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Radiative $VP\gamma$ transitions and η-η' mixing

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Abstract

A value for the η-η' mixing angle is extracted from the data on $VP\gamma$ transitions using simple quark-model ideas. The set of data covers *all* possible radiative transitions between the pseudoscalar and vector meson nonets. Two main ingredients of the model are the introduction of flavour-dependent overlaps for the various $q\bar{q}$ wave functions and the use of the quark-flavour basis to describe the η-η' system. In this basis the mixing angle is found to be $\phi_P = (37.7 \pm 2.4)^\circ$.

1 Introduction

Radiative transitions between pseudoscalar (P) and vector (V) mesons have been a classical subject in low-energy hadron physics for more than three decades. The study of the $VP\gamma$ couplings, governed by the magnetic dipole ($M1$) emission of a photon, played a major rôle when the basis of the quark model and $SU(3)$-symmetry were established, as well as when trying to understand their symmetry-breaking mechanisms. The pioneering work by Becchi and Morpurgo [1] successfully explained the specific $\omega \to \pi^0\gamma$ decay rate in terms of the u, d quark magnetic moments $\mu_{u,d}$, as deduced from the measurable magnetic moments of the nucleons $\mu_{p,n}$. Extension to other $V \to P\gamma$ and $P \to V\gamma$ radiative decays was soon performed exploiting $SU(3)$-symmetry relations as described, for instance, in the concise comment by Isgur [2] or in the review by O'Donnell [3] where symmetry-breaking effects are also discussed. Among the latter, those related to the η-η' system turn out to be particularly interesting and have recently heightened the theoretical activity on the $VP\gamma$ magnetic dipole transitions [4]–[13].

From the experimental point of view, the Novosibirsk CMD-2 [14] and SND [15] Collaborations have reported very recently accurate and consistent results on the various $V \to P\gamma$ radiative decays and, in particular, on the poorly known $\phi \to \eta'\gamma$ branching ratio. This latter e^+e^--annihilation result complements older data on the other two η'-meson radiative transitions, $\eta' \to \rho\gamma, \omega\gamma$, previously measured through $\gamma\gamma$-interactions. For the first time we have a well established and consistent set of data covering the $V \to P\gamma$ and $P \to V\gamma$ radiative decays with $V = \rho^0, \omega, \phi$ and $P = \pi^0, \eta, \eta'$, as shown by the current PDG edition [16]. This set of data is completed by the $\rho^+ \to \pi^+\gamma$, $K^{*+} \to K^+\gamma$ and $K^{*0} \to K^0\gamma$ transitions measured by the Primakoff-effect and thus affected by larger uncertainties. Globally, these experimental results represent an exhaustive and useful set of data covering *all* the twelve possible radiative transitions between the pseudoscalar and the vector meson nonets [16]. Moreover, the Frascati ϕ-factory DAΦNE [17] is expected to improve this situation quite soon.

Six of the just mentioned radiative transitions involve η or η' mesons and contain valuable information on the properties of the η-η' system and their mixing pattern. Interest on this issue has been recently renewed by Leutwyler *et al.* [18] in their analysis of the f_η and $f_{\eta'}$ decay constants governing the $\eta, \eta' \to \gamma\gamma$ transitions. In their effective chiral Lagrangian context, two mixing angles are required to express f_η and $f_{\eta'}$ in terms of the octet and singlet decay constants f_8 and f_0, the canonical treatment with one single mixing angle being recovered only in the good $SU(3)$ limit. Feldmann and collaborators [12, 13] have confirmed that a two mixing angle analysis is more coherent than the conventional one, but, on the other side, they have also reopened the possibility of a single angle description provided one abandons the octet-singlet basis and works in the quark-flavour basis. In this case one has

$$
\begin{aligned}
|\eta\rangle &\equiv \cos\phi_P |\eta_{NS}\rangle - \sin\phi_P |\eta_S\rangle \ , \\
|\eta'\rangle &\equiv \sin\phi_P |\eta_{NS}\rangle + \cos\phi_P |\eta_S\rangle \ ,
\end{aligned}
\tag{1}
$$

where $|\eta_{NS}\rangle \equiv |u\bar{u} + d\bar{d}\rangle/\sqrt{2}$ and $|\eta_S\rangle \equiv |s\bar{s}\rangle$ are the non-strange and the strange basis states. Most importantly, the two mixing angles now reduce to the one in Eq. (1), ϕ_P, not in the good $SU(3)$ limit but in the much safer approximation of perfect validity of the OZI-rule.

