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Lightly $\hat{\hat{g}}$-closed sets in intuitionistic fuzzy topological spaces

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Abstract: In this paper we introduce intuitionistic fuzzy lightly $\hat{\hat{g}}$-closed sets and intuitionistic fuzzy lightly $\hat{\hat{g}}$-open sets and study some of their properties with suitable examples are given.

Key words: Intuitionistic fuzzy topology, Intuitionistic fuzzy $\hat{\hat{g}}$-closed set and Intuitionistic fuzzy $L\hat{\hat{g}}$-closed set

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [17] and later Atanassov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [4] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. Recently many fuzzy topological concepts have been extended to intuitionistic fuzzy topological spaces. Further, several researchers find real-life applications in fuzzy topological spaces, soft fuzzy topological spaces and intuitionistic fuzzy topological space for example [2], [8], [9] and [13] and so on. In this paper we introduce intuitionistic fuzzy lightly $\hat{\hat{g}}$-closed sets and intuitionistic fuzzy lightly $\hat{\hat{g}}$-open sets and study some of their properties.

2. Preliminaries

Throughout this paper (X, τ) (briefly, X) will denote an intuitionistic fuzzy topological space or IFTS (X, τ). If $H < X$, $cl(H) = C(H)$ and $int(H) = I(H)$ will, respectively, denote the closure and interior of H in IFTS(X, τ). We given some definitions and note some fundamental results necessary for our present study.

Definition 2.1. An IFS A in an IFTS (X, τ) is said to be an

1. intuitionistic fuzzy regular closed set(IFRCS in short) if A = cl(int(A)), [6]
2. intuitionistic fuzzy semi closed set (IFSCS in short) if int(cl(A)) $\subseteq$ A, [3]
3. intuitionistic fuzzy α-closed set (IFαCS in short) if cl(int(cl(A))) $\subseteq$ A, [7]
4. intuitionistic fuzzy semi pre closed set (IFSPCS in short) if int(cl(int(A))) $\subseteq$ A. [16]

Definition 2.2. An IFS A in (X, τ) is said to be an

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1. intuitionistic fuzzy generalized closed set (IFGCS in short) if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is an IFOS in (X, τ), [14]
2. intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is an IFOS in (X, τ), [11]
3. intuitionistic fuzzy semi generalized closed set (IFSGCS in short) if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is an IFSOS in (X, τ), [15]
4. intuitionistic fuzzy generalized semi pre closed set (IFGSPCS in short) if spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is an IFOS in (X, τ), [10]
5. intuitionistic fuzzy α generalized closed set (IFαGCS in short) if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is an IFOS in (X, τ). [12]

The complements of the above mentioned intuitionistic fuzzy closed sets are called their respective intuitionistic fuzzy open sets.

Denote IFSGO(X), the set of all intuitionistic semi-generalized open sets of X.

3. IF lightly $\hat{\hat{g}}$-closed sets

Definition 3.1. Let H be an IFS in an IFTS (X,τ) is called an intuitionistic fuzzy lightly $\hat{\hat{g}}$-closed set (briefly, IFL$\hat{\hat{g}}$CS) if $H \subseteq G$, G is an IFSGOS $\Rightarrow C(I(H)) \subseteq G$.

The collection of all intuitionistic fuzzy lightly $\hat{\hat{g}}$-closed sets in X is denoted by IFL$\hat{\hat{g}}$C(X).

Example 3.1 *1. Consider $X=\{m,n\}$ with $\tau=\{0_\sim, A, 1_\sim\}$ is an IFT on X, where $A=\langle \alpha,(0.6,0.7),(0.4,0.3)\rangle$. Then $\theta_A(m)=0.6, \theta_A(n)=0.7, \beta_A(m)=0.4$ and $\beta_A(n)=0.3$. Let be an IFS $H=\langle \alpha,(0.4,0.3),(0.6,0.7)\rangle = A^c$. This verifies that H is an IFL$\hat{\hat{g}}$CS.*

2. Consider $X=\{m,n\}$ with $\tau=\{0_\sim, A, 1_\sim\}$ is an IFT on X, where $A=\langle \alpha,(0.6,0.7),(0.4,0.3)\rangle$. Then $\theta_A(m)=0.6, \theta_A(n)=0.7, \beta_A(m)=0.4$ and $\beta_A(n)=0.3$. Let be an IFS $H=\langle \alpha,(0.7,0.8),(0.3,0.2)\rangle$. This verifies that H is not an IFL$\hat{\hat{g}}$CS.

