



## TOTAL FATIGUE LIFE ESTIMATION OF AIRCRAFT STRUCTURAL COMPONENTS USING STRAIN ENERGY DENSITY METHOD

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**Abstract:** This paper is aimed at developing a suitable computation method for estimating the fatigue life of structural elements exposed to the load spectrum. The total fatigue life can be divided into two parts, until the appearance of the initial damage and the other part represents the remaining life, i.e. until the effective fracture. The conventional approach to estimating the total life requires that low-cycle fatigue characteristics of the material be used until the initial damage occurs, and dynamic characteristics of the material for the remaining life. In order to obtain a more efficient method, the Strain Energy Density (SED) method was used in this paper. The essence of this approach is to use the same low - cycle fatigue characteristics of the material to estimate the life expectancy and the remaining life. This work is focused to developing efficient computation method and software for total fatigue life of metal aircraft structural components. To obtain efficient computation method, here the same fatigue low cyclic material properties for crack initiation and crack growth are used together with finite element method (FEM) for stress analyzes. To validate quality computation methods and in-house software for fatigue life estimations computation results are compared with experiments. The results show that the predicted results agree well with the test data.

**Key words:** Aircraft structures, Fatigue, Crack initiation, Crack growth, Total fatigue life, FEM

### 1. INTRODUCTION

In the work related to fatigue behavior of real aircraft structural components under service loading, one of the fundamental issues is the evaluation and formulation of adequate relations which should describe fatigue life. Fatigue as a very complex process can be considered and analyzed as: 1) crack initiation phase and 2) crack propagation phase. In this paper we developed computational model for total fatigue life estimation of aircraft structural components.

Aircraft structural components and structural components of the other mechanical systems are components with geometrical discontinuities and the same are known as components with notches or notched structural components. As a rule, cracks appear at critical points which lie in the bases of notches. Because of that notches have important role in process of fatigue life investigation. Total fatigue life (TFL) of a particular aircraft structural component is sum of crack initiation and crack growth life (CIL and CGL), and mainly depends of number, form, size, position and arrangement

of its notches. CIL computation of aircraft structural components understands knowing: aircraft flight cycles, cyclic events in flight cycles, cyclic properties of material used or nominated for workmanship, stress-strain response at critical point or point of expected crack initiation and damages provoked by all cyclic events.

Stress-strain response at critical point (local stress-strain response) for all cyclic events and method of identification and counting of those events, have special importance. Local stress-strain response may be determined by strain gage measurement, by the finite element method (FEM) and by the methods that relate local stresses ( $\sigma_{loc}$ ) and strains ( $\epsilon_{loc}$ ) to nominal values. Identification and counting of cyclic events in aircraft flight cycles may be carried out using rain flow counting method, range pair method [1] and alike good method of reservoir [2]. Method of exact damages and CIL computation has not been developed so far. All known is based on numerous hypotheses (rules). Because of simplicity, Palmgreen-Miner's rule of linear damage accumulation is mostly in use [1-5]. For TFL computation, several investigators [6,7] have combined the two approaches by combining the computed results

for crack initiation and crack growth. The aim of this paper is to compute (to predict) TFL (TFL = CIL + CGL) of notched structural components using the same input parameters (cyclic properties) for CIL and CGL computation.

Methodologies of CIL and CGL computation of notched structural components based on criterions of low cycle fatigue (LCF) [1,8] and strain energy density (SED) [9] are described in this paper. The results obtained by computation and experimentally obtained results are compared.

## 2. CRACK INITIATION LIFE OF STRUCTURAL ELEMENTS

To estimate the age until the appearance of initial damage of structural elements subjected to cyclic loads and where material plasticization occurs in critical zones, it is necessary to use the material's cyclic behavior curves. For this purpose, the SWT relation was used. The Smith-Watson-Topper (SWT) relation for describing the low-cycle fatigue curve has the form:

$$P_{SWT} = \sqrt{\sigma_{\max} \frac{\Delta \varepsilon}{2} E} = \sqrt{(\sigma'_f)^2 (N_f)^{2b} + E \sigma'_f \varepsilon'_f (N_f)^{b+c}} \quad (2)$$

where the influence of mean stresses is included via the dependence

$$\sigma_{\max} = \sigma_m + \frac{\Delta \sigma}{2} \quad (2.2)$$

The notation  $P_{SWT}$  in (2.1) refers to the Smith-Watson-Topper parameter. The relation SWT (2.1) defines that there is no fatigue damage in situations where the value of the maximum stress,  $\sigma_{\max}$ , is zero or has a negative value, which is not entirely true.

