quantum reports

Article

Remembering George Sudarshan

Alberto Ibort [1,2,*] and Giuseppe Marmo [3]

1 ICMAT, Instituto de Ciencias Matematicas (CSIC-UAM-UC3M-UCM), Nicolás Cabrera, 1315, Campus de Cantoblanco, UAM, 28049 Madrid, Spain
2 Dipartimento de Matemáticas, Universitario Carlos III de Madrid, Avda. de la Universidad 30, Leganés, 28911 Madrid, Spain
3 Dipartimento di Fisica "E. Pancini", Università di Napoli Federico II and Sezione INFN di Napoli. Complesso Universitario di Monte S. Angelo, via Cintia, 80126 Naples, Italy; marmo@na.infn.it
* Correspondence: albertoi@math.uc3m.es

Received: 16 November 2019; Accepted: 27 November 2019; Published: 2 December 2019

Abstract: In these brief notes we want to render homage to the memory of E.C.G. Sudarshan, adding it to the many contributions devoted to preserve his memory from a personal point of view.

Keywords: Sudarshan; apology; no-interaction theorem

---

1. Sudarshan and the Foundations of Quantum Mechanics

E.C.G. Sudarshan has been one of the most innovative theoretical physicists of Indian origin in the second part of the 20th century. He has made epoch-making contributions to many areas of theoretical physics. We may quote, for instance, the V-A theory of weak interactions (formulated with his advisor R. Marshak [1]), which was the key for the formulation of electro-weak interactions. Indeed, as stressed by S. Weinberg [2], Sudarshan's formulation was essential for the development of a unified model of electromagnetic and weak interactions.

Following A. Einstein's famous quote: *Physical concepts are free creations of the human mind, and are not, however it may seem, uniquely determined by the external world (A. Einstein, The Evolution of Physics (1938) (co-written with Leopold Infeld))* we may agree that the intrinsic freedom of the researcher to provide interpretations of physical reality drinks heavily from the social, cultural and economical background of the individual. In this sense, Sudarshan's scientific work is a rare combination of western logic and precision with eastern imagination; Greek-type philosophy with an ancient Indian way of thought. Who else could have thought of what nowadays we call tachyons? [3,4].

We may characterize him as iconoclast, with marked absence of all-pervading cultural prejudices. These unique features of him, also make that his seminal work is often quoted but likely unread, often not really understood until considerable time has passed.

The deep common thread of his contribution has been an attempt to resolve the fundamental conflict between *being* and *becoming*, that is, between objects and processes: *All Natural Philosophy is characterized by one central problem: that is, to understand and describe the meaning of existence and the nature of change*.

An example of this was provided by the important contribution of Sudarshan to the formulation of quantum Zeno effect [5], by now experimentally verified, that finds applications in quantum information processing. It is difficult to find a better example of a physical situation where the tension between *being* and *becoming* is more apparent. But this peculiar way of thinking by Geoge has had an enormous impact in other areas of physics. For instance, modern quantum optics has build on the concept of coherent states, again a counter-intuitive quantum mechanical notions whose discovery constituted a leap forward in the understanding of Quantum Mechanics. In a few years, it is clearly realized the connection between coherent states and group theory. It is remarkable that George's ideas are constantly followed up by experimentalists thanks to the developments in experimental techniques.

2. Contributions to Physics

Apart from the already mentioned contributions, many other contributions where discussed by leading scientists in the IOP open-access Journal Conferences in occasion of his 75th birthday:

- *Sudarshan: Seven Science Quest*, Austin, Texas, US 6–7 November 2006. Journal of Physics: Conference Series, Volume 196, Number 1.
- *Particle and Fields: Classical and Quantum*, Jaca, Huesca, Spain 18–21 September 2006. Journal of Physics: Conference Series, Volume 87.

The Seven Quest that were referred to in the first paper are:

- V-A Symmetry.
- Spin and Statistics.
- Quantum Coherence.
- Quantum Zeno Effect.
- Tachyons.
- Open systems.

In the ten years elapsed since other topics were investigated, we would add: Entanglement and Tomography (see, for instance, [6–9] and referencers therein).

All of the Seven Quest are commented by Sudarshan himself in the two quoted open-access journal for conferences, therefore we would refer to these issues for additional information. As for Open Systems, we refer the reader to the article: *A brief History of the GKLS Equation*, by Darek Chruscinski and Saverio Pascazio [10].

