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Extemporized new sets in grill topological spaces

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Abstract: This aim of this research is to study the modern sort of open sets and introduces and examines a new class of $t^{\#}$-sets defined in terms of a Grill on X. Such sets' characterization and a few more characteristics are discovered.

Key words: $\mathcal{G}_t$-open sets, $\mathcal{G}_{\mathcal{R}}$-open sets, $\mathcal{G}_{t_\alpha}$-open sets and $\mathcal{G}_{\mathcal{R}_\alpha}$-open sets.

1. Introduction

The study of generalised types of closed sets has recently piqued the interest of many Topologists. Levine [4], for instance, pioneered a certain type of generalised closed sets. The definition of a type of generalised closed sets, which has been developed and studied, is in line with the trend.

Choquet [2] was the first to present the idea of a Grill. W.J. Thron has introduced [14] Proximity structure and grills. After him Roy and Mukherjee [11] published an article in on a common topology brought about by a grill topological spaces. In 2010 Hatir and Jafari [3] introduced on a few new sets classes and a fresh grill-based decomposition of continuity and finally M.O.Mustafa and Esmaeel [6] are discussed in their article about topological open grill separation axioms and closed sets in 2021.

Following investigations have shown that grills can be a very helpful tool for looking into a variety of topological issues.

The objective of this paper is to introduce and investigate a sophisticated class of $t^{\#}$-sets and $t^{\#}_{\alpha}$-sets defined in respect of a Grill on X. A description of previous sets is obtained along with some of their characteristics.

2. Preliminaries

The closure set and the inner set of D for a topological space (TS) (X, τ), $D \subseteq X$, and Grill Topological Space (GLTS) are referred to as $C(D)$ and $I(D)$ respectively throughout this work.

Theorem 2.1. *[11]*

It can be admitted that $(X, \tau, \mathcal{G})$ is a Grill Topological Space (GLTS). For $D, E \subseteq X$, the ensuing properties hold:

1. *$D \subseteq E$ indicates that $\Phi(D) \subseteq \Phi(E)$.*

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2. $\Phi(D \cup E) = \Phi(D) \cup \Phi(E)$.

3. $\Phi(\Phi(D)) \subseteq \Phi(D) = C(\Phi(D)) \subseteq C(D)$.

4. *If* $U \in \tau$ *then* $U \cap \Phi(D) \subseteq \Phi(U \cap D)$.

Theorem 2.2. *[11] If* D *and* E *are the subsets ox* X *in a space* $(X, \tau, \mathcal{G})$, *the beneath said answers are correct for the set operator* ψ.

1. $D \subseteq \psi(D)$,

2. $\psi(\phi) = \phi$ *and* $\psi(X) = X$,

3. $If D \subseteq E$, *then* $\psi(D) \subseteq \psi(E)$,

4. $\psi(D) \cup \psi(E) = \psi(D \cup E)$.

5. $\psi(\psi(D)) = \psi(D)$.

Definition 2.1. A subset D of space $(X, \tau, \mathcal{G})$ as

1. grill α-open (resp. $\mathcal{G}_\alpha$-open) [1] if $D \subseteq I(\psi(I(D)))$,

2. grill pre-open (resp. $\mathcal{G}_p$-open) [3] if $D \subseteq I(\psi(D))$.

3. grill semi-open (resp. $\mathcal{G}_s$-open) [1] if $D \subseteq \psi(I(D))$.

4. grill b-open (resp. $\mathcal{G}_b$-open) [1] if $D \subseteq I(\psi(D)) \cup \psi(I(D))$.

5. grill β-open (resp. $\mathcal{G}_\beta$-open) [1] if $D \subseteq C(I(\psi(D)))$.

Definition 2.2. [8] A subset D of GLTS $(X, \tau, \mathcal{G})$ is discovered to be

1. grill t-set (resp. $\mathcal{G}_t$-set) if $I(D) \subseteq I(\psi(D))$,

2. grill $\mathcal{R}$-set (resp. $\mathcal{G}_\mathcal{R}$-set) if $D = D_1 \cap D_2$, where D_1 is open and D_2 is $\mathcal{G}_t$-set.

3. On new sets in grill topological space

Definition 3.1. A subset H of a GLTS $(X, \mathcal{G}, I)$ is revealed to be

1. grill $t^\#$-set (resp, $\mathcal{G}_{t^\#}$-set) if $I(V) = \psi(I(V))$.

2. grill $t^\#_\alpha$-set (resp, $\mathcal{G}_{t^\#_\alpha}$-set) if $I(V) = \psi(I(\psi(V)))$.

3. grill $\mathcal{R}^\#$-set (resp, $\mathcal{G}_{\mathcal{R}^\#}$-set) if $V = R \cap S$, where R is open & S is $\mathcal{G}_{t^\#}$-set.

4. grill $\mathcal{R}^\#_\alpha$-set (resp, $\mathcal{G}_{\mathcal{R}^\#_\alpha}$-set) if $V = R \cap S$, where R is open & S is $\mathcal{G}_{t^\#_\alpha}$-set.