## 3. CRACK GROWTH MODEL BASED ON STRAIN ENERGY DENSITY METHOD

In this work fatigue crack growth method based on energy concept is considered and then it is necessary to determine the energy absorbed till failure. This energy can be calculated by using cyclic stress-strain curve. Function between stress and strain, as recommended by Ramberg-Osgood provides good description of elastic-plastic behavior of material, and may be expressed as:

$$\Delta \varepsilon = \frac{\Delta \sigma}{E} + 2 \left( \frac{\Delta \sigma}{2k'} \right)^{\frac{1}{n'}} \quad (3.1)$$

where  $E$  is the modulus of elasticity,  $\Delta \varepsilon/2$  is strain amplitude and  $\Delta \sigma/2$  is stress amplitude. Equation (3.1) enables the calculation of the stress-strain distribution by knowing low cyclic fatigue properties. As a result the energy absorbed till failure become [8,9]:

$$W_c = \frac{4}{1+n'} \sigma'_f \varepsilon'_f \quad (3.2)$$

where  $\sigma'_f$  is cyclic yield strength and  $\varepsilon'_f$  - fatigue ductility coefficient. Given the fact that strain energy density method is considered, the energy absorbed till failure must be determined after the energy concept is based on the following fact: The energy absorbed per unit growth of crack is equal to the plastic energy dissipated within the process zone per cycle. This energy concept is expressed by:

$$W_c \delta a = \omega_p, \quad (3.3)$$

where  $W_c$  is energy absorbed till failure,  $\omega_p$  - the plastic energy and  $a$  - the crack length. In equation (3.3) it is necessary just to determine the plastic energy dissipated in the process zone  $\omega_p$ . By integration of equation for the cyclic plastic strain energy density in the units of Joule per cycle per unit volume [8,9] from zero to the length of the process zone ahead of crack tip  $d^*$  it is possible to determine the plastic energy dissipated in the process zone  $\omega_p$ . After integration relation of the plastic energy dissipated in the process zone becomes:

$$\omega_p = \left( \frac{1-n'}{1+n'} \right) \frac{\Delta K_I^2 \psi}{E I_{n'}} \quad (3.4)$$

where  $\Delta K_I$  is the range of stress intensity factor,  $\psi$  - constant depending on the strain hardening exponent  $n'$ ,  $I_{n'}$  - the non-dimensional parameter depending on  $n'$ . Fatigue crack growth rate can be obtained by substituting Eq. (3.2) and Eq. (3.4) in Eq. (3.3):

$$\frac{da}{dN} = \frac{(1-n')\psi}{4E I_{n'} \sigma'_f \varepsilon'_f} (\Delta K_I - \Delta K_{th})^2, \quad (3.5)$$

where  $\Delta K_{th}$  is the range of threshold stress intensity factor and is function of stress ratio i.e.

$$\Delta K_{th} = \Delta K_{th0}(1-R)^\gamma, \quad (3.6)$$

$\Delta K_{th0}$  is the range of threshold stress intensity factor for the stress ratio  $R = 0$  and  $\gamma$  is coefficient (usually,  $\gamma = 0.71$ ). Finally number of cycles till failure can be determined by integration of relation for fatigue crack growth rate:

$$N = B \int_{a_0}^{a_c} \frac{da}{(\Delta K_I - \Delta K_{th})^2}, \quad B = \frac{4E I_{n'} \sigma'_f \varepsilon'_f}{(1-n')\psi} \quad (3.7)$$

and

$$\Delta K_I = Y S \sqrt{\pi a}, \quad (3.8)$$

Equation (3.7) enables us to determine crack growth life of different structural components. Very important fact is that equation (3.7) is easy for application since low cyclic

material properties ( $n'$ ,  $\sigma'_f$ ,  $\epsilon_f$ ) available in literature are used as parameters. The only important point is stress intensity factor which, depending on the geometry complexity and the type of loading, could be determined by using analytical and/or numerical approaches.