These recent findings have been ignored in many previous treatments of $VP\gamma$ transitions and the related η-η' mixing pattern. For this reason and because of the rich set of data we now have at our disposal, the purpose of this note is to present a detailed revision of those two issues.

2 A model for $VP\gamma$ $M1$ transitions

We will work in a conventional quark model context and assume that pseudoscalar and vector mesons are simple quark-antiquark S-wave bound states. All these hadrons are thus extended objects with characteristic spatial extensions fixed by their respective quark-antiquark P or V wave functions. In the pseudoscalar nonet, $P = \pi, K, \eta, \eta'$, the quark spins are antiparallel and the mixing pattern is given by Eq. (1). In the vector case, $V = \rho, K^*, \omega, \phi$, the spins are parallel and mixing is similarly given by $|\omega\rangle \equiv \cos\phi_V|\omega_{NS}\rangle - \sin\phi_V|\omega_S\rangle$ and $|\phi\rangle \equiv \sin\phi_V|\omega_{NS}\rangle + \cos\phi_V|\omega_S\rangle$, where $|\omega_{NS}\rangle$ and $|\omega_S\rangle$ are the analog non-strange and strange states, as before. We will work in the good $SU(2)$ limit with $m_u = m_d \equiv \bar{m}$ and with identical spatial extension of wave functions within each P and each V isomultiplet. $SU(3)$ will be broken in the usual manner taking constituent quark masses with $m_s > \bar{m}$ but also, and this is a specific feature of our approach, allowing for different spatial extensions for each P and V isomultiplet. Finally, we will consider that even if gluon annihilation channels may induce η-η' mixing, they play a negligible rôle in $VP\gamma$ transitions and thus fully respect the usual OZI-rule.

In our specific case of $VP\gamma$ $M1$ transitions, these generic statements translate into three characteristic ingredients of the model:

i) A $VP\gamma$ magnetic dipole transition proceeds via quark or antiquark spin-flip amplitudes proportional to $\mu_q = e_q/2m_q$. Apart from the obvious quark charge values, this effective magnetic moment breaks

$SU(3)$ in a well defined way and distinguishes photon emission from strange or non-strange quarks via $m_s > \bar{m}$.

ii) The spin-flip $V \leftrightarrow P$ conversion amplitude has then to be corrected by the relative overlap between the P and V wave functions. In older papers [2, 3] a common, flavour-independent overlap was introduced. Today, with a wider set of data, this new symmetry-breaking mechanism can be introduced without enlarging excessively the number of free parameters.

iii) Indeed, the OZI-rule reduces considerably the possible transitions and their respective VP wave-function overlaps: Z_S, Z_{NS} and Z_π characterize the $\langle \eta_S | \omega_S \rangle$, $\langle \eta_{NS} | \omega_{NS} \rangle = \langle \eta_{NS} | \rho \rangle$ and $\langle \pi | \omega_{NS} \rangle = \langle \pi | \rho \rangle$ spatial overlaps, respectively. Notice that distinction is made between the $|\pi\rangle$ and $|\eta_{NS}\rangle$ spatial extension due to the gluon or $U(A)_1$ anomaly affecting the second state, but not between the anomaly-free, non-strange vector states $|\rho\rangle$ and $|\omega_{NS}\rangle$. Independently, we will also need Z_K for the $\langle K | K^* \rangle$ overlap between strange isodoublets.

It is then a trivial task to write all the $VP\gamma$ couplings in terms of an effective $g \equiv g_{\omega_{NS}\pi\gamma}$:

$$
\begin{aligned}
g_{\rho^0\pi^0\gamma} &= g_{\rho^+\pi^+\gamma} = \tfrac{1}{3}\, g \ , \\
g_{\rho\eta\gamma} &= g\, z_{NS}\, \cos\phi_P \ , \\
g_{\eta'\rho\gamma} &= g\, z_{NS}\, \sin\phi_P \ , \\
g_{\omega\pi\gamma} &= g\, \cos\phi_V \ , \\
g_{\omega\eta\gamma} &= \tfrac{1}{3}\, g\, \left(z_{NS}\, \cos\phi_V \cos\phi_P - 2\, \tfrac{\bar{m}}{m_s}\, z_S\, \sin\phi_V \sin\phi_P \right) \ , \\
g_{\eta'\omega\gamma} &= \tfrac{1}{3}\, g\, \left(z_{NS}\, \cos\phi_V \sin\phi_P + 2\, \tfrac{\bar{m}}{m_s}\, z_S\, \sin\phi_V \cos\phi_P \right) \ , \\
g_{\phi\pi\gamma} &= g\, \sin\phi_V \ , \\
g_{\phi\eta\gamma} &= \tfrac{1}{3}\, g\, \left(z_{NS}\, \sin\phi_V \cos\phi_P + 2\, \tfrac{\bar{m}}{m_s}\, z_S\, \cos\phi_V \sin\phi_P \right) \ , \\
g_{\phi\eta'\gamma} &= \tfrac{1}{3}\, g\, \left(z_{NS}\, \sin\phi_V \sin\phi_P - 2\, \tfrac{\bar{m}}{m_s}\, z_S\, \cos\phi_V \cos\phi_P \right) \ , \\
g_{K^{*0}K^0\gamma} &= -\tfrac{1}{3}\, g\, z_K\, \left(1 + \tfrac{\bar{m}}{m_s} \right) \ , \\
g_{K^{*+}K^+\gamma} &= \tfrac{1}{3}\, g\, z_K\, \left(2 - \tfrac{\bar{m}}{m_s} \right) \ ,
\end{aligned}
\tag{2}
$$

Transition	$\Gamma_{\rm exp}$(keV)	$\Gamma_{\rm fit1}$(keV)	$\Gamma_{\rm fit3}$(keV)	$\Gamma_{\rm fit4}$(keV)	$\Gamma_{\rm fit5}$(keV)
$\rho^0 \to \pi^0\gamma$	102 ± 26	75 ± 4	67 ± 4	74 ± 4	71 ± 9
$\rho^+ \to \pi^+\gamma$	68 ± 7	74 ± 4	67 ± 4	74 ± 4	71 ± 8
$\rho^0 \to \eta\gamma$	36^{+12}_{-14}	46 ± 6	53 ± 3	49 ± 5	44 ± 6
$\eta' \to \rho^0\gamma$	60 ± 5	59 ± 10	58 ± 5	58 ± 13	58 ± 9
$\omega \to \pi^0\gamma$	717 ± 43	708 ± 36	637 ± 23	704 ± 35	720 ± 42
$\omega \to \eta\gamma$	5.5 ± 0.8	5.1 ± 0.8	5.9 ± 0.4	5.5 ± 0.6	5.2 ± 0.5
$\eta' \to \omega\gamma$	6.1 ± 0.8	6.7 ± 0.8	6.9 ± 0.6	6.4 ± 1.3	6.9 ± 0.8
$\phi \to \pi^0\gamma$	5.6 ± 0.5	5.6 ± 0.6	5.6 ± 0.5	5.6 ± 0.6	5.6 ± 0.8
$\phi \to \eta\gamma$	57.8 ± 1.5	58 ± 13	58.0 ± 6.7	58 ± 11	57.7 ± 6.9
$\phi \to \eta'\gamma$	$0.30^{+0.16}_{-0.14}$	0.37 ± 0.08	0.43 ± 0.04	0.25 ± 0.06	0.36 ± 0.03
$K^{*0} \to K^0\gamma$	116 ± 10	117 ± 13	124 ± 6	115 ± 11	
$K^{*+} \to K^+\gamma$	50 ± 5	50 ± 6	56 ± 4	50 ± 5	

Table 1: Comparison between the experimental values $\Gamma_{\rm exp}$ for the various $VP\gamma$ transitions taken from Ref. [16] and the corresponding predictions $\Gamma_{\rm fit1}$, $\Gamma_{\rm fit3}$, $\Gamma_{\rm fit4}$ and $\Gamma_{\rm fit5}$ from Eqs. (4), (7), (8) and (9), respectively.

where we have redefined $z_{NS} \equiv Z_{NS}/Z_\pi$, $z_S \equiv Z_S/Z_\pi$ and $z_K \equiv Z_K/Z_\pi$. The normalization of the couplings is such that $g_{\omega\pi\gamma} = g\cos\phi_V = 2(\mu_u + \mu_{\bar d})\,Z_\pi \cos\phi_V = e\,Z_\pi\cos\phi_V/\bar m$ and the decay widths are given by

$$\Gamma(V \to P\gamma) = \frac{1}{3}\frac{g^2_{VP\gamma}}{4\pi}|\mathbf{p}_\gamma|^3 = \frac{1}{3}\Gamma(P \to V\gamma)\ , \tag{3}$$

where $\mathbf{p}_\gamma$ is the final photon momentum.