George wrote over 500 papers, alone or with his students and collaborators all over the world. We list here the books written by ECG Sudarshan in collaboration with a number of collaborators:

- *Introduction to Elementary Particle Physics* with R. Marshak. Wiley Interscience, 1962.
- *Introduction to Quantum Optics* with John Klauder, W.A. Benjamin 1968.
- *Classical Dynamics: A Modern Perspective* with N. Mukunda, J. Wiley 1974.
- *100 Years of Planck Quantum* with Ian M. Duck. World Scientific, 1996.
- *W. Pauli and the Spin-Statistic Theorem* with Ian M. Duck. World Scientific, 1997.
- *Doubt and Certainty, conversations with Tony Rothman*. Perseus Book, 2000.
- *From Classical to Quantum Mechanics* with G. Esposito, G. Marmo. Cambridge UP, 2004.
- *Advanced Concepts in Quantum Mechanics* with G. Esposito, G. Marmo, G. Miele. Cambridge UP, 2015.

3. Some Biographical Traits

Ennackal Chandy George Sudarshan was born in Pallam, Kottayam District in the State of Kerala, India, on 16 September 1931, from Achamma, his mother Achamma, a school teacher, and his father, E.I. Chandy, who was revenue supervisor for the Kerala Government Service. He had two brothers and three sons. George attended the CMS College of Kottayam (1946–48), graduated B.Sc. at Madras, Christian College (1948–51) and got the Master of Arts Degree from Madras University (1952).

George Sudarshan joined the Tata Institute of Fundamental Research (TIFR) in Bombay in the spring of 1952 as a research student. He started collaborating with B. Peters in cosmic rays research, the greatest discovery he made by that time was that he could be a theoretician inspired by experimental research rather than being an experimentalist himself. The presence of a strong group in Mathematics at the Tata Institute allowed George Sudarshan to become acquainted with topology, modern algebra, theory of spinors and functional analysis.

Two years later, in 1954, Paul A.M. Dirac, a teacher of Bhabha at Cambridge in the 1930s, visited TIFR and gave a course of lectures on quantum mechanics. George collaborated with K.K. Gupta, a student of Bhabha, in the preparation of the lecture notes. This work gave him the occasion to be in close contact with Dirac, a formidable and unimaginably fortunate opportunity to learn the subject from the master himself. This experience shaped George's attitude to quantum mechanics all along his life, giving him the daring and courage to push and test the principles of the subject in many directions. ECG greatly admired Dirac, and the two remained lifelong friends.

Later George, in September 1955, went to Rochester for his Ph.D. Thesis with Bob Marshak (a former student of Hans A. Bethe). Here he started a life-long friendship with Susumo Okubo. Immediately after he went to Harvard University for two years as a post-doc with J. Schwinger.

With hindsight, one can say that some inspiration by Schwinger on George's later work may be found in the action principle (as enunciated by P. Weiss, is a variational principle which includes variations of the boundaries for the action integral); his presentation of classical mechanics with group theory, and, more generally, his pervasive use of group theory in many important papers. In his own words, in the opening of his book Classical Dynamics: A Modern Perspective, he says: This book is the public declaration of an "affair of the heart".

We owe much inspiration to three masters: Edmund Whittaker Paul Dirac, whose writings and words are an inspiration in all other endeavours in physics as well as in this; and Julian Schwinger, mentor to the senior author and the most extraordinary teacher of the spirit of dynamics.

In Harvard, George was in very good terms with Sheldon Glashow, another lifelong friend. From Boston, he went regularly back to Rochester, to continue the collaboration with Marshak and Okubo. Later, the group was joined by Steven Weinberg.

From Harvard, George moved back to Rochester (1959) as Assistant Professor, to become Full Professor in 1961. In 1963 he conceived a way to describe quantum states close to classical beams of light, the already mentioned coherent states, when he pointed out the importance of using the coherent states of the photon field [11].

After a one-year leave in Bern, he went to Syracuse as full professor in 1964. Here he gave birth to a group of particle physics (Syracuse, with the presence of Peter Bergmann, was mainly general relativity). In 1969, George moved again to join the University of Texas at Austin where he taught and continued his research until he passed away in 2017.

