Prediction of energy dissipation in violent sloshing flows simulated by Smoothed Particle Hydrodynamics

Calderon J., Marrone S., Martinez J., Antuono M., Colagrossi A.

Universidad Politécnica de Madrid, Spain
CNR-INM, Rome Italy

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 815044.

Outline

- Smoothed Particle Hydrodynamics (SPH) in brief
- SPH for sloshing flows and its current limitations
- A new SPH model for the SLOWD sloshing test case
- Energy balance and analysis of the fluid dissipation mechanisms
- Conclusions and perspectives

This project has received funding from the European Union's Horizon 2020
research and innovation programme under grant agreement No 815044.

The SPH model in brief

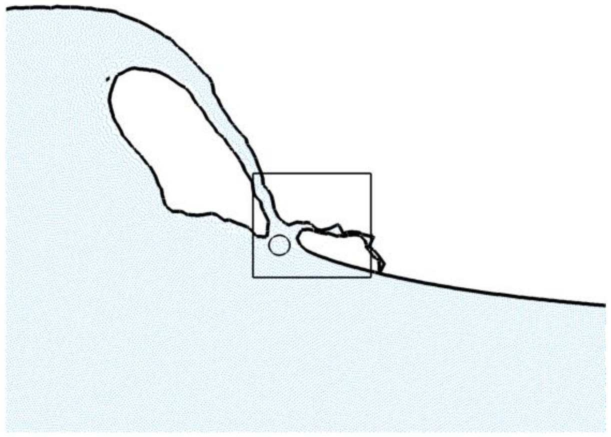
SPH is a meshless numerical method for discretizing Lagrangian PDEs

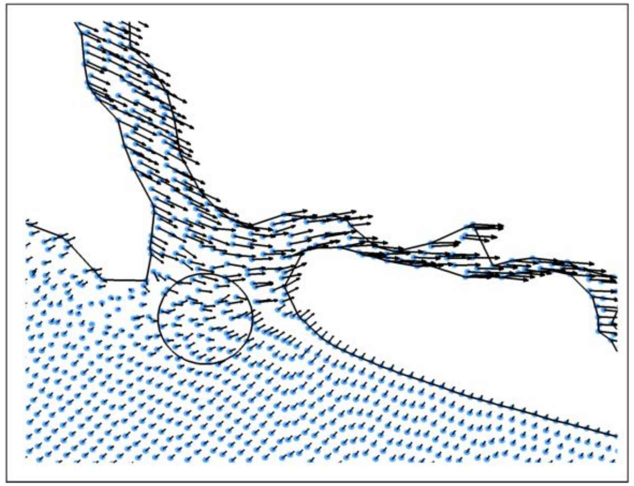
SPH for fluids

Meshless character: a fluid described through a set of unconnected fluid blobs called "particles" (small fluid volumes)

Lagrangian character: each blob is followed in the flow according to its own motion obeying fluid dynamics equations

Hamiltonian character: essential conservation properties (of mass, momentum, energy) are guaranteed

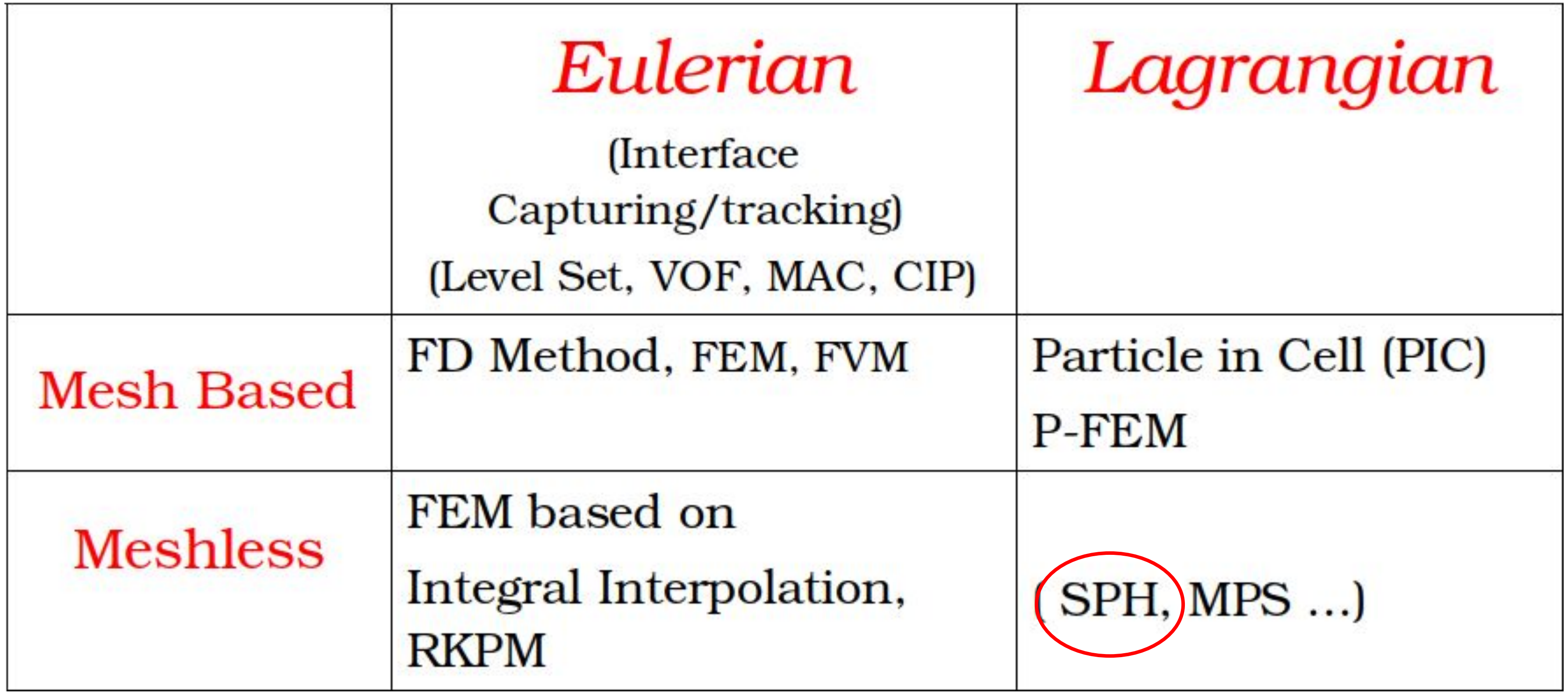




This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 815044.

The SPH model in brief

Positioning with respect to other numerical methods for free-surface flows:



	Eulerian (Interface Capturing/tracking) (Level Set, VOF, MAC, CIP)	*Lagrangian*
Mesh Based	FD Method, FEM, FVM	Particle in Cell (PIC) P-FEM
Meshless	FEM based on Integral Interpolation, RKPM	(SPH, MPS ...)

This project has received funding from the European Union's Horizon 2020
research and innovation programme under grant agreement No 815044.

The SPH model in brief

Mathematical model: **Lagrangian formulation of N-S equations** for a barotropic fluid

$$\begin{cases} \dfrac{\mathrm{D}\rho}{\mathrm{D}t} = -\rho\,\mathrm{div}(\boldsymbol{u}) \\ \rho\,\dfrac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} = -\nabla p + \mathrm{div}\,\mathbb{V} + \rho\,\boldsymbol{g} \\ \dfrac{\mathrm{D}\boldsymbol{r}}{\mathrm{D}t} = \boldsymbol{u}\,, \qquad p = F(\rho) \end{cases}$$

- Single phase approximation (only the liquid phase is modelled)
- Surface tension is neglected
- Liquid is modelled as a **Weakly-compressible** medium (Ma<0.1)

→ Linearized state equation

$$p = c_0^2\,(\rho - \rho_0)$$

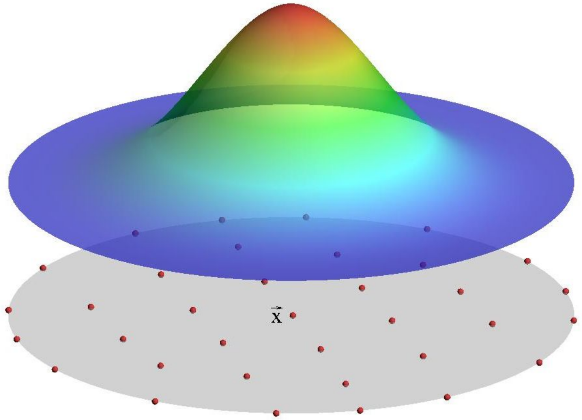
This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 815044.

The SPH model in brief

Standard discrete SPH equations

$$\begin{cases} \dfrac{D\rho_i}{Dt} = -\rho_i \sum_j \dfrac{m_j}{\rho_j}(\boldsymbol{u}_j - \boldsymbol{u}_i) \cdot \nabla_i W_{ij} \\ \dfrac{D\boldsymbol{u}_i}{Dt} = -\sum_j \dfrac{m_j}{\rho_i \rho_j}(p_j + p_i)\nabla_i W_{ij} + \mu \sum_j \dfrac{m_j}{\rho_i \rho_j}\pi_{ij}\nabla_i W_{ij} + \boldsymbol{g} \\ \dfrac{D\boldsymbol{r}_i}{Dt} = \boldsymbol{u}_i(t), \quad p = c_0^2(\rho - \rho_0) \end{cases}$$

Differential operators are computed as a **sum of particle interactions** weighted through a Kernel function ***W***



This project has received funding from the European Union’s Horizon 2020 research and innovation programme under grant agreement No 815044.

The SPH model in brief

Advanced model **δ-LES-SPH**: SPH with **LES filtering** for turbulence modelling

$$
\begin{cases}
\dfrac{D\rho_i}{Dt} = -\rho_i \sum_j (\boldsymbol{u}_j - \boldsymbol{u}_i)\cdot \nabla_i W(\boldsymbol{r}_{ij}) V_j + \sum_j \nu_{ij}^{\delta}\, \mathcal{D}_{ij}\cdot \nabla_i W(\boldsymbol{r}_{ij}) V_j, \\
\dfrac{D\boldsymbol{u}_i}{Dt} = \mathbf{g}_i - \dfrac{1}{\rho_i} \sum_j (p_i + p_j) \nabla_i W(\boldsymbol{r}_{ij}) V_j + \sum_j (\nu_i + \nu_{ij}^{T})\, \pi_{ij}\, \nabla_i W(\boldsymbol{r}_{ij}) V_j, \\
\dfrac{D\boldsymbol{r}_i}{Dt} = \boldsymbol{u}_i,\ p_i = c_0^2 (\rho_i - \rho_0)
\end{cases}
$$

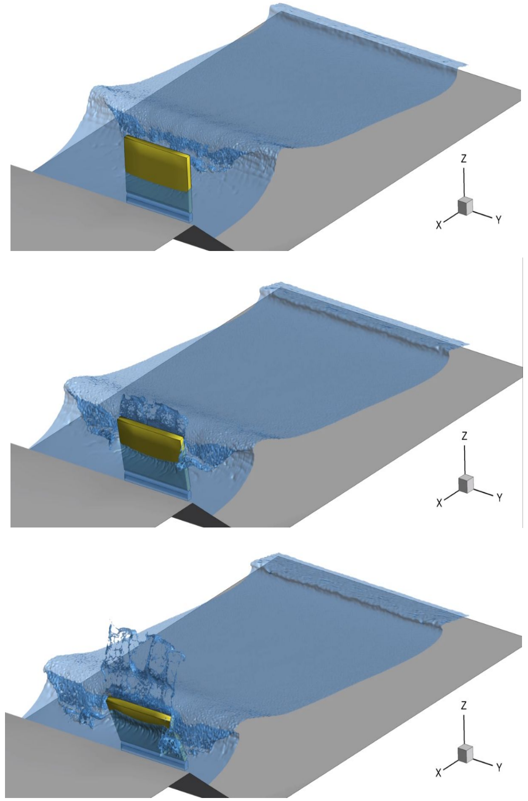
Classical Smagorinsky closure is adopted: $\nu_T = (C_S\, \sigma)^2 \ \|\widetilde{\mathbb{D}}\|,$

