LENGTH AND ORDER OF FUZZY AUTOCATALYTIC SET OF A PRESSURIZED WATER REACTOR

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ABSTRACT

The pressurized water reactor (PWR) is one type of nuclear power plant that generates electricity using heat from nuclear reactions which use steam produced from heated water to spin large turbines that generate electricity. The PWR contains two systems namely primary and secondary systems. The secondary system of a PWR in a previous study cannot give information regarding the catalytic interaction and relation amongst each and every parameter in the system and unable to present all the possible catalytic reactions between the variables in the system. In this research, several structures of Fuzzy Graph Type-3 of Fuzzy Autocatalytic Set (FACS) of a secondary system of a PWR, namely possible paths, fuzzy edge connectivity, length, detour distance, order, and size are explored. Omega algebra is used to determine all the possible paths for a FACS with respect to its fuzzy edge connectivity, length, and detour distance. An algorithm is developed and coded to find all the possible paths of the PWR. Furthermore, the algorithm is extended to obtain more characteristics of the FACS system. A sample data from a secondary system of a PWR is then applied to the algorithm to obtain a total of 317 possible cycle paths and 243 possible non-cycle paths at a maximum length of five. In addition, based on the patterns of non-cycle paths of FACS graphs, new characteristics of FACS are obtained and proven in the form of lemma, theorem and corollaries. In summary, the finding in this study can be used to characterize FACS of a PWR as well as other applications.

ABSTRAK

Reaktor air bertekanan (PWR) adalah satu jenis loji kuasa nuklear yang menjana elektrik dengan menggunakan haba daripada reaksi nuklear di mana wap yang dihasilkan oleh air yang dipanaskan memutarkan turbin besar yang menjana elektrik. PWR mempunyai dua sistem dinamai sistem utama dan sistem sekunder. Kajian terdahulu dalam sistem sekunder PWR tidak dapat memberi maklumat mengenai interaksi dan hubungan pemangkin dan tidak dapat menyampaikan semua tindak balas katalitik antara pembolehubah di dalam sistem. Dalam kajian ini, beberapa struktur bagi Graf Kabur Jenis-3 Set Automangkinan Kabur (FACS) dalam sistem sekunder PWR, iaitu semua laluan yang mungkin, keahlian kabur, panjang, jarak memutar, susunan dan saiz dikaji. Aljabar Omega digunakan untuk menentukan semua laluan yang mungkin untuk FACS berhubung dengan keahlian kabur, panjang dan jarak memutar. Algoritma dibangunkan dan dikodkan untuk mencari semua laluan yang mungkin dalam PWR. Tambahan pula, algoritma tersebut dikembangkan untuk memperoleh ciri-ciri sistem FACS yang lain. Sampel data dari sistem sekunder PWR digunakan ke atas algoritma untuk memperoleh sejumlah 317 laluan kitaran yang mungkin dan 243 laluan bukan kitaran yang mungkin dengan maksimum panjang laluan adalah lima. Sebagai tamabahan, berdasarkan corak laluan bukan kitaran yang mungkin dalam graf FACS, ciri baru untuk FACS diperoleh dan terbukti dalam bentuk lemma, teorem dan corollaries. Secara ringkasnya, hasil dapatan daripada kajian ini boleh digunakan untuk mencirikan PWR dalam FACS dan juga aplikasi lain.

