi^2D^3} - c^{(2)}_{e^2\phi^2D^3} \geq 0. \tag{4}$$

Several observations are now in order.

1. It is easy to prove that, for any pair of values $(c^{(1)}_{e^2\phi^2D^3}, c^{(2)}_{e^2\phi^2D^3})$ compatible with Eq. (4), there exists at least one well-defined (local and unitary) QFT which, at low-energies and at tree level, is described by the SMEFT where the *only* nonvanishing terms are the chosen $\mathcal{O}^{(1,2)}_{e^2\phi^2D^3}$ and, at most, other operators of the form $e^2\phi^2D^n$, with n some natural number.

2. Within any such QFT, let us compute the amplitude $\mathcal{A}(s,t)$, depending on the Mandelstam variables s and t, for $\varphi_i B \to \varphi_i B$ in the forward limit $t \to 0$, where crossing symmetry implies that $\mathcal{A}(s) = \mathcal{A}(-s)$.

Because of point 1, there are no tree-level contributions to this amplitude. Promoting s to a complex variable, the singularity structure of $\mathcal{A}(s)$ consists of a branch cut extending across the whole $\mathrm{Re}(s)$ axis due to the loops of the massless particles; see Fig. 2. Motivated by previous works on quantum gravity [44], we define the following observable:

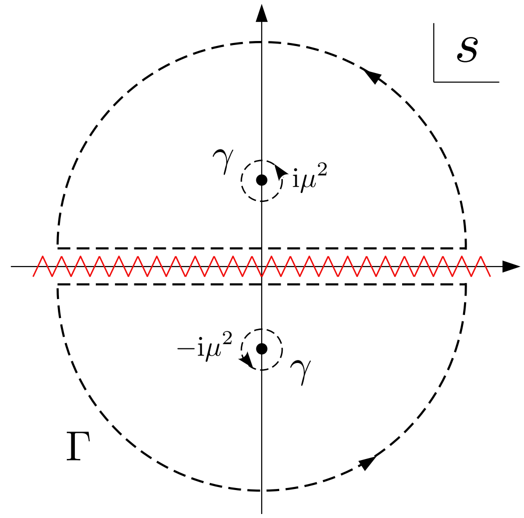

FIG. 2. Structure of the singularities of a two-to-two amplitude in the forward limit in the plane of the complex Mandelstam variable s at one loop.

$$\Sigma(\mu) \equiv \frac{1}{2\pi i}\int_\gamma \frac{\mathcal{A}(s)s^3}{(s^2+\mu^4)^3} = \frac{1}{2\pi i}\int_\Gamma \frac{\mathcal{A}(s)s^3}{(s^2+\mu^4)^3}, \tag{5}$$

where we have used the analyticity of $\mathcal{A}(s)$ to deform the contour of integration from γ to Γ in the second equality. Advocating the Froissart's bound [45], $\mathcal{A}(s)/s^3 \to 0$ at large s, the right-hand side of the equation can be computed explicitly, giving:

$$\begin{aligned}\Sigma(\mu) &= \frac{1}{\pi i}\int_0^\infty \frac{s^3}{(s^2+\mu^4)^3}\lim_{\epsilon\to 0}[\mathcal{A}(s+i\epsilon) - \mathcal{A}(s-i\epsilon)] \\ &= \frac{1}{\pi i}\int_0^\infty \frac{s^3}{(s^2+\mu^4)^3}\lim_{\epsilon\to 0}[\mathcal{A}(s+i\epsilon) - \mathcal{A}(s+i\epsilon)^*] \\ &= \frac{2}{\pi}\int_0^\infty \frac{\mathrm{Im}\mathcal{A}(s)s^3}{(s^2+\mu^4)^3} \geq 0.\end{aligned} \tag{6}$$

In the second equality, we have relied on the Schwarz's reflection principle $\mathcal{A}(s)^* = \mathcal{A}(s^*)$, while in the last one we have used the optical theorem, $\mathrm{Im}\mathcal{A}(s) \geq 0$.