4. Sudarshan and Classical Dynamics

There is yet a seminal contribution by George related to relativistic invariance and hamiltonian theories of interacting particles. George always had a deep love for the structure of classical mechanics, continuing with his own words in the preface of his book on classical mechanics together with N. Mukunda [12]:

This book is the public declaration of an "affair of the heart" that we have had with classical dynamics for most of our adult lives. Indian tradition recognizes ten stages for love, beginning with the beauty of form, through stages of closeness and of agony, to ultimate bliss. It is our belief that the beauty of form of classical dynamics is only her calling card and recognition of this beauty but the first step. We see classical dynamics not as part of physics, but as physics itself. We see her form pervading all of physics...

Consider things in context. From 1960, and for almost thirty years, George kept returning to the canonical description of relativistic particles. In the eighties there was a renewed interest in the canonical description of relativistic interacting particles (many specialists in relativity contributed to the field, among others Arthur Komar, Joshua N. Goldberg, Fritz Rohrlich). The activity was aimed at "evading" the no-interaction theorem which George, with his collaborators, had established much earlier. We shall try to sketch the meaning and results of the theorem.

When addressing the description of the dynamics of interacting particles together with the requirements of special relativity, there are new features with respect to the Newtonian description. The traditional specification of the instantaneous state of a system of N interacting particles is to give

N triplets of Cartesian coordinates $\mathbf{q}$ and their conjugate momenta $\mathbf{p}$. The time evolution would be described by a Hamiltonian function of the $3N$ classical pairs $\mathbf{q}_a, \mathbf{p}_a$.

In the Hamiltonian formalism, the irrelevance of time origin implies that $\partial H/\partial t = 0$. Moreover, irrelevance of orientation of the coordinate frame in 3-space implies the immutability of the angular momentum $\sum_a \mathbf{q}_a \wedge \mathbf{p}_a$; which, in turn, implies the rotation invariance of the Hamiltonian function H. The irrelevance of the space origin implies the invariance of the Hamiltonian under common transformation of the Cartesian coordinates and the corresponding invariance of the total momentum $\sum_a \mathbf{p}_a$.

Thus, we have a seven parameter set of canonical transformations implementing time translations, space translations and space rotations:

$$\begin{aligned} \{H, \mathbf{P}\} &= 0\,, \qquad \{H, \mathbf{J}\} = 0\,, \\ \{\mathbf{P}\cdot\mathbf{a}, \mathbf{P}\cdot\mathbf{b}\} &= 0\,, \qquad \{\mathbf{P}\cdot\mathbf{a}, \mathbf{J}\cdot\boldsymbol{\theta}\} = (\boldsymbol{\theta}\wedge\mathbf{a})\cdot\mathbf{P}\,, \\ \{\mathbf{J}\cdot\boldsymbol{\theta}, \mathbf{J}\cdot\boldsymbol{\varphi}\} &= (\boldsymbol{\varphi}\wedge\boldsymbol{\theta})\cdot\mathbf{J}\,. \end{aligned}$$

Poisson brackets are defined at a single time. For any dynamical variable F,

$$\frac{dF}{dt} = \{F, H\}\,.$$

All previous expressions look very non-relativistic. They employ a clock time t and dynamical variables determined simultaneously at the same time t. But special relativity tells us that distant simultaneity is not independent of the frame of reference.

Dirac showed that the invariance under changes of the relativistic frames may be implemented by adding boosts transformations to moving inertial frames. Thus, in any Hamiltonian relativistic theory there are ten generators which are canonically realized. The dynamical evolution is described by a Hamiltonian which is one of the ten generators in Dirac's generator formalism. To previous commutation relations we add:

$$\begin{aligned} \{\mathbf{K}\cdot\mathbf{u}, H\} &= -\mathbf{P}\cdot\mathbf{u}\,, \\ \{\mathbf{K}\cdot\mathbf{u}, \mathbf{P}\cdot\mathbf{a}\} &= -(\mathbf{u}\cdot\mathbf{a})\cdot H, \\ \{\mathbf{K}\cdot\mathbf{u}, \mathbf{K}\cdot\mathbf{v}\} &= (\mathbf{u}\wedge\mathbf{v})\cdot\mathbf{J}\,. \end{aligned}$$

This gives Dirac's "instant form" of the relativistic dynamics for any canonical system (see [13] for a discussion of Dirac's forms of relativistic dynamics in this issue). From these relations, we learn that if the Hamiltonian changes (because of interactions) also the boosts are obliged to change.