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LIST OF ABBREVIATIONS

ACS	-	Autocatalytic Set
FACS	-	Fuzzy Autocatalytic Set
FT3	-	Fuzzy graph Type-3
Max	-	Maximum
Min	-	Minimum
PWR	-	Pressurized Water Reactor

LIST OF SYMBOLS

C	-	Adjacency matrix
$C_{F_{ij}}$	-	Adjacency matrix of fuzzy graph
G_S	-	Crisp graph of secondary system of PWR
$d_{DG_{FT3}(V,E)}(v)$	-	Degree of vertex v of $DG_{FT3}(V,E)$
$DG(V,E)$	-	Directed graph
$d_{FT3}(v_i,v_j)$	-	Distance of path in fuzzy graph Type-3 in FACS
$\in$	-	Element of
$\geq$	-	Equal or greater than
G_{FACS}	-	FACS graph
G_F	-	Fuzzy graph of secondary system of PWR
h_F	-	Fuzzy heads
$h(e_i)$	-	Fuzzy head of the i^{th} edges
t_F	-	Fuzzy tails
w_F	-	Fuzzy weights
$G(V,E)$	-	Graph of G with vertex V and edge E
$>$	-	Greater than
G_F^i	-	i^{th} type fuzziness of fuzzy graph
$\ell(p)$	-	Length of path
$\ell_{FT3}(p)$	-	Length of path in fuzzy graph Type-3 in FACS
$\leq$	-	Less than or equal
$<$	-	Less than
$\vee$	-	Maximal
$\Delta(DG_{FT3}(V,E))$	-	Maximum degree of $DG_{FT3}(V,E)$
$\mu(e_i)$	-	Membership value for fuzzy edge connectivity for edge i
	-	

$\mu(\omega_{k(v_i,v_j)})$		Membership value of omega operation of k-operand from to v_j
$\mu(V)$	-	Membership value of vertices
$\rho(u,v)$	-	Membership value of vertex u to vertex v
	-	Minimum degree of $DG_{FT3}(V,E)$
$\wedge$	-	Minimal
$\notin$	-	Not element of
	-	Order of directed graph *FT3*
$O(G)$	-	Order of graph G
Ω_{FACS}	-	Omega algebra of G_{FACS}
ω	-	Omega
*	-	Omega operation of a secondary system in the PWR
$\omega_{k(v_i,v_j)}$	-	Omega operation of k-operand from to v_j
ω_n	-	Omega operation of n-operand
p	-	Path
$p(n)$	-	Paths of n vertices
E	-	Set of edges
Ω-algebra	-	Set of omega algebra
V	-	Set of vertices
$S(DG_{FT3}(V,E))$	-	Size of directed graph *FT3*
$S(G)$	-	Size of graph G
$\subset$	-	Subset of
e_i	-	The i^{th} edge
	-	The i^{th} vertex
$\cup$	-	Union

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CHAPTER 1

INTRODUCTION

1.1 Background of the Research

A graph has been used to model some systems (Balakrishnan and Ranganathan, 2012). Graph theory is one of the mathematical disciplines (Ashaari and Tahir, 2016) which provides a tool for studying interconnections among elements in natural and man-made systems (Noor Ainy *et al*., 2010, 2014). Graph theory was first introduced in 1736 by a Swiss Mathematician, Leonhard Euler when he dealt the famous Königsberg Bridge Problem (Harary, 1969). There are many branches of mathematics that have connections with graph theory such as group theory, matrix theory, probability, and topology (Balakrishnan and Ranganathan, 2012). A graph has numerous applications such as in system analysis, operational research, transportation, and economics (Blue *et al*., 1997).

Fuzzy set theory was introduced by Zadeh in 1965 by utilizing the concept of grade membership. Since then, numerous researchers have been concerned with the properties and applications of fuzzy sets (Gitman and Levine, 1970; Tamura *et. al*., 1971; Kandel and Yelowitz, 1974; Pathak and Pal, 1986; Keller and Tahani, 1992). This is due to the fact that certain aspects of reality such as complexity and ill-defined situations always escape most crisp mathematical models. Thus crisp models usually are inadequate in describing the whole process of systems.

A fuzzy set is a set where there are some uncertainties in the set (Blue *et al*., 1997). These uncertainties are presented in grade values in the interval [0, 1] while a crisp set is a set where its grade values element is either 0 or 1 only. A crisp set can be treated as a special case of a fuzzy set with its membership value is {0,1}.