So, $\Sigma(\mu)$ is positive, and from its very definition it can be computed within the EFT provided $\mu \ll \Lambda$. To order $\mathcal{O}(g_1^2)$, for any s in the neighborhood of $\pm i\mu^2$, we have:

$$\mathcal{A}(s) \sim -(\beta_8 s^2 + \beta_{12}s^4 + \cdots)\log\frac{s}{\Lambda^2}, \tag{7}$$

with β_8, β_{12}, etc. being respectively the beta functions of the dimension-8, dimension-12, etc. operators in the EFT, not present at tree level in the UV, that contribute to the amplitude at this order. In our case, β_8 is simply $-\dot{c}^{(1)}_{B^2\phi^2D^2}$, because this is the only Wilson coefficient entering the amplitude for $\varphi_i B \to \varphi_i B$.

From Eq. (7), we can compute $\Sigma(\mu)$ explicitly by using Cauchy's theorem:

$$\Sigma(\mu) = \sum_{r=\pm i\mu^2} \mathrm{Res}\frac{\mathcal{A}(s)s^3}{(s^2+\mu^4)^3}(s=r)$$
$$= \frac{1}{4\mu^4}[3\dot{c}^{(1)}_{B^2\phi^2D^2}\mu^4 + \mathcal{O}(\mu^8)] + \mathcal{O}\left(\mu^4 \log\frac{\mu}{\Lambda}\right). \quad (8)$$

Now, in the limit $\mu \to 0$, we have that:

$$\frac{4}{3}\Sigma(\mu) \approx \dot{c}^{(1)}_{B^2\phi^2D^2} \geq 0. \quad (9)$$

3. In the case at hand, $\dot{c}^{(1)}_{B^2\phi^2D^2}$ receives contributions only from loops involving the dimension-8 operators of the form $e^2\phi^2D^3$ and SM couplings; see Fig. 1. Loops involving pairs of $e^2\phi^2D^n$ operators (because of 1, other classes are absent), contain at least four external ϕ or fermions. Hence,

$$\dot{c}^{(1)}_{B^2\phi^2D^2} = \#_1 c^{(1)}_{e^2\phi^2D^3} + \#_2 c^{(2)}_{e^2\phi^2D^3} \geq 0. \quad (10)$$

4. The equation above must hold for any UV completion considered in points 1 and 2, and therefore, following 1, for all values of $c^{(1,2)}_{e^2\phi^2D^3}$ compatible with the positivity bound in Eq. (4). The only possibility is that

$$\dot{c}^{(1)}_{B^2\phi^2D^2} = -\delta(c^{(1)}_{e^2\phi^2D^3} + c^{(2)}_{e^2\phi^2D^3}), \qquad \delta \geq 0. \quad (11)$$

From here, it is clear that $\dot{c}^{(1)}_{B^2\phi^2D^2}$ vanishes in the case $c^{(1)}_{e^2\phi^2D^3} = c^{(2)}_{e^2\phi^2D^3}$, which is tantamount to say that $\tilde{\mathcal{O}}^{(1)}_{e^2\phi^2D^3}$ does not renormalize $\mathcal{O}^{(1)}_{B^2\phi^2D^2}$. A different way of looking at this consists in making the following change of basis:

$$\vec{c}_{e^2\phi^2D^3} = P_{e^2\phi^2D^3}\cdot \vec{\tilde{c}}_{e^2\phi^2D^3}, \qquad P_{e^2\phi^2D^3} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}, \quad (12)$$

where $\vec{c}_{e^2\phi^2D^3} = (c^{(1)}_{e^2\phi^2D^3} c^{(2)}_{e^2\phi^2D^3})$ and likewise for the tilde counterpart.

In this new basis, we have:

$$\dot{c}^{(1)}_{B^2\phi^2D^2} = (-\delta - \delta)\cdot P_{e^2\phi^2D^3}\cdot \vec{\tilde{c}}_{e^2\phi^2D^3}$$
$$= (\mathbf{0} - \delta)\cdot \vec{\tilde{c}}_{e^2\phi^2D^3}. \quad (13)$$

This zero in the ADM reflects the fact, mentioned at the beginning of this paper, that $\tilde{c}^{(1)}_{e^2\phi^2D^3}$ is not bounded by positivity (it can have either sign) while $\dot{c}^{(1)}_{B^2\phi^2D^2}$ must be positive.