3 Data fitting

The available experimental information on $\Gamma(V \to P\gamma)$ and $\Gamma(P \to V\gamma)$ partial widths is shown in the first column of Table 1 and has been taken exclusively from the recent PDG compilation [16]. A fit to these data with the couplings of our model, Eq. (2), leads to the predictions $\Gamma_{\rm fit1}$ listed in the second column of Table 1. The values of our seven free parameters are found to be

$$\begin{gathered} g = 0.70 \pm 0.02\ \mathrm{GeV}^{-1}\ , \quad m_s/\bar m = 1.24 \pm 0.07\ , \\ \phi_P = (37.7 \pm 2.4)^\circ\ , \quad \phi_V = (3.4 \pm 0.2)^\circ\ , \\ z_{NS} = 0.91 \pm 0.05\ , \quad z_S = 0.89 \pm 0.07\ , \quad z_K = 0.91 \pm 0.04\ . \end{gathered} \tag{4}$$

The quality of the fit is excellent, $\chi^2/d.o.f. = 3.2/5 \simeq 0.6$. The fitted values for the two mixing angles ϕ_P and ϕ_V are in good agreement with most results coming from other analyses using complementary information. Our values for g and $m_s/\bar{m}$ are also quite reasonable although their comparison with those from alternative studies is much more model dependent [19]. The three z's are specific of our approach and different from unity, the approximate value assumed in previous analyses. To further investigate this last issue, we have performed a second fit to the same twelve data fixing now $z_{NS} = 1$ and $z_S = z_K^2$. The quality of the fit, $\chi^2/d.o.f. = 6.8/7 \simeq 1$, is substantially reduced. This shows that allowing for different overlaps of quark-antiquark wave functions and, in particular, for those coming from the gluon anomaly affecting only the η and η' singlet component, has indeed some relevance.

As previously stated, a few of the twelve experimental data we are dealing with come from difficult Primakoff-effect analyses and could be affected by large uncertainties. The neutral and charged $K^* \to K\gamma$ transitions, for instance, have been measured only by one and two experimental groups respectively, and seem to need further confirmations. For these reasons, and also to allow later for easier comparison with work by other authors, we have performed a new fit ignoring the two $K^* \to K\gamma$ transition information. This new fit requires

$$
\begin{gathered}
g = 0.70 \pm 0.02 \ \mathrm{GeV}^{-1} \ , \\
\phi_P = (37.7 \pm 2.4)^\circ \ , \quad \phi_V = (3.4 \pm 0.2)^\circ \ , \\
z_{NS} = 0.91 \pm 0.05 \ ,
\end{gathered}
\tag{5}
$$

whereas $m_s/\bar{m}$ and z_S always appear in the combination $z_S\,\bar{m}/m_s$ fitted to 0.72 ± 0.04. The quality of the fit, $\chi^2/d.o.f. = 3.2/5 \simeq 0.6$, is as good as in the previous global fit and the results are practically identical. The adequacy of our treatment and the values of its main parameters are therefore insensitive to eventual modifications of future and desirable new data on $K^* \to K\gamma$ transitions.

From Eq. (2) one can immediately deduce the ratios

$$
\begin{gathered}
\frac{g_{\rho\eta\gamma}}{g_{\eta'\rho\gamma}} = \cot\phi_P \ , \\
\frac{g_{\omega\eta\gamma}}{g_{\eta'\omega\gamma}} \simeq \cot\phi_P \left(1 - 4\,\frac{\bar{m}}{m_s}\,\tan\phi_V\right) \ , \\
\frac{g_{\phi\eta\gamma}}{g_{\phi\eta'\gamma}} \simeq -\cot\phi_P \left(1 - 4\,\frac{m_s}{\bar{m}}\,\tan\phi_V\right) \ , \\
\frac{g_{K^{*0}K^0\gamma}}{g_{K^{*+}K^+\gamma}} = \frac{1+m_s/\bar{m}}{1-2m_s/\bar{m}} \ ,
\end{gathered}
\tag{6}
$$

where the two approximate expressions are remarkably accurate as a consequence of the results of our fits: $\tan\phi_V \ll 1, \sin 2\phi_P \simeq 1, z_{NS} \simeq z_S$. The value of $m_s/\bar{m}$ depends mainly on the fourth ratio in Eq. (6) involving only K^*–K transitions and its z_K independence was appreciated years ago by Sucipto and Thews [20]. The first three ratios are essential to fix the pseudoscalar nonet mixing angle ϕ_P. Again, they are practically z-independent, whereas they turn out to depend significantly on the ratio $f_\eta/f_{\eta'}$ in alternative approaches. This feature could have some relevance when extracting the value for ϕ_P and comparing with results from other authors, as we now proceed to discuss.