From fatigue crack growth relations (3.5) and (3.6) it can be seen that they require only mechanical and low-cyclic fatigue properties  $E$ ,  $\sigma'_f$ ,  $\epsilon'_f$  and  $n'$ , which presents great advantage by application of this method.

#### 4. NUMERICAL VALIDATION

The validity of the considered computation total fatigue life estimation method can only be assessed through a comparison with experimental data which is the focus of this section. In addition, stress intensity factors were obtained by using analytical and numerical approaches. In the case where numerical simulation was used for stress intensity factor for crack growth life estimation the evaluated polynomial expressions were used for corrective function which include geometry of structural element.

##### 4.1. Structural analysis of aircraft wing-fuselage joint

Computation estimates of the total life were made here, which include calculations until the appearance of initial damage, as well as an estimate of the residual life after the appearance of the initial crack, i.e. during the crack propagation. For this purpose, a part of the wing structure at the wing/fuselage junction was considered. In order to verify the computation procedure for the estimation of the life, representative parts of the wing were made in the form of "complex" test specimen, which represent a real part of the structure of the wing of the aircraft. Computation estimates of life, as well as tests, were performed with the same ("real") load spectrum. In the following discussions, only the essential part of the

research results will be presented, both in the domain of computation estimates and the results of fatigue tests. Fig. 1 shows part of aircraft wing-fuselage joint.



Figure 1. Part of aircraft wing-fuselage joint

The analysis of the stress state was performed using FEM [10]. For this purpose, due to the systematicity of the research, a linear analysis of FEM as well as an elastic-plastic analysis of FEM were applied. Elastic-plastic analysis was performed using FEM for the cyclic characteristics of the behavior of duraluminium, Table 2

Figure 2 shows the load spectrum of part of the wing-fuselage connection (one block corresponds to 50 hours of aircraft flight). Table 1 shows the load spectrum as well as the corresponding stresses for each load level in the spectrum. To determine the stress state for certain load levels, the elastic-plastic analysis of FEM was used. The material of the wing joint, that is, the part of the wing that is the subject of analysis, is made of duraluminium alloy 2024-T351, whose cyclic behavior curve of the material is shown in Table 2.