The reasoning we just followed, consisting in points 1–4, can be applied to any pair of dimension-8 interactions (the renormalizing and the renormalized ones), provided they are both restricted by positivity, with only one caveat. Let us consider the mixing of $e^2\phi^2D^2$ into ϕ^4D^4 operators. Point 1 before remains the same. In point 2, we should compute instead the amplitude for $\varphi_i\varphi_j \to \varphi_i\varphi_j$ which, for example for $i=1$, $j=2$, implies that $\dot{c}^{(2)}_{\phi^4D^4} \leq 0$ [46,47]. However, point 3 is no longer true, because in this case there are loops involving pairs of dimension-6 $e^2\phi^2D$ interactions that contribute to the aforementioned amplitude. This implies that the conclusions in 4 do not follow and hence, *a priori*, no constraints can be obtained for the mixing at hand.

We see that what makes the difference between the two cases is that, in the former one, the renormalized operator (of type $B^2\phi^2D^2$) does contain fields (B) not present in the renormalizing interaction (of type $e^2\phi^2D^3$). On general grounds, we can establish the following result:

(i) Let $\mathcal{O}_i$ be a tree-level dimension-8 operator with $c_i \geq 0$, and $\mathcal{O}_j$ any other dimension-8 operator such that $\mathcal{O}_i$ has fields not present in $\mathcal{O}_j$.

(ii) Then, the running of $\mathcal{O}_i$ by $\mathcal{O}_j$ fulfills $\dot{c}_i = \gamma_{ij}c_j \leq 0$.

(iii) If c_j is itself bounded by positivity, namely $c_j \geq 0$, then $\gamma_{ij} \leq 0$; otherwise, $\gamma_{ij} = 0$.

Notice that the only information about the UV stays in the tree-level origin of the operator $\mathcal{O}_i$. Embedding the EFT into a well-defined UV, as in point 1, is only a trick; after all, the ADM is an infrared object and therefore should be restricted on the basis of low-energy information only.

III. CONSTRAINED ANOMALOUS DIMENSION MATRIX

Let us use the previous result to derive different restrictions on the ADM of the SMEFT, summarized in Table I and explained below. To this aim, we work out first the relevant positivity constraints:

$$c^{(2)}_{\phi^4D^4} \geq 0, \qquad c^{(1)}_{\phi^4D^4} + c^{(2)}_{\phi^4D^4} \geq 0,$$
$$c^{(1)}_{\phi^4D^4} + c^{(2)}_{\phi^4D^4} + c^{(3)}_{\phi^4D^4} \geq 0; \quad (14)$$

$$-c^{(1)}_{B^2\phi^2D^2} \geq 0, \qquad -c^{(1)}_{W^2\phi^2D^2} \geq 0; \quad (15)$$

$$-c^{(1)}_{e^2\phi^2D^3} - c^{(2)}_{e^2\phi^2D^3} \geq 0,$$
$$-(c^{(1)}_{l^2\phi^2D^3} + c^{(2)}_{l^2\phi^2D^3} + c^{(3)}_{l^2\phi^2D^3} + c^{(4)}_{l^2\phi^2D^3}) \geq 0,$$
$$c^{(3)}_{l^2\phi^2D^3} + c^{(4)}_{l^2\phi^2D^3} - c^{(1)}_{l^2\phi^2D^3} - c^{(2)}_{l^2\phi^2D^3} \geq 0; \quad (16)$$

$$-c_{e^2B^2D} \geq 0, \qquad -c_{l^2B^2D} \geq 0,$$
$$-c_{e^2W^2D} \geq 0, \qquad -c^{(1)}_{l^2W^2D} \geq 0; \quad (17)$$

TABLE I. Structure of the SMEFT ADM in the subspace of operators constrained by positivity. The $+$ ($-$) implies that the corresponding entry is ≥ 0 (≤ 0). The nonbold zeros are trivial; see the text for details.