Interacting systems may be constructed by choosing $H = H_0 + V$, where H_0 is appropriate for a collection of free particles, and V is a rotationally and translationally invariant momentum-dependent "potential". Existence of invariant world lines for each particle can be encoded in the following relations:

$$\{\mathbf{K}\cdot\mathbf{u}, (\mathbf{q}_a)_j\} = (\mathbf{q}_a\cdot\mathbf{u})(\mathbf{v}_a)_j = \{(\mathbf{q}_a)_j, H\}(\mathbf{q}_a\cdot\mathbf{u})\,.$$

If we try to find out the interactions which are compatible with the world line condition (WLC) expressed above, we find that there are none, all accelerations must vanish if we have canonical realizations of the relativistic transformation group satisfying the WLC.

In the early eighties, the collaboration of Giuseppe Marmo with George Sudarshan started. Marmo joined Sudarshan, Mukunda, Balachandran, Nilsson and Zaccaria in the analysis of this problem and they provided a new proof of the no-interaction theorem based on a Lagrangian approach [14].

The Lagrangian proof of the no-interaction theorem. When going from the Hamiltonian to the Lagrangian formalism we face some novel problems. Hamiltonian equations, which are explicit differential equations:

$$\frac{dq_a}{dt} = \frac{\partial H}{\partial p_a}\,, \qquad \frac{dp_a}{dt} = -\frac{\partial H}{\partial q_a}$$

where q_a denote now a generic set of configuration coordinates and p_a the corresponding conjugate momenta, are replaced by Euler–Lagrange equations:

$$\frac{dq_a}{dt} = v_a\,, \qquad \frac{d}{dt}\frac{\partial L}{\partial v_a} = \frac{\partial L}{\partial q_a}\,,$$

which are implicit differential equations. Spelling them out we get:

$$\frac{\partial^2 L}{\partial v_r \partial v_s}\frac{dv_s}{dt} + \frac{\partial^2 L}{\partial v_r \partial q_s}\frac{dq_s}{dt} = \frac{\partial L}{\partial q_r}\,,$$

To put them into normal form we need:

$$\det\left(\frac{\partial^2 L}{\partial v_r \partial v_s}\right) \neq 0\,.$$

Whenever we have to deal with implicit differential equations, we will face difficulties with the notion of symmetries and constants of the motion (a convenient presentation of these issues will be done in [15]). However to implement the action of space translations and rotations we have no problems because they are implemented by point transformations. We encounter problems with the implementation of boosts because they cannot be realized as "point transformation". They do not respect the tangent bundle structure of the carrier space. Moreover, when the Lagrangian function is degenerate, there are no Poisson brackets directly available. By handling these various problems, it was found that the no-interaction theorem applies also for degenerate Lagrangians, as long as there are no second-class constraints.

From a better understanding of the proof it was possible to see how to evade the no-interaction theorem. However, by requiring a notion of separability for far apart particles, a novel and more profound no-interaction theorem was found [16]. The curious result was reminiscent of the EPR "paradox" in quantum theory [17,18] but the indicated circle of ideas seemed to point out that correlations between distant objects need not always involve transport of material influences. It might rather depend upon the indecomposable nature of the dynamical system itself. This was brought about by the imposition of the apparently innocent WLC. We have the feeling that a deeper analysis of the conditions and the reasons for the theorem to work would enlighten substantially the current problem of causality in quantum mechanics.

As a conclusion from the results of the new version of the no-interaction theorem we may take alternative points of view at this juncture. One can accept the result and proceed to a field formalism as the sole vehicle for particle interactions (this is the road taken in [15]). One could abandon differential equations of motion and replace them by integro-differential equations instead (perhaps these two methods are not essentially different).

A third way is to seek a more geometrical and explicitly invariant formalism from the start. In these descriptions one uses an explicitly invariant set of configuration variables to describe the system and guarantee that the system does describe particles interacting by imposing suitable constraints. In this geometrical constraint approach, one has an enlargement of Dirac's original framework: It is necessary to consider dynamical choices of temporal evolution variables rather than the kinematical choices offered by Dirac in his various forms of relativistic dynamics (see [13] for further discussion on these issues).

Author Contributions: Conceptualization, G.M.; writing-review and editing, G.M. and A.I.; funding acquisition, G.M. and A.I.