In 1975, Rosenfeld introduced the concept of a fuzzy graph by applying the idea of fuzziness to a graph. Yeh and Bang (1975) presented some special cases of graph fuzziness where the vertices are crisp and the edges are fuzzy. Then, Blue *et al*. (1997, 2002) introduced the taxonomy of fuzzy graphs to generalize various types of fuzziness as follows.

Type 1: Fuzzy sets of graphs
Type 2: Crisp vertices set and fuzzy edges
Type 3: Crisp vertices and edges with fuzzy connectivity
Type 4: Fuzzy vertices set and crisp edges
Type 5: Crisp graph with fuzzy weight

Tahir *et al*. (2010) refined the taxonomy where a fuzzy graph is a graph G_F satisfying one of the fuzziness (G_F^i of the i^{th} type) or any of its combination as below.

1. $G_F^1 = \{G_{1_F}, G_{2_F}, G_{3_F}, \dots, G_{n_F}\}$ where fuzziness is on G_{i_F} for $i = 1,2,3, \dots, n$.
2. $G_F^2 = \{V, E_F\}$ where the edge set is fuzzy.
3. $G_F^3 = \{V, E(t_F, h_F)\}$ where both the vertices and edges are crisp, but the edges have fuzzy heads and tails.
4. $G_F^4 = \{V_F, E\}$ where the vertex set is fuzzy.
5. $G_F^5 = \{V, E(w_F)\}$ where both the vertices and edges are crisp but the edges have fuzzy weights.

The significant examples of the implementation of fuzzy graph are the modeling of clinical waste incineration process (Tahir *et al*., 2010), evaporation process in a boiler system (Noor Ainy *et al*., 2014), and pressurized water reactor (PWR) in a nuclear power plant (Ashaari *et al*., 2015a,c). In this study, we devote on PWR in a nuclear power plant.

Nuclear power plant system is one of the most frequently mentioned areas of thermal energy. It is a power station that generates electricity using heat from nuclear reactions which used steam produced from heated water to spin the large turbines that generate electricity. One type of nuclear power plant is PWR (Nelson, 1982).

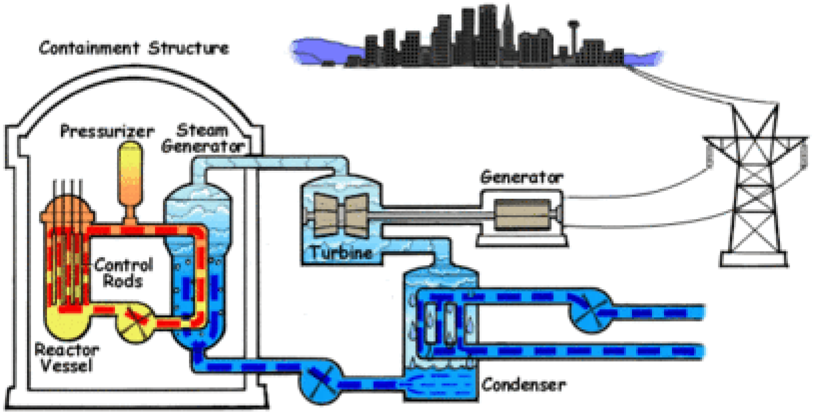

Figure 1.1 Diagram of pressurized water reactor (PWR)

The PWR contains two systems namely primary and secondary systems (Ahmet and Hasbi, 2001) as shown in Figure 1.2.