	$c^{(1)}_{\phi^4 D^4}$	$c^{(2)}_{\phi^4 D^4}$	$c^{(3)}_{\phi^4 D^4}$	$\tilde{c}^{(1)}_{e^2\phi^2 D^3}$	$\tilde{c}^{(2)}_{e^2\phi^2 D^3}$	$\tilde{c}^{(1)}_{l^2\phi^2 D^3}$	$\tilde{c}^{(2)}_{l^2\phi^2 D^3}$	$\tilde{c}^{(3)}_{l^2\phi^2 D^3}$	$\tilde{c}^{(4)}_{l^2\phi^2 D^3}$	$c_{e^4 D^2}$	$c^{(1)}_{l^4 D^2}$	$c^{(2)}_{l^4 D^2}$	$c^{(1)}_{l^2 e^2 D^2}$	$c^{(2)}_{l^2 e^2 D^2}$
$c^{(1)}_{B^2\phi^2 D^2}$	+	+	+	**0**	−	**0**	−	**0**	−	0	0	0	0	0
$c^{(1)}_{W^2\phi^2 D^2}$	+	+	+	0	0	**0**	−	**0**	−	0	0	0	0	0
$\tilde{c}^{(2)}_{e^2\phi^2 D^3}$	+	+	+	×	×	**0**	−	**0**	−	−	0	0	**0**	−
$\tilde{c}^{(2)}_{l^2\phi^2 D^3}$	+	+	+	**0**	−	×	×	×	×	0	−	−	**0**	−
$\tilde{c}^{(4)}_{l^2\phi^2 D^3}$	+	+	+	**0**	−	×	×	×	×	0	−	−	**0**	−
$c_{e^2 B^2 D}$	0	0	0	**0**	−	0	0	0	0	−	0	0	**0**	−
$c_{l^2 B^2 D}$	0	0	0	0	0	**0**	−	**0**	−	0	−	−	**0**	−
$c_{e^2 W^2 D}$	0	0	0	**0**	−	0	0	0	0	0	0	0	**0**	−
$c^{(1)}_{l^2 W^2 D}$	0	0	0	0	0	**0**	−	**0**	−	0	−	−	0	0
$c^{(2)}_{l^2 e^2 D^2}$	0	0	0	**0**	−	**0**	−	**0**	−	−	−	−	×	×

$$
-c_{e^4 D^2} \geq 0, \qquad -c^{(2)}_{l^4 D^2} \geq 0, \qquad -(c^{(1)}_{l^4 D^2} + c^{(2)}_{l^4 D^2}) \geq 0,
$$
$$
-c^{(2)}_{l^2 e^2 D^2} \geq 0. \tag{18}
$$

Some of these relations were previously obtained in Refs. [30,43,47–50].

Next, we focus on the renormalization of $B^2\phi^2 D^2$ by $\phi^4 D^4$. In light of Eq. (15), and following the previous discussion, we have that $\dot{c}^{(1)}_{B^2\phi^2 D^2} \geq 0$ and that this inequality must hold for all values of $c^{(1)}_{\phi^4 D^4}, c^{(2)}_{\phi^4 D^4}, c^{(3)}_{\phi^4 D^4}$ compatible with Eq. (14). Therefore:

$$
\begin{aligned}
\dot{c}^{(1)}_{B^2\phi^2 D^2} &= \alpha(c^{(1)}_{\phi^4 D^4} + c^{(2)}_{\phi^4 D^4} + c^{(3)}_{\phi^4 D^4}) + \beta(c^{(1)}_{\phi^4 D^4} + c^{(2)}_{\phi^4 D^4}) \\
&\quad + \gamma c^{(2)}_{\phi^4 D^4} + \cdots \\
&= (\alpha+\beta) c^{(1)}_{\phi^4 D^4} + (\alpha+\beta+\gamma) c^{(2)}_{\phi^4 D^4} + \alpha c^{(3)}_{\phi^4 D^4} + \cdots,
\end{aligned} \tag{19}
$$

with α, β, $\gamma \geq 0$. The ellipses indicate Wilson coefficients of other operator classes, which can be turned to zero. We conclude that

$$
\gamma_{c^{(1)}_{B^2\phi^2 D^2}, c^{(2)}_{\phi^4 D^4}} \geq \gamma_{c^{(1)}_{B^2\phi^2 D^2}, c^{(1)}_{\phi^4 D^4}} \geq \gamma_{c^{(1)}_{B^2\phi^2 D^2}, c^{(3)}_{\phi^4 D^4}}; \tag{20}
$$

all them being non-negative. This result is what is shown in the first three entries of Table I, though the relation between the size of the three ADM elements themselves is not specified there.