5. strong grill $\mathcal{R}$-set (resp, $\mathcal{G}_{\mathcal{S}\mathcal{R}}$-set) if $V = R \cap S$, where S is $\mathcal{G}_t$-set and R is open , $I(\psi(S)) = \psi(I(S))$.

Remark 3.1. *For a GLTS* $(X, \mathcal{G}, I)$,

1. *if* L *is open* $\Longrightarrow$ L *is* $\mathcal{G}_{\mathcal{R}^\#}$*-set.*

2. *if* L *is* $\mathcal{G}_{t^\#}$*-set* $\Longrightarrow$ L *is* $\mathcal{G}_{\mathcal{R}^\#}$*-set.*

Remark 3.2. *In every part of the Remark 3.1 the converse part is not necessarily true as given in the next upcoming Examples.*

Example 3.1. *Let* $X = \{2, 4, 6\}$ *and* $\tau = \{\phi, \{2\}, \{4\}, \{2, 4\}, X\}$.

If $\mathcal{G} = \{\{4\}, \{4, 6\}, X\}$.

1. $\{6\}$ *is* $\mathcal{G}_{\mathcal{R}\#}$*-set but not open.*
2. $\{4\}$ *is* $\mathcal{G}_{\mathcal{R}\#}$*-set but not* $\mathcal{G}_{t\#}$*-set.*

Remark 3.3. *These correlations are displayed in the representation.*

$$\begin{array}{ccc} \textbf{open} & & \\ \downarrow & & \\ \mathcal{G}_{\alpha}\textbf{-open} & \longrightarrow & \mathcal{G}_{p}\textbf{-open} \\ \downarrow & & \downarrow \\ \mathcal{G}_{s}\textbf{-open} & \longrightarrow & \mathcal{G}_{b}\textbf{-open} \\ & & \downarrow \\ & & \mathcal{G}_{\beta}\textbf{-open} \end{array}$$

Proposition 3.1. *If* R *and* S *are* $\mathcal{G}_{t\#}$*-sets, then* $R \cap S$ *is* $\mathcal{G}_{t\#}$*-set.*

Proof.

Let R and S be $\mathcal{G}_{t\#}$-sets. $I(R \cap S) \subseteq I(R \cap S) \subseteq \psi(I(R \cap S)) = \psi(I(R) \cap I(S)) \subseteq \psi(I(R)) \cap \psi(I(S)) = I(R) \cap I(S)$ (by guess) $= I(R \cap S)$.

Thus $I(R \cap S) = \psi(I(R \cap S))$ and hence $R \cap S$ is $\mathcal{G}_{t\#}$-set. □

Theorem 3.1. *The subsequent properties are equivalent for a subsets* H *of a grill topological space*

1. B *is open,*
2. B *is* $\mathcal{G}_s$*-open &* $\mathcal{G}_{\mathcal{R}\#}$*-set.*

Proof.

(1) implies (2): (1) of Remark 3.1 and (2) of Remark 3.3 come after each other.

(2) implies (1): Shown B is $\mathcal{G}_{\mathcal{R}\#}$-set. So $B = R \cap S$ where R is open and $I(S) = \psi(I(S))$. Then $B \subseteq R = I(R)$. Also B is $\mathcal{G}_s$-open implies $B \subseteq \psi(I(B)) \subseteq \psi(I(S)) = I(S)$ by guess. Thus $B \subseteq I(R) \cap I(S) = I(R \cap S) = I(R)$ and hence B is open. □

Remark 3.4. *For a GLTS, the notions of* $\mathcal{G}_{\mathcal{R}\#}$*-sets and of* $\mathcal{G}_s$*-open sets are autonomous.*

Example 3.2. *In Example 3.1,*

1. $\{4, 6\}$ *is* $\mathcal{G}_s$*-open set but not* $\mathcal{G}_{\mathcal{R}\#}$*-set.*
2. $\{6\}$ *is* $\mathcal{G}_{\mathcal{R}\#}$*-set but not* $\mathcal{G}_s$*-open.*

Remark 3.5. *For a grill topological space* $(X, \mathcal{G}, I)$,

1. *if* L *is open* $\Longrightarrow$ L *is* $\mathcal{G}_{\mathcal{R}^{\#}_{\alpha}}$*-set.*

2. *if L is $\mathcal{G}_{t_\alpha^\#}$-set $\Longrightarrow$ L is $\mathcal{G}_{\mathcal{R}_\alpha^\#}$-set.*

Remark 3.6. *The converse in the every part of Remark 3.5 is as noted in the next two examples, need not be accurate.*

Example 3.3. *In Example 3.1,*

1. *$\{6\}$ is not open but $\mathcal{G}_{\mathcal{R}_\alpha^\#}$-set.*
2. *$\{4\}$ is not $\mathcal{G}_{t_\alpha^\#}$-set but $\mathcal{G}_{\mathcal{R}_\alpha^\#}$-set.*

Proposition 3.2. *If R and S are $\mathcal{G}_{t_\alpha^\#}$-sets of a space $(X, \mathcal{G}, I)$, then $R \cap S$ is $\mathcal{G}_{t_\alpha^\#}$-set.*

Proof.