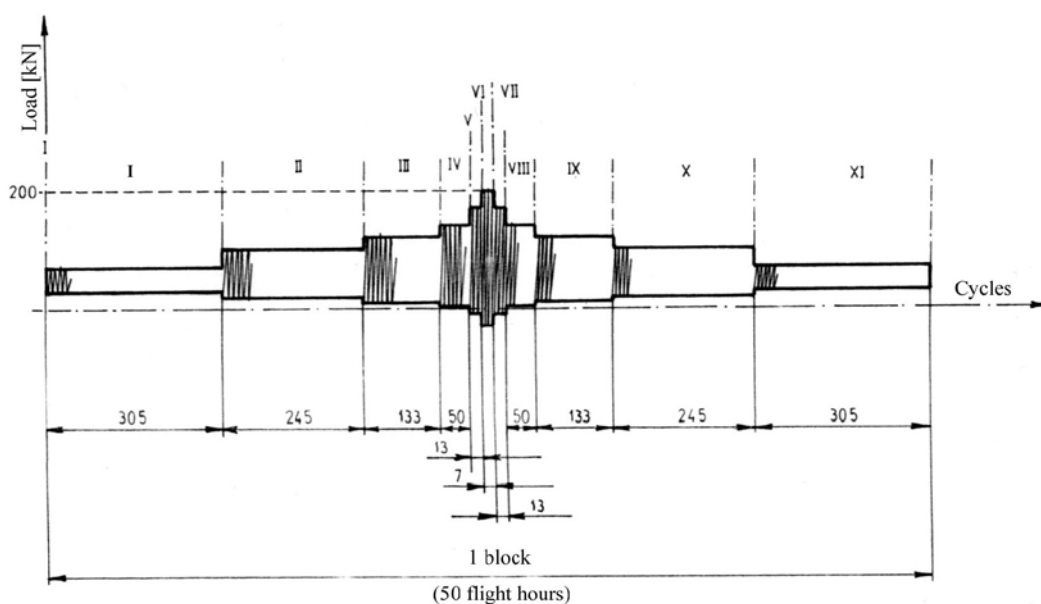


Figure 2. Load spectrum in the wing-fuselage connection (per one screw in the wing-fuselage connection)

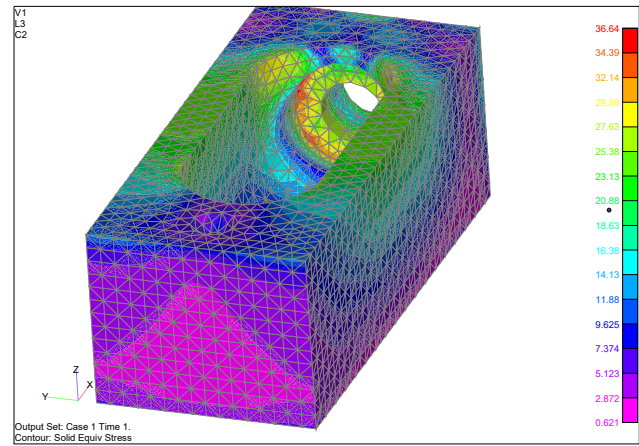
**Table 1.** Load spectrum and corresponding stresses in the critical position in the wing/fuselage connection section

Load level	$n_i$	$F_{\min}$ [kN]	$F_{\max}$ [kN]	$\sigma_{\min}$ [MPa]	$\sigma_{\max}$ [MPa]
I	305	13.3	33.3	58.8	147.2
II	245	9.30	46.6	41.0	205.5
III	133	6.0	60.0	28.0	279.8
IV	50	3.0	73.3	13.3	313.8
V	13	-4.6	86.6	20.3	336.5
VI	7	-14.0	100.0	61.9	366.4
VII	13	-4.6	86.6	20.3	336.5
VIII	50	3.0	73.3	13.3	313.8
IX	133	6.0	60.0	28.0	279.8
X	245	9.3	46.6	41.0	205.5
XI	305	13.3	33.3	58.8	147.2

Figure 3 shows the finite element model of the wing part at the point of connection of the wing to the fuselage of the aircraft. When it comes to structural elements with complex geometric shapes and loads, it is necessary to first determine the potentially critical zone in which the occurrence of damage under the effect of cyclic loads is first expected. For this purpose, the finite element method (FEM) was used in this case. When it comes to estimating the age until the appearance of initial damage, a precise calculation of stress states is required, especially in the critical zone of the structure. Two methods can be used to determine the elastic-plastic stress state: a) Using the linear analysis of FEM, cyclic material behavior curves and Neuber curves whose intersection determines the elastic-plastic stress state; b) Using elastic-plastic FEM analysis for all load levels within the spectrum, Fig. 3. The elastic-plastic analysis of the FEM was used for the computation assessment of the life until the appearance of initial damage, and the computation results are given in table 1. Neuber's approach based on the use of linear FEM analysis is used here only as a control computation.

**Table 2.** Low-Cyclic material properties of duraluminium 2024 T351

Property	Value
Modulus of elasticity, E [MPa]	70430.0
Cyclic strength coefficient, $K'$ [MPa]	794
Cyclic strain hardening exponent, $n'$	0.0919
Fatigue strength coefficient, $\sigma'_f$ [MPa]	740.0
Fatigue strength exponent, b	-0.0701
Fatigue ductility coefficient, $\epsilon'_f$	0.334
Fatigue ductility exponent, c	-1.2

**Figure 3.** Elastic-plastic FEM analysis in the part of the wing structure for the maximum load in the spectrum (Level VI in the spectrum)