The next **0** and $-$ in the first row of Tab. I follows from our previous discussion on the renormalization of $B^2\phi^2 D^2$ by $e^2\phi^2 D^3$; see Eq. (13). Reasoning alike for $l^2\phi^2 D^3$, we define:

$$
\vec{c}_{l^2\phi^2 D^3} = P_{l^2\phi^2 D^3} \cdot \vec{\tilde{c}}_{l^2\phi^2 D^3},
$$
$$
P_{l^2\phi^2 D^3} = \begin{bmatrix} -2 & 1 & -1 & 1 \\ 2 & 0 & 1 & 0 \\ 0 & -1 & -1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \tag{21}
$$

with $\vec{c}_{l^2\phi^2 D^3} = (c^{(1)}_{l^2\phi^2 D^3} c^{(2)}_{l^2\phi^2 D^3} c^{(3)}_{l^2\phi^2 D^3} c^{(4)}_{l^2\phi^2 D^3})$ and similarly for the tilde counterpart. In this case, taking into account Eq. (16), we have that $\tilde{c}^{(2)}_{l^2\phi^2 D^3} \leq 0$ and $\tilde{c}^{(4)}_{l^2\phi^2 D^3} \leq 0$, while the first and third ones are unconstrained. Accordingly, $\tilde{c}^{(1)}_{l^2\phi^2 D^3}$ and $\tilde{c}^{(3)}_{l^2\phi^2 D^3}$ cannot renormalize any Wilson coefficient with definite-sign beta function, which explains the zeros in their respective columns in Table I.

Following the same logic for the rest of the interactions (no more changes of basis are needed), we complete Table I. The crosses entries $\times$ in the table cannot be constrained on the basis of positivity. We note in passing that $\tilde{c}^{(1)}_{e^2\phi^2 D^3}$, $\tilde{c}^{(1)}_{l^2\phi^2 D^3}$, etc. do renormalize other operators, obviously not restricted by positivity, as it can be checked by explicit computation. For example:

$$
\dot{\tilde{c}}^{(1)}_{l^2\phi^2 D^3} \propto (g_1^2 - 3Y_e^2)\tilde{c}^{(1)}_{e^2\phi^2 D^3}. \tag{22}
$$

Two more comments are still in order. First, note that the $+$ ($-$) entries in Table I indicate that the corresponding entries are positive (negative) irrespective of the actual values of the SM couplings, meaning that they could be proportional to combinations like for example $g_2^2 + Y_e^2$, but not $g_2 Y_e$ or $g_2^2 - Y_e^2$; for example the latter can have arbitrary sign depending on whether one takes the limit $g_2 \ll Y_e$ or $g_2 \gg Y_e$. (This contrasts with what occurs to most of the other anomalous dimensions; check simply

Eq. (22) above.) So the table provides more information than one could naively expect.

And second, on top of the definite signs, there exist inequalities between different ADM entries. We have already commented on some of them; see Eq. (20). The rest are

$$\gamma_{c^{(1)}_{W^2\phi^2D^2},c^{(2)}_{\phi^4D^4}} \geq \gamma_{c^{(1)}_{W^2\phi^2D^2},c^{(1)}_{\phi^4D^4}} \geq \gamma_{c^{(1)}_{W^2\phi^2D^2},c^{(3)}_{\phi^4D^4}}; \quad (23)$$

$$\gamma_{c^{(1)}_{e^2\phi^2D^3},c^{(2)}_{\phi^4D^4}} \geq \gamma_{c^{(1)}_{e^2\phi^2D^3},c^{(1)}_{\phi^4D^4}} \geq \gamma_{c^{(1)}_{e^2\phi^2D^3},c^{(3)}_{\phi^4D^4}}; \quad (24)$$

$$\gamma_{\tilde{c}^{(2)}_{l^2\phi^2D^3},c^{(2)}_{\phi^4D^4}} \geq \gamma_{\tilde{c}^{(2)}_{l^2\phi^2D^3},c^{(1)}_{\phi^4D^4}} \geq \gamma_{\tilde{c}^{(2)}_{l^2\phi^2D^3},c^{(3)}_{\phi^4D^4}}; \quad (25)$$

$$\gamma_{\tilde{c}^{(4)}_{l^2\phi^2D^3},c^{(2)}_{\phi^4D^4}} \geq \gamma_{\tilde{c}^{(4)}_{l^2\phi^2D^3},c^{(1)}_{\phi^4D^4}} \geq \gamma_{\tilde{c}^{(4)}_{l^2\phi^2D^3},c^{(3)}_{\phi^4D^4}}; \quad (26)$$