Funding: This research was funded by the Spanish Ministry of Economy and Competitiveness, through the Severo Ochoa Programme for Centres of Excellence in RD (SEV-2015/0554). A.I. and F.C. would like to thank partial support provided by the MINECO research project MTM2017-84098-P and QUITEMAD++, S2018/TCS-A4342. G.M. would like to thank the support provided by the Santander/UC3M Excellence Chair Programme 2019/2020.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Sudarshan, E.C.G.; Marshak, R.E. The Nature of the Four-Fermion Interaction. In Proceedings of the Conference on Mesons and Newly Discovered Particles, Padua, Venice, Italy, 22–27 September 1957; reprinted in *The Development of Weak Interaction Theory*; Kabir, P.K., Ed.; Gordon and Breach: New York, NY, USA, 1963; pp. 119–128.
2. Weinberg, S. Essay: Half a Century of the Standard Model. *Phys. Rev. Lett.* **2018**, *121*, 220001. [CrossRef] [PubMed]
3. Bilaniuk, O.M.P.; Deshpande, V.K.; Sudarshan, E.C.G. "Meta" relativity. *Am. J. Phys.* **1962**, *30*, 718. [CrossRef]
4. Arons, M.E.; Sudarshan, E.C.G. Lorentz invariance, local field theory, and faster-than-light particles. *Phys. Rev.* **1968**, *173*, 1622. [CrossRef]
5. Misra, B.; Sudarshan, E.C.G. The Zeno's paradox in quantum theory. *J. Math. Phys.* **1977**, *18*, 756. [CrossRef]
6. Asorey, M.; Facchi, P.; Manko, V.I.; Marmo, G.; Pascazio, S.; Sudarshan, E.C.G. Generalized Tomographic Maps. *Phys. Rev.* **2008**, *A 77*, 042115. [CrossRef]
7. Modi, K.; Sudarshan, E.C.G. Role of preparation in quantum process tomography. *Phys. Rev.* **2010**, *A 81*, 052119. [CrossRef]
8. Ibort, A.; Lopez-Yela, A.; Manko, V.I.; Marmo, G.; Simoni, A.; Ventriglia, F.; Sudarshan, E.C.G. On the tomographic description of classical fields. *Phys. Lett.* **2012**, *A 376*, 1417–1425. [CrossRef]
9. Manko, V.I.; Marmo, G.; Sudarshan, E.C.G.; Zaccaria, F. Positive maps of density matrix and a tomographic criterion of entanglement. *Phys. Lett.* **2004**, *A 327*, 353–364. [CrossRef]
10. Chruscinski, D.; Pascazio, S. Gorini-Kossakowski-Lindblad-Sudarshan Master Equation 40 years after. In Proceedings of the 48th Symposium on Mathematical Physics, Torun, Poland, 10–12 June 2016.
11. Sudarshan, E.C.G. Equivalence of Semiclassical and Quantum Mechanical Descriptions of Statistical Light Beams. *Phys. Rev. Lett.* **1963**, *10*, 277. [CrossRef]
12. Sudarshan, E.C.G.; Mukunda, N. *Classical Dynamics: A Modern Perspective*; John Wiley and Sons: New York, NY, USA, 1974.
13. Ciaglia, F.M.; di Cosmo, F.; Ibort, A.; Marmo, G. Description of Relativistic Dynamics with World Line Condition. *Quantum Rep.* **2019**, *1*, 181–192. [CrossRef]
14. Marmo, G.; Mukunda, N.; Sudarshan, E.C.G. Relativistic Particle Dynamics—Lagrangian Proof of the No-Interaction Theorem. *Phys. Rev. D* **1984**, *30*, 2110. [CrossRef]
15. Asorey, M.; Falceto, F.; Ibort, A.; Marmo, G. *Classical Field Theory: A Geometric Approach*; Springer: Berlin/Heidelberg, Germany, 2020.
16. Balachandran, A.; Dominici, D.; Marmo, G.; Mukunda, N.; Nilsson, J.; Samuel, J.; Sudarshan, E.C.G.; Zaccaria, F. Separability in Relativistic Hamiltonian Particle Dynamics. *Phys. Rev. D* **1982**, *26*, 3492. [CrossRef]
17. Bohm, D. *Quantum Theory*; Prentice Hall: New York, NY, USA, 1951.
18. Einstein, A.; Podolsky, B.; Rosen, N. Can Quantum-Mechanical Descripton of Physical Reality be Considered Complete? *Phys. Rev.* **1935**, *47*, 777–780. [CrossRef]