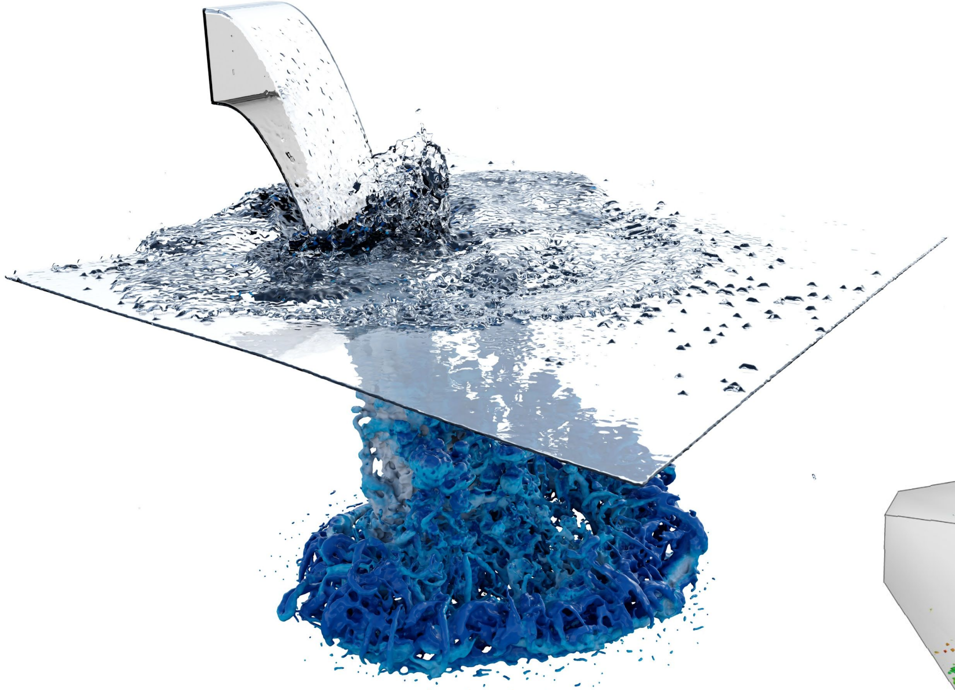
"Smoothed Particle Hydrodynamics method from a large eddy simulation perspective."
Di Mascio et al. Physics of Fluids (2017)

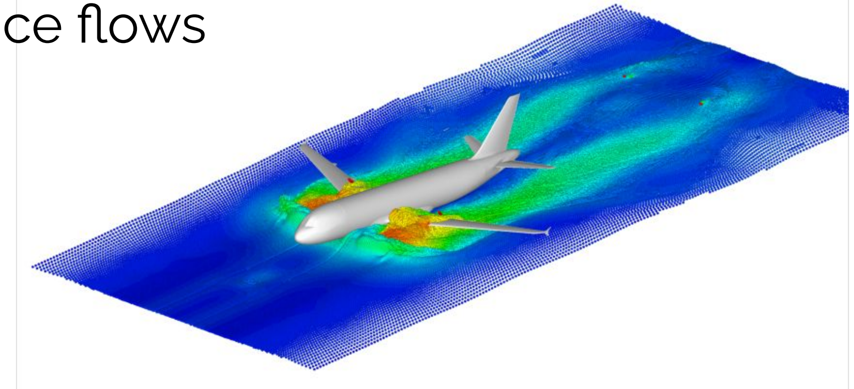
This project has received funding from the European Union’s Horizon 2020
research and innovation programme under grant agreement No 815044.

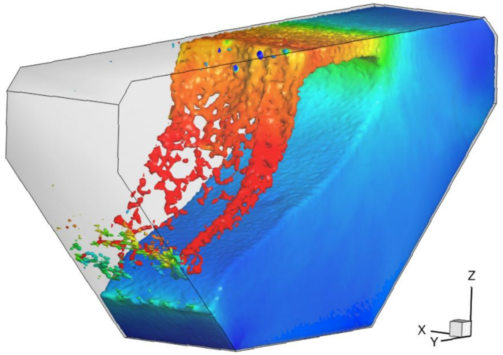
The SPH model in brief

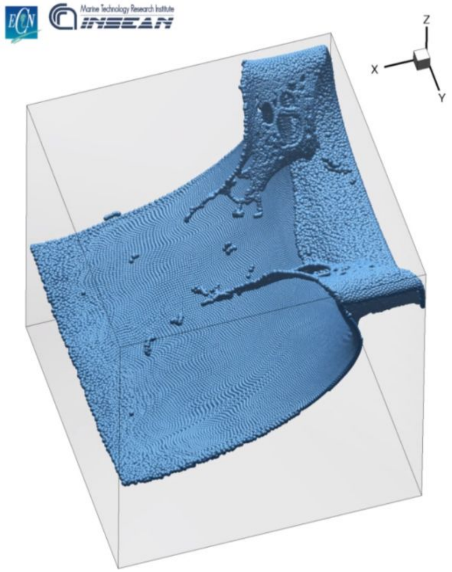
This model is particularly effective in simulating free-surface flows







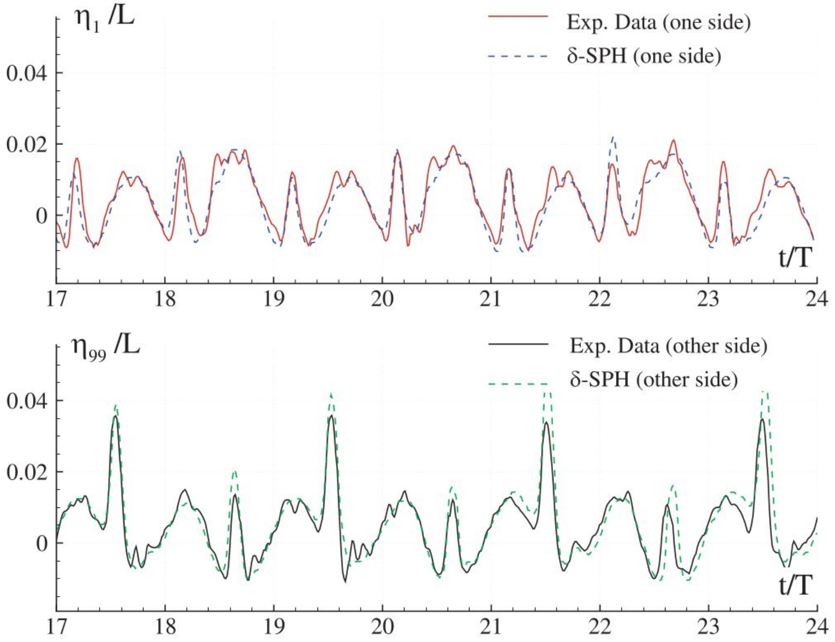


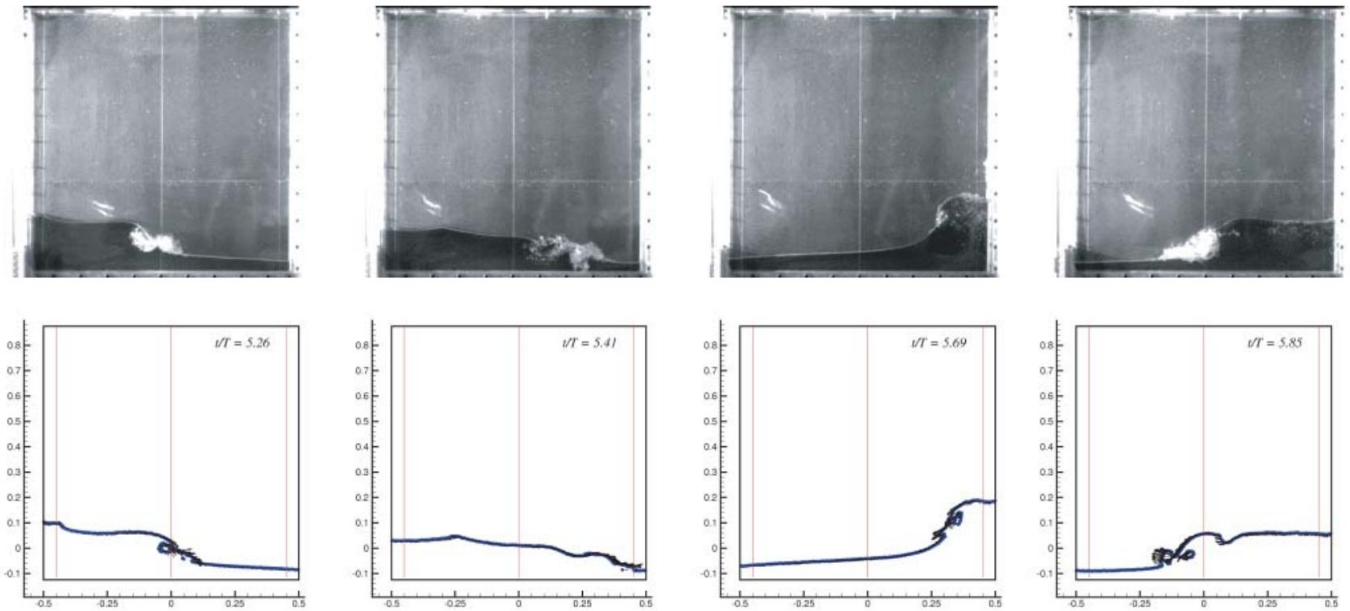


This project has received funding from the European Union's Horizon 2020
research and innovation programme under grant agreement No 815044.