and

$$\left|\gamma_{\tilde{c}^{(2)}_{l^2\phi^2D^3},c^{(1)}_{l^4D^2}}\right| \leq \left|\gamma_{\tilde{c}^{(2)}_{l^2\phi^2D^3},c^{(2)}_{l^4D^2}}\right|, \quad (27)$$

$$\left|\gamma_{\tilde{c}^{(4)}_{l^2\phi^2D^3},c^{(1)}_{l^4D^2}}\right| \leq \left|\gamma_{\tilde{c}^{(4)}_{l^2\phi^2D^3},c^{(2)}_{l^4D^2}}\right|, \quad (28)$$

$$\left|\gamma_{c_{l^2B^2D},c^{(1)}_{l^4D^2}}\right| \leq \left|\gamma_{c_{l^2B^2D},c^{(2)}_{l^4D^2}}\right|, \quad (29)$$

$$\left|\gamma_{c^{(1)}_{l^2W^2D},c^{(1)}_{l^4D^2}}\right| \leq \left|\gamma_{c^{(1)}_{l^2W^2D^3},c^{(2)}_{l^4D^2}}\right|, \quad (30)$$

$$\left|\gamma_{c^{(2)}_{l^2e^2D^2},c^{(1)}_{l^4D^2}}\right| \leq \left|\gamma_{c^{(2)}_{l^2e^2D^2},c^{(2)}_{l^4D^2}}\right|. \quad (31)$$

IV. CONCLUSIONS

We have derived a number of restrictions on the anomalous dimensions of the SMEFT at dimension 8, relying uniquely on the crossing symmetry, analyticity and positivity of the imaginary part of two-to-two scattering amplitudes in the forward limit.

This way, restricting to the electroweak sector of the SMEFT with only one flavor, and in the appropriate basis of operators, we have found 52 elements of the ADM that must have definite sign (either nonpositive or nonnegative), as well as 24 non-trivial zeros. Moreover, we have found inequalities involving the aforementioned anomalous dimensions themselves.

We can envisage different future directions. To start with, it would be desirable to cross-check our results by explicit calculation. (Those concerning operators with only e, B and ϕ will be given in a longer companion paper.) Also, we can envision applying these findings to phenomenological studies where the running of dimension-8 operators might be important [20,22,31,51,52]. Likewise, it would be interesting to extend these results to the full SMEFT (that means, including colour and flavor) as well as to the LEFT [53] and other EFTs, with the aim of understanding better the quantum structure of these theories.

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APPENDIX: NOTATION AND CONVENTIONS

Within this paper, we ignore color and flavor. We use the following notation for the SM fields: e and l represent the right- and left-handed leptons; B and W refer to the $U(1)_Y$ and $SU(2)_L$ gauge bosons, respectively, and g_1 and g_2 are the corresponding gauge couplings; $\phi = (\varphi_1 + \mathrm{i}\varphi_2, \varphi_3 + \mathrm{i}\varphi_4)^T$ stands for the Higgs doublet. Thus, the relevant SM Lagrangian reads:

$$\begin{aligned}\mathcal{L}_{\mathrm{SM}} = &-\frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \bar{l}\mathrm{i}\not{D}l + \bar{e}\mathrm{i}\not{D}e \\ &+ (D_\mu\phi)^\dagger(D^\mu\phi) + \mu_\phi^2|\phi|^2 - \lambda_\phi|\phi|^4 - (\bar{l}\phi Y_e e + \mathrm{H.c.}),\end{aligned} \quad (\mathrm{A1})$$

where Y_e is the Yukawa coupling and $\tilde{\phi} = \mathrm{i}\sigma_2\phi^*$ with σ_I being the Pauli matrices; $I = 1, 2, 3$.