Let R and S be $\mathcal{G}_{t_\alpha^\#}$-sets. $I(R\cap S) \subseteq I(R\cap S) \subseteq I(\psi(R\cap S)) \subseteq \psi(I(\psi(R\cap S))) \subseteq \psi(I(\psi(R))) \cap \psi(I(\psi(S))) = I(R) \cap I(S)$ (by guess) $= I(R\cap S)$.

Then $I(R\cap S) = \psi(I(\psi(R\cap S)))$ and subsequently, $R\cap S$ is $\mathcal{G}_{t_\alpha^\#}$-set. □

Remark 3.7. *The notions of $\mathcal{G}_\beta$-open sets and $\mathcal{G}_{\mathcal{R}_\alpha^\#}$-sets is a grill topological spaces are independent.*

Example 3.4. *Let $X = \{2, 4, 6, 8\}$ and $\tau = \{\phi, \{8\}, \{2, 6\}, \{2, 6, 8\}, X\}$.*

If $\mathcal{G} = \{\{6\}, \{6, 8\}, X\}$.

1. *$\{6, 8\}$ is $\mathcal{G}_\beta$-open set but not $\mathcal{G}_{\mathcal{R}_\alpha^\#}$-set.*
2. *$\{6\}$ is $\mathcal{G}_{\mathcal{R}_\alpha^\#}$-set but not $\mathcal{G}_\beta$-open.*

Theorem 3.2. *The following properties are equivalent for a subset H of a space $(X, \mathcal{G}, I)$.*

1. *T is open,*
2. *T is $\mathcal{G}_\beta$-open and a $\mathcal{G}_{\mathcal{R}_\alpha^\#}$-set.*

Proof.

(1) implies (2): (1) of Remark 3.5 and (2) of Remark 3.3 come after each other.

(2)implies (1) : Given T is a $\mathcal{G}_{\mathcal{R}_\alpha^\#}$-set. So $T = R\cap S$ where R is open and S is $\mathcal{G}_{t_\alpha^\#}$-set. Then $T \subseteq R = I(R)$. Also H is $\mathcal{G}_\beta$-open implies $T \subseteq \psi(I(\psi(T))) \subseteq \psi(I(\psi(S))) = I(S)$ since S is $\mathcal{G}_{t_\alpha^\#}$-set.

Thus $T \subseteq I(R) \cap I(S) = I(R\cap S) = I(T)$ and thus T is open. □

Remark 3.8. *The subsequent relations are true for a subset H of a space $(X, \mathcal{G}, I)$.*

1. *T is open $\Longrightarrow$ T is $\mathcal{G}_\mathcal{R}$-set.*
2. *T is $\mathcal{G}_t$-set T with $I(\psi(T)) = \psi(I(T))$ $\Rightarrow$ T is $\mathcal{G}_{\mathcal{R}^\#}$-set.*

Proof.

Proof follows straight, since the Definition of $\mathcal{G}_{\mathcal{SR}}$-set. □

Remark 3.9. *As mentioned in the next example, the converse of Remark 3.8(1) is not true.*

Example 3.5. *In Example 3.1, the set* $\{4,6\}$ *is not open but* $\mathcal{G}_{\mathcal{SR}}$*-set.*

Proposition 3.3. *In a space* $(X,\mathcal{G},I)$*, each* $\mathcal{G}_{\mathcal{SR}}$*-set is a* $\mathcal{G}_{\mathcal{R}}$*-set.*

Proof.

Proof follows from the note on that $\mathcal{G}_t$-set T with $I(\psi(T)) = \psi(I(T))$ is $\mathcal{G}_t$-set, which is a $\mathcal{G}_{\mathcal{R}}$-set by of Definition 2.2. □

Theorem 3.3. *In a subset* T *of a space* $(X,\mathcal{G},I)$*, the following properties are equivalent:*

1. T *is open;*
2. T *is* $\mathcal{G}_b$*-open and* $\mathcal{G}_{\mathcal{SR}}$*-set.*

Proof.

(1) implies (2): (1) of Remark 3.8 and (2) of Remark 3.3 come after each other.

(2) implies (1): Given T is $\mathcal{G}_{\mathcal{SR}}$-set. So $T = R \cap S$ where R is open and S is $\mathcal{G}_t$-set with $I(\psi(S)) = \psi(I(S))$. Then $T \subseteq R = I(R)$. Also T is $\mathcal{G}_b$-open implies $T \subseteq I(\psi(T)) \cup \psi(I(T)) \subseteq I(\psi(S)) \cup \psi(I(S)) = I(S)$ by guess. Thus $T \subseteq I(R) \cap I(S) = I(R \cap S) = I(T)$ and hence T is open.

□

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