SPH for sloshing flows and its current limitations

SPH has been thoroughly validated on simulation of sloshing flows



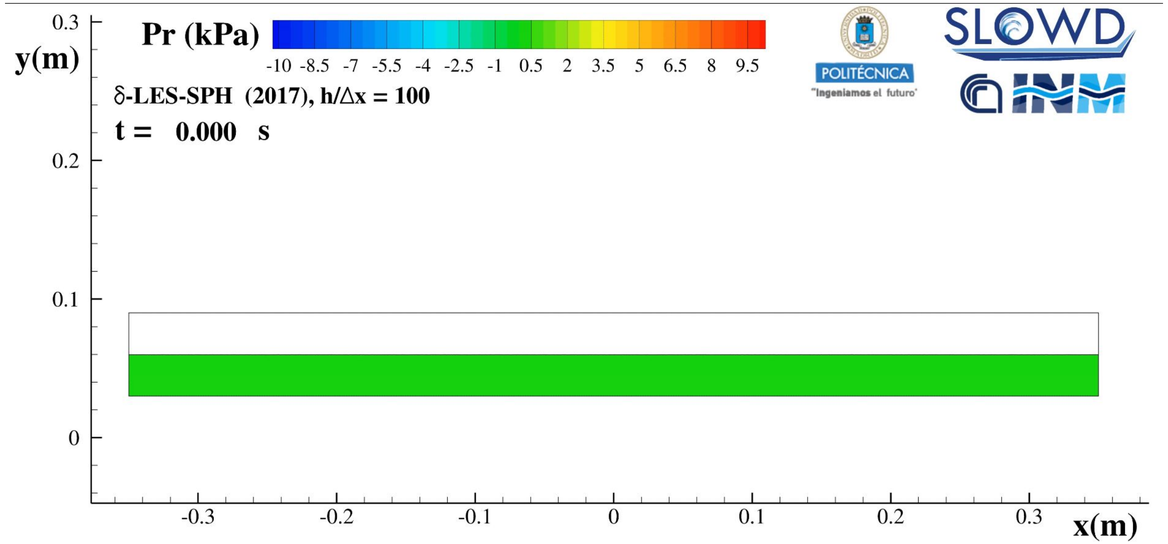


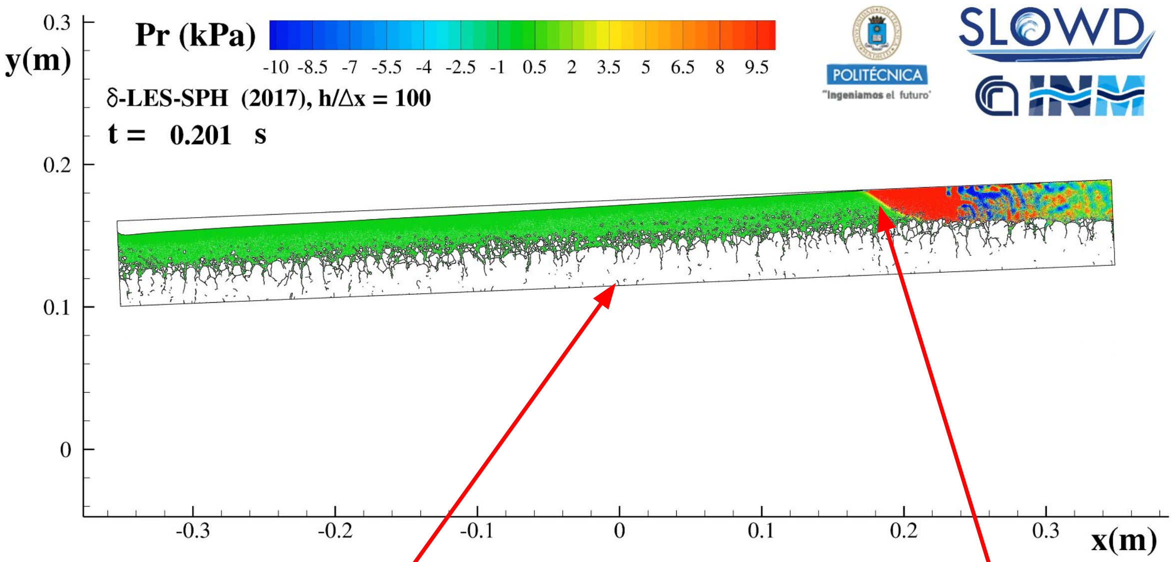
Numerical and experimental investigation of nonlinear shallow water sloshing.
Bouscasse et al . International Journal of Nonlinear Sciences and Numerical Simulation, 14(2), 2013

This project has received funding from the European Union’s Horizon 2020 research and innovation programme under grant agreement No 815044.

SPH for sloshing flows and its current limitations

However, for very high vertical accelerations, intense negative pressure develops and "**numerical cavitation**" occurs:





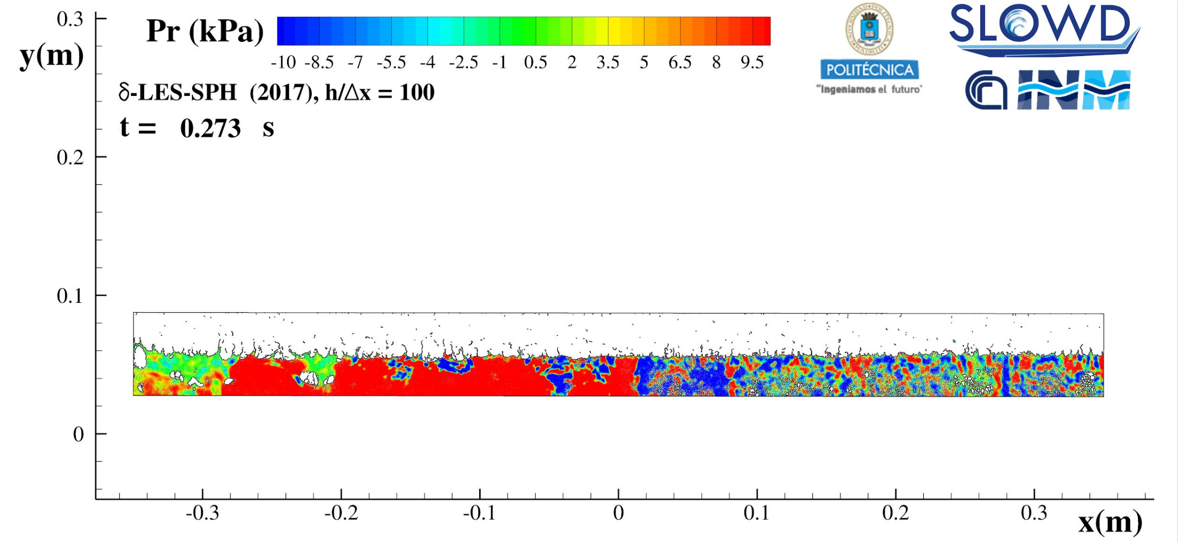
Detachment of the fluid from the bottom, linked to the negative pressure value (tensile instability)

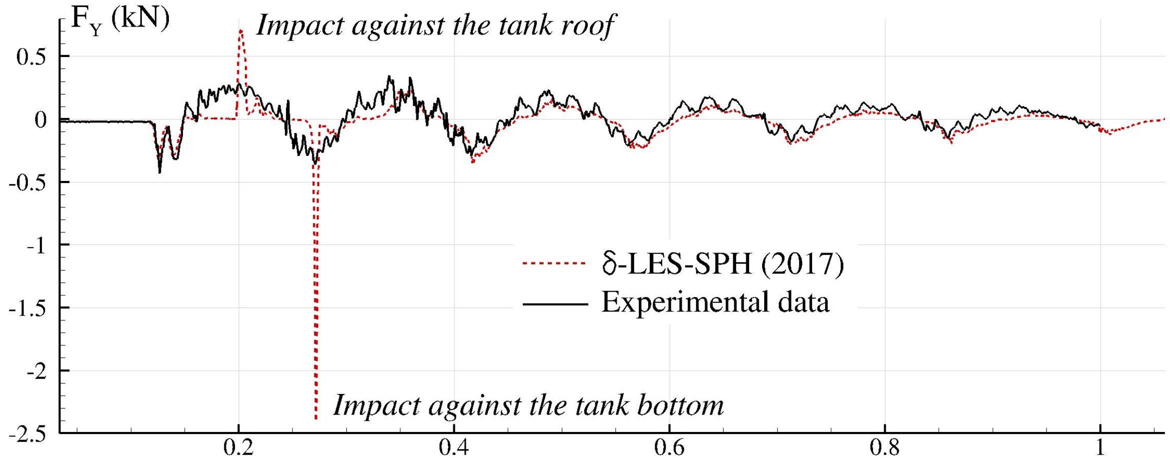
Violent pressure impact on the tank roof.

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 815044.

SPH for sloshing flows and its current limitations

However, for very high vertical accelerations, intense negative pressure develops and “**numerical cavitation**” occurs:

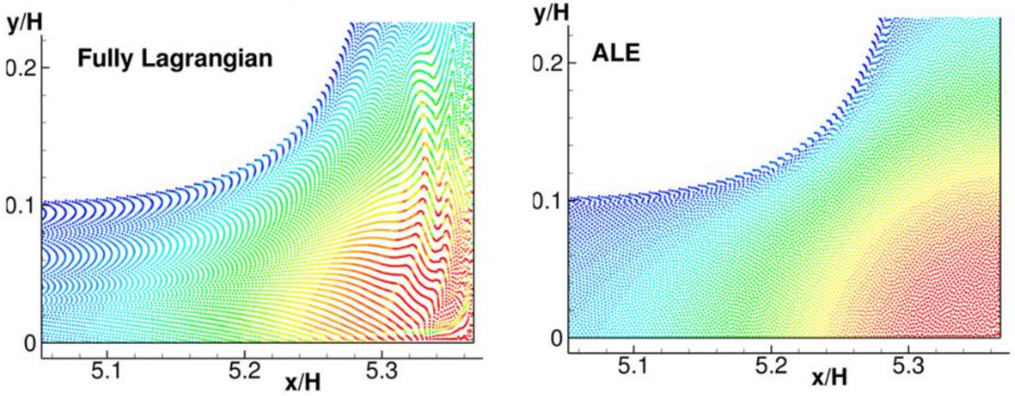




This project has received funding from the European Union’s Horizon 2020 research and innovation programme under grant agreement No 815044.

A new SPH model

δ-LES-SPH model rewritten in an **Arbitrary Lagrangian Eulerian** framework (ALE)



Particles motion is made **quasi-Lagrangian** in order to keep particles uniformly distributed

$$\left\{\begin{aligned} \frac{d\tilde\rho_i}{dt} &= -\,\tilde\rho_i\sum_j\big[(\tilde{\boldsymbol u}_j+\delta\tilde{\boldsymbol u}_j)-(\tilde{\boldsymbol u}_i+\delta\tilde{\boldsymbol u}_i)\big]\cdot\nabla_i W_{ij}\,V_j \;+ \\ &\qquad \sum_j(\tilde\rho_j\delta\tilde{\boldsymbol u}_j+\tilde\rho_i\delta\tilde{\boldsymbol u}_i)\cdot\nabla_i W_{ij}\,V_j \;+\; \sum_j \delta_{ij}\,\boldsymbol\psi_{ji}\cdot\nabla_i W_{ij}\,V_j \\ \frac{d\tilde{\boldsymbol u}_i}{dt} &= -\frac{1}{\tilde\rho_i}\sum_j P_{ij}\nabla_i W_{ij}\,V_j \;+\; \frac{\rho_0}{\tilde\rho_i}\,K\sum_j \alpha_{ij}\,\pi_{ij}\nabla_i W_{ij}\,V_j \;+ \\ &\qquad \frac{\rho_0}{\tilde\rho_i}\sum_j(\tilde{\boldsymbol u}_j\otimes\delta\tilde{\boldsymbol u}_j+\tilde{\boldsymbol u}_i\otimes\delta\tilde{\boldsymbol u}_i)\cdot\nabla_i W_{ij}\,V_j \\ \frac{d\tilde{\boldsymbol x}_i}{dt} &= \tilde{\boldsymbol u}_i+\delta\tilde{\boldsymbol u}_i\,, \qquad \tilde p_i = F(\tilde\rho_i)\,, \qquad V_i = \frac{m_i}{\tilde\rho_i}\,, \end{aligned}\right.$$

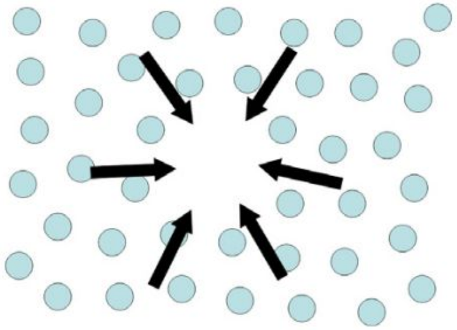
"SPH accuracy improvement through the combination of a quasi-Lagrangian shifting transport velocity and consistent ALE formalisms."
Oger et al. Journal of Computational Physics 2016

This project has received funding from the European Union’s Horizon 2020 research and innovation programme under grant agreement No 815044.