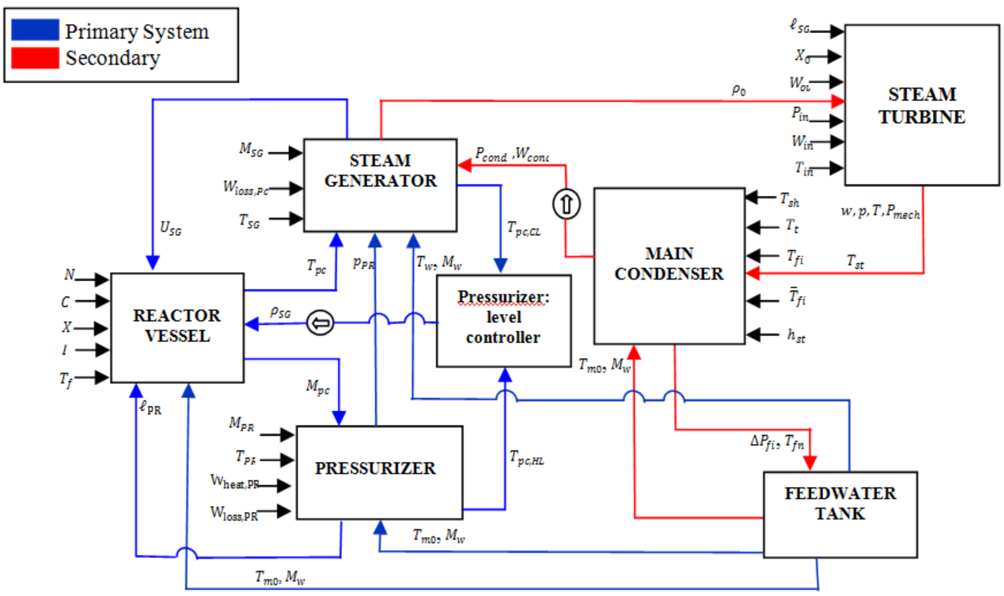

Figure 1.2 The Pressurized Water Reactor (PWR) schematic diagram (Ashaari *et al.*, 2015c)

The primary system of PWR consists of three main components namely reactor vessel, pressurizer, and steam generator while for a secondary system of PWR they are a steam generator, steam turbine, and main condenser. Both systems operate simultaneously with a separate flow system. The steam generator is known as the bridge between the primary and secondary systems where it is a place for phase changes from water to steam (Ashaari *et al*., 2015b). The heat generated from nuclear fission in the primary system is transferred to a steam generator through moderator where the phase change from water into steam is formed in the secondary system. Next, the system is passed on to the main turbine via the secondary system circuit where electricity is generated. Then, the steam is sent to the main condenser to remove excess heat from the steam, which allows the steam to condense. Lastly, the water is piped back to the system generator for reuse and the process is repeated until the PWR reactor is shut down. This study is on secondary systems of PWR.

In addition, the concept of an autocatalytic set (ACS) was introduced in 1971 (Kauffman, 1971; Eigen, 1971; and Rossler, 1971) as a set of catalytically integrated molecules. Then, Jain and Krishna (1998) presented ACS in terms of a graph. An autocatalytic set is described by a directed graph with nodes that represent species and the directed links represent catalytic interactions among them. A link from node j to node i indicate that species j is a catalyst for i.

Ashaari *et al*. (2015c) described the process in a secondary system of PWR using autocatalytic set. He identified the chemical reactions which occur in the process. There are seven important compounds namely, corrosion, moderator, sulphuric acid, boric acid, sodium hydroxide, chlorides, and nitrogen with twenty-one edges that represent the catalytic relationship among the compounds.

Subsequently, Tahir *et al*. (2010) introduced the fuzzy autocatalytic set (FACS) as a result of combining graph, ACS and fuzzy where these works were suitable in describing the dynamics of the process. Afterward, FACS is used in modeling the secondary system of PWR in a nuclear power plant (Ashaari *et al*., 2015c).

This research is undertaken to further explore the fuzzy graph of Type-3 of the secondary system of PWR in terms of possible paths of ACS, fuzzy edge connectivities of FACS and the relationship between these paths as a notion of length, distance, order, and size.

1.2 Statement of the Problem

The secondary system of PWR by Ashaari *et al.*, 2015 cannot give information regarding the catalytic interaction and relation amongst each and every parameter in the system. In addition, the study was also unable to present all the possible catalytic reactions between variables of the system.