Proposition 3.1. *Let H be in an IFTS (X,τ), the following statements are true.*

1. *If H is an IFCS then H is an IFL$\hat{\hat{g}}$CS.*
2. *If H is an IF$\hat{\hat{g}}$CS then H is an IFL$\hat{\hat{g}}$CS.*
3. *If H is an IFRCS then H is an IFL$\hat{\hat{g}}$CS.*

Proof. 1. Let H be an IFCS in (X,τ). Let G be an IFSGOS such that $H \subseteq G$. Since $C(H)=H, C(I(H)) \subseteq C(H)=H$. Then $C(I(H)) \subseteq H \subseteq G$ whenever $H \subseteq G$ and G is IFSGOS. It is follows that H is an IFL$\hat{\hat{g}}$CS in X.

2. Let H be an IF$\hat{\hat{g}}$CS in (X,τ). Let G be an IFSGOS such that $H \subseteq G$. Since H is an IF$\hat{\hat{g}}$CS, $C(H) \subseteq G$. Since $C(I(H)) \subseteq C(H)$, then $H \subseteq G$, G is an IFSGOS $\Rightarrow C(I(H)) \subseteq G$. This verifies that H is an IFL$\hat{\hat{g}}$CS in X.

3. Let H be an IFRCS in (X, τ). Let G be an IFSGOS in (X, τ) such that $H \subseteq G$. Since H is an IFRCS, $C(I(H)) = H \subseteq G$. Thus we have $H \subseteq G$, G is an IFSGOS $\Rightarrow C(I(H)) \subseteq G$. Which verifies that H is an IFL$\hat{\hat{g}}$CS.

Remark 3.1. *The following example shows that converse of Proposition 3.1 is not true in general.*

Example 3.2. *Consider $X = \{m, n\}$ and $\tau = \{0_\sim, A, 1_\sim\}$ is an IFT on X, where $A = \langle \alpha, (0.3, 0.4), (0.7, 0.6) \rangle$. Then $\theta_A(m) = 0.3, \theta_A(n) = 0.4, \beta_A(m) = 0.7$ and $\beta_A(n) = 0.6$. Let be an IFS $H = \langle \alpha, (0.2, 0.3), (0.8, 0.7) \rangle$ is an IFL$\hat{\hat{g}}$CS but not an IFCS.*

2. *Consider $X = \{m, n\}$ and $\tau = \{0_\sim, A, 1_\sim\}$ is an IFT on X, where $A = \langle \alpha, (0.4, 0.3), (0.6, 0.7) \rangle$. Then $\theta_A(m) = 0.4, \theta_A(n) = 0.3, \beta_A(m) = 0.6$ and $\beta_A(n) = 0.7$. Let be an IFS $H = \langle \alpha, (0.3, 0.2), (0.7, 0.8) \rangle$. This verifies that H is an IFL$\hat{\hat{g}}$CS but not an IF$\hat{\hat{g}}$CS.*

3. *Consider $X = \{m, n\}$ and $\tau = \{0_\sim, A, 1_\sim\}$ is an IFT on X, where $A = \langle \alpha, (0.6, 0.7), (0.4, 0.3) \rangle$. Then $\theta_A(m) = 0.6, \theta_A(n) = 0.7, \beta_A(m) = 0.4$ and $\beta_A(n) = 0.3$. Let be an IFS $H = \langle \alpha, (0.4, 0.3), (0.6, 0.7) \rangle$. This verifies that H is an IFL$\hat{\hat{g}}$CS but not an IFRCS.*

Proposition 3.2. *If H is an IFL$\hat{\hat{g}}$CS in an IFTS (X, τ), then $I(H)$ is an IFGSPCS.*