A new SPH model

Motion is corrected through a Particle Shifting Technique

$$\delta\hat{\boldsymbol{u}}_i = -M_a\,\ell\,c_0 \sum_j \left[1 + R \left(\frac{W_{ij}}{W(\Delta x)} \right)^n \right] \nabla_i W_{ij} V_j$$



$$\begin{cases} \dfrac{d\tilde{\rho}_i}{dt} = -\tilde{\rho}_i \displaystyle\sum_j \left[(\tilde{\boldsymbol{u}}_j + \boxed{\delta\tilde{\boldsymbol{u}}_j}) - (\tilde{\boldsymbol{u}}_i + \boxed{\delta\tilde{\boldsymbol{u}}_i}) \right] \cdot \nabla_i W_{ij}\, V_j + \\ \qquad \boxed{\displaystyle\sum_j (\tilde{\rho}_j \delta\tilde{\boldsymbol{u}}_j + \tilde{\rho}_i \delta\tilde{\boldsymbol{u}}_i) \cdot \nabla_i W_{ij}\, V_j} + \displaystyle\sum_j \delta_{ij}\, \boldsymbol{\psi}_{ji} \cdot \nabla_i W_{ij}\, V_j \\ \dfrac{d\tilde{\boldsymbol{u}}_i}{dt} = -\dfrac{1}{\tilde{\rho}_i} \displaystyle\sum_j P_{ij} \nabla_i W_{ij}\, V_j + \dfrac{\rho_0}{\tilde{\rho}_i} K \displaystyle\sum_j \alpha_{ij}\, \pi_{ij} \nabla_i W_{ij}\, V_j + \\ \qquad \boxed{\dfrac{\rho_0}{\tilde{\rho}_i} \displaystyle\sum_j (\tilde{\boldsymbol{u}}_j \otimes \delta\tilde{\boldsymbol{u}}_j + \tilde{\boldsymbol{u}}_i \otimes \delta\tilde{\boldsymbol{u}}_i) \cdot \nabla_i W_{ij}\, V_j} \\ \dfrac{d\tilde{\boldsymbol{x}}_i}{dt} = \tilde{\boldsymbol{u}}_i + \boxed{\delta\tilde{\boldsymbol{u}}_i} \qquad \tilde{p}_i = F(\tilde{\rho}_i)\,, \qquad V_i = \dfrac{m_i}{\tilde{\rho}_i}\,, \end{cases}$$

This project has received funding from the European Union’s Horizon 2020 research and innovation programme under grant agreement No 815044.

A new SPH model

LES-Model for
weakly-compressible SPH

$$\alpha_{ij} = \frac{\mu}{\rho_0} + 2\,\frac{\nu_i^T\,\nu_j^T}{\nu_i^T + \nu_j^T} \qquad \delta_{ij} = 2\,\frac{\nu_i^\delta\,\nu_j^\delta}{\nu_i^\delta + \nu_j^\delta}$$

$$\nu_i^T = (C_S\,\ell)^2\,\|\widetilde{\mathbb{D}}_i\| \qquad \nu_i^\delta = (C_\delta\,\ell)^2\,\|\widetilde{\mathbb{D}}_i\|$$

$$\left\{\begin{aligned}
\frac{d\tilde{\rho}_i}{dt} &= -\,\tilde{\rho}_i \sum_j \left[(\tilde{\boldsymbol{u}}_j + \delta\tilde{\boldsymbol{u}}_j) - (\tilde{\boldsymbol{u}}_i + \delta\tilde{\boldsymbol{u}}_i)\right] \cdot \nabla_i W_{ij}\, V_j \,+ \\
&\qquad \sum_j (\tilde{\rho}_j \delta\tilde{\boldsymbol{u}}_j + \tilde{\rho}_i \delta\tilde{\boldsymbol{u}}_i) \cdot \nabla_i W_{ij}\, V_j + \boxed{\sum_j \delta_{ij}\, \boldsymbol{\psi}_{ji} \cdot \nabla_i W_{ij}\, V_j} \\
\frac{d\tilde{\boldsymbol{u}}_i}{dt} &= -\frac{1}{\tilde{\rho}_i} \sum_j P_{ij} \nabla_i W_{ij}\, V_j \,+ \boxed{\frac{\rho_0}{\tilde{\rho}_i} K \sum_j \alpha_{ij}\, \pi_{ij} \nabla_i W_{ij}\, V_j} + \\
&\qquad \frac{\rho_0}{\tilde{\rho}_i} \sum_j (\tilde{\boldsymbol{u}}_j \otimes \delta\tilde{\boldsymbol{u}}_j + \tilde{\boldsymbol{u}}_i \otimes \delta\tilde{\boldsymbol{u}}_i) \cdot \nabla_i W_{ij}\, V_j \\
\frac{d\tilde{\boldsymbol{x}}_i}{dt} &= \tilde{\boldsymbol{u}}_i + \delta\tilde{\boldsymbol{u}}_i\,, \qquad \tilde{p}_i = F(\tilde{\rho}_i)\,, \qquad V_i = \frac{m_i}{\tilde{\rho}_i}\,,
\end{aligned}\right.$$

This project has received funding from the European Union's Horizon 2020
research and innovation programme under grant agreement No 815044.

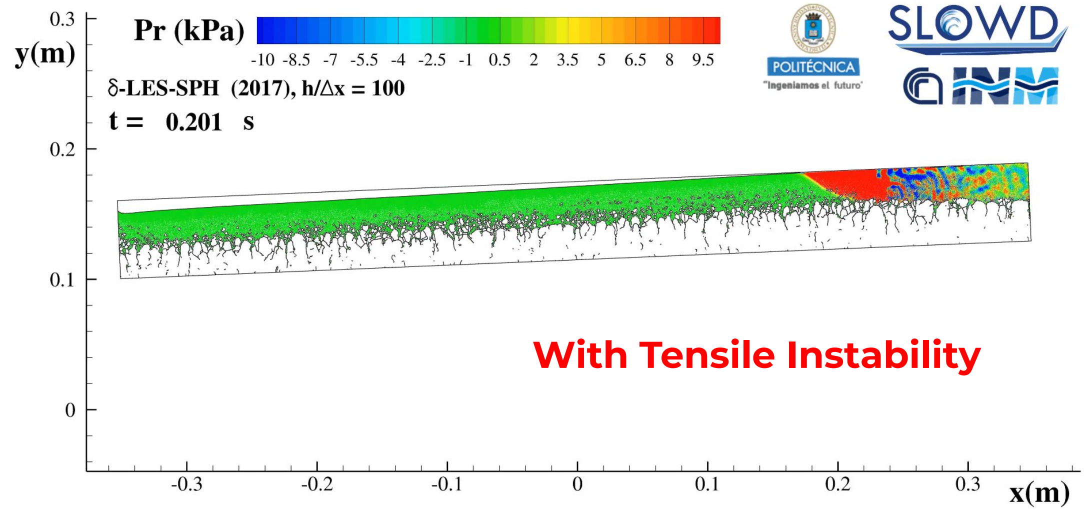
A new SPH model

- **Accuracy has been improved**
- **Tensile instability has been removed**
- **Numerical dissipation has been reduced**
- **The evaluation of energy dissipation terms is more complex due to the new particles interaction terms**

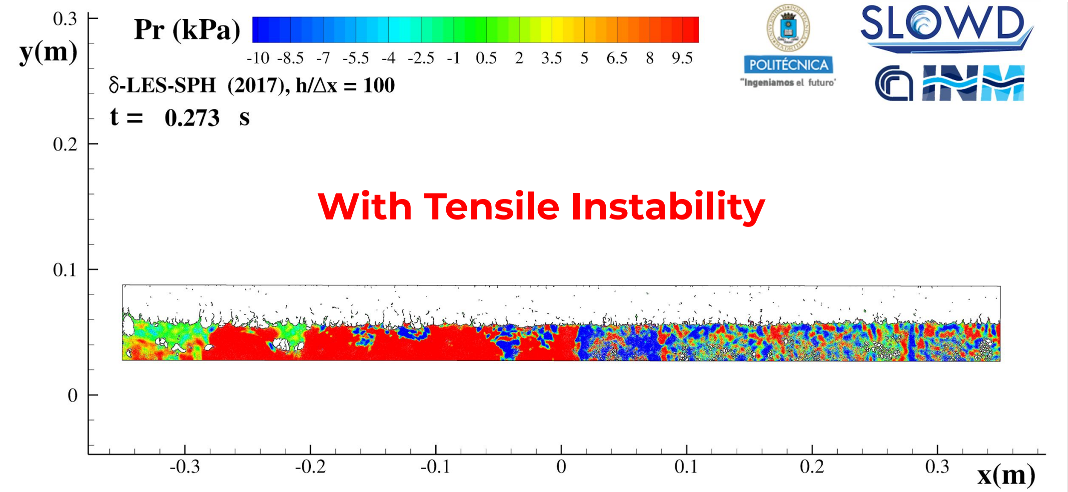
$$
\left\{
\begin{aligned}
\frac{d\tilde{\rho}_i}{dt} &= -\,\tilde{\rho}_i \sum_j \left[(\tilde{\boldsymbol{u}}_j + \delta\tilde{\boldsymbol{u}}_j) - (\tilde{\boldsymbol{u}}_i + \delta\tilde{\boldsymbol{u}}_i)\right] \cdot \nabla_i W_{ij}\, V_j \, + \\
&\qquad \sum_j (\tilde{\rho}_j \delta\tilde{\boldsymbol{u}}_j + \tilde{\rho}_i \delta\tilde{\boldsymbol{u}}_i) \cdot \nabla_i W_{ij}\, V_j \, + \sum_j \delta_{ij}\, \boldsymbol{\psi}_{ji} \cdot \nabla_i W_{ij}\, V_j \\
\frac{d\tilde{\boldsymbol{u}}_i}{dt} &= -\frac{1}{\tilde{\rho}_i} \sum_j P_{ij} \nabla_i W_{ij}\, V_j \, + \, \frac{\rho_0}{\tilde{\rho}_i}\, K \sum_j \alpha_{ij}\, \pi_{ij} \nabla_i W_{ij}\, V_j \, + \\
&\qquad \frac{\rho_0}{\tilde{\rho}_i} \sum_j (\tilde{\boldsymbol{u}}_j \otimes \delta\tilde{\boldsymbol{u}}_j + \tilde{\boldsymbol{u}}_i \otimes \delta\tilde{\boldsymbol{u}}_i) \cdot \nabla_i W_{ij}\, V_j \\
\frac{d\tilde{\boldsymbol{x}}_i}{dt} &= \tilde{\boldsymbol{u}}_i + \delta\tilde{\boldsymbol{u}}_i \, , \qquad \tilde{p}_i \, = \, F(\tilde{\rho}_i) \, , \qquad V_i \, = \, \frac{m_i}{\tilde{\rho}_i} \, ,
\end{aligned}
\right.
$$

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 815044.