The main interest in this research is to observe the interaction between parameters in terms of the possible paths, fuzzy edge connectivity, length, detour distance, order, and size for FACS of the secondary system of PWR. The problem statement of this research is formulated as a research question given in the following:

How can we determine the possible paths, order, and size for ACS and FACS of the secondary system of PWR?

1.3 Objectives of the Study

The main objectives of this research are given as follows:

(a) To find the possible paths and fuzzy edge connectivity of FACS in a secondary system of PWR.

(b) To determine the length and fuzzy detour distance of the path in the FACS in a secondary system of PWR.

(c) To determine the order and size of FACS in a secondary system of PWR.

1.4 Scope of the Study

This study is focused to determine the possible paths, fuzzy edge connectivity, the length of the path, fuzzy detour distance of path, order, and size in fuzzy graph Type-3 of FACS for a secondary system of PWR in Ashaari *et al*. (2015c).

1.5 Significance of the Study

The purpose of this study is to observe the catalytic interaction and relation amongst each and every parameter in the secondary system of PWR, not only among the pair of parameters by presenting all the possible catalytic reactions between variables of the system. An algorithm is developed and coded to find all the possible paths and expended to be used to more FACS system. Besides that, some new definitions and theorems are stated and proven, respectively.

1.6 Research Methodology

In this research, the catalytic interaction and relation amongst each and every parameter in the secondary system of PWR are studied. Some structures of fuzzy graph Type-3 of Fuzzy Autocatalytic Set (FACS), namely possible paths, fuzzy edge connectivity, length, detour distance, order, and size are explored. Omega algebra is used to determine all the possible paths for a FACS with respect to its fuzzy edge connectivity, length, and distance. These structures are then applied on FACS of a secondary system of pressurized water reactor (PWR).

An algorithm is developed and coded to find all the possible paths of the PWR and then expended to be used to more FACS system. Some new definitions are stated and the results obtained are then used to develop a new theorem. Figure 1.3 illustrates the research framework of this study.