4 Comparison with other approaches

As just stated, several recent analyses of $VP\gamma$ transitions [4, 5] introduce symmetry-breaking terms in such a way that the various $VP\gamma$ amplitudes turn out to be dependent on the corresponding P decay constant f_P. The clear advantage of this procedure is that valuable information on $P \to \gamma\gamma$ transitions can be treated within the same context. A serious drawback, as already mentioned in the Introduction, is that the complicated two-angle dependence of $f_{\eta,\eta'}$ on $f_{8,0}$ is hard to take into account and usually ignored (an exception is the recent treatment of $\phi \to \eta\gamma, \eta'\gamma$ transitions via QCD sum rules in Ref. [21]). A drastic solution for this problem could consist in fixing all f_P's to the same unbroken value, thus accepting that one has no control on this part of the symmetry-breaking mechanism and that the final results are just a rough estimate. This is equivalent to the treatment in Refs. [6, 7] or to the present one fixing all the z's to unity. In this case, a fit to the twelve experimental entries of Table 1 leads to the estimates $\Gamma_{\rm fit3}$ listed in its third column. The quality of the fit now decreases to $\chi^2/d.o.f. = 10.4/8 \simeq 1.3$ but the values of the main parameters are quite consistent with our previous ones:

$$\phi_P = (35.6 \pm 1.8)^\circ \ , \quad m_s/\bar{m} = 1.27 \pm 0.05 \ . \tag{7}$$

Other authors [8, 9] have proposed different symmetry-breaking mechanisms inspired in earlier work by O'Donnell [3]. Rather than assuming the nonet symmetry ordinarily associated to quark model ideas, Benayoun *et al.* [8] include a nonet symmetry breaking parameter, x, in their approach. The expressions for their coupling constants follow from those in Eq. (2) once we put $z_{NS} = z_S = 1$ and substitute $\cos\phi_P$ and $-\sin\phi_P$ in the couplings involving an η meson by X_η^{NS} and X_η^S, respectively, with $X_\eta^{NS} = \cos\phi_P\,(1 + 2x + \sqrt{2}(1-x)\tan\phi_P)/3$, $X_\eta^S = -\sin\phi_P\,(2 + x + \sqrt{2}(1-x)\cot\phi_P)/3$; the couplings involving an η' meson can then be obtained from the latter substituting $\cos\phi_P$ and $-\sin\phi_P$ by $\sin\phi_P$ and $\cos\phi_P$, respectively, as required by Eq. (1). The four ratios (6) follow then at leading order of symmetry-breaking. Also, a global fit leads again to the excellent results $\Gamma_{\rm fit4}$ listed in the fourth column of Table 1 ($\chi^2/d.o.f. = 3.1/6 \simeq 0.5$) and to the values

$$\phi_P = (40.0 \pm 2.8)^\circ \ , \quad m_s/\bar{m} = 1.25 \pm 0.06 \ . \tag{8}$$

Finally, we discuss another source of $SU(3)$-breaking corrections suggested in other recent treatments by Frère *et al.* [10, 11] and by Feldmann *et al.* [12, 13]. It consists in the introduction of different annihilation constants, f_V, for the various vector mesons, quite in line with the different f_P's simultaneously used in the pseudoscalar sector, both accounting for the values of the respective wave functions at the origin. In a sense, the $f_V f_P$ factor appearing in the corresponding $VP\gamma$ transition (see Refs. [12, 13] for details) are then related to our Z factor accounting similarly for the wave-function overlap. The independent symmetry-breaking factor $m_s/\bar{m}$ —which is essential to adjust the ratio between the two K^*–K transitions— is not contemplated in Refs. [12, 13], thus precluding the immediate possibility of an acceptable global fit. The other first three ratios in Eq. (6) are easily reproduced and an excellent fit is obtained if the two kaonic channels are excluded, as shown in the final column of Table 1. The quality of the fit is $\chi^2/d.o.f. = 2.9/3 \simeq 1$ and the mixing angle is