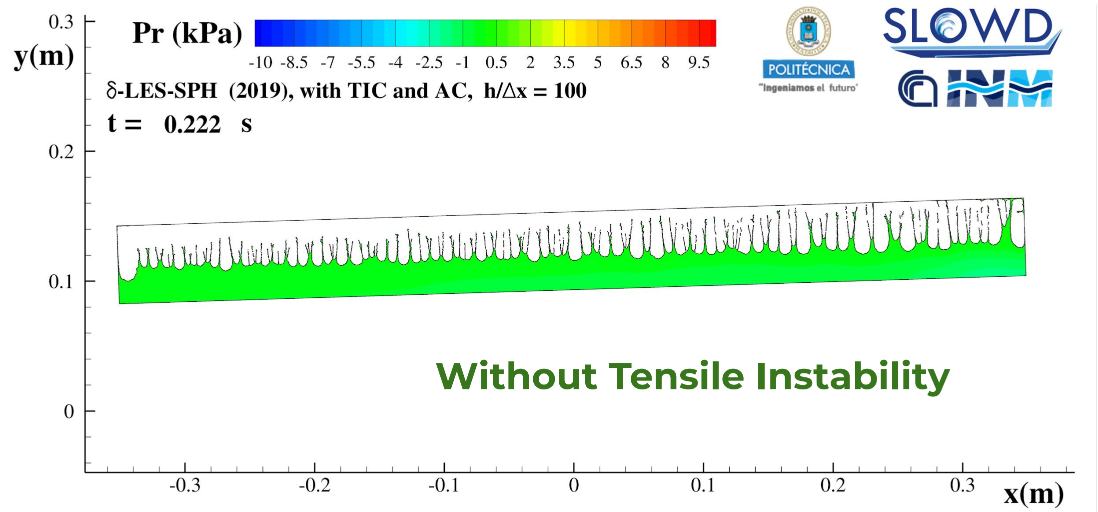
Solutions without Tensile Instability



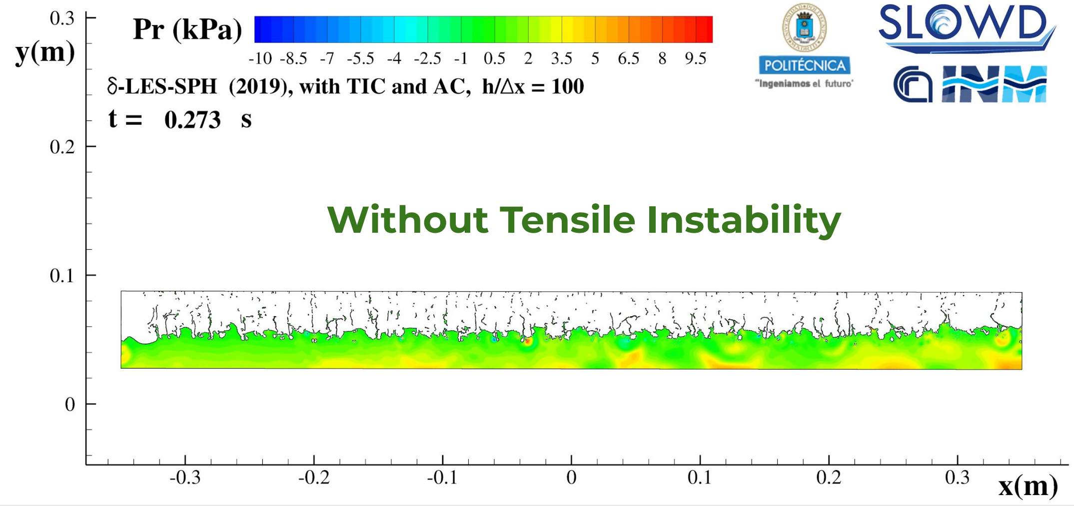
With Tensile Instability



With Tensile Instability



Without Tensile Instability

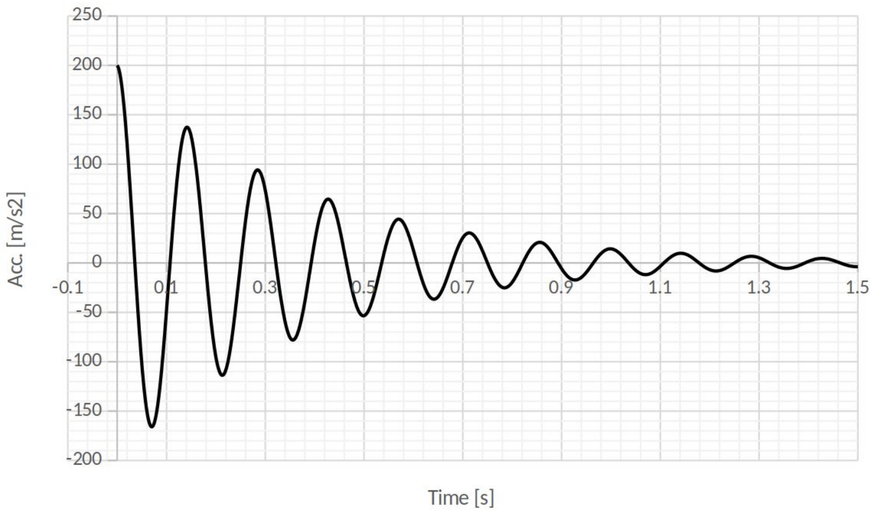


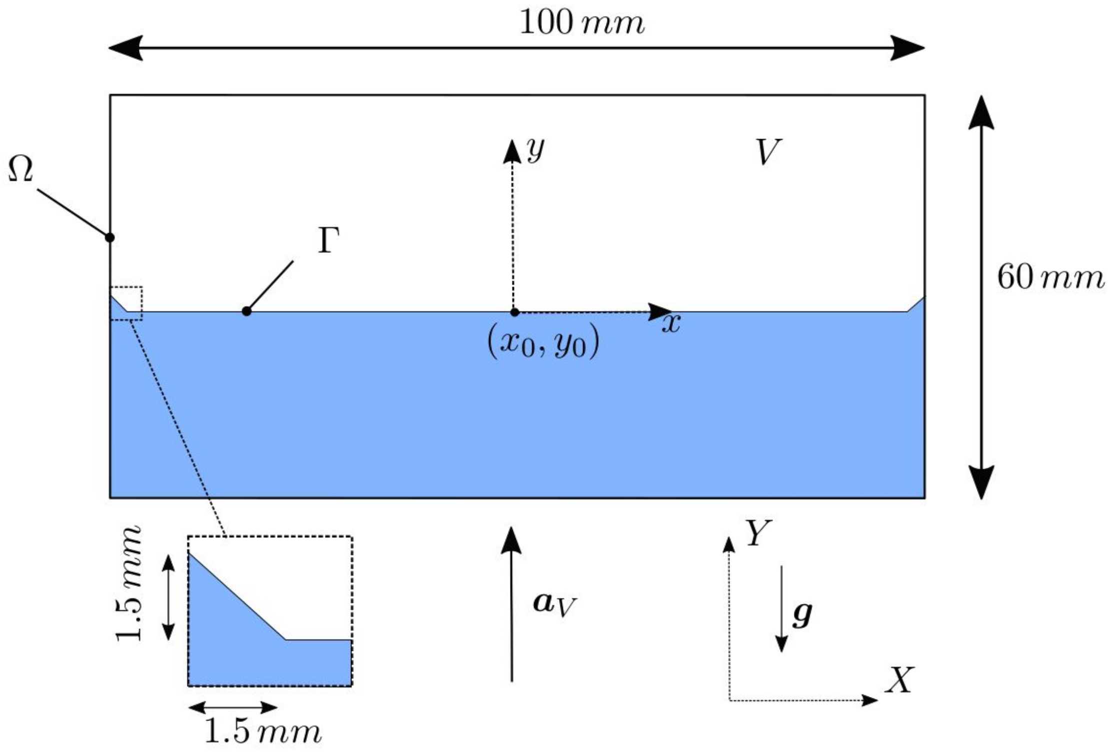
Without Tensile Instability

Benchmarking

Analytical law of the tank motion

$$a(t) = a_0 e^{\lambda t} cos(\omega t)$$

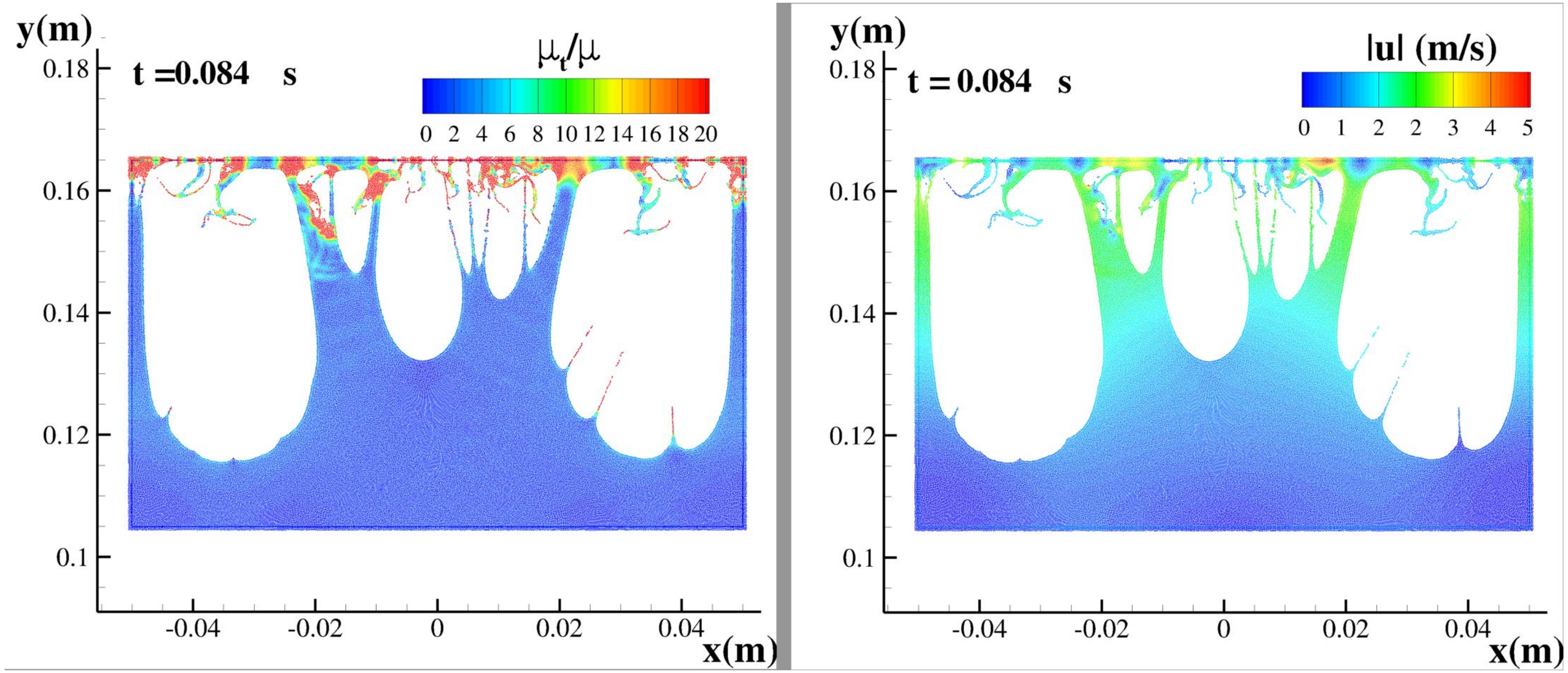




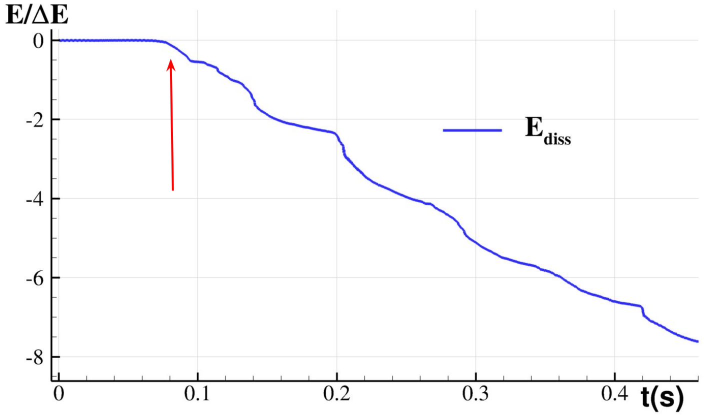
A **simplified liquid meniscus** at wall is added
→ initial flow instability is triggered

This project has received funding from the European Union’s Horizon 2020 research and innovation programme under grant agreement No 815044.