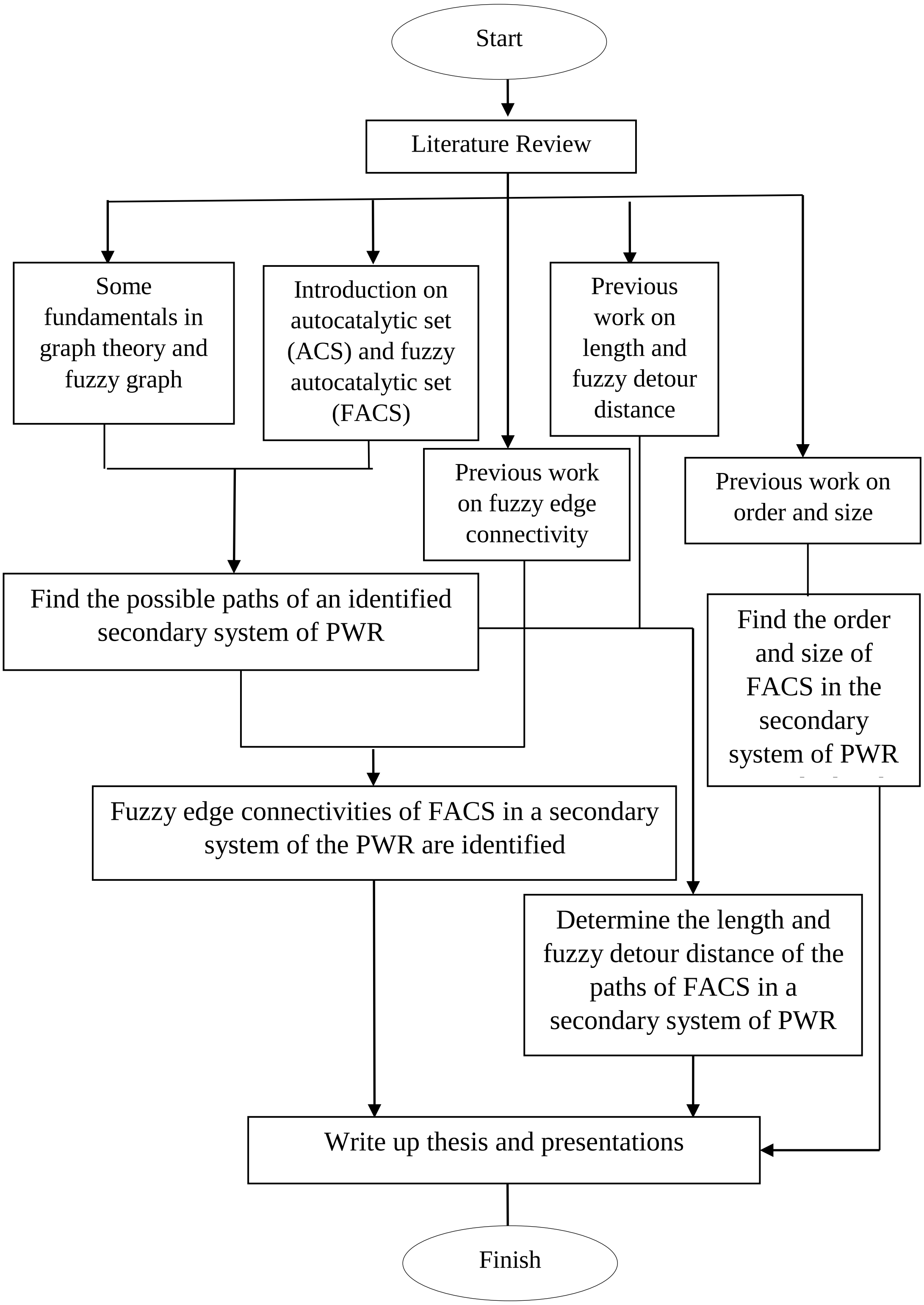

Figure 1.3 Research Framework

1.7 Summary and Outline of the Thesis

The first chapter provides the general background and information regarding the research, the statement of the problem, the objectives of the study, the scope of the study, the significance of the study, research methodology, and conclusion.

Chapter 2 presents a review of relevant published literature of the research topics which includes the definitions and basics of the graph (crisp graph and fuzzy graph). Apart from that, a previous works regarding autocatalytic set (ACS), fuzzy autocatalytic set (FACS), membership value of fuzzy edge connectivity, omega algebra of FACS, fuzzy graph type-3 (FT3)-length of path in ACS, fuzzy detour fuzzy graph type-3 (FT3)-distance in FACS, order, and size of a fuzzy graph are provided in this chapter.

In Chapter 3, an algorithm is developed and coded to find the possible paths and the membership value of fuzzy edge connectivity of a secondary system of the PWR. In addition, some new lemmas, theorem, and corollaries related to the possible paths are stated and proven, respectively.

Chapter 4 begins with the definitions of Fuzzy FT3-Length and Fuzzy Detour FT3-Distance in terms of omega algebra. Then, an algorithm is developed and coded to find the Fuzzy FT3-Length of the secondary system of PWR and the results are used in finding the Fuzzy Detour FT3-Distance.

In Chapter 5, definitions of order and size of fuzzy graph Type-3 are presented and some propositions are stated and proven. The definitions are used in finding the order and size of the graph of a secondary system of PWR. Chapter 6 concludes the study and suggests some open problems.

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Indexed Journal

1. **Zainab, M**. and Tahir, A. (2017). Possible Paths in Autocatalytic and Fuzzy Autocatalytic Sets. *International Journal of Pure and Applied Mathematics*, 115(1), 43-58. doi: 10.12732/ijpam.v115i1.4. **(Indexed by SCOPUS)**

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