### Initial fatigue life estimation

For a given load block, Fig. 2, as well as the stress states determined for each load level, the number of blocks until the appearance of initial damage can be determined. SWT (Swish-Watson-Topper) relation, given by equation (2.1), was used to estimate the fatigue life until the appearance of initial damage, expressed through the number of blocks  $N_{bl}$ . The last relation takes into account the effects of medium stresses on the estimate of remaining life, and experience has shown that it is the most accurate for life estimates with the influence of medium stresses. Finally, to estimate the lifetime until the appearance of initial damage, the elastic-plastic analysis of FEM was used to determine the stress states in conjunction with the SWT (Swish-Watson-Topper) relation. Using this approach, the number of blocks until the appearance of initial damage,  $N_{bl,i}$ , was determined. The computation initial fatigue life until the appearance of initial damage, expressed through the number of blocks, using previous relations, is given in Table 3.

**Table 3.** Estimated initial fatigue life to failure of the wing-fuselage connection according to (SWT) and Morrow

Podaci o opterećenju				Podaci o materijalu				Broj blokova SWT	
2024 T351 Naziv: 2024 T351				Naziv Materijala: 2024 T351				480.1830472088	
Smin1	58.800	Smax1	147.200	n1	305	Kprim:	794	Broj blokova Morrow	505.5855210519
Smin2	41.000	Smax2	205.500	n2	245	E:	70430	n prim:	0.0919
Smin3	28.000	Smax3	279.800	n3	133	Epsilonf:	0.334	b:	-0.0701
Smin4	13.300	Smax4	313.800	n4	50	Sigmaf:	740	c:	-1.2
Smin5	20.300	Smax5	336.500	n5	13	Kt:	1.49	Broj nivoa	11
Smin6	61.900	Smax6	366.400	n6	7				
Smin7	20.300	Smax7	336.500	n7	13				
Smin8	13.300	Smax8	313.800	n8	50				
Smin9	28.000	Smax9	279.800	n9	133				
Smin10	41.000	Smax10	205.500	n10	245				
Smin11	58.800	Smax11	147.200	n11	305				
Smin12	0.000	Smax12	0.000	n12	0				
Smin13	0.000	Smax13	0.000	n13	0				
Smin14	0.000	Smax14	0.000	n14	0				
Smin15	0.000	Smax15	0.000	n15	0				
Smin16	0.000	Smax16	0.000	n16	0				
Smin17	0.000	Smax17	0.000	n17	0				
Smin18	0.000	Smax18	0.000	n18	0				
Smin19	0.000	Smax19	0.000	n19	0				
Smin20	0.000	Smax20	0.000	n20	0				
Smin21	0.000	Smax21	0.000	n21	0				