Flow evolution

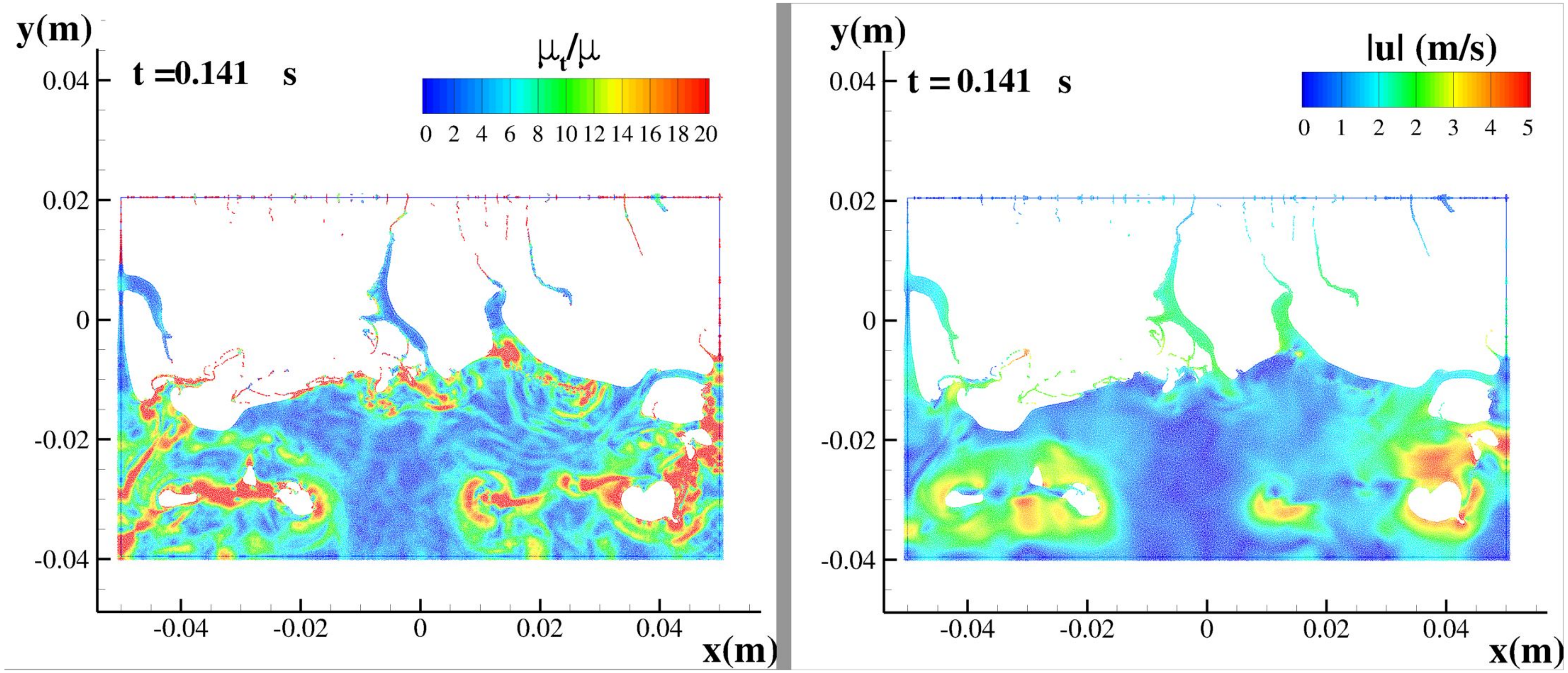


Dissipated energy

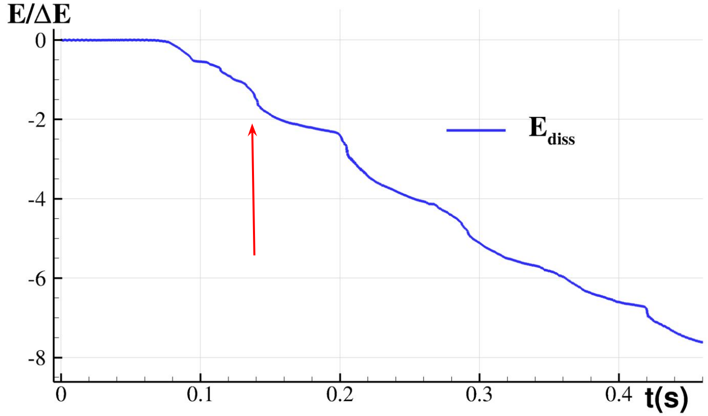


This project has received funding from the European Union’s Horizon 2020 research and innovation programme under grant agreement No 815044.

Flow evolution

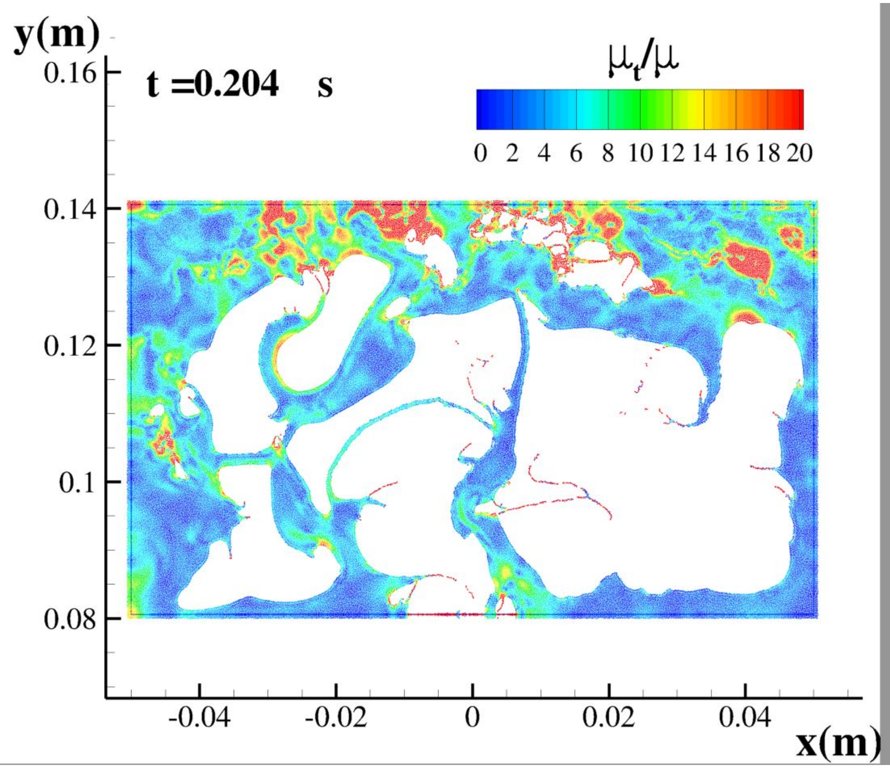


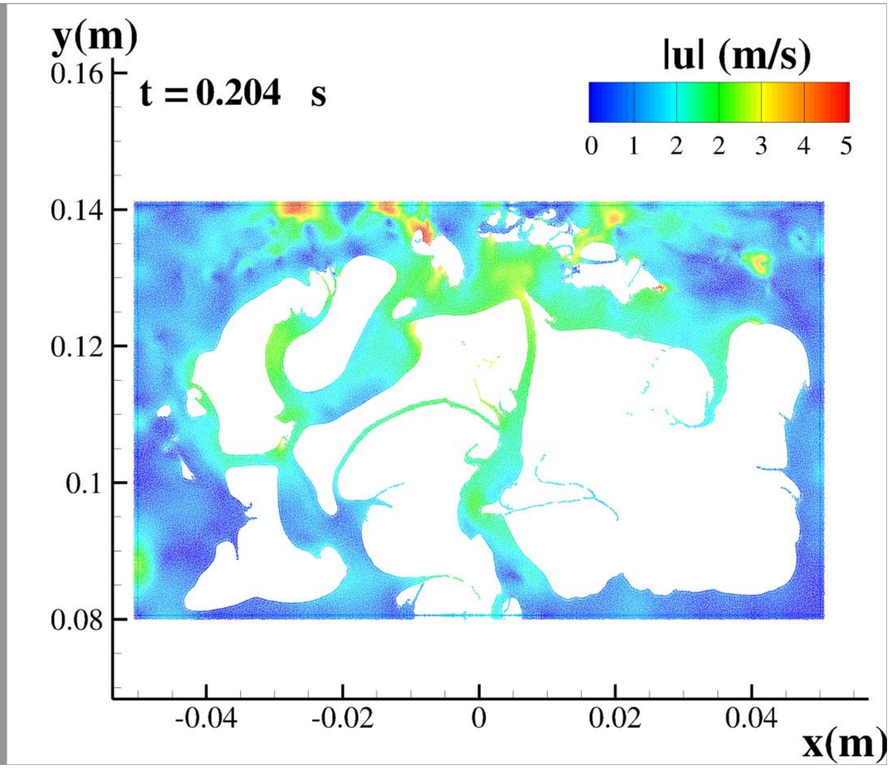
Dissipated energy



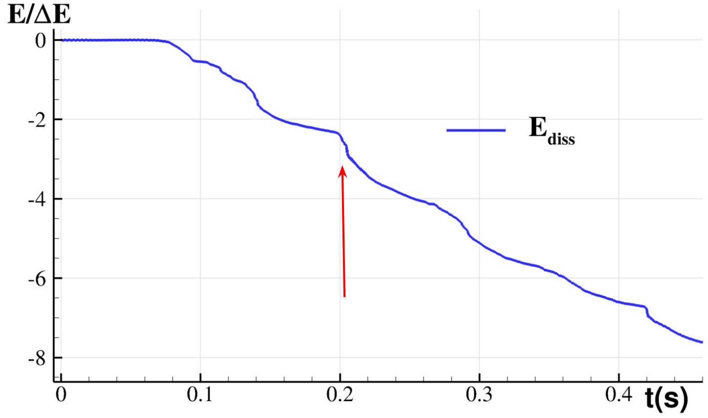
This project has received funding from the European Union’s Horizon 2020 research and innovation programme under grant agreement No 815044.

Flow evolution





Dissipated energy



This project has received funding from the European Union’s Horizon 2020 research and innovation programme under grant agreement No 815044.

Energy budget

$$-\mathcal{P}_{ext} + \dot{\mathcal{E}}_M + \dot{\mathcal{E}}_C = \mathcal{P}_V + \mathcal{P}_V^{turb} + \mathcal{P}_N \leq 0$$

Power related to **external forces**

Time derivative of the fluid **mechanical energy**

Time derivative of reversible **compressible energy**

$$\mathcal{P}_C := -\int_{\Omega} p\,\mathrm{div}(\boldsymbol{u})\,dV$$

→ generally negligible

Power related to laminar and turbulent **viscous dissipation**

Power related to **numerical dissipation**

A. Colagrossi, B. Bouscasse, S. Marrone, *Energy decomposition analysis for viscous free-surface flows*, Phys. Rev. E, Vol. **92**, pp. 053003-13, (2015).

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 815044.