In the absence of lepton-number violation, the SMEFT Lagrangian reads:

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \frac{1}{\Lambda^2}\sum_i c_i^{(6)}\mathcal{O}_i^{(6)} + \frac{1}{\Lambda^4}\sum_j c_j^{(8)}\mathcal{O}_j^{(8)} + \cdots, \quad (\mathrm{A2})$$

where $\Lambda \gg 100$ GeV represents the cutoff below which the SMEFT is no longer a valid theory, and the ellipses encode higher-dimensional operators. The first sum runs over a basis of dimension-6 interactions [12], while the second does it over the dimension-8 counterpart [26,54]. In this work, we are mostly interested in the dimension-8 Wilson coefficients, which we parametrize using the notation of Ref. [26],[1] from where the field content of the interactions

[1]The only change we make in this respect is that, for the second l^4D^2 operator, we consider the more commonly used

$$\mathcal{O}^{(2)}_{l^4D^2} = D_\nu(\bar{l}\gamma^\mu\sigma_I l)D^\nu(\bar{l}\gamma_\mu\sigma_I l).$$

is apparent. (For example, the class of operators with two e fields, one B boson, one Higgs and one derivative is named $e^2B\phi^2D$.) The dependence of dimension-8 operators on the energy scale $\tilde{\mu}$ is governed by the corresponding beta functions, which at one loop read:

$$\beta_{c_i} = \dot{c}_i^{(8)} = 16\pi^2\tilde{\mu}\frac{dc_i^{(8)}}{d\tilde{\mu}} = \gamma_{ij}c_j^{(8)} + \gamma'_{ijk}c_j^{(6)}c_k^{(6)}, \quad \text{(A3)}$$

where we have indicated explicitly that both β_{c_i} and $\dot{c}_i$ name the same object.

A notable part of the ADM γ has been already computed explicitly in Refs. [30,32,33], but most is still missing. Likewise for γ'; see Refs. [29,31,33]. In this work, we want rather to unveil certain correlations (some known, some others not previously anticipated) between different entries in γ without relying on explicit calculations.