	Maksim	Minima	sig1	sig2	eps1	eps2	E1	BrojCiklusa	DeltaN/Nf	Izabrani test
	147.2	58.8	219.2395	87.582719	0.0031145	0.00124	0.00209	5522804.3	18107.555	Morrow
	205.5	41	305.1526	60.047720	0.0043629	0.00088	0.00291	3501810.8	14293.105	Morrow
	279.8	28	398.5146	23.342555	0.0062108	0.00088	0.00398	971327.44	7303.2139	Morrow
	313.8	13.3	428.6573	-19.01452	0.0073078	0.00094	0.00449	125191.51	2503.8303	Morrow
	336.5	20.3	445.1848	-25.82688	0.0081647	0.00147	0.00486	53123.145	4086.3957	Morrow
	366.4	61.9	464.2674	10.641242	0.0095030	0.00306	0.00542	42304.004	6043.4292	Morrow
	336.5	20.3	445.1848	-25.82688	0.0081647	0.00147	0.00486	53123.145	4086.3957	Morrow
	313.8	13.3	428.6573	-19.01452	0.0073078	0.00094	0.00449	125191.51	2503.8303	Morrow
	279.8	28	398.5146	23.342555	0.0062108	0.00088	0.00398	971327.44	7303.2139	Morrow
	205.5	41	305.1526	60.047720	0.0043629	0.00088	0.00291	3501810.8	14293.105	Morrow
	147.2	58.8	219.2395	87.582719	0.0031145	0.00124	0.00209	5522804.3	18107.555	Morrow
	147.2	58.8	219.2395	87.582719	0.0031145	0.00124	0.00209	3413680.8	11192.396	SWT
	205.5	41	305.1526	60.047720	0.0043629	0.00088	0.00291	2376607.9	9700.4406	SWT
	279.8	28	398.5146	23.342555	0.0062108	0.00088	0.00398	688602.92	5177.4655	SWT
	313.8	13.3	428.6573	-19.01452	0.0073078	0.00094	0.00449	123980.54	2479.6108	SWT
	336.5	20.3	445.1848	-25.82688	0.0081647	0.00147	0.00486	65790.842	5060.8340	SWT
	366.4	61.9	464.2674	10.641242	0.0095030	0.00306	0.00542	63862.278	9123.1825	SWT
	336.5	20.3	445.1848	-25.82688	0.0081647	0.00147	0.00486	65790.842	5060.8340	SWT
	313.8	13.3	428.6573	-19.01452	0.0073078	0.00094	0.00449	123980.54	2479.6108	SWT
	279.8	28	398.5146	23.342555	0.0062108	0.00088	0.00398	688602.92	5177.4655	SWT
	205.5	41	305.1526	60.047720	0.0043629	0.00088	0.00291	2376607.9	9700.4406	SWT
	147.2	58.8	219.2395	87.582719	0.0031145	0.00124	0.00209	3413680.8	11192.396	SWT

$N_{bl, i=480}$  (SWT) (Computation estimate of the life until the appearance of the initial damage)

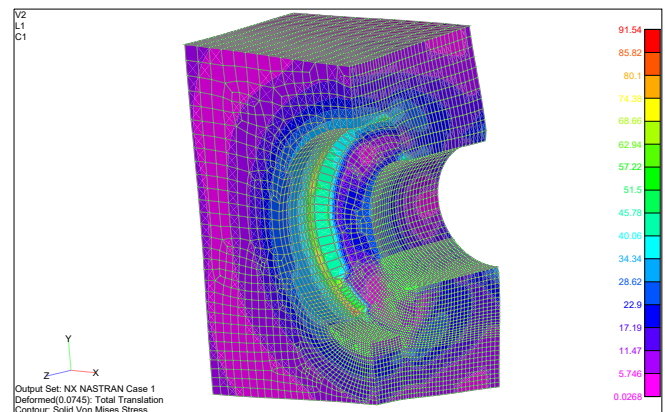
$N_{bl, i=505}$  (Morrow) (Computation estimate of the life until the appearance of the initial damage)

### 4.3 Residual fatigue life estimation

In addition to the estimation of the age until the appearance of the initial damage, the results of which are for the part of the wing, loaded with the load spectrum Fig. 2, determined in the previous point and shown in Table 1, it is necessary to make a computation estimate of the residual fatigue life, that is, during the crack growth. To estimate the residual life, it is necessary to assume initial damage in the form of surface cracking. Based on the maximum stress value, determined by elastic-plastic analysis of FEM, the critical position is defined, Fig. 3. This is where the initial damage is defined in the form of an initial surface crack. The calculation estimate of the remaining life is based on the knowledge of the stress intensity factor in an analytical form. Since it is a complex construction, there are no known analytical expressions for SIF. For this purpose, the finite element method was used to determine the analytical expression for SIF. 3-D singular finite elements around the assumed surface crack line were used to determine the SIF. The complete procedure for defining analytical expressions for SIF was performed using FEM.

As mentioned before, in order to establish an analytical expression for SIF, FEM was again used, assuming several depths of surface cracks. For this purpose, singular finite elements around the tip of the crack were

used. Figure 4 shows the stress state for one "depth" of crack.