Energy budget

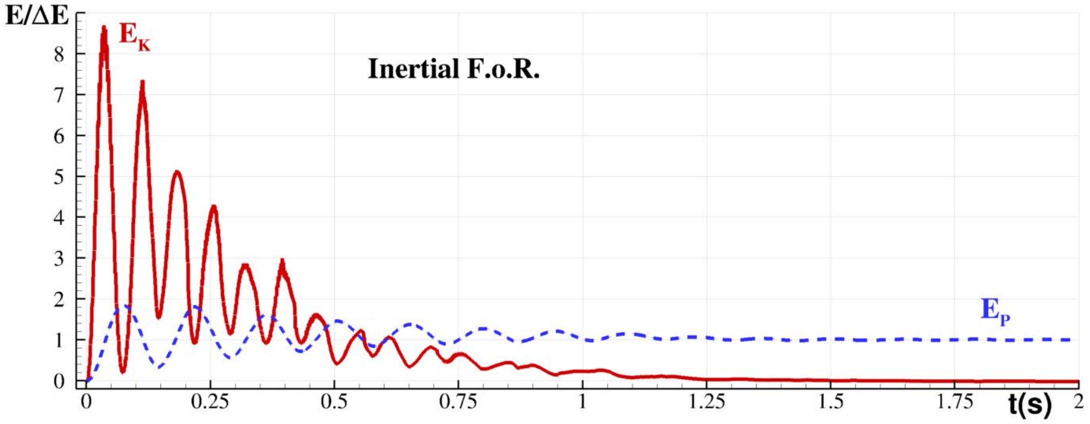
Dependency on the Frame of Reference of the simulation:

$$-\mathcal{P}_{ext} + \dot{\mathcal{E}}_M + \underbrace{\dot{\mathcal{E}}_C = \mathcal{P}_V + \mathcal{P}_V^{turb}}_{\boxed{\text{Invariant for rigid motions of the F.o.R.}}} + \mathcal{P}_N \leq 0$$

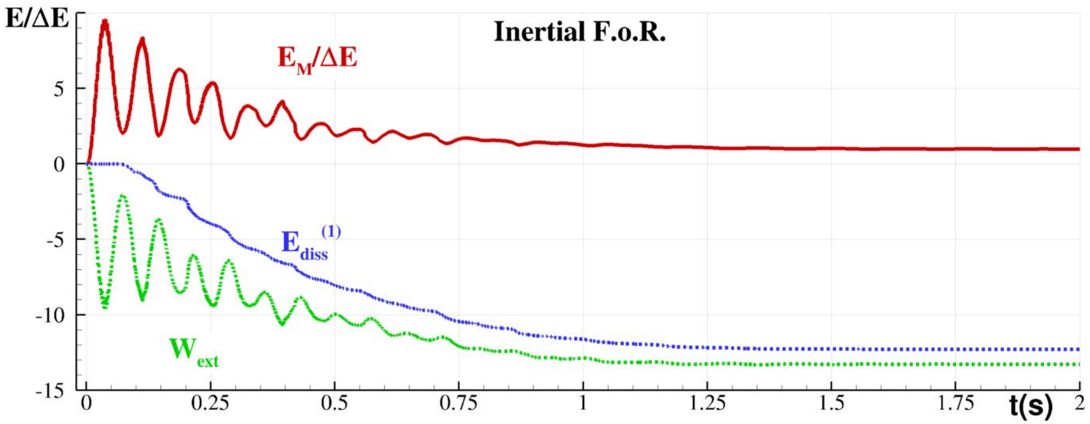
This project has received funding from the European Union’s Horizon 2020
research and innovation programme under grant agreement No 815044.

Energy budget

Simulation in the **inertial** Frame of Reference



Kinetic and Potential Energy

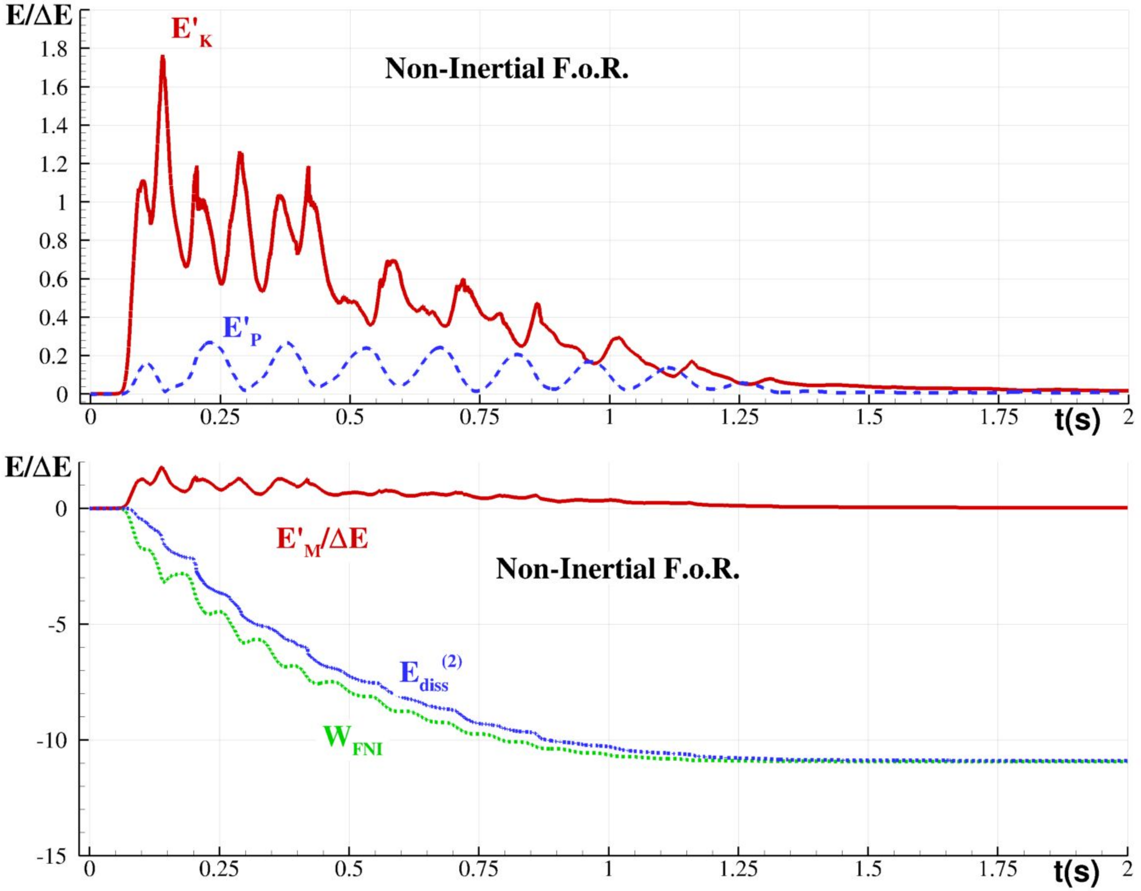


Mechanical energy (**EM**) and Work done by solid walls (**Wext**)

$$\overbrace{-\mathcal{P}_{ext} + \dot{\mathcal{E}}_M + \dot{\mathcal{E}}_C}^{\mathbf{E}_{\mathbf{diss}}^{(1)}} = \mathcal{P}_V + \mathcal{P}_V^{turb} + \mathcal{P}_N$$

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 815044.

Energy budget



Simulation in the **non-inertial** Frame of Reference

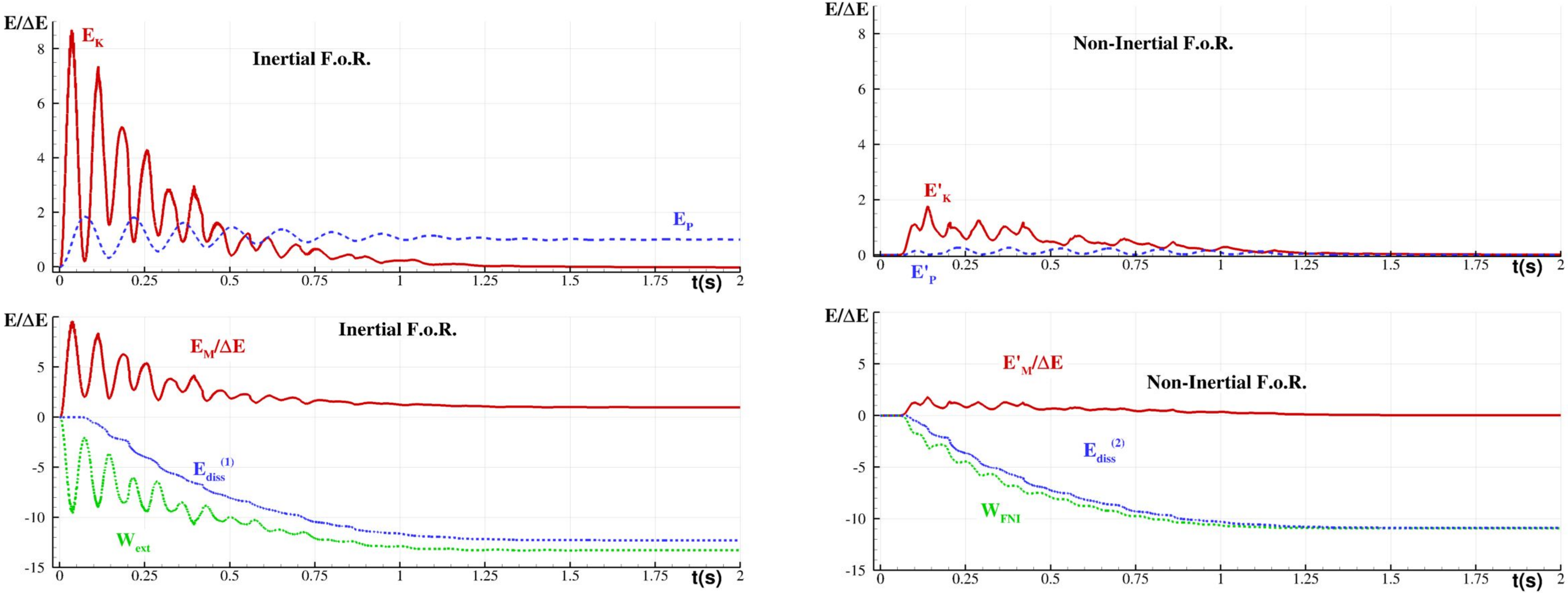
Kinetic and Potential Energy

Mechanical energy (**EM**) and Work done by non-inertial Forces (**FNI**)

$$\overbrace{-\mathcal{P}_{ext} + \dot{\mathcal{E}}_M + \dot{\mathcal{E}}_C}^{E_{diss}^{(2)}} = \mathcal{P}_V + \mathcal{P}_V^{turb} + \mathcal{P}_N$$

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 815044.

Energy budget

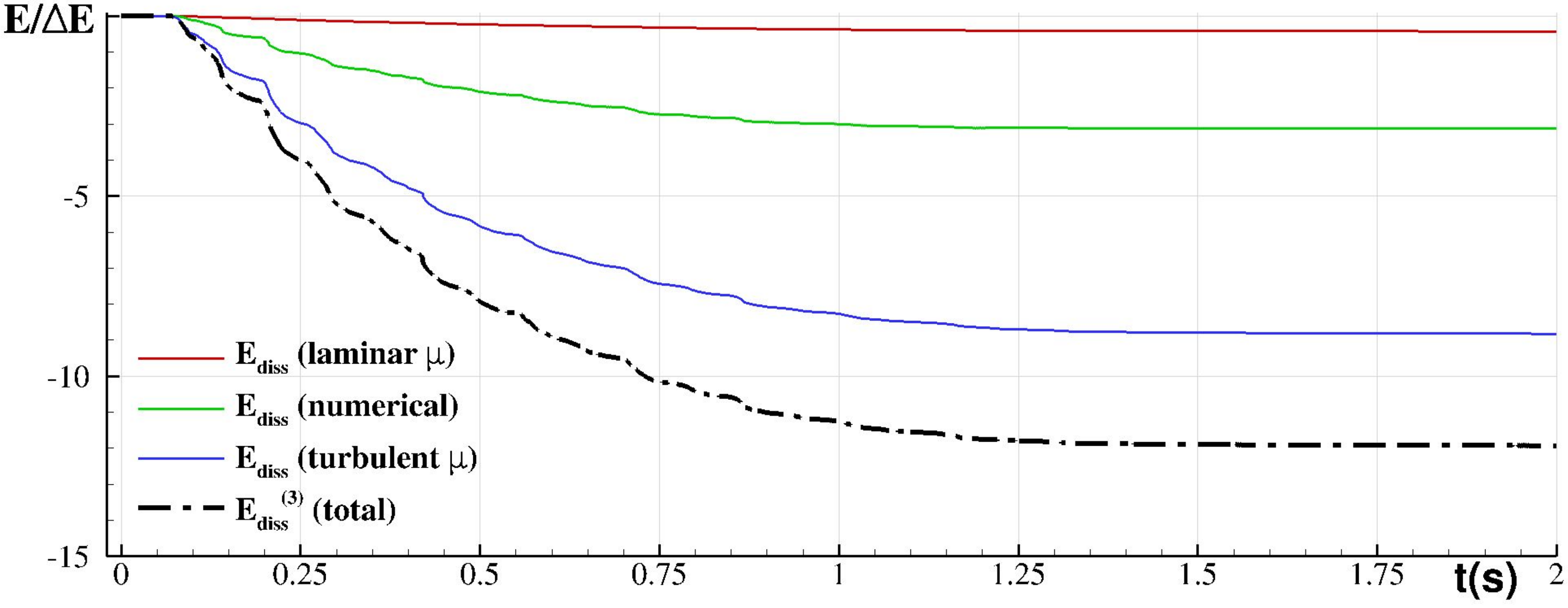


This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 815044.