Proof. Let H be an IFL$\hat{\hat{g}}$CS in (X, τ). Let G be IFOS in X such that $H \subseteq G$. Then $I(H) \subseteq I(G) = G$. It is known that every IFOS is an IFSGOS. Since H is IFL$\hat{\hat{g}}$CS in X, $C(I(H)) \subseteq G$. We have $I(C(I(H))) \subseteq I(G) = G$ and $I(H) \subseteq G$. It implies that $spcl(I(H)) = I(H) \cup I(C(I(I(H)))) = I(H) \cup I(C(I(H))) \subseteq G \cup G = G$. Thus $I(H)$ is an IFGSPCS in X.

Remark 3.2. *The following example shows that converse of Proposition 3.2 is not true in general.*

Example 3.3. *Consider $X = \{m, n\}$ and $\tau = \{0_\sim, A, 1_\sim\}$ is an IFT on X, where $A = \langle \alpha, (0.7, 0.6), (0.3, 0.4) \rangle$. Then $\theta_A(m) = 0.7, \theta_A(n) = 0.6, \beta_A(m) = 0.3$ and $\beta_A(n) = 0.4$. Let be an IFS $H = \langle \alpha, (0.8, 0.7), (0.2, 0.3) \rangle$. This verifies that $I(H) = \langle \alpha, (0.7, 0.6), (0.3, 0.4) \rangle = A$ is an IFGSPCS but H is not an IFL$\hat{\hat{g}}$CS in X.*

Remark 3.3. *The following example shows that the family of IFGCS and the family of IFL$\hat{\hat{g}}$CS are independent of each other.*

Example 3.4. *Consider $X = \{m, n\}$ and $\tau = \{0_\sim, A, 1_\sim\}$ is an IFT on X, where $A = \langle \alpha, (0.2, 0.3), (0.7, 0.6) \rangle$. Then $\theta_A(m) = 0.2, \theta_A(n) = 0.3, \beta_A(m) = 0.7$ and $\beta_A(n) = 0.6$. Let be an IFS $H = \langle \alpha, (0.8, 0.7), (0.1, 0.2) \rangle$. This verifies that H is an IFGCS but H is not an IFL$\hat{\hat{g}}$CS in X.*

Example 3.5. *Consider $X = \{m, n\}$ and $\tau = \{0_\sim, A, 1_\sim\}$ is an IFT on X, where $A = \langle \alpha, (0.6, 0.7), (0.3, 0.2) \rangle$. Then $\theta_A(m) = 0.6, \theta_A(n) = 0.7, \beta_A(m) = 0.3$ and $\beta_A(n) = 0.2$. Let be an IFS $H = \langle \alpha, (0.4, 0.3), (0.5, 0.6) \rangle$. This verifies that H is an IFL$\hat{\hat{g}}$CS but H is not an IFGCS in X.*

Remark 3.4. *The following example shows that the family of IFSCS and the family of IFL$\hat{\hat{g}}$CS are independent of each other.*

Example 3.6. *Consider* $X = \{m, n\}$ *and* $\tau = \{0_\sim, A, 1_\sim\}$ *is an IFT on* X*, where* $A = \langle \alpha, (0.3, 0.4), (0.6, 0.5)\rangle$*. Then* $\theta_A(m) = 0.3, \theta_A(n) = 0.4, \beta_A(m) = 0.6$ *and* $\beta_A(n) = 0.5$*. Let be an IFS* $H = \langle \alpha, (0.4, 0.5), (0.5, 0.4)\rangle$*. This verifies that* H *is an IFSCS but* H *is not an IFL*$\hat{\hat{g}}$*CS in* X*.*

2. *Consider* $X = \{m, n\}$ *and* $\tau = \{0_\sim, A, 1_\sim\}$ *is an IFT on* X*, where* $A = \langle \alpha, (0.4, 0.3), (0.5, 0.6)\rangle$*. Then* $\theta_A(m) = 0.4, \theta_A(n) = 0.3, \beta_A(m) = 0.5$ *and* $\beta_A(n) = 0.6$*. Let be an IFS* $H = \langle \alpha, (0.3, 0.2), (0.6, 0.7)\rangle$*. This verifies that* H *is an IFL*$\hat{\hat{g}}$*CS but* H *is not an IFSCS in* X*.*