**Figure 4.** Distribution of the stress state for a crack depth of 3.5 mm

For the analysis of crack propagation, i.e. for the estimation of the residual fatigue life, the strain energy density (SED) method was used, using the relation (3.5). The estimate of the remaining life, during the crack propagation, expressed through the number of blocks is:  $N_{bl,p}=52$  (Computation estimate of the residual, i.e. during the crack propagation). So, the SED method was used to estimate the residual life, described in detail in section 3 of this paper, on the one hand, while establishing analytical expressions for SIF using FEM, on the other hand.

## 5. CONCLUSION

This paper deals with research in the domain of estimating the total fatigue life of aircraft structures under the effect of cyclic loads. The goal is certainly to provide an efficient calculation method. For this purpose, the strain energy density method (SED) was developed for crack propagation analysis. Unlike the conventional method, which uses the dynamic characteristics of the material for calculation, this method uses the low-cycle fatigue characteristics of the material, which are also used in the estimation of the life until the appearance of initial damage. This, in turn, reduces the additional experimental tests required for the experimental determination of the dynamic characteristics of the material. Since the finite element method (FEM) is used in the work for the analysis of the stress state, it enables complex structures such as aircraft structures, which makes this approach not only efficient but also general. In order to differ from the conventional approach where low-cycle material properties are used to estimate the fatigue life until the occurrence of initial damage and dynamic properties to estimate the residual life, here low-cycle fatigue properties of the material are used in both cases. The total fatigue life of the part of the wing-fuselage connection of the aircraft (shown in Fig. 1) until the effective failure of 807 load blocks was experimentally determined. The computation estimate determined the total fatigue life to effective failure of 557 blocks. The obtained difference between computation results and experiment is 18.6%.

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## References

- [1] BANNANTINE, J.A., COMER, J., HANDROCK, J.: *Fundamentals of Material Fatigue Analysis*, Prentice-Hall, Englewood Cliffs, New Jersey, 1990.
- [2] KOSTAES, D.: *Fatigue Behaviour and Analysis*, Talat Lecture 2401, Technische Universität München, EAA – European Aluminium Association, 1994.
- [3] PALMGREN, A.: *Die Lebensdauer von Kugellagern*, Verfahrenstechnik, Berlin, 68 (1924) 339-341.
- [4] MINER, M.A.: *Cumulative Damage in Fatigue*, Journal of Applied Mechanics, 67 (1945) A159-A164.
- [5] FATEMI, A., L. YANG, L.: *Cumulative Fatigue Damage and Life Prediction Theories: A Survey of the State of the Art for Homogeneous Materials*, International Journal of Fatigue, 20 (1) (1998) 9-34.
- [6] MAKSIMOVIĆ, S.: *Fatigue Life Analysis of Aircraft Structural Components*, Scientific Technical Review, Vol. LV, No.1, 2005.
- [7] BOLJANOVIĆ, S., MAKSIMOVIĆ, S.: *Fatigue crack growth modeling of attachment lugs*, International Journal of Fatigue, 58(1) (2014) 66-74.
- [8] MAKSIMOVIĆ, K., POSAVLJAK, S., MAKSIMOVIĆ, M., VASOVIĆ MAKSIMOVIĆ, I., and BALAC, M.: *Total Fatigue Life Estimation of Aircraft Structural Components Under General Load Spectra*; The Editor(s) (if applicable) and The Author(s), under exclusive license to Springer Nature Switzerland AG 2021. N. Mitrovic et al. (Eds.): CNNTech 2020, LNNS 153, pp. 394–412, 2021. Print ISBN 978-3-030-58361-3; Online ISBN 978-3-030-58362-0; [https://doi.org/10.1007/978-3-030-58362-0\\_23](https://doi.org/10.1007/978-3-030-58362-0_23)
- [9] MAKSIMOVIĆ M., VASOVIĆ MAKSIMOVIĆ, I., MAKSIMOVIĆ, K., TRIŠOVIĆ, N., MAKSIMOVIĆ, S.: *Residual Life Estimation of Cracked Aircraft Structural Components*; FME Transactions; 46 (1) (2018) 124-128; doi:10.5937/fmet1801124M; Faculty of Mechanical Engineering, Belgrade
- [10] MSC/NASTRAN finite element software code: *Theoretical Manuals*.