$$\mathcal{P}_{ext} + \dot{\mathcal{E}}_M + \dot{\mathcal{E}}_C = \mathcal{P}_V$$

Energy budget

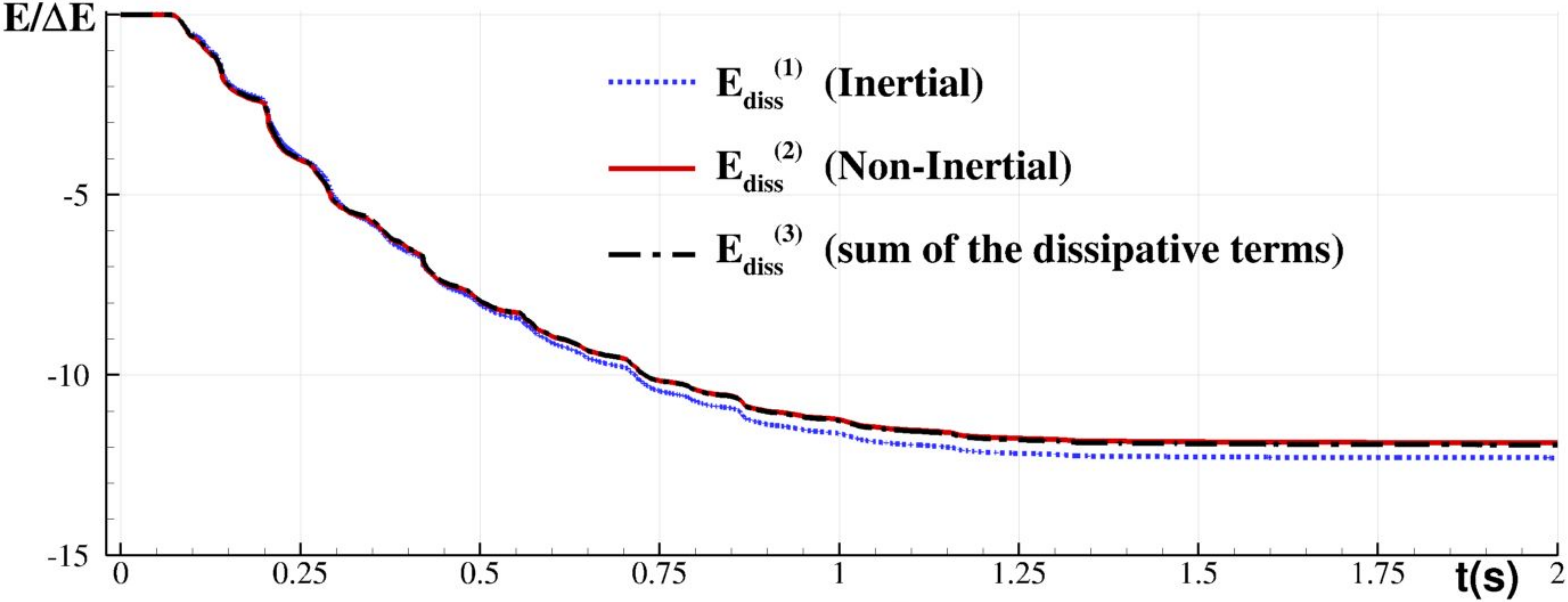
Purely dissipative terms



$$-\mathcal{P}_{ext} + \dot{\mathcal{E}}_M + \dot{\mathcal{E}}_C = \overbrace{\mathcal{P}_V + \mathcal{P}_V^{turb} + \mathcal{P}_N}^{E_{diss}^{(3)}}$$

This project has received funding from the European Union’s Horizon 2020 research and innovation programme under grant agreement No 815044.

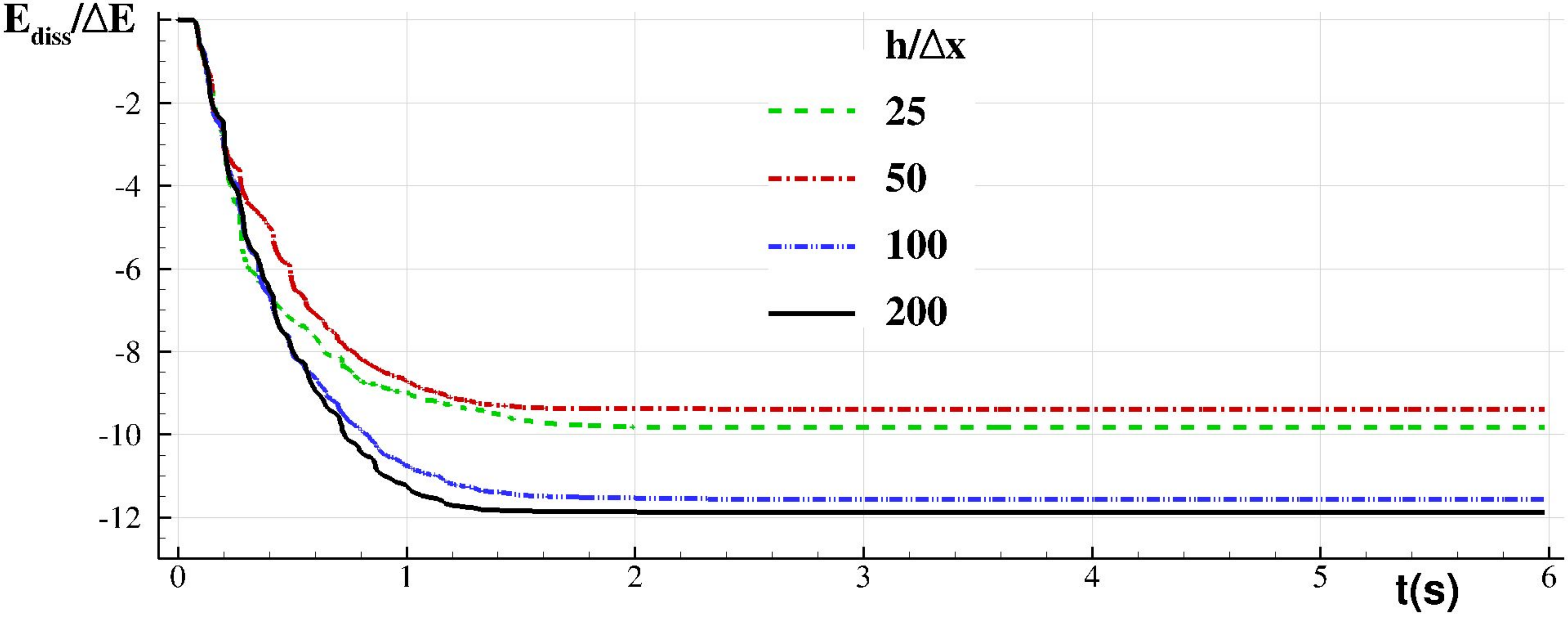
Energy budget



$$\boxed{\mathcal{P}_{ext} + \dot{\mathcal{E}}_M + \dot{\mathcal{E}}_C} = \boxed{\mathcal{P}_V + \mathcal{P}_V^{turb} + \mathcal{P}_N}$$

This project has received funding from the European Union’s Horizon 2020 research and innovation programme under grant agreement No 815044.

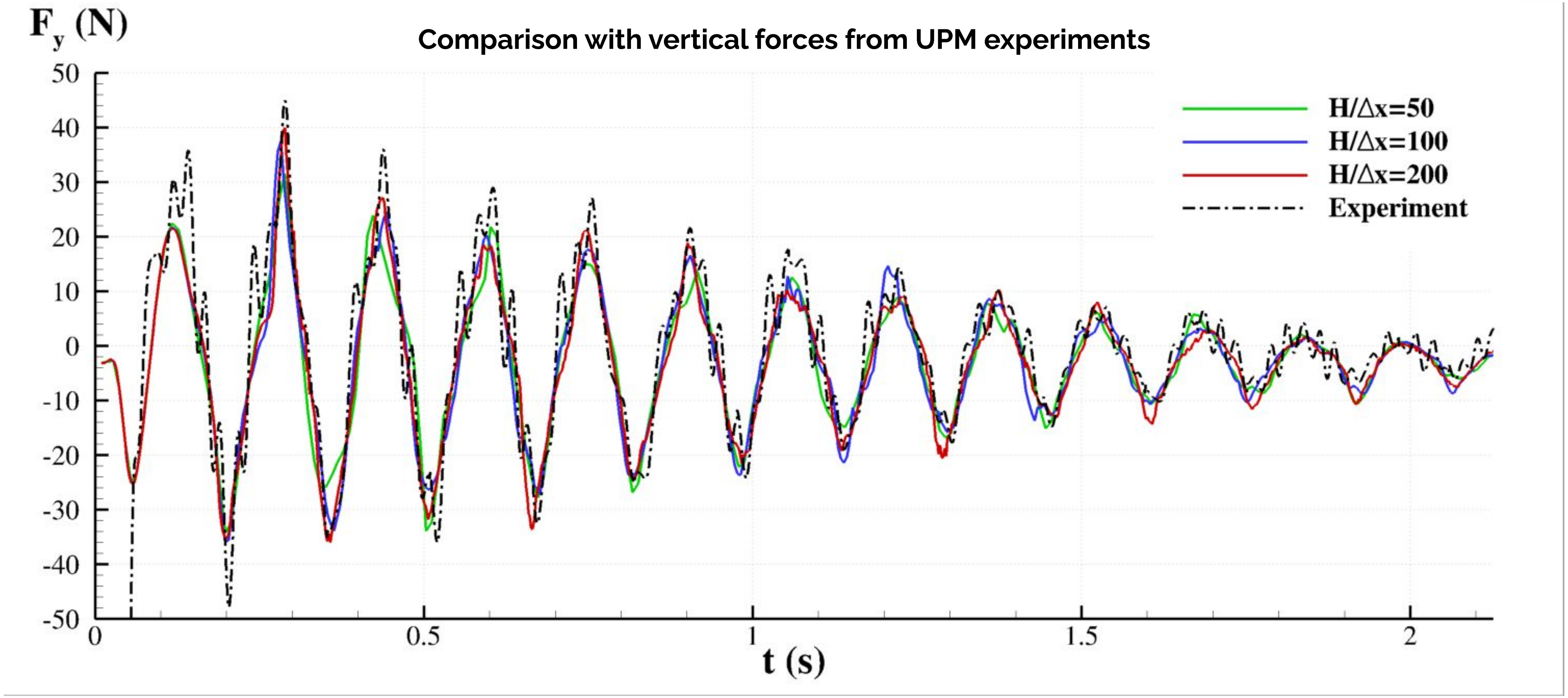
Energy budget



This project has received funding from the European Union’s Horizon 2020 research and innovation programme under grant agreement No 815044.

Flow evolution

Comparison with vertical forces from UPM experiments



This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 815044.

Conclusions

- the SPH model has been applied to a violent sloshing flow in pure heave motion
- standard SPH schemes exhibit tensile instability issues due to intense negative pressure in the first acceleration stage
- a new SPH model has been proposed to eliminate this issue and increase accuracy
- closure of the different energy terms has been shown for different choices of the frame of reference
- most of the energy drops seem related to impact stage
- an in-depth validation is still needed due to the complexity of the problem

This project has received funding from the European Union's Horizon 2020
research and innovation programme under grant agreement No 815044.

Thanks for your attention

This project has received funding from the European Union’s Horizon 2020
research and innovation programme under grant agreement No 815044.

Thank you!

The research leading to these results has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 815044.