Theorem 3.1. *An IFS* H *of an IFTS* (X, τ)*. If* H *is both IFCS and IF*α*GCS and IFO(X) = IFSGO(X), then* H *is an IFL*$\hat{\hat{g}}$*CS in* X*.*

Proof. Let H be an IFS in an IFTS (X, τ). Given H is both IFCS and IFαGCS. Since H is an IFαGCS, we have $\alpha C(H) \subseteq G$ whenever $H \subseteq G$ and G is an IFOS in X. Since H is an IFCS, we have $C(I(H)) \subseteq C(I(C(H))) \subseteq H \cup C(I(C(H))) = \alpha C(H)$. We have $C(I(H)) \subseteq G$ whenever $H \subseteq G$ and G is an IFOS in X. Also given IFO(X) = IFSGO(X), then $C(I(H)) \subseteq G$ whenever $H \subseteq G$ and G is an IFSGOS in X. Hence H is an IFL$\hat{\hat{g}}$CS in X.

Theorem 3.2. *An IFS* H *of an IFTS* (X, τ)*. If* H *is both IFOS and IFL*$\hat{\hat{g}}$*CS, then* H *is an IFGCS in* X*.*

Proof. Since H is an IFOS in (X, τ). Then $H = I(H)$. Let G be an IFOS in X such that $H \subseteq G$. Then $H \subseteq G$ and G is an IFSGOS in X. Since H is IFL$\hat{\hat{g}}$CS in (X, τ). We have $C(I(H)) = C(H) \subseteq G$. This proves that H is an IFGCS in X.

Theorem 3.3. *An IFS* H *of an IFTS* (X, τ)*. If* H *is both IFOS and IFL*$\hat{\hat{g}}$*CS, then* H *is an IFCS in* X*.*

Proof. Since H is an IFOS and an IFL$\hat{\hat{g}}$CS in (X, τ). Then $H \subseteq H$ and H is an IFSGOS in X. Given H is an IFL$\hat{\hat{g}}$CS in X, $C(I(H)) \subseteq H$. Also given H is an IFOS, $C(H) \subseteq H$. Thus $C(H) = H$ and H is an IFCS in X.

Corollary 3.1. *An IFS* H *of an IFTS* (X, τ)*. If* H *is both IFOS and IFL*$\hat{\hat{g}}$*CS, then* H *is both IFROS and IFRCS in* X*.*

Proof. Since H is both IFOS and IFL$\hat{\hat{g}}$CS in (X, τ). By Theorem 3.3, H is an IFCS. Since H is both IFOS and IFCS, $C(I(H)) = C(H) = H$ and $I(C(H)) = I(H) = H$. Thus H is both IFROS and IFRCS in X.

4. IF lightly $\hat{\hat{g}}$-open sets

Definition 4.1. An IFS H of an IFTS (X, τ) is called an intuitionistic fuzzy lightly $\hat{\hat{g}}$-open set (briefly, IFL$\hat{\hat{g}}$OS) if its complement H^c is an IFL$\hat{\hat{g}}$CS in (X, τ).

The family of all intuitionistic fuzzy lightly $\hat{\hat{g}}$-open sets in X is denoted by IFL$\hat{\hat{g}}$O(X).

Example 4.1. *Consider* $X = \{m, n\}$ *and* $\tau = \{0_\sim, A, 1_\sim\}$ *is an IFT on* X*, where* $A = \langle \alpha, (0.6, 0.7), (0.4, 0.3)\rangle$*. Then* $\theta_A(m) = 0.6, \theta_A(n) = 0.7, \beta_A(m) = 0.4$ *and* $\beta_A(n) = 0.3$*. Let be an IFS* $H = \langle \alpha, (0.4, 0.3), (0.6, 0.7)\rangle = A^c$*. This verifies that* H *is an IFL*$\hat{\hat{g}}$*OS.*