$$\phi_P = (38.1 \pm 2.5)^\circ \ . \tag{9}$$

5 Conclusions

The old and widespread belief that simple quark-model ideas are quite appropriate to describe $VP\gamma$ transitions has been confirmed by the present analysis. Indeed, the rather solid set of data now available covering *all* possible $VP\gamma$ transitions between the pseudoscalar and vector meson nonets has been shown to be easily described in terms of the basic model implemented with various symmetry-breaking mechanisms. Distinction among the latter seems feasible by comparing future and more accurate measurements of $\Gamma(\phi \to \eta'\gamma)$ with the various predictions shown in Table 1. However, quite independently of the details of these mechanisms one can safely conclude from Eqs. (4), (5), (8) and (9) that the value of the η-η' mixing angle ϕ_P deduced from $VP\gamma$ data has to be in the range 37.5°–39.5°.

More specifically, we propose the value $\phi_P = (37.7 \pm 2.4)^\circ$ following from our own treatment of $SU(3)$-breaking effects. This treatment, in line with the recent approach by Feldmann *et al.* [12, 13] emphasizing the rôle played by the non-strange and strange components of the η and η' mesons, circumvents the difficulties encountered in other η-η' mixing analyses. Moreover, $SU(3)$-breaking effects originated by the flavour-dependence in the various VP wave-function overlaps are taken into account. This flavour-dependence turns out to be relevant and contains useful information on the spatial extension of the P and V mesons.

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References

[1] C. Becchi and G. Morpurgo, Phys. Rev. **140**, 687B (1965).

[2] N. Isgur, Phys. Rev. Lett. **36**, 1262 (1976).

[3] P.J. O'Donnell, Rev. Mod. Phys. **53**, 673 (1981).

[4] A. Bramon, A. Grau and G. Pancheri, Phys. Lett. B **344**, 240 (1995).

[5] M. Hashimoto, Phys. Lett. B **381**, 465 (1996); Phys. Rev. D **54**, 5611 (1996).

[6] A. Bramon and M.D. Scadron, Phys. Lett. B **234**, 346 (1990).

[7] A. Bramon, R. Escribano and M.D. Scadron, Phys. Lett. B **403**, 339 (1997); Eur. Phys. J. C **7**, 271 (1999).

[8] M. Benayoun *et al.*, Phys. Rev. D **59**, 114027 (1999).

[9] E. Gedalin, A. Moalem and L. Razdolskaya, nucl-th/0006073.

[10] P. Ball, J.-M. Frère and M. Tytgat, Phys. Lett. B **365**, 367 (1996).

[11] R. Escribano and J.-M. Frère, Phys. Lett. B **459**, 288 (1999).

[12] T. Feldmann, P. Kroll and B. Stech, Phys. Lett. B **449**, 339 (1999).

[13] T. Feldmann, Int. J. Mod. Phys. A **15**, 159 (2000).

[14] R.R. Akhmetshin *et al.*, Phys. Lett. B **460**, 242 (1999);
R.R. Akhmetshin *et al.*, Phys. Lett. B **473**, 337 (2000).

[15] M.N. Achasov *et al.*, Eur. Phys. J. C **12**, 25 (2000);
M.N. Achasov *et al.*, Nucl. Phys. A **675**, 213 (2000).

[16] D.E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).

[17] See, for example, J. Lee-Franzini, in *The Second DaΦne Physics Handbook*, edited by L. Maiani, G. Pancheri and N. Paver (INFN-LNF publication 1995), p. 761.

[18] H. Leutwyler, Nucl. Phys. Proc. Suppl. **64**, 223 (1998);
R. Kaiser and H. Leutwyler, hep-ph/9806336.

[19] Not invoking the $U(1)_A$ anomaly but using instead a pseudoscalar γ_5 coupling with constituent mass quark loops predicts $g = 0.62$ GeV^{-1}. See R. Delbourgo, D. Liu and M. D. Scadron, Int. J. Mod. Phys. A **14**, 4331 (1999).

[20] E. Sucipto and R.L. Thews, Phys. Rev. D **36**, 2074 (1987).

[21] F. De Fazio and M.R. Pennington, JHEP **7**, 51 (2000).