[1] M. E. Machacek and M. T. Vaughn, Nucl. Phys. **B222**, 83 (1983).
[2] M. E. Machacek and M. T. Vaughn, Nucl. Phys. **B236**, 221 (1984).
[3] M. E. Machacek and M. T. Vaughn, Nucl. Phys. **B249**, 70 (1985).
[4] I. Jack and H. Osborn, J. Phys. A **16**, 1101 (1983).
[5] M.-x. Luo, H.-w. Wang, and Y. Xiao, Phys. Rev. D **67**, 065019 (2003).
[6] F. Lyonnet, I. Schienbein, F. Staub, and A. Wingerter, Comput. Phys. Commun. **185**, 1130 (2014).
[7] F. Lyonnet and I. Schienbein, Comput. Phys. Commun. **213**, 181 (2017).
[8] M.-x. Luo and Y. Xiao, Phys. Lett. B **555**, 279 (2003).
[9] R. M. Fonseca, M. Malinský, and F. Staub, Phys. Lett. B **726**, 882 (2013).
[10] I. Schienbein, F. Staub, T. Steudtner, and K. Svirina, Nucl. Phys. **B939**, 1 (2019); **B966**, 115339(E) (2021).
[11] L. Sartore and I. Schienbein, Comput. Phys. Commun. **261**, 107819 (2021).
[12] B. Grzadkowski, M. Iskrzynski, M. Misiak, and J. Rosiek, J. High Energy Phys. 10 (2010) 085.
[13] I. Brivio and M. Trott, Phys. Rep. **793**, 1 (2019).
[14] E. E. Jenkins, A. V. Manohar, and M. Trott, J. High Energy Phys. 01 (2014) 035.
[15] E. E. Jenkins, A. V. Manohar, and M. Trott, J. High Energy Phys. 10 (2013) 087.
[16] R. Alonso, E. E. Jenkins, A. V. Manohar, and M. Trott, J. High Energy Phys. 04 (2014) 159.
[17] R. Alonso, H.-M. Chang, E. E. Jenkins, A. V. Manohar, and B. Shotwell, Phys. Lett. B **734**, 302 (2014).
[18] Y. Liao and X.-D. Ma, J. High Energy Phys. 11 (2016) 043.
[19] S. Davidson, M. Gorbahn, and M. Leak, Phys. Rev. D **98**, 095014 (2018).
[20] M. Ardu and S. Davidson, J. High Energy Phys. 08 (2021) 002.
[21] C. Hays, A. Martin, V. Sanz, and J. Setford, J. High Energy Phys. 02 (2019) 123.
[22] T. Corbett, A. Helset, A. Martin, and M. Trott, J. High Energy Phys. 06 (2021) 076.
[23] T. Corbett, J. Desai, O. J. P. Éboli, M. C. Gonzalez-Garcia, M. Martines, and P. Reimitz, Phys. Rev. D **107**, 115013 (2023).
[24] A. Azatov, R. Contino, C. S. Machado, and F. Riva, Phys. Rev. D **95**, 065014 (2017).
[25] M. Chala, C. Krause, and G. Nardini, J. High Energy Phys. 07 (2018) 062.
[26] C. W. Murphy, J. High Energy Phys. 10 (2020) 174.
[27] G. Durieux, G. Durieux, M. McCullough, M. McCullough, E. Salvioni, and E. Salvioni, J. High Energy Phys. 12 (2022) 148; 02 (2023) 165(E).
[28] M. Chala and A. Titov, Phys. Rev. D **104**, 035002 (2021).
[29] M. Chala, G. Guedes, M. Ramos, and J. Santiago, SciPost Phys. **11**, 065 (2021).
[30] S. Das Bakshi, M. Chala, A. Díaz-Carmona, and G. Guedes, Eur. Phys. J. Plus **137**, 973 (2022).
[31] K. Asteriadis, S. Dawson, and D. Fontes, Phys. Rev. D **107**, 055038 (2023).
[32] M. Accettulli Huber and S. De Angelis, J. High Energy Phys. 11 (2021) 221.
[33] A. Helset, E. E. Jenkins, and A. V. Manohar, J. High Energy Phys. 02 (2023) 063.
[34] T. Hahn and M. Perez-Victoria, Comput. Phys. Commun. **118**, 153 (1999).
[35] T. Hahn, Comput. Phys. Commun. **140**, 418 (2001).
[36] A. Carmona, A. Lazopoulos, P. Olgoso, and J. Santiago, SciPost Phys. **12**, 198 (2022).
[37] Z. Bern, L. J. Dixon, D. C. Dunbar, and D. A. Kosower, Nucl. Phys. **B435**, 59 (1995).
[38] Z. Bern, L. J. Dixon, D. C. Dunbar, and D. A. Kosower, Nucl. Phys. **B425**, 217 (1994).
[39] C. Cheung and C.-H. Shen, Phys. Rev. Lett. **115**, 071601 (2015).
[40] N. Craig, M. Jiang, Y.-Y. Li, and D. Sutherland, J. High Energy Phys. 08 (2020) 086.
[41] J. Elias-Miro, J. R. Espinosa, and A. Pomarol, Phys. Lett. B **747**, 272 (2015).
[42] A. Adams, N. Arkani-Hamed, S. Dubovsky, A. Nicolis, and R. Rattazzi, J. High Energy Phys. 10 (2006) 014.
[43] X. Li and S. Zhou, Phys. Rev. D **107**, L031902 (2023).
[44] M. Herrero-Valea, R. Santos-Garcia, and A. Tokareva, Phys. Rev. D **104**, 085022 (2021).
[45] M. Froissart, Phys. Rev. **123**, 1053 (1961).
[46] M. Chala and J. Santiago, Phys. Rev. D **105**, L111901 (2022).

[47] G. N. Remmen and N. L. Rodd, J. High Energy Phys. 12 (2019) 032.
[48] Q. Bi, C. Zhang, and S.-Y. Zhou, J. High Energy Phys. 06 (2019) 137.
[49] G. N. Remmen and N. L. Rodd, Phys. Rev. Lett. **125**, 081601 (2020).
[50] J. Gu, L.-T. Wang, and C. Zhang, Phys. Rev. Lett. **129**, 011805 (2022).
[51] G. Panico, A. Pomarol, and M. Riembau, J. High Energy Phys. 04 (2019) 090.
[52] S. Alioli *et al.*, in 2022 Snowmass Summer Study (2022), arXiv:2203.06771.
[53] E. E. Jenkins, A. V. Manohar, and P. Stoffer, J. High Energy Phys. 03 (2018) 016.
[54] H.-L. Li, Z. Ren, J. Shu, M.-L. Xiao, J.-H. Yu, and Y.-H. Zheng, Phys. Rev. D **104**, 015026 (2021).