2. *Consider* $X = \{m,n\}$ *and* $\tau = \{0_\sim, A, 1_\sim\}$ *is an IFT on* X, *where* $A = \langle \alpha, (0.6, 0.7), (0.4, 0.3)\rangle$. *Then* $\theta_A(m) = 0.6, \theta_A(n) = 0.7, \beta_A(m) = 0.4$ *and* $\beta_A(n) = 0.3$. *Let be an IFS* $H = \langle \alpha, (0.3, 0.2), (0.7, 0.8)\rangle$. *This verifies that* H *is not an IFL*$\hat{\hat{g}}$*OS.*

Theorem 4.1. *An IFS* H *of an IFTS* (X, τ) *is IFL*$\hat{\hat{g}}$*OS* $\iff$ $G \subseteq I(C(H))$ *whenever* $G \subseteq H$ *and* G *is an IFSGCS.*

Proof. Necessity: Given H is an IFL$\hat{\hat{g}}$OS in X. Then H^c is an IFL$\hat{\hat{g}}$CS. Let G be an IFSGCS such that $G \subseteq H$. Then G^c is an IFSGOS such that $H^c \subseteq G^c$. Since H^c is an $IFL\hat{\hat{g}}$CS, then $C(I(H^c)) \subseteq G^c$. Thus $G \subseteq I(C(H))$.

Sufficiency: Assuming that $G \subseteq I(C(H))$ whenever $G \subseteq H$ and G is IFSGCS. Then G^c is an IFSGOS such that $H^c \subseteq G^c$ and $(I(C(H)))^c \subseteq G^c$. This implies $C(I(H^c)) \subseteq G^c$. Hence H^c is an $IFL\hat{\hat{g}}$CS. This proves that H is an IFL$\hat{\hat{g}}$OS.

Proposition 4.1. *In an IFTS* (X, τ), *every an IFOS is an IFL*$\hat{\hat{g}}$*OS.*

Proof. Since H is an IFOS in (X, τ). Then H^c is an IFCS in X. By Proposition 3.1(1), since every IFCS is an IFL$\hat{\hat{g}}$CS in X. Therefore H^c is an IFL$\hat{\hat{g}}$CS in X. Which proves that H is an IFL$\hat{\hat{g}}$OS in X.

Remark 4.1. *The following example shows that converse of Proposition 4.1 is not true in general.*

Example 4.2. *Consider* $X = \{m,n\}$ *and* $\tau = \{0_\sim, A, 1_\sim\}$ *is an IFT on* X, *where* $A = \langle \alpha, (0.3, 0.4), (0.6, 0.7)\rangle$. *Then* $\theta_A(m) = 0.3, \theta_A(n) = 0.4, \beta_A(m) = 0.6$ *and* $\beta_A(n) = 0.7$. *Let be an IFS* $H = \langle \alpha, (0.8, 0.7), (0.2, 0.3)\rangle$. *Which is an IFL*$\hat{\hat{g}}$*OS but not an IFOS.*

Proposition 4.2. *In an IFTS* (X, τ), *every an IF*$\hat{\hat{g}}$*OS is an IFL*$\hat{\hat{g}}$*OS.*

Proof. Let A be an IF$\hat{\hat{g}}$OS in (X, τ). Then A^c is an IF$\hat{\hat{g}}$CS in X. By Proposition 3.1(2), we have every IF$\hat{\hat{g}}$CS is an IFL$\hat{\hat{g}}$CS in X. Therefore A^c is an IFL$\hat{\hat{g}}$CS in X. Hence A is an IFL$\hat{\hat{g}}$OS in X.

Remark 4.2. *The following example shows that converse of Proposition 4.2 is not true in general.*

Example 4.3. *Consider* $X = \{m,n\}$ *and* $\tau = \{0_\sim, A, 1_\sim\}$ *is an IFT on* X, *where* $A = \langle \alpha, (0.4, 0.3), (0.6, 0.7)\rangle$. *Then* $\theta_A(m) = 0.4, \theta_A(n) = 0.3, \beta_A(m) = 0.6$ *and* $\beta_A(n) = 0.7$. *Let be an IFS* $H = \langle \alpha, (0.7, 0.8), (0.3, 0.2)\rangle$. *Which follows that* H *is an IFL*$\hat{\hat{g}}$*CS but not an IF*$\hat{\hat{g}}$*CS.*

Proposition 4.3. *In an IFTS* (X, τ), *every IFROS is an IFL*$\hat{\hat{g}}$*OS.*

Proof. Let A be an IFROS in (X, τ). Then A^c is an IFRCS in X. By Proposition 3.1(3), we have every IFRCS is an IFL$\hat{\hat{g}}$CS in X. Therefore A^c is an IFL$\hat{\hat{g}}$CS in X. Hence A is an IFL$\hat{\hat{g}}$OS in X.

Remark 4.3. *The following example shows that converse of Proposition 4.3 is not true in general.*

Example 4.4. *Consider* $X = \{m,n\}$ *and* $\tau = \{0_\sim, A, 1_\sim\}$ *is an IFT on* X, *where* $A = \langle \alpha, (0.6, 0.7), (0.4, 0.3)\rangle$. *Then* $\theta_A(m) = 0.6, \theta_A(n) = 0.7, \beta_A(m) = 0.4$ *and* $\beta_A(n) = 0.3$. *Let be an IFS* $H = \langle \alpha, (0.6, 0.7), (0.4, 0.3)\rangle$. *Which follows that* H *is an IFL*$\hat{\hat{g}}$*OS but not an IFROS.*

Proposition 4.4. *An IFL$\hat{\hat{g}}$OS H of an IFTS (X,τ). Then $C(H)$ is an IFGSPOS.*

Proof. Let H be an IFL$\hat{\hat{g}}$OS in (X,τ). Then H^c is an IFL$\hat{\hat{g}}$CS in X. By Theorem 3.2, then $I(H^c)$ is an IFGSPCS in X. Hence $(I(H^c))^c = C((H^c)^c) = C(H)$ is an IFGSPOS in X.

Remark 4.4. *The following example shows that converse of Proposition 4.4 is not true in general.*

Example 4.5. *Consider $X = \{m,n\}$ and $\tau = \{0_\sim, A, 1_\sim\}$ is an IFT on X, where $A = \langle \alpha, (0.7, 0.6), (0.3, 0.4)\rangle$. Then $\theta_A(m) = 0.7, \theta_A(n) = 0.6, \beta_A(m) = 0.3$ and $\beta_A(n) = 0.4$. Let be an IFS $H = \langle \alpha, (0.2, 0.3), (0.8, 0.7)\rangle$. Then $C(H) = \langle \alpha, (0.3, 0.4), (0.7, 0.6)\rangle = A^c$ is an IFGSPOS but H is not an IFL$\hat{\hat{g}}$OS in X.*

Remark 4.5. *The following example shows that an IFGSPOS is not an IFL$\hat{\hat{g}}$OS in general.*

Example 4.6. *Consider $X = \{m,n\}$ and $\tau = \{0_\sim, A, 1_\sim\}$ is an IFT on X, where $A = \langle \alpha, (0.3, 0.4), (0.7, 0.6)\rangle$. Then $\theta_A(m) = 0.3, \theta_A(n) = 0.4, \beta_A(m) = 0.7$ and $\beta_A(n) = 0.6$. Let be an IFS $H = \langle \alpha, (0.2, 0.3), (0.8, 0.7)\rangle$. This verifies that H is an IFGSPOS but H is not an IFL$\hat{\hat{g}}$OS in X.*

Remark 4.6. *The following example shows that the family of IFGOS and the family of IFL$\hat{\hat{g}}$OS are independent of each other.*

Example 4.7. *Consider $X = \{m,n\}$ and $\tau = \{0_\sim, A, 1_\sim\}$ is an IFT on X, where $A = \langle \alpha, (0.2, 0.3), (0.7, 0.6)\rangle$. Then $\theta_A(m) = 0.2, \theta_A(n) = 0.3, \beta_A(m) = 0.7$ and $\beta_A(n) = 0.6$. Let be an IFS $H = \langle \alpha, (0.1, 0.2), (0.8, 0.7)\rangle$. This verifies that H is an an IFGOS but H is not an IFL$\hat{\hat{g}}$OS in X.*

Example 4.8. *Consider $X = \{m,n\}$ and $\tau = \{0_\sim, A, 1_\sim\}$ is an IFT on X, where $A = \langle \alpha, (0.6, 0.7), (0.3, 0.2)\rangle$. Then $\theta_A(m) = 0.6, \theta_A(n) = 0.7, \beta_A(m) = 0.3$ and $\beta_A(n) = 0.2$. Let be an IFS $H = \langle \alpha, (0.5, 0.6), (0.4, 0.3)\rangle$. This verifies that H is an IFL$\hat{\hat{g}}$OS but H is not an IFGOS in X.*

Remark 4.7. *The following example shows that the family of IFSOS and the family of IFL$\hat{\hat{g}}$OS are independent of each other.*

Example 4.9. *Consider $X = \{m,n\}$ and $\tau = \{0_\sim, A, 1_\sim\}$ is an IFT on X, where $A = \langle \alpha, (0.3, 0.4), (0.6, 0.5)\rangle$. Then $\theta_A(m) = 0.3, \theta_A(n) = 0.4, \beta_A(m) = 0.6$ and $\beta_A(n) = 0.5$. Let be an IFS $H = \langle \alpha, (0.5, 0.4), (0.4, 0.5)\rangle$. This verifies that H is an IFSOS but H is not an IFL$\hat{\hat{g}}$OS in X.*

Example 4.10. *Consider $X = \{m,n\}$ and $\tau = \{0_\sim, A, 1_\sim\}$ is an IFT on X, where $A = \langle \alpha, (0.4, 0.3), (0.5, 0.6)\rangle$. Then $\theta_A(m) = 0.4, \theta_A(n) = 0.3, \beta_A(m) = 0.5$ and $\beta_A(n) = 0.6$. Let be an IFS $H = \langle \alpha, (0.6, 0.7), (0.3, 0.2)\rangle$. This verifies that H is an IFL$\hat{\hat{g}}$OS but H is not an IFSOS in X.*

Theorem 4.2. *An IFS H of an IFTS (X,τ). If H is both an IFOS and an IFαGOS and IFO(X) = IFSGO(X), then H is an IFL$\hat{\hat{g}}$OS in X.*

Proof. Since H be an IFS in an IFTS (X,τ). Since H is both an IFOS and an IFαGOS. Then H^c is both IFCS and IFαGCS. Also given IFO(X) = IFSGO(X). By Theorem 3.1, then H^c is an IFL$\hat{\hat{g}}$CS in X. This proves that H is an IFL$\hat{\hat{g}}$OS in X.

Theorem 4.3. *An IFS H of an IFTS (X,τ). If H is both IFCS and IFL$\hat{\hat{g}}$OS, then H is an IFGOS in X.*

Proof. Since H is an IFS in an IFTS (X, τ). Since H is both IFCS and IFL$\hat{\hat{g}}$OS. Hence H^c is both IFOS and IFL$\hat{\hat{g}}$CS. By Theorem 3.2, then H is an IFGCS in X. This proves that H is an IFGOS in X.

Theorem 4.4. *An IFS H of an IFTS (X, τ). If H is both IFCS and IFL$\hat{\hat{g}}$OS, then H is an IFOS in X.*

Proof. Since H is an IFS in an IFTS (X, τ). Since H is both IFCS and IFL$\hat{\hat{g}}$OS. Therefore H^c is both IFOS and IFL$\hat{\hat{g}}$CS. By Theorem 3.3, then H is an IFCS in X. This proves that H is an IFOS in X.

Corollary 4.1. *An IFS H of an IFTS (X, τ). If H is both IFCS and IFL$\hat{\hat{g}}$OS, then H is both IFROS and IFRCS in X.*

Proof. Since H is both IFCS and IFL$\hat{\hat{g}}$OS in (X, τ). Then by Corollary 3.1, H is an IFOS. Since H is both IFOS and IFCS, H is both IFROS and IFRCS in X.

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